# SOME NEW POLYHEDRA WITH VERTEX DEGREE 4 AND/OR 5 ONLY 

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Abstract. A table of 4- and 5-hedra of orders up to and including 22 is given.

In 1981 we reported on the number of polyhedral graphs [5]. That work was a byproduct of the search for the lowest-order squared square, which was found in March 1978 [3]. The squaring problem is closely related to the theory of 3 -connected planar graphs, as was first shown by Brooks, Smith, Stone, and Tutte [1] in 1940. In 1962 we developed the necessary techniques for computer manipulation of 3 -connected planar graphs. These techniques were reported in [2]. The set of 4 - and 5 -hedra is a subset of the set of 3 -connected planar graphs. In that paper, a code for 3-connected graphs was introduced in which the essential properties of planarity are preserved.

It is assumed that the graph is drawn on the sphere. The vertices are numbered arbitrarily from 1 to $K$, where $K$ is the number of vertices of the graph. The sides or meshes are numbered arbitrarily from 1 to $M$, where $M$ is the number of sides (or meshes). The boundary contains a set of vertices. A code of a side is obtained as follows: while walking in the positive sense along the boundary of the side, starting with $V_{i}$, we encounter $V_{j}, V_{k}, V_{l}, \ldots$, until we return to $V_{i}$. The sequence $V_{i}, V_{j}, V_{k}, V_{l}, \ldots, V_{i}$ is a code of the side.
Example. A possible code of side 1 of the reference graph is 12651 , as can be seen from Figure 1; but we can also take 26512,65126 , or 51265.

A code of the graph is the sequence of codes of all its sides, separated by zeros. At the end, two more zeros are added.

Example. A code of the reference graph is as follows:

$$
126510236203563034530154101432100
$$

In case we deal with more than nine vertices, it is more convenient to code the vertices with (capital letters, where $A=1, B=2, C=3$, etc.

Example. The above code of the reference graph reads:

$$
A B F E A 0 B C F B 0 C E F C 0 C D E C 0 A E D A 0 A D C B A 00
$$

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Figure 1. Reference graph


Figure 2. Ancestor graph
All 3-connected graphs can be generated starting with the ancestor graph consisting of eight edges, using a theorem of Tutte [6]. See Figure 2.

Tutte considered the set $S_{B}$ of 3-connected planar graphs having $B$ edges.
Let $s \in S_{B}$ and let $s^{\prime}$ be its dual. Then if $s$ is not a wheel, at least one of the graphs $s$ or $s^{\prime}$ can be constructed from an element $\sigma$ of $S_{B-1}$ by addition of an edge joining two vertices of $\sigma$. A wheel is a planar graph with an even number of vertices $(B)$, with one vertex of degree $1 / 2 B$, and $B-1$ vertices of degree 3. See also Figure 3.



Figure 3. Low-order wheels
The process of adding wires is referred to as the generation process. The generation process produces many duplicates. It is therefore necessary to develop an identification algorithm by which graphs can uniquely be identified. The basic ideas for identifying graphs were given in [2]. The identification problem, including the calculation of the order of the automorphism group, was completely solved in 1978 [4].

As reported in [5], we generated and identified all 3-connected planar graphs
of orders 9 up to and including 22. Those of orders 23 and 24 were only generated for graphs with 10 vertices. The results have been stored on magnetic tape. Very recently, Dr. King from the chemistry department of New Georgia University asked us to search our tapes for graphs with the property that either the original or its dual consists of vertices with degree 4 and/or 5 only. We found 40 such graphs which are listed in Table 1.

First, the code is given, then the order of the automorphism group, next an identification of selfduality ( $0=$ not selfdual, $1=$ selfdual ) and finally the identification number. For explanation of the identification number we refer to [4].

Example. The reference graph
$A B F E A 0 B C F B 0 C E F C 0 C D E C 0 A E D A 0 A D C B A 000002100000000075523$

Table 1. 4- and 5-hedra

```
EFAEOADEAOEDCEOFECFOCBFCOFBAFODABDOBCDBOO
004800000000000000075537
FGBFOEFBEOBAEBOFEDFOGFDGOGDCGOGCBGODEADOCDACOABCAOO
002000000000000003777423
GIIBGOFGBAFOGFEGOHGEHOHEDCHOHCBHOFADFODEFDOCDACOBCABOO
001600000000001706424537
GHCGOFGCFOFCBPOBAFBOGFEGOHGEHOHEDHOHDBCHOEFAEODEADOBDABOO
000400000000001746324563
GHCGOFGCFOFCBFOGFEGOHGEHOHEDHOHDCHOFBAFOAEFAODEADODABDOBCDBOO
000800000000001533672741
HICHOGHCBGOGBAGOHGFHOIHFEIOIEDIOCDBCOIDCIOFGAFOAEFAODEABDOO
001200000000741503014537
HICHOGHCGOGCBGOHGFHOIHFEIOIEDIOIDCIOFGBFOFBAFOAEFAODEADOCDABCOO
000200000000365656020437
HICHOGHCBGOGBAGOHGFHOIHFEIOIEDIOIDCIOFGAFOEFAEOABEAODEBDOCDBCOO
000400000000751523015117
HIDCHOGHCGOGCBGOHGFHOIHFIOIFEIOIEDIOGBAGOFGAFOEFAEODEABDOBCDBOO
000400000000761462425117
HIDCHOGHCGOGCBGOHGFHOIHFIOIFEIOIEDIOGBAGOFGAFOEFAEOBEABODEBDOBCDBOO
000800000000670726546063
HICHOGHCGOGCBGOHGFHOIHFEIOIEDIOIDCIOFGBFOFBAFOAEFAODEADOBDABOCDBCOO
000200000000761261566216
UDIOHIDCHOHCBHOIHGIOIGFJOJFEJOJEDJOGHBGOFGBAFOEFAEODEACDOBCABOO
000400000740640443024537
ЛHJOJHCJOGJCBAGOUGIOIGFIOHIFEDHODBCDOCHDCOEABEOBDEBOFGAFOAEFAOO
002000000740640502424537
UDIOHIDCHOHCBHOIHGFIOIFJOJFEJOJEDJOGHBGOGBAGOFGAEFODEACDOBCABOO
001600000740640504405537
HICHOGHCGOGCBGOHGFHOHFEHOEIHEOIEDIOIDCIOFGBFOFBAFOAEFAODEADOBDABOCDBCOO
001200000000654566533330
UCIOHGICHOHCBHOHBAHOIGFEIOJIEJOJEDJOJDCJOGHAGOAFGAOBFABODFBCDOEFDEOO
000200000352652601444253
```


## Table 1 (continued)

## IHCIOJCIOGJCBCOGBAGOJGFJOEDHEOUFIOFEHIFODBCDOCHDCODEABDOFGAFOAEFAOO 000200000354647402024563 <br> IHCIOJICFOGCGOGCBAGOJGFJOIFEIOEDHIEOHDBHOBCHBODEADOABDAOFGAFOAEFAOO 000400000362632502424613

UCIOHGICHOHCBHOHBAHOIGEIOJEJOJEDJOJDCJOGHAGOAFEGAOBFABOFBCDFODEFDOO 000200000370715402020537

IHCIOJCJOGJCGOGCBAGOJGFJOUFEDIODHIDOHDBHOBCHBOEABEOBDEBOFGAFOAEFAOO 000200000724525406020537

UDIOHIDHOHDCHOHCBHOIHGFIOJIFJOJFEJOJEDJOGHBGOGBAGOFGAEFODEACDOBCABOO 000200000750650422124536

UDIOHIDHOHDCHOHCBHOIHFIOJFJOFGEJFOJEDJOGFHBGOGBAGOEGAEODEACDOBCABOO 000100000744660601250137

UDIOHIDHOHDCHOHCBHOIHGIOJGFJOJFEJOJEDJOGHBGOFGBAFOEFAEODEACDOBCABOO 000100000750650601222527

IJDHIOHDCHOHCBHOBIHBOGIBGOGBAGOIGFIOJFJOJFEJOJEDJOFGAFOEFAEOABCEAOCDECOO 0001000000000000000570550563320107
UDIOHIDHOHDCHOCBACOHCAHOIHGFIOJFJOJFEJOJEDJOGHAGOGABGOBFGBOEFBEOBCDEBOO 0004000000000000000272354553504212
IICIOJCJOGJCBGOGBAGOJGFJOFEDFOUFIOFDHIFOHDBHOBCHBODEADOABDAOGAEGOEFGEOO 0002000000000000000370750522540037
UDIOHIDHOITDCHOIHGFIOJFJOJFEJOJEDJOGHCGOGCBGOGBAGOAFGAOEFAEOBEABODEBCDOO 0002000

000000000000570550532720423
IIICIOJCJOGJCGOGCBAGOJGFJOJFEJOEDUEODHIDOHDBHOBCHBOEABEOBDEBOFGAFOAEFAOO (0004000000000000000664325251361007

UDIOHIDHOHDCHOCBACOHCAHOIHGFIOJFJOJFEJOJEDJOGHAGOGABGOBFGBOEFBCEOCDECOO 0002000000000000000664524316710431

UDIOHIDHOHDCHOIHGIOJIGFJOJFEJOJEDJOHCBHOGHBGOGBAGOFGAFOAEFAOCEABCODECDOO 0002000000000000000670350334720423

UDIOHIDHOHDCBHOIHGIOIGFIOFJIFOJFEJOJEDJOGHBGOFGBAFOEFAEOCEACOABCAODECDOO 0004000000000000000670351524601117

UDIOHIDHOHDCHOHCBHOIHGIOJGFJOJFEJOJEDJOGHBGOFGBAFOEFAEOEACEOCDECOBCABOO 0004000000000000000750650621524613

UDIOHIDHOHDCHOIHFIOJFJOFGEJFOJEDJOHCBHOGFHBGOGBAGOEGAEOCEACOABCAODECDOO 0002000000000000000760630522544253

JKEDJOUDIOIDCBIOJGJOKJGKOGHFKGOKFEKOHGIBHOHBAHOFHAFOEFACEODECDOBCABOO 0004000000000001700640120243212633
JDJODCJOKJCKOCDBACOHKCAHOKHGKOGFEGOEUKGEOIEBIOBDIBOEFABEOHAFHOFGHFOO 0002000000000001700640300302424537
JDJOKJDKOHKDCBHOHBAHOKHGKOJKGFEJOUEIOIECIODICDOFABFOBCEFBOGHAGOAFGAOO 0004000000000001700640301201015537
UDIOHIDHOHDCHOHCBHOIHGIOJGJOJGFEJOJEDJOGHBGOFGBFOBAFBOEFAEODEACDOBCABOO 0001000000000000000760530326510233
JDHJOHIJHOHDCHOHCBHOBIHBOGIBGOGBAGOIGFIOJFJOJFEJOJEDJOFGAFOEFAEOABCEAOCDECOO 0002000000000000000730710522517322
UDHIOHDCHOHCBHOBIHBOGIBGOGBAGOIGFIOJFJOJFEJOJEDJOFGAFOEFAEOABCAOCEACOCDECOO 0002000000000000000750612433465603

JDHJOHIJHOHDCHOHCBHOBIHBOGIBGOGBAGOIGFIOJFJOJFEJOJEDJOFGAFOEFAEOABCAOCEACOCDECOO 0016000000000000000662325261662552

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