CORRIGENDA

CHARLES CHUI & TIAN XIAO HE, *On the dimension of bivariate superspline spaces*, Math. Comp. **53** (1989), 219–234.

We are grateful to Carla Manni for pointing out an error in the proof of Lemma 1. This necessitates corrections in the statement of Lemma 1 and its consequences. In addition to the notations used in the paper, we set

$$\beta_d^r = \left[\binom{d-r+1}{2} - 2\binom{r+1}{2} + \binom{(r-(d-2r)_+ + 1)_+}{2}\right]_+,$$

$$\gamma_d^r = \left[\binom{d+2}{2} - \binom{2r+2}{2}\right]_+,$$

$$\sigma_d^r(n) = \begin{cases} \displaystyle\sum_{j=r+1}^{d-r} [r+j+1-nj]_+ & \text{for } d \geq 3r+1, \\ \displaystyle\sum_{j=r+1}^{d-r} [r+j+1-n(j+d-3r-1)]_+ & \text{for } 2r+1 \leq d \leq 3r, \\ 0 & \text{otherwise.} \end{cases}$$

The correct formulation of Lemma 1 is then as follows.

Lemma 1. *Let Δ_0 be a rectilinear grid partition, as described. Then*

$$\dim \widehat{S}_d^r(\Delta_0) = \binom{d+2}{2} + [n_0\beta_d^r + \sigma_d^r(N_0) - \gamma_d^r]_+.$$

In particular, it is clear that for $d \geq 4r+1$ or $d \leq 2r$ the formulation of this lemma reduces to the formulation of Lemma 1 in the original paper. For $2r+1 \leq d \leq 4r$, the mistake in the proof of Lemma 1 occurs in the formulation of equation (24). A correct statement is the following. For $d \geq 3r+1$, we have

$$\begin{aligned} \operatorname{rank} H' &= \min\left(\sum_{j=r+1}^{d-2r} \operatorname{rank} H_j + \sum_{j=d-2r+1}^{d-r} \operatorname{rank}[B_{r+1}^j \cdots B_{j-1}^j \overline{H}_j],\ n_0\beta_d^r\right) \\ &= \min\left(\sum_{j=r+1}^{d-r} (r+j+1-[r+j+1-N_0 j]_+),\ n_0\beta_d^r\right) \\ &= n_0\beta_d^r - [n_0\beta_d^r - \gamma_d^r + \sigma_d^r(N_0)]_+. \end{aligned}$$

For $2r < d < 3r+1$, we have

$$\begin{aligned}\operatorname{rank} H' &= \min\left(\operatorname{rank}\overline{H}_{r+1} + \sum_{j=r+2}^{d-r}\operatorname{rank}[D^j_{r+1}\cdots D^j_{j-1}\overline{H}_j],\ n_0\beta^r_d\right)\\ &= n_0\beta^r_d - [n_0\beta^r_d - \gamma^r_d + \sigma^r_d(N_0)]_+.\end{aligned}$$

The above reformulation of Lemma 1 entails changes in the statements of Lemma 2, Theorems 1, 2, 3, 4 and three of the corollaries. The correct statements of these results are given below.

Lemma 2. *Let Δ_1 be a rectilinear grid partition and $\varepsilon = 0$ or 1 as described. Then*

$$\dim\widehat{S}^r_d(\Delta_1) = \binom{d+2}{2} + E_I\beta^r_d - \varepsilon[\gamma^r_d - \sigma^r_d(N_0)] + (\varepsilon[\gamma^r_d - \sigma^r_d(N_0)] - n_0\beta^r_d)_+.$$

Theorem 1. *Let*

$$D^r_d = \dim\widehat{S}^r_d(\Delta) - \binom{d+2}{2} - E_I\beta^r_d + V_I\gamma^r_d.$$

Then

$$\sum_{i=1}^{V_I}\sigma_i \le D^r_d \le \sum_{i=1}^{V_I}\tilde{\sigma}_i,$$

where $\sigma_i = \sigma^r_d(e_i) + (\gamma^r_d - \sigma^r_d(e_i) - d_i\beta^r_d)_+$, $\tilde{\sigma}_i = \sigma^r_d(\tilde{e}_i) + (\gamma^r_d - \sigma^r_d(\tilde{e}_i) - \tilde{d}_i\beta^r_d)_+$ with d_i and e_i denoting the number of edges and the number of edges with different slopes attached to the vertex A_i respectively, and $\tilde{d}_i$ and $\tilde{e}_i$ denoting the number of edges and the number of edges with different slopes attached to the vertex A_i but not A_j, $j < i$, respectively, where $i = 1, \dots, V_I$. In particular, for $d \ge 3r+1$ and all $\tilde{e}_i \ge 2$,

$$\dim\widehat{S}^r_d(\Delta) = \binom{d+2}{2} + E_I\beta^r_d - V_I\gamma^r_d.$$

Theorem 2. *Let*

$$F^r_d(n) := (n\beta^r_d + \sigma^r_d(n) - \gamma^r_d)_+.$$

Then

$$\dim\widehat{S}^r_d(\Delta_c) = \binom{d+2}{2} + L\beta^r_d + \sum_{i=1}^{V_I}F^r_d(l_i).$$

Corollary 1. *Let $d \ge 3r+1$. Then*

$$\begin{aligned}\dim\widehat{S}^r_d(\Delta^{(2)}_{mn}) &= mn\left[6\binom{d-r+1}{2} - 12\binom{r+1}{2} - 2\binom{d+2}{2} + 2\binom{2r+2}{2}\right]\\ &\quad + (m+n)\left[5\binom{d-r+1}{2} - 10\binom{r+1}{2} - \binom{d+2}{2} + \binom{2r+2}{2}\right]\\ &\quad + \binom{2r+2}{2} + 4\binom{d-r+1}{2} - 8\binom{r+1}{2}.\end{aligned}$$

Corollary 2. *Let* $\Delta_{mn}^{(1)}$ *be a uniform type-*1 *triangulation of* Ω_R. *Then*

$$\dim \widehat{S}_d^r(\Delta_{mn}^{(1)}) = \binom{d+2}{2} + mn[3\beta_d^r - \gamma_d^r + \sigma_d^r(3)] + (2m+2n+1)\beta_d^r.$$

Corollary 3. *Let* $\Delta_{mn}^{(2)}$ *be a uniform type-*2 *triangulation of* Ω_R. *Then*

$$\dim \widehat{S}_d^r(\Delta_{mn}^{(2)}) = \binom{d+2}{2} + mn[6\beta_d^r - 2\gamma_d^r + \sigma_d^r(2) + \sigma_d^r(4)]$$
$$+ (m+n)[5\beta_d^r - \gamma_d^r + \sigma_d^r(2)] + 4\beta_d^r - \gamma_d^r + \sigma_d^r(2).$$

In the following, we need the notations:

$$\beta_d^{r,\rho} = \left[\binom{d-r+1}{2} - 2\binom{\rho-r+1}{2} + \binom{(\rho - r - (d-\rho)_+ + 1)_+}{2}\right]_+,$$

$$\gamma_{d,\rho} = \left[\binom{d+2}{2} - \binom{\rho+2}{2}\right]_+,$$

$$\sigma_d^{r,\rho}(n) = \begin{cases} \displaystyle\sum_{j=\rho-r+1}^{d-r} (r+j+1-nj)_+ & \text{for } d > 2\rho - r, \\ \displaystyle\sum_{j=\rho-r+1}^{d-r} [r+j+1-n(j+d-2\rho+r-1)]_+ & \text{for } \rho < d \le 2\rho - r, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3. *Let*

$$D_d^{r,\rho} = \dim S_d^{r,\rho}(\Delta) - \binom{d+2}{2} + V_I\gamma_{d,\rho} - E_I\beta_d^{r,\rho}.$$

Then

$$\sum_{i=1}^{V_I} \sigma_i^\rho \le D_d^{r,\rho} \le \sum_{i=1}^{V_I} \tilde{\sigma}_i^\rho,$$

where $\sigma_i^\rho = \sigma_d^{r,\rho}(e_i) + (\gamma_{d,\rho} - \sigma_d^{r,\rho}(e_i) - d_i\beta_d^{r,\rho})_+$, $\tilde{\sigma}_i^\rho = \sigma_d^{r,\rho}(\tilde{e}_i) + (\gamma_{d,\rho} - \sigma_d^{r,\rho}(\tilde{e}_i) - \tilde{d}_i\beta_d^{r,\rho})_+$, *and* e_i, d_i, $\tilde{e}_i$ *and* $\tilde{d}_i$ *have been defined in Theorem* 1.

Theorem 4. *Let* $\rho \ge r$ *and*

$$F_d^{r,\rho}(n) := (n\beta_d^{r,\rho} + \sigma_d^{r,\rho}(n) - \gamma_{d,\rho})_+.$$

Then

$$\dim S_d^{r,\rho}(\Delta_c) = \binom{d+2}{2} + L\beta_d^{r,\rho} + \sum_{i=1}^{V_I} F_d^{r,\rho}(l_i).$$

Charles K. Chui
Tian Xiao He