REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

10[65-01, 65Dxx, 65Fxx, 65Gxx, 65Hxx, 65Kxx].—GÜNTHER HÄMMERLIN & KARL-HEINZ HOFFMANN, *Numerische Mathematik*, Grundwissen Mathematik, vol. 7, Springer, Berlin, 1989, xii+448 pp., 24 cm. Price: Softcover DM 38.00.

This is an introductory text, roughly at the first-year graduate level, covering some basic topics of numerical analysis, not including, however, differential and integral equations. The topics are organized in nine chapters, entitled: Computing, Linear Systems of Equations, Eigenvalues, Approximation, Interpolation, Splines, Integration, Iteration, and Linear Optimization. Every chapter has several subsections, each with its own set of exercises. In line with the objectives of the series *Grundwissen Mathematik*, the authors have made an attempt to emphasize cross connections with other disciplines of mathematics, notably the theory of normed linear spaces, and to place the subject in historical perspective by providing numerous historical and biographical notes. While the treatment, on the whole, follows standard patterns, there are topics rarely found in texts at this level, for example, a discussion of complexity measures for algorithms, the Peano representation of the remainder term in polynomial interpolation, multidimensional splines, and the recent polynomial algorithms of Khachiyan and Karmarkar in linear programming. On the other hand, no detailed analysis is given of rounding errors in the basic algorithms of numerical linear algebra.

The reviewer noted only very few typographical errors and found the exposition carefully and expertly done. A few minor inaccuracies, nevertheless, might be corrected in future editions: On p. 174, Rodrigues's formula for Legendre polynomials is (incorrectly) attributed to Gauss. In connection with Bernstein's example on p. 223, the sequence of polynomials interpolating $f(x) = |x|$ on equally spaced points in $[-1, 1]$ is said to diverge for all $0 \leq |x| < 1$, whereas it actually converges (nontrivially) at $x = 0$. The Pólya-Steklov theory does not cover, as claimed on p. 336, convergence of Gaussian quadrature rules on infinite intervals. Finally, on p. 104, the biographical note on Gerhard Hessenberg (1847–1925) seems out of place, since Hessenberg matrices are named after K. Hessenberg who, in his 1941 T. H. Darmstadt dissertation, developed a similarity transformation of an arbitrary matrix to "Hessenberg form" (cf. [1, pp. 314ff]).

In spite of these minor blemishes, the book is a welcome addition to the literature on basic numerical analysis and should especially appeal to mathematically mature students (who are conversant with the German language).

W. G.

1. R. Zurmühl, *Matrizen*, Springer, Berlin, 1950.

11[65M05, 65M10, 65M20, 76-08].—F. W. Wubs, *Numerical Solution of the Shallow-Water Equations*, CWI Tract 49, Centre for Mathematics and Computer Science, Amsterdam, 1988, iv+115 pp., 24 cm. Price Dfl17.80.

This tract is the summary of a research project on the shallow water equations from 1983–1988. The tract consists of two parts. The first part describes the numerical model used and its implementation on the Cyber 205 computer. The second part is a reprint of two papers. One is by Wubs on the stabilization of explicit methods by smoothing. The other paper is by Van der Houwen, Sommeijer, and Wubs on residual smoothing. The implicit smoothing used in the first paper is similar to that proposed both by Lerat and Jameson for fluid dynamic problems. It has also been used by Chima and Jorgenson for time-dependent problems. Here, Wubs describes some theory for this residual smoothing and considers applications to the shallow water equations. The second paper is an extension of the first; the authors derive smoothers for both hyperbolic and parabolic equations.

In the first section a staggered grid is introduced and the two velocity components and height are defined at different locations. A Cartesian grid is used, and so general boundaries are approximated by polygonal approximations. There is no discussion of using body-fitted coordinates. Both second-order and fourth-order central differences are constructed for this staggered mesh. Special formulas are needed near the boundaries. The finite difference equations are integrated in time by a Runge-Kutta scheme, and there is a short description of the stability theory, including the effects of residual smoothing. There is a detailed description of the vectorization of the code and its implementation on the Cyber 205. Finally, results are presented for the Taranto bay in Italy and for the Anno Friso Polder in the Netherlands.

The tract is mainly of interest for people in oceanography. It is nice, however, to see the interplay between this field and fluid dynamics in the use of residual smoothing. This may encourage additional interaction between these close fields. Finally, the description of the vectorization complements that described in other papers and will give the reader details of the implementation on the Cyber 205, though that machine is no longer being produced.

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12[68T05, 65K10, 92-04].—EMILE AARTS & JAN KORST, *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinatorial Optimization and Neural Computing*, Wiley Interscience, New York, 1989, xii+272 pp., 24 cm. Price $49.95.

This book is an excellent introduction for mathematicians and physicists to the subjects of simulated annealing and Boltzmann machines. Furthermore, the discussion of Boltzmann machines provides a rigorous foundation with which to penetrate the very trendy subjects of "neural computing" and "neural networks". The book is divided into two sections, the first concentrating on the simulated annealing algorithm, and the second on aspects of Boltzmann machines, especially those pertaining to parallel and neural computation.

The authors motivate the simulated annealing algorithm as a method for solving problems of combinatorial optimization. These problems are generally considered to be very hard to solve, and in particular, all of the examples of combinatorial optimization problems in the book are from the class of NP-complete problems. The simulated annealing algorithm is then presented using the conceptual analogy of the algorithm to metallurgical annealing. The presentation is very general and only requires a minimization problem with a well-defined objective function, C, over a finite and discrete solution space that has a neighborhood structure. Within this mathematical framework the simulated annealing algorithm consists of proposing a neighboring configuration. The proposal is accepted if it either decreases the objective function, or, when the proposed configuration increases the objective function, a uniformly distributed random number chosen in $[0, 1]$ is greater than the value of $e^{-\Delta C/c}$. This is essentially the well-known Metropolis algorithm, where the constant c in the Boltzmann factor is the simulated annealing analog of temperature. This procedure, augmented with a sequence of c values going to zero, making certain that the algorithm reaches the equivalent of thermal equilibrium at every value of c, constitutes the simulated annealing algorithm. The decreasing sequence of c values is called a cooling schedule.

The discussion of the simulated annealing algorithm then continues with practical considerations, implementations of the algorithm for the NP-complete examples, analytic results, and numerical examples. There seems to be an extensive body of results concerning the global asymptotic convergence properties of the algorithm, and two very important results are presented in great detail. The first is an asymptotic result with the assumption of thermal equilibrium at each value of c. The second, more impressive result, shows that the global asymptotic convergence of the algorithm with a given cooling schedule is possible with only a finite number of iterations at each value of c. This result is based on the analysis of the simulated annealing algorithm as a finite-state Markov chain, and requires only elementary results from the theory of finite-state Markov processes.

The extensive discussion of the simulated annealing algorithm serves as strong motivation for the second section, a discussion of Boltzmann machines. A Boltzmann machine is an interconnected network of elements whose state is either 0 or 1. These binary units are bidirectionally connected with strengths that can take arbitrary positive or negative values. Implicit in a set of connection strengths is the consensus function of the Boltzmann machine which is the sum of the product of the connection strengths and the states of the interconnected units. The computational task of the Boltzmann machine is to maximize its consensus function. This is accomplished with an algorithm analogous to simulated annealing. A unit is chosen for a proposed change in state. This change is accepted if either the consensus function increases or if $1/(1+e^{-\Delta C/c})$ is smaller than a chosen uniformly distributed random number in the interval $[0, 1]$. Decreasing values of c are then used to "cool" the Boltzmann machine into a near optimal configuration.

This definition of Boltzmann machines shows the clear analogy with the simulated annealing algorithm, and so the homologous asymptotic convergence results for Boltzmann machines that are presented next are predictable. One can augment the definition of the Boltzmann machine to allow the choice of proposed units for transition to be done in a concurrent manner for implementation on a parallel device. However, asymptotic convergence for these parallel Boltzmann machines is still an open problem. Implementational and numerical aspects of Boltzmann machines for the solution of the examples from combinatorial optimization are then presented, concluding the discussion of optimization.

The parallel implementation of the Boltzmann machines leads very naturally into the subject of neural computing. First, the problem of classification for Boltzmann machines is addressed. Here a simple example of the classification of digits in a digital display is carefully presented. Its implementation is with a Boltzmann machine with two layers of units, an input and an output layer. The famous problem of classification for the exclusive-or function is then shown to confound a simple two-layered Boltzmann machine, motivating the consideration of hidden units. With hidden units come the ambiguities in the assignment of connection strengths in Boltzmann machines. This leads to the consideration of learning algorithms for iteratively determining connection strengths that will result in a consensus function with local maxima which correspond to the desired classification groupings.

A very elegant theorem for learning in a Boltzmann machine is then described. Given a current equilibrium distribution of local maxima, q', and a desired equilibrium, q, the divergence function $D(q|q')$ is defined. Minimization of this function in the space of connection strength is shown to be equivalent to determining the optimal set of connection strengths for the desired equilibrium distribution. In the special case where the Boltzmann machine has no hidden units, it is proven that $D(q|q')$ is a strictly convex function with a single local minimum. This implies that a steepest descent approach to the minimization of the divergence function is guaranteed to converge. If a Boltzmann machine does have hidden units, $D(q|q')$ is no longer guaranteed to be convex, and heuristic approaches to its minimization are presented.

All in all, the presentation of the material in this book is very balanced. Rigorous results are presented, and an indication of what the authors believe to be the important open problems in the field are included. The Boltzmann machine serves as a fairly rigorous intellectual springboard into the much less rigorous field of neural networks and neural computing. For myself, I found this book an intellectually comforting introduction to this seemingly chaotic new discipline, which clearly marks out the firm ground and the quicksand.

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13[65-01, 65Fxx, 65Kxx].—PHILIPPE G. CIARLET, *Introduction to Numerical Linear Algebra and Optimisation*, Cambridge University Press, Cambridge, 1989, xiv+436 pp., $22\frac{1}{2}$ cm. Price $29.95.

This is what appears to be a straight translation of the French original, entitled "Introduction à l'analyse numérique matricielle et à l'optimisation", except that the exercises, which originally were published separately, are now incorporated in the same volume at the end of each subsection. For a review of the original text, see [1].

W. G.

1. V. Thomée, Review 5, Math. Comp. **42** (1984), 713–714.

14[65-00, 65-01, 65-04, 41-00, 41-01, 33-00].—B. A. POPOV & G. S. TESLER, *Computation of Functions on Electronic Computers–Handbook* (in Russian), Naukova Dumka, Kiev, 1984, 599 pp., 21 cm. Price 1 Ruble, 90 Kopecks.

For the user of modern computers or calculators of all sizes, the computation of values of elementary functions—and even of some special functions—has become a simple and common task. This fact, however, should not make us forget that a good deal of mathematics has had to be developed over the last few decades in order to establish the methods which ensure that these computations can be performed in a fast and accurate manner. Several handbooks have been published in English which treat the numerical computation of functions under different aspects, for example the manual of Cody and Waite [1], and the translation from the Russian of the handbook of Lyusternik et al. [2], for elementary functions, or the various books by Luke, in particular [3]. More than twenty years after the appearance of [2], Popov and Tesler have published, in Russian, another handbook for the computation of functions. Although some of the information contained in it can also be found elsewhere, its collection in one place and its concise presentation together with a treatment of the underlying theory, is undoubtedly valuable. This is even more true when one takes into account that many of the original papers published in Russian are not likely to be easily accessible elsewhere. It should be noted, however, that the authors were apparently unaware of the book by Cody and Waite [1].

The handbook consists essentially of two quite distinct parts. One part (Chapter 1 and Appendices 2 and 3) treats theoretical aspects of procedures which are useful for the approximation of functions, or is concerned with error analysis. The other part (Chapters 2 to 6) consists essentially of a collection of formulae, algorithms, and expansions (with their corresponding coefficients, graphs, etc.) for different classes of functions. Apart from the few short sections of text, a knowledge of Russian is not essential for understanding this part of the handbook.

In Chapter 1 (168 pages), a number of approximation methods are presented including approximation by polynomials or rational functions (e.g., Padé approximation), continued fractions, iterative processes, and piecewise approximation (e.g., splines). Nonlinear approximation and approximation by asymptotic series are also considered, as well as questions of economization and convergence acceleration. A fairly large part of this chapter is devoted to the presentation of a method which the authors call "expansion by residual values", which seems to have been developed mainly by one of the authors. Examples and tables of various kinds are interspersed in the text to illustrate the theory and facilitate applications.

Chapter 2 (109 pages) deals with the algorithmic computation of elementary functions, and Chapter 3 (61 pages) with the so-called "integral" functions, i.e., those which are obtained by nonelementary integration of elementary function, such as the error function. The gamma function is also discussed in this chapter, as well as some functions useful in statistics (the χ^2-, F-, and Student's t-distributions). Chapter 4 (62 pages) is concerned with the family of Bessel functions (of integer and fractional order). Chapter 5 (36 pages) discusses the elliptic integrals and elliptic functions (Jacobi, Weierstrass, and Theta). The last Chapter 6 (39 pages) gives information about some classes of orthogonal polynomials (Jacobi, Gegenbauer, Legendre, Laguerre, Hermite, and Chebyshev). In connection with Chebyshev polynomials, methods for the economization of power series and the Lanczos τ-method are discussed. Euler and Bernoulli polynomials complete this chapter.

It is important to note that most of the computational methods presented in Chapters 2 to 6 refer to functions of real variables. Those algorithms for the elementary functions which are actually implemented on a large number of computers (in particular on those in use in the socialist countries) are given in Appendix 1. Appendix 2 discusses questions of error analysis and Appendix 3 problems of optimization. Appendix 4 gives some tables of useful constants.

An impressive bibliography of more than 400 references, about 60 percent of them referring to publications in Russian, and an index, complete the volume.

Unfortunately, access to this useful handbook is likely to be difficult outside the Soviet Union. In view of the amount of information it contains, one could imagine that a (perhaps updated) English edition would be a valuable complement to the already existing handbooks.

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1. W. J. Cody and W. Waite, *Software manual for the elementary functions*, Prentice-Hall, Englewood Cliffs, N.J., 1980.
2. L. A. Lyusternik, O. A. Chervonenkis, and A. R. Yanpol′skii, *Handbook for computing elementary functions*, Pergamon Press, Oxford, 1965.
3. Y. L. Luke, *Mathematical functions and their approximations*, Academic Press, New York, 1975.

15[68Mxx, 68Q20, 65V05, 65-04].—Jean-Michel Muller, *Arithmétique des ordinateurs—Opérateurs et fonctions élémentaires*, Etudes et recherches en informatique, Masson, Paris, 1989, 214 pp., 25 cm. Price 215 FF.

This book provides a good introduction to the subject of computer arithmetic, covering most of the major areas in an easy manner. The style is pleasantly narrative—but like many narrative novels—it leaves the reader somewhat disappointed at its lack of depth and detail in places. There are chapters on each of the important topics—Boolean logic, number representations, addition, multiplication, division, and elementary function evaluation. Each has its strengths and its weaknesses.

There are also major omissions. Most notably, perhaps, are the very scant treatment of noninteger representations and arithmetic, and the consequent almost total absence of a discussion of errors in computer arithmetic. Another serious lack, since the book is purported to be a text for courses in this subject, is that there are *no* exercises for the student/reader.

Nonetheless, the overall blend of mathematical theory with some of the practicalities of hardware implementation of algorithms is pleasing. I just wish it could have been expanded to a full and comprehensive treatment of these important topics.

The first chapter (9 pages) provides the reader with a very brief review of Boolean algebra and the various logic gates that are used in integrated circuits.

The inclusion of this review is an attractive feature in helping the book to be self-contained. It also allows subsequent algorithmic discussions to be accompanied by practical considerations and circuit diagrams. This is followed by an all-too-brief introduction to complexity theory, which I found both interesting and frustrating. A more extensive treatment could have been followed up in the subsequent chapters in discussing the efficiency of the algorithms.

Chapter 2 (29 pages) introduces various number representations, concentrating largely on complement forms of integer representation and redundant systems. The floating-point system is given just three pages, with an extra two for rounding errors. There is a short section on new representation systems for real arithmetic—but it is too short. For example, the BCD system is included among the integer representations, but there is no mention of Hull's CADAC real arithmetic system, which uses the decimal base. The work of Matula and Kornerup on arithmetics based on continued fraction representations is also not mentioned. The bibliography of this chapter is somewhat patchy, relying almost entirely on publications of the ACM and the IEEE Computer Society. Surely, for example, Kahan deserves some mention somewhere in connection with the IEEE floating-point standard. The discussion of redundant representations is followed up throughout the rest of the book; some reference at this point to its utility in bit-serial and pipelined arithmetic would help to justify the longest section of this chapter.

Chapter 3 (31 pages) gives a very good treatment of the design of various adders, such as ripple carry, conditional sum, carry skip and carry look-ahead adders, including the Brent and Kung implementation. The use of carry save adders is left until their use within multiplication algorithms in the next chapter. The propagation of carries is discussed both theoretically and algorithmically. The efficiency of the different algorithms is also considered. The final section deals with addition of redundant representations—but still with no motivation for such representations.

Chapters 4 (54 pages) and 5 (32 pages) deal in a similar thorough way with multiplication and division. The multiplication algorithms include decomposition, pipelining and in-line algorithms, which finally provide a reason for devoting so much space to redundant representations. Also included are Wallace and Dadda trees; that is, the use of a tree of carry save adders or half-adders. The theoretical and algorithmic discussions are easily read and convincing. Division is also treated thoroughly—restoring, nonrestoring, in-line and iterative algorithms are all covered efficiently, although within the confined space of this book it seems scarcely necessary to treat Newton's iteration and a special case of it as separate sections. The bibliography for both these chapters is extensive.

The final chapter (39 pages) on elementary function evaluation amounts to a very good treatment of CORDIC-type algorithms, including the scheme for the complex exponential function. What is disappointing is the short shrift given to other methods. Many computer routines are still based on clever use of series expansions and rational functions. The latter topic is reduced to one theorem and 3/4 page. Series are subsumed into polynomial approximation, which consists of a brief introduction to Legendre and Chebyshev polynomials. Whilst this is interesting material, it has little relation to the subject at hand, since no algorithms are presented for such approximations to any of the elementary functions, and there is nothing on the efficient evaluation of either Chebyshev or Legendre series. This efficient evaluation is arguably the principal reason that Chebyshev expansions are genuine *practical* approximation tools.

Overall, I found this book something of a curate's egg.

I cannot envisage using it as a teaching text for its lack of exercises, nor will it form an important reference book in my library, since the treatment too often left me with more questions. What it covers in detail, the basic arithmetic algorithms, are dealt with very well and the style is certainly easy. I just wish it had been twice as long so that the reader is left with more answers and fewer questions. Maybe this implies its likely role—as an introduction for a research student which will prompt him/her to ask some of the questions and perhaps find some of the answers.

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16[62-07, 62A10].—MURRAY AITKEN, DOROTHY ANDERSON, BRIAN FRANCIS & JOHN HINDE, *Statistical Modelling in GLIM*, Oxford Statistical Science Series, Vol. 4, Clarendon Press, Oxford, 1989, xi+374 pp., $23\frac{1}{2}$ cm. Price \$75.00 hardcover, \$35.00 paperback.

This text provides a statistically capable reader with an opportunity to view the interplay between the theory of maximum likelihood estimation using analysis of deviance principles and data analytic techniques available with the GLIM3, Generalized Linear Interactive Modelling, statistical package. For each of a variety of useful exponential family models, the authors present a concise summary of the likelihood approach to model fitting and parameter estimation. An analysis of a specific data set typically follows. Included in this analysis are suitable GLIM commands to implement the chosen model, examples of GLIM output, especially analysis of deviance tables and likelihood ratio tests based on the differences between deviances, discussions about model selection, simplification and adequacy, and graphical tools, available as GLIM macros, that greatly facilitate the entire modelling procedure.

For a data analyst, the strength of the text is in the examples, especially when the authors discuss choices between competing models, where the principle of *parsimony* (see Section 2.1) is applied when formal statistical methods can no longer distinguish between alternative models. The reader may not always agree with the authors' final model selection, but will benefit from observing the data analytic approach taken and the GLIM tools used to reach the decision. This is particularly important when the competing models involve different error distributions and link functions (see Sections 2.4 and 2.5) and the models themselves become more complex (see Chapter 6).

The text will benefit an audience with a mixed statistical background, especially those data analysts who require a more extensive theoretical framework than that provided by classical regression theory. However, the reader should be familiar with standard normal model regression theory and have some knowledge of maximum likelihood model fitting, along with an understanding of the log likelihood function and likelihood ratio test. The theoretically weak reader should follow the discussion in Appendix 1 on maximum likelihood fitting of exponential family regression models, especially the sections devoted to the *profile likelihood* which is used extensively throughout the text. The reader wanting further theoretical detail should consult the recommended text *Generalized Linear Models* by McCullagh and Nelder.

With its obvious dependence on GLIM modelling capabilities, the reader should have access to some version of GLIM. The examples provided do allow the text to be used in a teaching environment without access to the appropriate software, although at a considerable reduction in the text's value. Chapter 1 of the text is devoted to an overview of GLIM command syntax and output features. By itself, this chapter and the appendices on GLIM directives and structures are sufficient for a new user to cope with the examples and macros presented within the text. However, it is certain that any user should have the GLIM manual available for reference, so as to maximize the practical, not just statistical, advantages of the text. This is especially true if the user intends to embark on macro writing, or even to fully understand the details of the macros supplied within the text.

Chapter 2 introduces modelling and inference using maximum likelihood techniques in exponential families. The definition of the generalized linear model through its three primary elements: the probability distribution of the dependent variable; the linear regression function involving the explanatory variables; and the *link* function between the linear predictor and the mean of the dependent variable, is given in Section 2.4. The likelihood function and maximum likelihood estimation of the parameters is developed in Section 2.5, while in Section 2.6 there is a discussion of the nature and potential weaknesses of standard errors of regression parameters based on the estimated expected information matrix that GLIM provides. The likelihood ratio test for choosing among models (Section 2.7), model simplification methods (Section 2.8) and model adequacy and diagnostic checking (Section 2.10) are especially valuable parts of the text. The reader is also referred to Section 2.14, where the authors mention the minor difference between the GLIM definition of *deviance* and the definition used throughout the text.

While Chapter 3 presents topics involving the well-known normal regression model and analysis of variance, the great strength of this chapter is its presentation of the Box-Cox transformation family and model selection using the BOXCOX macro for construction and plotting of the *profile log likelihood*. Theory is given in Section 3.1, then GLIM modelling directives and model comparison in the next section. The authors make the excellent point that the transformations apply to the skewed distribution of the response variable and are to be distinguished from the link function. Examples of the latter appear in Section 3.3. Regression models for prediction, the mean squared error criterion for model selection, cross-validation techniques and the PRESS statistic are discussed in Sections 3.4–3.6. The analyses of designed factorial experiments and cross-classified observational data are discussed in Sections 3.11 and 3.12. Techniques given for handling unequally replicated cross-classified data are especially useful, in particular the modification to a two-parameter Box-Cox family of transformations and corresponding profile log likelihood function, or the addition of small positive constants to empty cells.

Chapter 4 looks at binomial response data, beginning with the well-known logit, probit and complementary log-log transformations, the idea of the log-odds ratio, and model evaluation methods for such data (Sections 4.1–4.3). The remaining sections of this chapter discuss more complex contingency tables for binary data, with detailed examples of modelling with GLIM, especially with the large example in Section 4.8. The reader's attention is also drawn to Section 4.9, where the problem of overdispersion due to omitted variables is briefly discussed.

The natural extension of the binary response model to a multicategory response appears in Chapter 5. Section 5.4 summarizes the theory and GLIM fitting of the multinomial logit model. However, since GLIM lacks a specific multinomial error distribution, the fitting is achieved by exploiting the known relationship between the multinomial and Poisson distributions, and using GLIM's available Poisson error distribution. Sections 5.5 and 5.6 provide the necessary details, while direct fitting of a Poisson model and overdispersion problems with such fits are discussed in the early sections of this chapter. The topic of ordered response categories is addressed in Section 5.7.

In Chapter 6, the emphasis switches to continuous responses and role of the exponential distribution, and many related and derived distributions, in survival analysis. The hazard function is introduced in Section 6.2. GLIM fitting of the exponential distribution is performed in Sections 6.3 and 6.4, and a comparison made with the normal family and Box-Cox transformations in Section 6.5. Censoring in survival analysis is discussed throughout Sections 6.6–6.8. Most of Sections 6.9–6.19 are devoted to short introductions to a variety of competing survival distributions, including gamma, Weibull, extreme and reversed extreme value, piecewise exponential, logistic, log-logistic and lognormal distributions. The Cox proportional hazards model is briefly discussed in Section 6.15. The reader will benefit from Section 6.20, where GLIM procedures are used to help decide among different survival distributions. A number of published papers listed in the references will provide the more advanced reader with GLIM modelling methods for a greater variety and complexity of survival models.

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17[76-06, 76T05].—JOSÉ FRANCISCO RODRIGUES (Editor), *Mathematical Models for Phase Change Problems*, Internat. Ser. Numer. Math., Vol. 88, Birkhäuser, Basel, 1989, x+410 pp., 24 cm. Price $76.00.

These are the proceedings of a workshop held at Óbidos, Portugal, October 1–3, 1988. There are 20 contributions, organized in three chapters entitled: Generalized Phase Changes, Stefan Problems, and Miscellaneous Problems. While the emphasis is on mathematical modeling, several contributors also address computational issues.

W. G.

18[65-06, 65Dxx].—TOM LYCHE & LARRY L. SCHUMAKER (Editors), *Mathematical Methods in Computer Aided Geometric Design*, Academic Press, Boston, 1989, xv+611 pp., 23 $\frac{1}{2}$ cm. Price $49.95.

This volume grew out of an international conference on the topic of the title, held at the University of Oslo, Norway, June 16–22, 1988. Its content is accurately described on the back cover of the book: "The volume contains survey papers as well as full-length research papers. The mathematical objects discussed include univariate and multivariate splines, algebraic curves, rational curves and surfaces, Bézier curves and surfaces, and finite elements. The topics treated include scattered data interpolation, geometry processing, convexity and shape preservation, subdivision, knot insertion and removal, knot selection for parametric curves, geometric continuity, and cardinal interpolation."

W. G.

19[68U30, 68N05].—DAVID V. CHUDNOVSKY & RICHARD D. JENKS (Editors), *Computer Algebra*, Lecture Notes in Pure and Applied Mathematics, Vol. 113, Marcel Dekker, New York and Basel, 1989, ix+240 pp., 25 $\frac{1}{2}$ cm. Price: Softcover $99.75.

The book consists of a collection of articles related to the Conference on *Computer Algebra as a Tool for Research in Mathematics and Physics*, held at New York University in April 1984. This conference was the first of the *Computers & Mathematics* series, the latest of which was held at Massachusetts Institute of Technology in July 1989. The papers cover a diverse range of subjects with the emphasis on either computer use to carry out mathematical investigations with very complex expressions, or on the description of certain such computer systems.

The article by D. Chudnovsky and G. Chudnovsky deals with how to use computers to investigate diophantine approximation via functional approximation methods; in particular, Padé approximations. The paper constitutes about one third of the entire volume. A short article by H. Cohn reports on computing certain Hilbert modular equations. G. Andrews investigates, using the Scratchpad system, summation-product identities arising in various contexts; among them, the famous Rogers-Ramanujan identities. R. Askey points to difficult multidimensional definite integrals of special functions for the need of using computers to handle the complicated algebra necessary in their proofs. D. Ford and J. McKay give a brief account of how one would determine the Galois group of a rational polynomial. The paper by L. Auslander and A. Silberberg and the paper by J. Cooley describe the application of the Scratchpad systems in the search for better convolution algorithms.

The systems described are the Cayley system for finite group theory in the article by M. Slattery, who exhibits several example sessions; the special-purpose system POLYPAK for computing very high-order series solutions of differential equations in celestial mechanics, described by its designer D. Schmidt; and a Fortran library by R. Riley (called the PNCRE system) for manipulating finitely generated subgroups of $SL_2(\mathbf{C})$.

J. Davenport's paper discusses several issues arising in data abstraction and representation when building a computer algebra system. The book ends with a transcript (taken by Davenport) of a panel discussion on "The Potential of Computer Algebra as a Research Tool" held during the conference. The panel members were R. Askey, M. E. Fisher, J. McCarthy, J. Moses, and J. Schwartz. As a participant of the conference and the panel discussion, I found it quite useful to have a written record of what was said then—four years ago—about this field.

Overall, the book has the flavor of a conference proceedings (with an index of key terms), and certainly provides glimpses into the possibilities of using computers for doing mathematics. I find it a good addition to my collection of works on computer algebra.

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20[18–04, 55–04, 55Q05, 68Q40].—Martin C. Tangora (Editor), *Computers in Geometry and Topology*, Lecture Notes in Pure and Applied Mathematics, Vol. 114, Dekker, New York and Basel, 1989, viii+317 pp., $25\frac{1}{2}$ cm. Price \$99.75.

These lecture notes contain fourteen papers arising from the Conference on Computers in Geometry and Topology, held on March 24–28, 1986, in Chicago.

The papers, though varying in tone and flavor, are all concerned with the problems one encounters when trying to do calculations in topology or homological algebra. Usually in mathematics, and especially in topology, the interesting canonical invariants that one wishes to determine are given in terms of certain obstruction groups like cohomology or homotopy groups. These groups are in turn defined to be the quotient of some very infinite object by another. The actual computation of these invariants poses therefore new and, as it turns out, interesting problems.

It is, for a start, shown in the first paper, by D. J. Anick, the only paper with a genuine computer science flavor, that the subject matter is difficult: he shows that computing the homotopy groups $\pi_n(X) \otimes \mathbf{Q}$ of finite simply connected CW-complexes X is at least as hard as any problem that can be solved nondeterministically in polynomial time. The difficulties one encounters, in theory and in practice, are vividly illustrated by three papers that are concerned with calculations pertaining to the higher homotopy groups of the spheres, and by several other papers concerned with explicit computations: regarding knots, immersions of $\mathbf{P}^n(\mathbf{R})$ in Euclidean spaces, bifurcation theory and the classifying space of the generalized dihedral group of order 16.

The volume contains furthermore some papers on general-purpose algorithms to compute cohomology groups, syzygies, etc. and discussions of computer algebra packages suitable for "topological calculations". Finally, there are two papers by Milnor and by Handler, Kauffman and Sandin on the Mandelbrojt set.

R. S.

21[11A15, 11Y55].—Samuel S. Wagstaff, Jr., *Table of all Carmichael numbers* $< 25 \cdot 10^9$, 38 computer output sheets deposited in the UMT file.

This table of the 2163 Carmichael numbers $< 25 \cdot 10^9$ was placed in the UMT file in connection with the paper [2]. It has not been reviewed until now for reasons too complicated to set down here. It is reviewed now because of Jaeschke's UMT table described in the following review.

Wagstaff's table has 14 columns. Col. 1 has the sequential number of the Carmichael number (CN) from 1 to 2163. Col. 2 is the CN from CN = 561 to CN = 24991309729. Cols. 3–12 give a characterization of the CN for each of the first ten prime bases: $p = 2$ through $p = 29$. If the CN is not a *strong pseudoprime* to the base p (and this is usually the case), the respective column states "weak". See, e.g., Definition 44 in [3, p. 227]. If p divides CN, the column is left blank. If

$$\mathrm{CN} = t \cdot 2^s + 1,$$

with t odd, and if

$$(*) \qquad p^t \equiv 1 \pmod{\mathrm{CN}},$$

the column states "ST+1", while if CN is a strong pseudoprime to the base p but $(*)$ does not hold, the column states "ST−1".

Col. 13 lists the number (3 through 7) of prime factors in CN, and Col. 14 gives the factorization of CN. The reviewer easily checked that the three Carmichaels $< 25 \cdot 10^9$ that are "acceptable Perrin composites" [1] are in the table as CN #1353, #1375 and #2142. But the fourth Carmichael that is an acceptable Perrin composite is beyond this table, since it equals $43234580143 = 223 \cdot 5107 \cdot 37963$. See the next review.

D. S.

1. G. C. Kurtz, Daniel Shanks, and H. C. Williams, *Fast primality tests for numbers less than* $50 \cdot 10^9$, Math. Comp. **46** (1986), 691–701.
2. Carl Pomerance, J. L. Selfridge and Samuel S. Wagstaff, Jr., *The pseudoprimes to* $25 \cdot 10^9$, Math. Comp. **35** (1980), 1003–1026.
3. Daniel Shanks, *Solved and unsolved problems in number theory*, 3rd ed., Chelsea, New York, 1985.

22[11A15, 11Y55].—GERHARD JAESCHKE, *Table of all Carmichael numbers* $< 10^{12}$, 21 computer output sheets deposited in the UMT file.

This table of the 8238 Carmichael numbers (CN) $< 10^{12}$ was placed in the UMT file in connection with the paper [1]. They are listed 395 per page in five columns and 79 rows. No other information is given; compare the elaborate detail in the previous review. Thus, even to determine that 43234580143, which is mentioned in the previous review, is CN #2652, requires a moderate effort. The present table, therefore, supersedes Wagstaff's table only in part.

Three points about [1] may be mentioned here. A CN may be defined as a number that satisfies

$$a^{\mathrm{CN}} \equiv a \pmod{\mathrm{CN}}$$

for *all* integers a. This is both simpler and more general than the definition given in [1]. Even a casual glance at the table shows that most (?) of the CN end in the decimal digit 1. This has long been known. In [1], the CN are analyzed $(\mathrm{mod}\,12)$ but not $(\mathrm{mod}\,10)$. Swift's earlier UMT table of the 646 CN $< 10^9$ has an "Author's summary" [2] wherein CN that are products of three primes are also analyzed.

As submitted, each page of the present table had a two-inch solid black band at the top of the page. After determining that there was no information here, the reviewer boldly sliced off this top with a paper trimmer. This (a) reduced the space requirement of the table in the UMT file and (b) enabled the reviewer to appropriately celebrate the 200th anniversary of the French Revolution.

D. S.

1. Gerhard Jaeschke, *The Carmichael numbers to* 10^{12}, Math. Comp. **55** (1990), 361–367.
2. J. D. Swift, *Table of Carmichael numbers to* 10^9, Review **13**, Math. Comp. **29** (1975), 338–339.