REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

20[65–01, 65Mxx, 65Nxx].—MYRON B. ALLEN III, ISMAEL HERRERA & GEORGE F. PINDER, *Numerical Modeling in Science and Engineering*, Wiley, New York, 1988, x+418 pp., 24 cm. Price $39.95.

This book seeks to give a unified treatment of numerical modeling by combining material from continuum mechanics, partial differential equations (PDE's) and numerical methods for PDE's. The book is intended to serve as a text for a year-long graduate course for engineers, scientists and applied mathematicians. The basic idea of the book, namely, to first derive the equations from physical principles, then consider fundamental mathematical questions such as existence and qualitative properties of solutions, and finally devise numerical methods (finite difference and finite element methods), is of course sound. It should help the student to gain a better understanding of numerical methods if a proper theoretical background is given, and the theoretical studies are well motivated if the usefulness and importance is shown of numerical modeling in applications. However, such a unified approach requires a very careful selection of material to be successful.

The present book starts with a chapter where the basic PDE's in continuum mechanics (fluid mechanics and elasticity) are derived. Then there follow an introductory chapter on numerical methods and three chapters on steady state (elliptic), dissipative (parabolic) and nondissipative (hyperbolic) problems involving both mathematical and numerical aspects. In the final chapter some nonlinear problems are considered. The style of the book is informal or "nonmathematical"; for example, no theorems with precise assumptions are stated. To my taste, the mathematical level of the text is too low to be really suitable for a graduate course. For instance, variational principles for Poisson's equation and finite elements are discussed without the notions of the H^1-norm and completeness, which seems unnecessarily restrictive, since these concepts are now common also in the engineering literature. Mostly standard material concerning finite difference methods is presented in the chapters on elliptic, parabolic and hyperbolic problems. The book also contains material on finite element methods for these problems, but here the presentation is not adequate. For instance, in Chapter 2, second-order differential operators are applied to piecewise linear functions which are only continuous, and it is stated that the error in the energy norm using piecewise linear basis functions is $O(h^2)$. Such things should be very confusing to the student and defeat the stated ambition of the authors to increase the understanding of the subject.

0025-5718/89 $1.00 + $.25 per page

Chapter 5 contains a section on finite elements for hyperbolic problems, which is a very active area of research, where rapid progress has been made in the last years, for example on problems in several dimensions and nonlinear problems. However, this development is not touched upon in the present book.

To sum up, I find the basic idea of the book to be very natural but I think it should be possible to give a presentation which would be more precise mathematically and more up-to-date numerically, without becoming more difficult to read.

CLAES JOHNSON

Department of Mathematics
Chalmers University of Technology
and University of Göteborg
S-412 96 Göteborg, Sweden

21[65–02, 76–02].—OLIVIER PIRONNEAU, *Méthodes des Éléments Finis pour les Fluides*, Collection Recherches en Mathématiques Appliquées, Vol. 7, Masson, Paris, 1988, 199 pp., 24 cm. Price FF 175.00.

This short monograph appears in a relatively new collection edited by P. G. Ciarlet and J.-L. Lions. Although the collection accepts texts in both French and English, most titles are in French. Such an outlet for high-quality research is especially welcome for Ph.D. level teaching where there is a definite lack of this kind of material. In this respect, the book by O. Pironneau is exemplary. It presents an excellent summary of classical problems of fluid mechanics, including a discussion of the physical limitations of models. It also presents a fairly large number of methods for convection-diffusion problems which are probably the most difficult ones in fluid simulation at high Reynolds number. The results are in no way complete, but they leave the reader with a real sense of what the issues are.

The treatment of incompressibility is well done, even if the repertoire of elements is a little short. There is a treatment of less standard boundary conditions, which can also be very useful for beginners; this kind of material is alway very hard to find.

Navier-Stokes equations, both for incompressible and compressible flow, are considered. Again one should not look for complete results but for a broad picture sketching the main facts.

The book contains a collection of information which can nowhere else be found in one place. It should definitely be compulsory reading for beginners in flow simulation. Experienced readers will enjoy the practical presentation and will discover links between facts and methods they may not have thought about before.

MICHEL FORTIN

Département de Mathématiques
Université de Laval
Cité Universitaire
Quebec, Quebec G1K 7P4, Canada

22[65–04, 65M60, 65N30].—I. M. SMITH & D. V. GRIFFITHS, *Programming the Finite Element Method*, 2nd ed., Wiley, Chichester, 1988, xii+469 pp., 23 $\frac{1}{2}$ cm. Price $49.95.

The major objective of this text is to present the reader with an understanding of how finite element concepts are realized in computer software. This is accomplished rather effectively by means of a software "building block" technique whereby relatively complex programs may be assembled from a library of relatively simple, special purpose subroutines, each capable of carrying out some important step in the finite element analysis process. This device allows the reader a rather clear view of the overall process, since programs can be written quite compactly making use of building block subroutines.

The author very thoughtfully starts out by assembling subroutines into relatively narrow, simple programs and progresses to broader, more complex programs by modifications and additions.

The building block subroutines are categorized as either "black box" routines or "special purpose" routines. Black box routines are those associated with standard matrix handling procedures, and, as the name implies, little explanation of the bases of these routines is provided. Special purpose routines, on the other hand, accomplish some significant step in finite element analysis. Although a theoretical basis of each of these routines is presented, these discussions often seemed too abbreviated to provide adequate background for the finite element novice. It is recommended that reading of this text either be preceded by reading of an introductory finite element text or accompanied by a thorough classroom presentation of the finite element method.

A number of example programs are presented in the various chapters of the text. These presentations are enhanced by the availability of a summary of the functions of each special purpose subroutine in an appendix and a listing of each of these subroutines in another appendix. The reader may have to bounce back and forth in these various areas of the text as he/she reads, but this becomes relatively easy as familiarity with the arrangement increases. It will likely be important to many readers that all subroutines are available for a nominal charge on tape or diskettes for many computational environments. Fortran 77 is the language of choice for all subroutines and programs. Reading of programs and subroutines is made easier by use of mnemonics for subroutine names as well as variable names.

In general, this book is clearly written and well presented with few typographical errors or awkward constructions. Some readers may dislike use of examples from both solid and fluid mechanics. Also some examples make use of inefficient analysis procedures.

The positive aspects of the book far outweigh the negative aspects. It presents the broad picture of the basis of finite element software very clearly. The detailed picture of the basis of various finite element concepts is sometimes left incomplete or fuzzy from the point of view of the novice.

This book will make a fine addition to the library of anyone interested in finite element computer programming. It is viewed as complementary to the more detailed presentations of the finite element concepts found in other texts.

V. J. MEYERS

School of Civil Engineering
Purdue University
West Lafayette, Indiana 47907

23[35L65, 35L67, 65N05, 65N30, 76G15, 76H05].—S. P. SPEKREIJSE, *Multigrid Solution of the Steady Euler Equations*, CWI Tract 46, Centrum voor Wiskunde en Informatica (Centre for Mathematics and Computer Science), Amsterdam, 1988, vi+153 pp., 24 cm. Price Dfl.24.20.

The subject at hand is the numerical solution of the compressible inviscid equations of fluid dynamics, the Euler equations. Only steady state solutions are desired, though knowledge of properties of the unsteady Euler equations is required to derive suitable methods. These equations are of great interest in aerodynamics and turbomachinery applications. One goal of numerical algorithm designers in the field of computational fluid dynamics is to produce efficient, robust solvers that can be used as design tools on a routine basis. The designer must be able to vary some parameter, for example a geometrical quantity, in a specified parameter set, and study how the solution varies. The millennium is not yet upon us, though it is perhaps in sight for problems in two space dimensions.

This compact book (153 pages) presents a large amount of material in a remarkably clear and concise fashion. From a derivation of the Euler equations and general material on the Riemann problem for hyperbolic systems in Chapter 1, the author moves to a long chapter concerned with upwind discretizations of the Euler equations. The key ingredient is an approximate Riemann solver; the author prefers Osher's. All the modern apparatus of upwind schemes is clearly presented here: numerical flux functions, approximate Riemann solvers, total-variation diminishing schemes, monotonicity, and limiters. Noteworthy is the author's new, weaker, definition of monotonicity for a two-dimensional scheme, which evades the negative result of Goodman and Leveque that total-variation diminishing schemes in two space dimensions are at most first-order accurate. The new definition of monotonicity still rules out interior maxima and minima (for scalar equations) but does not preclude second-order accuracy. The third chapter gives a very brief description of the multigrid algorithm and shows some numerical solutions of a first-order accurate discretization. Practitioners in the field have found in the past few years that multigrid algorithms converge well for first-order accurate discretizations of steady fluid flow equations. The convergence is helped along by a large amount of numerical viscosity. Unfortunately, this large amount of numerical viscosity leads to unacceptable inaccuracy in the computed solutions. Multigrid algorithms applied directly to second-order discretizations (which would give acceptable accuracy) tend to converge slowly, if they converge at all. This motivates the fourth and last chapter, in which defect correction is used. This allows one to produce a second-order accurate solution, even though only problems corresponding to first-order accurate discretizations need to be solved. It seems that this is the first time defect correction has been used in the solution of any problem in computational fluid dynamics. Perhaps the use of defect correction here will spark more interest in the technique.

There are just a few, easily corrected, typographical errors in the displayed equations of this book. More numerous, though perhaps no more numerous than is the norm nowadays, are the misspelled English words. The book does not claim to be a comprehensive account of the numerical solution of the Euler equations, nor of upwind methods for the Euler equations (central differences and the competitors to Osher's scheme are not discussed). Statements like "Nowadays, finite volume schemes are almost universally used for shock capturing codes" could be misleading to a novice. There are many working codes which are based on finite difference methods and which produce very good results for problems with shocks. The expert reader who has had trouble with convergence to steady state may look at the convergence plots in Chapter 4 of this book and wonder if the algorithm would produce solutions accurate to machine zero if iterated indefinitely. One who wanted to play the devil's advocate could look at some of the plots in Chapter 3 and claim they showed a convergence rate which is not independent of grid size (the method converging slower on finer grids).

These caveats noted, this book is the best single reference for a person trying to construct a multigrid code for the steady Euler equations. In this reviewer's opinion, a novice in the field of computational fluid dynamics, armed only with this book, could produce a working code. This is a tribute to the book's completeness and clarity.

DENNIS C. JESPERSON

NASA Ames Research Center
MS 202A-1
Moffett Field, California 94035

24[65–02, 65N30].—D. LEGUILLON & E. SANCHEZ-PALENCIA, *Computation of Singular Solutions in Elliptic Problems and Elasticity*, Wiley, Chichester, 1987, 200 pp., 24 cm. Price $42.95.

The monograph is focused on the explicit computation of the singular solutions near corners and interfaces in plane elasticity theory for multimaterials.

These problems are governed by the elasticity system with piecewise constant coefficients. Indeed, the constitutive law is different in each component of the material under consideration. The jumps in the coefficients are across the interfaces between the different components. These interfaces are usually assumed to be straight, or polygonal, lines. One can also view these problems as transmission problems across these interfaces.

The general theory shows that the solutions to these problems behave as

$$r^\alpha u(\theta) + \text{more regular terms}$$

in polar coordinates, where r is the distance to the singular point under consideration, which is either one corner of the boundary, or a corner of an interface, or even a point where an interface meets the boundary.

The corresponding stresses behave as $r^{\alpha-1}$ and therefore may become infinite as r goes to 0 when $\operatorname{Re}\alpha < 1$. Although this is in clear contradiction with the assumption that the strains remain small, which legitimizes the linear theory, it turns out in practice that it provides a good hint of where damage may occur. The smaller α is, the likelier is the damage.

The general theory of singular solutions is not provided in this monograph. However, assuming that the data, volume loads and surface loads, vanish in the vicinity of the corner or singular point under consideration, the authors give a very straightforward and illuminating analysis that shows the form of the singular solution there.

The analysis of the case of one single material shows that α is the solution of a rather complicated transcendental equation. The corresponding transcendental equations for multimaterials seem quite out of reach by classical analysis. This is why it is sensible to rely on numerical computation for producing approximate values of the corresponding α.

Two approaches to the actual calculation of α are described. They are based on the fact that the above u is the solution of a second-order boundary value problem for a system of two ordinary differential equations that depend quadratically on α.

In the first approach, the boundary value problem is discretized directly by the finite element method using piecewise linear functions. The approximate problem reduces to finding the zeros of a determinant $D(\alpha)$ that depends analytically on α. The method is quite cheap, yet accurate.

In the second approach, one performs a preliminary reduction of the order with respect to α, by writing the boundary value problem as a first-order system. Then one faces the more common problem of finding eigenvalues and eigenvectors to a boundary value problem for a system of ordinary differential equations. Again, this is approximated by piecewise linear elements. One is left with the problem of finding the eigenelements for a large matrix that is nonsymmetric in most cases. This method is more expensive and requires heavier equipment, but it also allows one to calculate u approximately.

The whole monograph is very clearly written. Many illustrative examples are provided. It will certainly be very helpful to engineers.

PIERRE GRISVARD

Laboratoire de Mathématiques
Université de Nice
F-06034 Nice Cédex, France

25[49-01, 49H05].—M. J. SEWELL, *Maximum and Minimum Principles—A Unified Approach, with Applications*, Cambridge Univ. Press, Cambridge, 1987, xvi+468 pp., 23 $\frac{1}{2}$ cm. Price \$79.50 hardcover, \$34.50 paperback.

This is a text on operational variational methods. It concentrates on how to associate extremum principles with differential equations and applied problems. The book requires a minimal mathematical background and provides a large number of examples. The book has many exercises, so it could be used as a text in a mathematical methods course.

The organization is somewhat unusual. The first chapter treats mostly finite-dimensional saddle-point problems, with one small section on saddle-point problems for initial value problems. Chapter 2 is entitled "Duality and Legendre transformations" and includes some singularity theory and a number of interesting applications. Again, most of the chapter is finite-dimensional, with some infinite-dimensional examples interspersed.

The heart of the book is in Chapter 3, where the author shows how to associate dual variational problems with a saddle functional. This is done without any treatment of minimax theorems. Most of the examples involve ordinary differential equations and include variational inequalities and problems with nonstandard side conditions. Chapter 4 describes how to derive bounds on linear functionals of solutions of variational problems and develops some formal methods for finding variational principles for initial value problems.

The last chapter (Chapter 5) is 90 pages long and is entirely devoted to a theory of variational principles arising in the mechanics of solids and fluids. Much of this section reflects the author's own research and is a unique contribution to the literature.

In the first volume of Courant and Hilbert, the calculus of variations was described as being calculus on function spaces. Sewell's text provides a calculus, but not an analysis. It tries to present the theory without using the concepts and tools of real, or functional, analysis. Such an attitude was necessary in Courant's time. It is not defensible today. Most of the texts on finite-dimensional optimization theory referenced here use more analysis than this book does.

Equally surprising for a text in applied mathematics, it does not treat approximate, or numerical, methods for finding solutions. There is no discussion of second derivative conditions and no treatment of stability theory. It concentrates entirely on first derivative conditions and on setting up extremum principles. It does not study the type of critical point occurring at the solution. For the computational implementation of variational principles, it is crucial to know the type of a critical point. Algorithms for computing local minima are based on different considerations than those for finding saddle points.

This book is not directed at mathematicians. It does not define a Hilbert space, nor does it mention Banach spaces, Lebesgue or Sobolev spaces, or (metric) completeness. The author ignores many analytical details, and there is no discussion of existence questions. There is no use of the insights of convex analysis, or of minimax theory, both of which could have simplified much of the exposition. The Legendre transformation, described at length in Chapter 2, is a way to compute the convex conjugate of a given functional in convex analysis. It is the properties of this conjugate functional, not the Legendre transformation itself, that seem to be essential in much of duality theory. The author ignores the modern theory of the calculus of variations and the large volume of literature which has been written in a more sophisticated mathematical language; especially the contributions of the French, Italian and Soviet schools. Names that are missing from the bibliography include those of Brezis, Fichera, Mikhlin and Vainberg.

On the other hand, the examples and applications are of considerable interest. The book might be an appropriate introduction to these fascinating topics for someone, with an interest in applications, who is unwilling to learn elementary functional analysis. Euler would feel completely at home with this text.

GILES AUCHMUTY

Department of Mathematics
University of Houston
Houston, Texas 77204-3476

26[65–02, 65Kxx, 90Cxx].—R. FLETCHER, *Practical Methods of Optimization*, 2nd ed., Wiley, Chichester, 1987, xiv+436 pp., $23\frac{1}{2}$ cm. Price $53.95.

The development of optimization methods played a particularly vigorous role in the surge of numerical mathematics following the advent of electronic computing. Constrained optimization took center stage, first starting in the late 1940's with the development of linear and nonlinear programming by Dantzig, Charnes, Frisch, Zoutendijk, just to name a few. In the late 1950's Davidon's variable metric method, its subsequent analysis and independent work by Fletcher and Powell, yielded an entirely new perspective on the endeavor of unconstrained optimization, which at the time—hard as it is to comprehend now—was largely considered routine. Both aspects of optimization have been enriched by progress in numerical linear algebra, and their methodologies are indeed unthinkable without it.

This flourishing field, as described by one of its foremost pioneers, is the topic of this meticulously crafted book. It provides a concise, coherent, and no-nonsense description of the major practical techniques, together with evaluations and recommendations based on the author's extensive experience. His discussions touch on many alternatives and extensions to key ideas, placing them also in historical context. Experience and experiment are stressed throughout, and the text is correspondingly interspersed with illuminating numerical examples. Key theorems are stated and proved rigorously. A rich variety of exercises at the end of each chapter deepens the understanding and furnishes important additional detail. The entire material is supported by a carefully selected, yet complete, bibliography. There are only very occasional lapses into technical jargon, my favorite being "all pivots > 0 in Gaussian elimination without pivoting" (page 15). All in all, a highly recommended reading for those acquainted with the fundamentals of numerical linear algebra and multivariate calculus.

The division into two parts, on "unconstrained" and "constrained" optimization, respectively, reflects two different flavors of methodology. Each part had previously been published as separate volumes [1], [2], with the first part reprinted twice. The second edition, now combining both volumes, deepens the descriptions of key subjects in the first part and expands the subject matter of the second. Thus, in the first part, the Dennis-Moré characterization of superlinear convergence in nonlinear systems has been included and the important Chapter 2 has been substantially reorganized. In the second part, a section on "network programming", as well as a brief discussion of Karmarkar's interior point method, have been added. The reader will also find many new and stimulating questions in the problem sections. The second edition offers definitely more than the first edition, while retaining the focus and the crispness of the previous exposition.

Chapters 1 through 6 form the first part. The emphasis is on finding "local" minimizers using local differential information, explicitly in the form of separately computed gradients, or implicitly by utilizing difference information. The thorny problem of determining "global" minimizers is only occasionally mentioned. Chapter 1 contains introductory material. The important Chapter 2 surveys the general structure of the methods and pays well-placed attention to the principles and algorithms for "line search", a feature in many of the subsequently discussed methods. Chapters 3 and 4 on Newton, quasi-Newton, and conjugate gradient methods form the core of the first part on unconstrained optimization. The final two chapters round out that material, featuring among others: trust regions, Levenberg-Marquardt techniques, the Newton-Raphson method and Davidenko continuation methods for solving systems of nonlinear equations, the theory of superlinear convergence.

Chapters 7 through 14 form the second part. Here "constrained optimization" is first and foremost the optimization of functions, generally excluding the large field of combinatorial optimization. Chapter 7 is again introductory. Chapter 8 features linear programming with emphasis on the simplex method and some of its variations such as "product form" and LU-factoring of the basis and Dantzig-Wolfe decomposition. Lagrange multipliers, first- and second-order optimality conditions, convexity and duality are introduced in Chapter 9 preparatory to the description of classical nonlinear programming in Chapters 10 through 12: quadratic programming, linearly constrained optimization, zigzagging, penalty and barrier functions, sequential quadratic programming, feasible directions. Integer programming is only briefly discussed, stressing branch-and-bound techniques. It shares Chapter 13 with sections on geometric programming and optimal flows in networks. The final Chapter 14 offers a unique self-contained treatment of nondifferentiable or, rather, piecewise differentiable optimization.

The book is a classic and invaluable for the practitioner as well as the student of the field.

CHRISTOPH WITZGALL

Center for Computing and Applied Mathematics
National Institute of Standards and Technology
Gaithersburg, Maryland 20899

1. R. FLETCHER, *Practical Methods of Optimization, Vol. 1: Unconstrained Optimization*, Wiley, Chichester, 1980.

2. R. FLETCHER, *Practical Methods of Optimization, Vol. 2: Constrained Optimization*, Wiley, Chichester, 1981.

27[90–01, 90B99].—PANOS Y. PAPALAMBROS & DOUGLASS J. WILDE, *Principles of Optimal Design—Modeling and Computation*, Cambridge Univ. Press, Cambridge, 1988, xxi+416 pp., 26 cm. Price $49.50.

For anyone interested in modeling, model building, and in particular, optimization models and the interaction between optimization and the modeling process, this book is a must. It combines classical optimization theory with new ideas of monotonicity and model boundedness that provide valuable information for determining the most efficient and correct formulation of a model. It is an easy-to-read book with clear, modern notation, well-drawn graphs, and an easy-to-follow organization. It contains just enough theory and proofs to be complete, without being overburdened, and follows each concept with examples, applications, and realistically designed engineering design problems. Although aimed at the engineering design student, it is a valuable addition to the libraries of operations researchers, economists, numerical analysts, and computer scientists. The engineering flavor might at first be discouraging, but the discomfort quickly vanishes as the down-to-earth treatment of the topics comes through.

There are eight chapters in the book. Each has a concise introduction to set the stage for the concepts to be presented, and a summary to tie all the ideas together and reinforce what was introduced. The chapters are organized so as to create in the reader an appreciation for models, from their definition to their use.

Chapter 1 is an excellent introduction to mathematical modeling and the different types of models. It defines the design optimization problem and most of the other ideas and issues that are addressed throughout the book, including feasibility and boundedness, design space topography, data modeling, solution, and computation. The concept of a "good" model is introduced (one that represents reality in a simple but meaningful manner), and the limitations of modeling are also touched on.

The second chapter, on model boundedness, introduces the concepts of verification and simplification, which all too often are overlooked in modeling literature and practice. The authors stress the importance of applying the techniques of this chapter *before* performing any detailed computation, and note that in so doing, one obtains reductions in model size, tighter formulations, more robust models, and even formulation error detection. In this, the authors could not be more correct. As computers and computing become cheaper and more powerful, more researchers are relying on modeling and computational experiments to test new ideas. Problems which could not be solved before are now being addressed with success. The ideas introduced in this chapter are critical to this process and are the same ones used so successfully, for example, to solve large-scale integer programming problems. (See, e.g., [1], [2].)

Chapter 3, on interior optima, provides the mathematical foundation for local iterative methods. Its concise treatment is especially useful for those who want not only to understand the theory but also to use the methods to solve problems. Optimality conditions are presented here, along with convexity, gradient methods, Newton-type methods, and stabilization using modified Cholesky factorization. Unfortunately, there is no discussion about interior point methods for solving linear programming problems.

Boundary optima are the subject of Chapter 4. The discussion proceeds from feasible directions to sensitivity analysis. Also included are discussions of reduced gradients, Lagrange multipliers, the generalized reduced gradient method, gradient projection, and Karush-Kuhn-Tucker conditions. After the general case is treated, linear programming is developed at the end of the chapter, an approach I found appealing.

In Chapter 5, on model reduction, the ideas from Chapters 2, 3, and 4 are combined into a well-developed approach to the idea of producing as tight a formulation as is possible. Two new concepts are introduced to help in this search for a minimal formulation, a rigorous Maximal Activity Principle, and a heuristic Coincidence Rule. I found this theoretical approach to model reduction very appealing.

Global bound construction, the topic of Chapter 6, is the logical next step in the progression toward developing useful models. Constructing tight bounds, as the authors note, not only saves effort, but avoids accepting suboptimal solutions. Many techniques for constructing bounds are introduced here, from simple lower bounds to geometric inequalities, to unconstrained geometric programming. Combining these techniques into a process for model reduction using branch and bound is also discussed.

Chapter 7, on local computation, contains descriptions of various numerical algorithms for solving nonlinear programming problems. It is introduced with a brief section on choosing a method from among the large number of generally accepted techniques that are available. It contains sections on convergence, termination criteria, single variable minimization, Quasi-Newton methods, differencing, scaling, active set strategies, and Penalty and Barrier methods.

The real action is in Chapter 8, Principles and Practice. It begins with a review of the modeling techniques and approaches presented in the previous chapters. The goal is "to point out again the intimacy between modeling and computation that was explored first in Chapter 1." An optimization checklist is also provided for the novice modeler to use as a prompt while gaining more experience.

RICHARD H. F. JACKSON

Center for Manufacturing Engineering
National Institute of Standards and Technology
Gaithersburg, Maryland 20899

1. K. L. HOFFMAN & M. W. PADBERG, *Techniques for Improving the Linear Programming Representation of Zero-One Programming Problems*, George Mason Univ. Tech. Rep., 1988.

2. H. CROWDER, E. L. JOHNSON & M. PADBERG, "Solving large-scale zero-one linear programming problems," *Oper. Res.*, v. 31, 1983, pp. 803–834.

28[65–01, 65Fxx].—DARIO BINI, MILVIO CAPOVANI & ORNELLA MENCHI, *Metodi Numerici per l'Algebra Lineare*, Zanichelli, Bologna, 1988, x+514 pp., 24 cm. Price L. 48 000 paperback.

An excellent treatment of numerical methods in linear algebra, this text is destined to become the standard work on the subject in the Italian language. It contains seven chapters, of which the first three are introductory, providing the necessary tools of matrix algebra, eigenvalue theory and estimation, and norms. Chapter 4 discusses the major direct methods, Chapter 5 the more important iterative methods, for solving linear algebraic systems. Methods for computing eigenvalues and eigenvectors of symmetric and nonsymmetric matrices are the subject of Chapter 6. The final chapter deals with least squares problems and related matters.

The exposition, throughout, is crisp and to the point. There is a healthy balance between theoretical analysis, the study of error propagation and complexity, and practical experimentation. Each chapter is provided with a superb collection of exercises—many supplied with solution sketches—and with historical notes and bibliographies. For future editions, however, the authors may wish to pay more attention to the correct spelling of names.

W. G.

29[68T99, 65D07, 65D10].—ANDREW BLAKE & ANDREW ZISSERMAN, *Visual Reconstruction*, The MIT Press Series in Artificial Intelligence, The MIT Press, Cambridge, Mass., 1987, ix+225 pp., 23 $\frac{1}{2}$ cm. Price \$25.00.

This book appears in a series on artificial intelligence, and seems to be aimed primarily at computer scientists. However, mathematicians interested in computer vision will certainly want to look at it, and it may be of some peripheral interest to numerical analysts and approximation theorists interested in optimization and/or splines.

The subject is *visual reconstruction*, which is defined by the authors to be the process of reducing visual data to stable descriptions. The visual data may be thought of as coming from photoreceptors, spatio-temporal filters, or from depth maps obtained by stereopsis or optic rangefinders. Stability in this setting refers to the desire that the representation should be invariant to certain distortions such as sampling grain, optical blurring, optical distortion and sensor noise, rotation and translation, perspective distortions, and variation in photometric conditions.

The bulk of the book is devoted to the use of certain variational methods (called here weak strings, weak rods, and weak plates) for detecting edges (discontinuities in value or slope) of functions and surfaces. The methods are a form of penalized least squares, where the penalty includes a measure of smoothness of the function (typically an integral of a derivative) which is reminiscent of spline theory. These problems are analyzed using variational methods, and solved by discretizing them and applying an appropriate optimization method. The optimization method discussed here is referred to as the *graduated non-convexity algorithm*, and is designed to work on the nonconvex problems arising here. Convergence properties and the optimality of the algorithm are discussed.

The book includes about 150 references, mostly in the computer science literature. The authors seem to assume that the reader is familiar with much of this literature; results are often referred to without explanation. Readers not familiar with such things as "pontilliste depth map", "hyperacuity", "cyclopean space", or "data-fusion machine" may find the going hard. The authors have elected to "avoid undue mathematical detail", and despite several appendices, I expect that most mathematicians will not be fully satisfied.

L. L. S.

30[01A45].—A. W. F. EDWARDS, *Pascal's Arithmetical Triangle*, Oxford Univ. Press, New York, 1987, xii+174 pp., 24 cm. Price $37.50.

As an impressive culmination of meticulous research into original sources, this definitive study constitutes the first full-length history of the Arithmetical Triangle, reputedly the most celebrated of all number patterns. Its origins are herein traced to Pythagorean arithmetic, Hindu combinatorics, and Arabic algebra.

The elements of this triangle evolved historically in three equivalent forms; namely, figurate numbers, combinatorial numbers, and binomial coefficients. The different aspects of these numbers were first combined by Blaise Pascal in his *Traité du triangle arithmétique*, written in 1654 and printed posthumously in 1665. He was the first to consider the properties of this array as pure mathematics, deduced from the fundamental addition relation independently of any binomial or combinatorial application.

Dr. Edwards discusses in detail this creative treatise and shows, in particular, how it influenced Wallis, Newton, and Leibniz in the early development of analysis. Special attention is given to Wallis's ingenious derivation of his well-known infinite product by means of interpolation in an analogous triangular array of reciprocals of figurate numbers.

The modern theory of probability originated in the extensive correspondence between Pascal and Fermat concerning the previously unsolved problems called, respectively, the Problem of Points and the Gambler's Ruin. The solutions obtained by Pascal and Fermat have been described in detail in two previously published papers by the author, which are herein reprinted as appendices. Dr. Edwards establishes that the basic solution of the first problem is properly attributable to Pascal instead of Fermat, contrary to tradition.

The origins of the binomial and multinomial distributions are also traced to Pascal's treatise, as revealed in the correspondence of Leibniz, John Bernoulli, De Moivre, and Montmort.

A concluding chapter is devoted to a discussion of James Bernoulli's famous work, *Ars conjectandi*, which is chiefly noted for containing the first limit theorem in probability. Apparently, James Bernoulli independently derived the binomial distribution, unaware of Pascal's treatise, to which his attention was first drawn just before his death in 1705 by Leibniz.

Each of the ten chapters of this carefully written book is supplemented by explanatory notes and bibliographic references. In addition, there is a comprehensive list of nearly two hundred sources and an index of the names of more than one hundred persons appearing in this historical account.

This scholarly book should be of special interest to students and teachers of the history of mathematics and statistics.

J. W. W.

31[65F10, 65N20].—A. HADJIDIMOS (Editor), *Iterative Methods for the Solution of Linear Systems*, North-Holland, Amsterdam, 1988, 291 pp., 27 cm. Price \$78.00/Dfl. 160.00.

This is a reissue in book form of *Journal of Computational and Applied Mathematics*, v. 24, 1988, nos. 1 & 2. It provides a useful cross section of current work in the area of the title, including topics such as convergence theory, parameter selection, preconditioning, rectangular and infinite systems, implementation on machines with parallel architectures, and software packages.

W. G.

32[65–06, 68–06].—JAMES MCKENNA & ROGER TEMAM (Editors), *ICIAM '87: Proceedings of the First International Conference on Industrial and Applied Mathematics*, SIAM, Philadelphia, 1988, xx+376 pp., 26 cm. Price \$56.50.

The conference in the title of these proceedings, co-organized by four sister societies of applied mathematics, GAMM, IMA, SIAM and SMAI from Germany, England, the United States and France, was held in Paris on June 29—July 3, 1987. Part I contains the welcoming and opening addresses. Sixteen invited papers make up Part II. The authors and their titles are: Wolfgang Alt, Modelling of motility in biological systems; Karl Johan Åström, Stochastic control theory; Michael Atiyah, Topology and differential equations; Robert Azencott, Image analysis and Markov fields; Philippe G. Ciarlet, Modeling and numerical analysis of junctions between elastic structures; D. G. Crighton, Aeronautical acoustics: mathematics applied to a major industrial problem; Carl de Boor, What is a multivariate spline?; Yves Genin, On a duality relation in the theory of orthogonal polynomials and its application in signal processing; W. Hackbusch, A new approach to robust multi-grid solvers; John Hopcroft, Model driven simulation systems; Peter D. Lax, Mathematics and computing; L. Lovász, Geometry of numbers: an algorithmic view; Andrew Majda, Vortex dynamics: numerical analysis, scientific computing, and mathematical theory; F. Natterer, Mathematics and tomography; P. Perrier, Numerical flow simulation in aerospace industry; M. J. D. Powell, A review of algorithms for nonlinear equations and unconstrained optimization. A similar variety of topics is treated in 69 minisymposia whose abstracts appear in Part III. A list of contributed papers in Part IV, and an author index, conclude the volume. Full-length versions of 29 contributions by Dutch authors have previously been published in [1].

W. G.

1. A. H. P. VAN DER BURGH & R. M. M. MATTHEIJ (Editors), *Proceedings of the First International Conference on Industrial and Applied Mathematics (ICIAM 87): Contributions from the Netherlands*, CWI Tract, Vol. 36, Centre for Mathematics and Computer Science, Amsterdam, 1987. [Review **21**, *Math. Comp.*, v. 50, 1988, p. 649.]

33[65–06].—ROLAND GLOWINSKI, GENE H. GOLUB, GÉRARD A. MEURANT & JACQUES PÉRIAUX, *First International Symposium on Domain Decomposition Methods for Partial Differential Equations*, SIAM, Philadelphia, 1988, x+431 pp., 26 cm. Price $48.50.

This is the proceedings of a symposium on the subject of the title of this volume. It consists of 22 papers by invited speakers. The papers presented were not subject to the refereeing process. The topics of the articles include theoretical foundations, applications to physical problems, related techniques such as block relaxation and computer implementation of decomposition algorithms.

J. H. B.

34[33–06, 33A65, 41–06, 42C05].—M. ALFARO, J. S. DEHESA, F. J. MARCELLAN, J. L. RUBIO DE FRANCIA & J. VINUESA (Editors), *Orthogonal Polynomials and Their Applications*, Lecture Notes in Math., vol. 1329, Springer-Verlag, Berlin, 1988, xv+334 pp., 24 cm. Price $28.60.

These are the proceedings of the Second International Symposium on Orthogonal Polynomials and their Applications held in Segovia, Spain, September 22–27, 1986. (For the first symposium, see [1].) The volume contains nine invited lectures, covering analytic and group-theoretic aspects of orthogonal polynomials and applications to approximation theory. In addition, there are 13 contributed papers and a collection of open problems.

W. G.

1. C. BREZINSKI, A. DRAUX, A. P. MAGNUS, P. MARONI & A. RONVEAUX (eds.), *Polynômes Orthogonaux et Applications*, Lecture Notes in Math., vol. 1171, Springer, Berlin, 1985. (Review **23**, *Math. Comp.*, v. 49, 1987, pp. 305–306; Corrigendum, *ibid.*, v. 50, 1988, p. 359.)

35[65–06, 68–06].—GARRY RODRIGUE (Editor), *Parallel Processing for Scientific Computing*, SIAM, Philadelphia, 1989, 428 pp., $25\frac{1}{2}$ cm. Price $45.00.

These are the proceedings of the Third SIAM Conference on Parallel Processing for Scientific Computing held in Los Angeles, California, December 1–4, 1987. The papers (and in some cases abstracts only) of 14 invited lectures and 76 contributed talks are arranged in seven parts: Matrix Computations, Numerical Methods, Differential Equations, Scientific Applications, Languages, Software Systems, and Architectures.

W. G.