REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

29[65–01, 65Mxx, 65Nxx].—DANIEL EUVRARD, *Résolution Numérique des Équations aux Dérivées Partielles: Différences Finies, Éléments Finis, Problèmes en Domaine non Borné*, 3rd ed., Masson, Paris, 1994, xiv + 329 pp., 24 cm. Price: Softcover F 182.

This is the third edition of a textbook, in French, intended for students in physics, mechanics, and engineering, on the numerical solution of partial differential equations. It has grown out of courses given by the author over more than 20 years at a number of different universities in France, most recently at the University of Paris VI.

Since it is not intended for students with a good mathematical background, it avoids mathematical rigor, but the choice of material and the presentation are nevertheless strongly influenced by the mathematically oriented school of numerical analysis originating with Professor J. L. Lions, which is also acknowledged in the introduction of the book.

The author emphasizes the point of view that in order to be able to reasonably solve a PDE numerically, one has to know the properties of this equation and its solutions. He therefore spends about as much time on the continuous problems as on their discretization, deriving exact solutions for specific examples where these can be easily found, and using these to elucidate general properties. These examples replace a general theory, and are used to mctivate properties of numerical methods.

The book is divided into three parts headed I. Finite Differences, II. Finite Elements, and III. Numerical Problems in Unbounded Domains. In Part I, which makes up about half of the book, the five chapters are concerned with Laplace's equation, the heat equation, the wave equation, Burgers's equation, and the Navier-Stokes equations. Basic material, and concepts such as consistency, stability, and convergence are discussed as well as discrete maximum principles and energy arguments, and the standard finite difference methods are presented.

In Part II, making up about one fourth of the book, there are three chapters on Basic Finite Elements, Advanced Finite Elements, and Applications to Elasticity Problems. Again, the material is standard, with emphasis on practically useful elements, and on applications.

Part III, finally, also has three chapters, entitled The Boundary Element Method, Applications to Wave Propagation, and Alternative Methods. The

emphasis in this part is on integral representation with singular kernels of solutions. In the first of the chapters, the exterior Neumann problem is discretized using collocation with piecewise constants and a polygonal approximation of the domain. In the second, the ideas are applied to problems in acoustics and hydrodynamics, and in the final chapter, the method of coupling of finite elements and integral representations is discussed.

Even though I would personally have preferred a somewhat bigger dose of mathematical analysis in a text on numerical methods for PDEs for the type of students targeted (and we do include that in the course for engineering students in our university), I have quite a bit of sympathy for the presentation and the list of topics covered in the present book.

V.T.

30[49–02, 49M15, 49N99, 90C30].—R. Bulirsch & D. Kraft (Editors), *Computational Optimal Control*, Internat. Ser. Numer. Math., Vol. 115, Birkhäuser, Basel, 1994, x + 382 pp., 24 cm. Price $94.00.

This book is a collection of selected papers of the ninth IFAC Workshop on Control Applications of Optimization held in Munich in September 1992. There are 30 papers ranging from 6 to 19 pages in length. The collection is divided into five sections. The first section contains four invited papers surveying the field of computational optimal control. Two of the papers describe the transcription of optimal control problems into nonlinear programming problems and discuss sequential quadratic programming (SQP) methods for solving them. The other two papers survey optimal control problems that have been studied in robotics and aerospace applications.

There are five papers in the second section of the book which discusses the theoretical aspects of optimal control and nonlinear programming. These papers discuss recent work on methods of solving boundary value problems, synthesizing adaptive optimal controls, reduced SQP methods, and time-optimal control of mechanical systems. Unfortunately, several of the proofs of the theoretical results have been omitted and are referenced in "forthcoming papers" or theses (that generally take some time to obtain).

Section three contains eight papers presenting algorithms used for optimal control computations. Algorithms discussed include SQP methods, backward procedures for calculating the solvability sets in differential games, repetitive optimization, and interior-point methods. While most of these papers outline algorithms for a class of problems and subsequently illustrate their use on example problems, a couple of the papers are more specialized in that they discuss algorithms for solving particular problems (time-optimal control of a type-2 third-order system and space shuttle reentry with uncertain air density).

In Section four, there are four papers detailing available software and recent efforts in producing software for optimal control calculations. Approaches used in these software packages include symbolic differentiation of equations, using a symbolic manipulation language to generate optimization routines for numeric solution, and interfaces for allowing the user to view the status of numerical results as well as to change design parameters. Examples are given in each of these papers to give the reader a flavor for how the software packages operate.

Finally, nine papers comprise Section five, describing applications of optimal control to a variety of fields such as aeronautics, robotics, and biology. These papers give a sampling of ways optimal control or optimization enter into different disciplines. One paper, discussing an object-oriented approach to developing control systems, does not really describe an application of optimal control and actually seems out of place in this computational optimal control book.

The writing in general is good, but as most of the authors are from non-English speaking countries, there are occasionally awkward sentence structures and grammatical mistakes. The referencing and bibliographic styles vary from paper to paper. Further, there are no section numbers used, but section numbers are occasionally referenced; undoubtedly, section numbers were used in the original conference papers and were removed in the compilation for this book.

Overall, this is a good collection of papers to give the reader an overview of recent theoretical and applied studies in optimal control and optimization. This book is thus recommended as a library holding. However, since detailed proofs of some of the theoretical results are missing, and since the book only gives an overview rather than a comprehensive coverage of any particular application area, most researchers will probably find it difficult to justify purchasing the book for their individual libraries.

Lucy Y. Pao

Department of Electrical Engineering and Computer Science
Northwestern University
Evanston, IL 60208

31[65–01, 65F10, 65N55].—Wolfgang Hackbusch, *Iterative Solution of Large Sparse Systems of Equations*, Applied Mathematical Sciences, Vol. 95, Springer, New York, xxii + 429 pp., 24 cm. Price $59.00.

When systems of partial differential equations are discretized, by finite difference methods or finite element methods, corresponding systems on discrete finite-dimensional spaces arise. Also, if this methodology is applied to a linear system of differential equations, a discrete *linear* system is obtained. Hence, such systems can, in principle, be solved by standard elimination methods, well known to any student of linear algebra. Furthermore, it is relatively easy to develop general computer codes for these procedures. However, the necessary amount of work required increases dramatically with the size of the systems. Therefore, in order to solve systems with up to a million unknowns, which often is required in modern scientific computing, one has to search for more efficient algorithms.

Since differential operators are local operators, the corresponding discrete operators will inherit a similar property. Hence, we are led to the study of sparse systems, i.e., systems where only a few of the unknowns are present in each equation. Iterative methods, where a converging sequence of approximations of the solution of the system is generated, are very well suited for large sparse systems. Therefore, such methods have become the dominant class of methods for large systems arising from partial differential equations.

The thesis of D. M. Young [4], from 1950, is often referred to as the beginning of the modern development of iterative methods for the linear systems arising

from discretizations of elliptic differential equations. Since then, there has been a steadily increasing activity in this field, leading up to the development of very effective algorithms like multigrid methods and domain decomposition methods in the 1970s and the 1980s. The purpose of the present book is to describe the recent state of the theory for iterative methods.

In the first chapter of the book the linear operator which arises from the simplest discretization of the Poisson equation, i.e., the five-point operator, is studied. In particular, the relation between the difference operator and possible matrix representations of the operator is discussed. This simple example, which is referred to throughout the book, is essential for the presentation since it introduces a family of systems which depend fundamentally on a discretization parameter. Hence, through this example, the author is able to illustrate the main ingredients in the development of elliptic solvers.

In the second chapter a recapitulation of linear algebra is given, while Chapters 3–7 are mostly devoted to the classical iterations (Jacobi, Gauss-Seidel and SOR) and other classical procedures like alternating directions and Chebyshev methods. This part of the book can be described as an updated version of Varga's book [2] from 1962. In Chapter 8 the general concept of "preconditioning" is introduced, and this chapter ends with a discussion of preconditioners derived from incomplete factorizations. The theory of the conjugate gradient method and its variants is discussed in Chapter 9.

The two last chapters cover multigrid methods and domain decomposition methods. These chapters have probably been the most difficult to write since these areas are heavily influenced by research from the last years. For other references on these topics we refer, for example, to the book of Bramble [1] on multigrid methods and the survey article by Xu [3].

The chapter on multigrid methods contains a motivating example in one space dimension and a careful analysis of two-grid methods. Furthermore, results for general multigrid methods are derived from properties of two-grid methods.

The final chapter on domain decomposition is mostly devoted to additive and multiplicative Schwarz iterations, i.e., domain decomposition methods with overlapping subdomains. In particular, attention is focused on the use of these algorithms on parallel computers.

The book is written in a precise mathematical style with theorems, proofs and algorithms. Furthermore, applications and computational issues are frequently discussed. Throughout the book, Pascal procedures of the suggested algorithms are given and numerical results are presented. The book also contains many exercises, which makes it potentially useful as a textbook in a graduate course on iterative methods.

The author deserves a lot of credit for writing a complete book on iterative methods, which starts with elementary matrix theory and ends with a discussion of multigrid methods and domain decomposition methods for elliptic problems. However, a potential textbook in this area also has to include some necessary background on differential equations and discretization techniques. In my opinion, this book would have been improved if the first chapter had been expanded in order to give more of this background. For example, by presenting indefinite problems, discrete Stokes' problems and discrete integral equations in the introductory chapter, it would have been easier for the author to explain the nec-

essary relations between the iterative methods and properties of the systems. In particular, this would have benefited the readers with a weak background in differential equations. Furthermore, it appears to me that in order to fully understand the two final chapters, it is necessary to have a background in partial differential equations far beyond what is given in the book. Still, I am of the opinion that the author has written a book which will be very influential for the development of computational mathematics for many years to come.

RAGNAR WINTHER

Department of Informatics
University of Oslo
Blindern
0316 Oslo, Norway

1. J. H. Bramble, *Multigrid methods*, Pitman Research Notes in Mathematics, 1993.

2. R. S. Varga, *Matrix iterative analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1962.

3. J. Xu, *Iterative methods by space decomposition and subspace correction*, SIAM Rev. **34** (1992), 581–613.

4. D. M. Young, *Iterative methods for solving partial differential equations of elliptic type*, Thesis, Harvard University, 1950.

32[31B30, 35J99, 46N40, 65–02, 65D32].—S. L. SOBOLEV, *Cubature Formulas and Modern Analysis: An Introduction* (Translated from the Russian), Gordon and Breach, Philadelphia, PA, 1992, xvi + 379 pp., $23\frac{1}{2}$ cm. Price $120.00.

This book is an abridged translation from the Russian of S. L. Sobolev's 1974 work 'An Introduction to the Theory of Cubature Formulas'. The cover says that the book has been revised and updated. Comparison with the Russian original suggests that the main revision has been the condensation of the first 13 chapters (pp. 1–644) of the original into the first two chapters (pp. 1–199) of the present volume. The claim that the work has been updated is hardly sustainable, given that the most recent reference is dated 1974, and that the material on cubature in Chapters 3 through 8 is a near verbatim translation of Chapters 14 through 19 of the original.

A cubature formula is an approximate expression for an integral over a domain $\Omega \subset R^n$, of the form

$$(1) \qquad \int_\Omega \phi(x)\, dx \approx \sum_{k=1}^{N} c_k \phi(x^{(k)}),$$

where $x^{(1)}, \ldots, x^{(n)}$ (the 'nodes') are points in Ω, and $c_1, \ldots, c_n$ are real numbers, usually required to be positive.

Cubature (or multidimensional quadrature) has many applications in physics, chemistry, statistics, and other fields, but this book is not directed at people with practical problems: it contains essentially no explicit formulas, and does not discuss any practical issues. Rather, its focus is on the theory of 'optimal' cubature formulas of a certain kind (this notion being explained below), and, as the subtitle suggests, on certain related areas of modern analysis. (The analysis literally comes first: not until p. 205 is cubature defined!)

An optimal formula, roughly speaking, is one that, for a given value of N, is 'best' in the sense that its error functional has the smallest norm for all

functions ϕ in an appropriate function class. For Sobolev, not unnaturally, the function spaces are the Sobolev spaces $L_2^m(\Omega)$ or $L_2^m(R^n)$ of functions whose (generalized) derivatives of orders up to m are square integrable. He also generally insists that the formula be exact for all polynomials of degree $\leq m - 1$, as a result of which two functions are considered to be equivalent if they differ only by a polynomial of degree $\leq m - 1$. The norm in this space is then

$$(2) \qquad \|\phi\|_m = \left(\sum_{|\alpha|=m} \frac{m!}{\alpha!} k \int (D^\alpha \phi)^2 dx \right)^{1/2},$$

where the integrals are over Ω or R^n, and where (in the usual notation for partial differential equations) $\alpha = (\alpha_1, \ldots, \alpha_n)$, $\alpha! = \alpha_1! \cdots \alpha_n!$ and $|\alpha| = |\alpha_1| + \cdots + |\alpha_n|$. An optimal formula is then one for which the norm $\|l\|$ of the error functional l for the integral over Ω is as small as possible.

One deficiency of the book, arising from the 20-year delay of its appearance in English, is that it ignores the very large amount of work that has been done, by Russian and other authors, on optimal formulas in other settings, for example in spaces of functions with bounded L_2 norms of mixed derivatives up to the same order in each variable.

The core of the argument in the setting of this book is that, provided we assume $m > n/2$, the error functional $l(\phi)$ is a bounded linear functional on L_2^m, so that the Riesz representation theorem allows $l(\phi)$ to be represented as an inner product in L_2^m,

$$(3) \qquad l(\phi) = (\phi, \psi)_m = \int \sum_{|\alpha|=m} \frac{m!}{\alpha!} D^\alpha \phi D^\alpha \psi \, dx,$$

for some $\psi \in L_2^m$. Repeated integration by parts leads to the characterization of ψ as the solution of the polyharmonic equation

$$(4) \qquad \Delta^m \psi = (-1)^m \left[\chi_\Omega - \sum_k c_k \delta(\cdot - x_k) \right],$$

with χ_Ω the characteristic function of Ω. Since $\|l\| = \|\psi\|_m$, the question of minimizing $\|l\|$ can now be tackled by methods of potential theory and partial differential equations.

Since equation (4) is obtained by integration by parts, the absence of boundary terms clearly requires some boundary assumptions on ψ or ϕ if Ω is a bounded region, but the precise setting that is intended at each point of the argument is not always clear.

The analysis nevertheless is of impressive weight, as one would expect from an author of Sobolev's standing. The book can therefore be expected to be of some interest to analysts with an interest in partial differential equations.

Should the book be required reading for numerical analysts with an interest in cubature? Perhaps, but we suspect that they will find it hard going, and the effort arguably not worthwhile. It has to be said, first, that the book is marred by many instances of careless presentation, illustrated by three typographical errors in the very first page. (The first is: 'The product of an $m \times n \ldots$ and a $k \times l$ matrix is ... ') While some of the faults are trivial, similar errors occur in

material that is often far from trivial, and even the preliminary chapters need to be read carefully, because it is here that the notation (much of it nonstandard) is established.

Errors aside, some readers may find it difficult to establish exactly what the assumptions are—even the 1-dimensional example in Chapter 6 stops short of presenting the actual quadrature formulas obtained with this approach. Instead, it refers the reader to work by Shamalov that is unavailable in the West. In the 1-dimensional setting sharper and more explicit results can be found, for example, in the survey [1] (not among the references of the original or the translation).

As noted above, a key question is: what restrictions, if any, are placed on the behavior of ϕ and ψ at the boundary of Ω? This is surprisingly hard to pin down, because the setting at the start of the cubature discussion seems to be $L_2^m(\Omega)$ but then shifts to $L_2^m(R^n)$.

Finally, our friends in numerical analysis who master this book will, we think, be inclined to question the practical value of the optimal rules produced by the methods of this book. Indeed, the author is admirably frank about the difficulties that face optimal cubature: 'Unfortunately, this choice [of function space setting] is partly dictated by the desire to obtain the problem subject to the method of investigation planned by the author, since the natural formulation itself may turn out to be too difficult.' It should also be mentioned that the optimality discussion in this book does not consider in full generality the choice of nodes: instead, guided by results for the periodic case, 'we ... restrict ourselves to the case when the system of nodes ... is a parallelepipedal lattice, and [we] will only change the parameters of this lattice.' It is generally accepted that for nonperiodic smooth functions on bounded regions the cubature points should in some way be concentrated towards the boundary, and this is recognized in Chapter 5 through the ad hoc addition of extra cubature nodes near the boundary. While this is done in such a way as to preserve the optimal order of convergence, many readers will feel that the treatment of boundary effects remains unsatisfactory.

Ian H. Sloan
Christoph Schwab

School of Mathematics
University of New South Wales
Sydney 2052
Australia

Department of Mathematics and Statistics
University of Maryland Baltimore County
Baltimore, MD 21228-5398

1. S. M. Nikolskii, *Quadrature formulas* (with an appendix by N. P. Korneichuk), 3rd ed., "Nauka", Moscow, 1979. (Russian)

33[11–02, 11Y40].—Michael E. Pohst, *Computational Algebraic Number Theory*, DMV Seminar, Band 21, Birkhäuser, Basel, 1993, x + 88 pp., 24 cm. Price: Softcover $26.50.

This 88-page booklet is based on lectures given by the author in the DMV seminar on Computational Number Theory held at Schloß Mickeln, Düsseldorf,

August 1990. It contains a discussion of the fundamental computational problems of algebraic number theory: the computation of the ring of integers of a number field and the computation of the unit group and the ideal class group of this ring. All algorithms presented are based on calculations with integral lattices, in particular on the Lenstra-Lenstra-Lovász lattice reduction algorithm.

The text does not contain full proofs of the basic theorems of algebraic number theory. For these, the reader, unfortunately, is often referred to the idiosyncratic book by Pohst and Zassenhaus [1]. However, in contrast to that book, the discussion of the computational aspects of the theory is here very clear and pleasant to read. The examples presented, which are computed using the "Kant" computer algebra package, are impressive. They represent the state of the art and give the reader some feeling for the practical side of the problem.

I highly recommend this short text to anyone interested in the computational aspects of algebraic number theory.

R.S.

1. M. Pohst and H. Zassenhaus, *Algorithmic algebraic number theory*, Cambridge Univ. Press, Cambridge, 1989.