LARGEST KNOWN TWIN PRIMES

B. K. PARADY, JOEL F. SMITH, AND SERGIO E. ZARANTONELLO

ABSTRACT. $663777 \cdot 2^{7650} \pm 1$, $571305 \cdot 2^{7701} \pm 1$ and $1706595 \cdot 2^{11235} \pm 1$ are twin primes.

A search for large twin prime pairs of the form $3 \cdot a \cdot 2^n \pm 1$, where *a* is odd, was performed at the Amdahl Benchmark Center in Santa Clara, California. Three large prime pairs were found. The search consisted of the following steps:

1. An initial selection of approximately 8,000,000 prime pair candidates of the form $3 \cdot a \cdot 2^n \pm 1$ was made. These were obtained from a subset of n in the range $7500 \le n \le 11240$ and a subset of odd a in the range a < 750,000.

2. Both the +1 case and the -1 case were sieved with factors up to $5.8 \cdot 10^9$. After sieving, 36,194 candidates remained.

3. Using a Lucasian test [1], 224 primes of the form $3 \cdot a \cdot 2^n - 1$ were found.

4. For these primes, the corresponding integer of the form $3 \cdot a \cdot 2^n + 1$ was tested using Proth's theorem [2]. This produced the three largest known twin prime pairs.

The first two prime pairs were found on May 30, 1989, and the largest pair on June 7, 1989. The Lucasian primality tests each take six seconds for the smaller numbers and ten seconds for the larger numbers on the Amdahl 1200. The values of n > 7500 were chosen so that any twin prime pairs found would be the largest known, exceeding those discovered by Dubner and Atkin [3, p. 203].

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The first author is now with Intergraph Corporation, 2400 Geng Rd., Palo Alto, California 94303.

The third author is now with Fujitsu America, Inc., M/S B2-7, 3055 Orchard Dr., San Jose, California 95134-2022.

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Amdahl Corporation (M/S 153), 1250 Easy Arques Ave., Sunnyvale, California 94088-3470 $\,$