Mathematics of Computation



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Mathematics of Computation

Special Issue

Dedicated to EUGENE ISAACSON April 1989

Dedication

The Editors of Mathematics of Computation are pleased to dedicate this issue to Eugene Isaacson on the occasion of his 70th birthday, June 14, 1989.

Professor Isaacson served as an Associate Editor from 1975 to the present time and was Chairman of the Editorial Committee from 1966 to 1975. His wise council has been invaluable to the journal.

Gene Isaacson received his Ph.D. from New York University in 1949 under the direction of Professor K. O. Friedrichs, having previously been on the staff of the Mathematical Tables Project and having worked for Professor Richard Courant as a research assistant. Gene's thesis grew out of scientific work connected with World War II and involved the determination of water waves on a beach as a function of the angle of inclination of the beach. His solution generalized work of Friedrichs, Lewy, and Stoker and, interestingly, led to a new proof of the law of quadratic reciprocity in number theory. He continued working on problems of fluid flow and gas dynamics, and in the process developed a strong interest in the numerical solution of partial differential equations. His work in this area involving parabolic equations, and his joint work with Courant and Rees on hyperbolic equations, is still widely known today. In 1949 Isaacson joined the Institute for Mathematics and Mechanics—as the Courant Institute was known at that time—at New York University and has been on the faculty at NYU since that time. He played an important role in developing the Institute's computational activities and computing facilities. He was instrumental in the acquisition of the first computer by the Courant Institute in 1952 and was in charge of the first large computational project done on it-the prediction of the flood stage at the Hanford Nuclear Reactor that would result from a failure in the Grand Coulee Dam. Over the years, Gene has had many excellent Ph.D. students. With his former colleague, Dr. Herbert B. Keller, he authored the influential text Analysis of Numerical Methods.

The papers in this issue are by a selection of well established as well as younger numerical analysts, many of whom have been associates of Professor Isaacson in some capacity. The Editors are aware that there are others who would have liked to honor Gene with their contributions. To those who could not be included, we apologize. Finally, we want to thank the authors of the papers for making this special issue possible. We are especially pleased that Gene's sons, David and Eli Isaacson, are among the contributors.

> James H. Bramble, Walter Gautschi, and John E. Osborn for the editors



EUGENE ISAACSON

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