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PRIMITIVE CENTRAL IDEMPOTENTS OF FINITE GROUP RINGS OF SYMMETRIC GROUPS

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ABSTRACT. Let p be a prime. We denote by S_n the symmetric group of degree n, by A_n the alternating group of degree n and by $\mathbb{F}_p$ the field with p elements. An important concept of modular representation theory of a finite group G is the notion of a block. The blocks are in one-to-one correspondence with block idempotents, which are the primitive central idempotents of the group ring $\mathbb{F}_qG$, where q is a prime power. Here, we describe a new method to compute the primitive central idempotents of $\mathbb{F}_qG$ for arbitrary prime powers q and arbitrary finite groups G. For the group rings $\mathbb{F}_pS_n$ of the symmetric group, we show how to derive the primitive central idempotents of $\mathbb{F}_pS_{n-p}$ from the idempotents of $\mathbb{F}_pS_n$. Improving the theorem of Osima for symmetric groups we exhibit a new subalgebra of $\mathbb{F}_pS_n$ which contains the primitive central idempotents. The described results are most efficient for $p = 2$. In an appendix we display all primitive central idempotents of $\mathbb{F}_2S_n$ and $\mathbb{F}_4A_n$ for $n \leq 50$ which we computed by this method.

INTRODUCTION AND NOTATION

Let p be a prime, let $q = p^s$ for some $s \in \mathbb{N}$ and let G be a finite group. For the finite field with q elements we write $\mathbb{F}_q$, and $\mathbb{F}_qG$ denotes the group ring of G over $\mathbb{F}_q$. We use S_n and A_n for the symmetric and alternating group of degree n, respectively. We write $\mathbb{Z}_m$ for a cyclic group of order m, and $\mathbb{Z}_m \wr S_i$ for the wreath product of this group with S_i. We use $\mathrm{C}_G(g)$ for the centralizer of $g \in G$, and $\mathrm{C}_G(U)$ for the centralizer of $U \subseteq G$. The centre of a group ring FG is denoted by $Z(FG)$ and the radical of the centre by $Rad(Z(FG))$. We use $\mathrm{Irr}\, G$ for the set of irreducible characters of G defined over the field $\mathbb{C}$ of complex numbers and we write $\mathrm{ord}(g)$ for the order of an element $g \in G$. The symbols $\mathbb{Z}$ and $\mathbb{N}$ denote the integers and the natural numbers, respectively.

A big part of modular representation theory deals with blocks. There are several possibilities to characterize blocks, for instance by the block idempotents, i.e. the primitive central idempotents of $\mathbb{F}_qG$. Therefore it is important to find methods to compute the primitive central idempotents.

The usual method to compute primitive central idempotents is described in [8], Lemma 16.6. For this method it is necessary to compute the character table of G over the field $\mathbb{C}$ of complex numbers first. For the symmetric group S_{50} it is known that there are 204226 characters over $\mathbb{C}$, but there are only 5 primitive central idempotents of $\mathbb{F}_2S_{50}$. Due to the vast amount of data it is not possible to compute

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the character table of S_{50}, but it is possible to compute the 5 primitive central idempotents of $\mathbb{F}_2S_{50}$ using the algorithm described in [12], 2.21. Thus we use the first part to give a description of this algorithm, which works for all finite group rings.

The specialization to symmetric and alternating groups allows us to speed up the algorithm. We do this by proving theoretical results about the group rings $\mathbb{F}_pS_n$ and their primitive central idempotents. To state the results we need the following notation: It is well known that the conjugacy classes of S_n can be indexed by the partitions of n. We write $\mu = 1^{\alpha_1}, ..., n^{\alpha_n}$ for the partition

$$\mu = (\underbrace{1, ..., 1}_{\alpha_1}, \underbrace{2, ..., 2}_{\alpha_2}, ...)$$

of n. The fact that μ is a partition of n is abbreviated by $\mu \vdash n$. C_μ is the conjugacy class of S_n belonging to μ, and C_μ^+ denotes the class sum of C_μ in $\mathbb{F}_pS_n$. The n-tuple of multiplicities $(\alpha_1, ..., \alpha_n)$ appearing in μ is called the cycle type of the element $\sigma \in C_\mu$. We define

$$W(\mu) := \sum_{i=2}^{n} i \cdot \alpha_i$$

and call it the *essential weight* of the partition μ. For our purpose it is convenient to ignore the parts equal to 1 in the partition, because an element like $(1, 2, 3) \in S_3$ is also an element of bigger symmetric groups. So we usually write $\mu = 2^{\alpha_2}, ..., n^{\alpha_n}$ for a partition and the corresponding class C_μ is a class of an arbitrary symmetric group S_n with $n \geq W(\mu)$ depending on the context, i.e. C_2 denotes the conjugacy class of transpositions in every symmetric group S_n, $n \geq 2$. If we want to emphasize that C_μ is a class of a certain symmetric group S_n we write $C_\mu|_{S_n}$ and also $C_\mu^+|_{S_n}$ for the class sum in $\mathbb{F}_pS_n$. The class multiplication coefficients $c_{\lambda\mu\nu} \in \mathbb{F}_p$ are defined via

$$C_\lambda^+ C_\mu^+ = \sum_{\nu \vdash n} c_{\lambda\mu\nu} C_\nu^+.$$

Here, the $c_{\lambda\mu\nu}$ depend on n, but to keep notation simple we usually suppress the n. In cases of ambiguity we write $c_{\lambda\mu\nu}|_{S_n}$ for the coefficient of $C_\nu^+|_{S_n}$ in $C_\lambda^+|_{S_n} \cdot C_\mu^+|_{S_n}$.

For an element

$$B = \sum_{g \in G} a_g g$$

of a group ring $\mathbb{F}_qG$ the support is the set $\operatorname{supp} B := \{g \in G \mid a_g \neq 0\}$. The theorem of Osima mentioned above states that the support of primitive central idempotents of $\mathbb{F}_qG$ consists of p'-elements, i.e. elements of an order which is not divisible by p. Usually these elements and the corresponding class sums are called p-regular, but in symmetric groups this expression is used for partitions $\mu = 1^{\alpha_1}, ..., n^{\alpha_n}$, where $\alpha_i < p$ for $i = 1, ..., n$. Thus we avoid using this expression for the classes and use the term 'p'-conjugacy class' instead for the conjugacy classes of p'-elements. The corresponding partitions are usually called p-class regular, but we prefer to call them p'-partitions. Furthermore we define p-near-regular partitions and conjugacy classes: $\mu = 1^{\alpha_1}, ..., n^{\alpha_n}$ is called p-near-regular, when $\alpha_i < p$ for $i = 2, ..., n$ and the corresponding conjugacy classes are the p-near-regular classes. A theorem of Murray ([14], Corollary 5) states that the vector space

$$Z_{p'}^G := \langle C^+ \mid C \text{ is } p' - \text{conjugacy class of } G \rangle$$

is a subalgebra of the centre of $\mathbb{F}_qG$, if G is a symmetric group. We also use the vector spaces

$$Z^{S_n}_{p-reg} := \langle C^+_\mu \mid \mu \text{ is } p-\text{regular partition of } n\rangle$$

and

$$Z^{S_n}_{p-nreg} := \langle C^+ \mid C \text{ is } p-\text{near-regular conjugacy class of } S_n\rangle.$$

Now we can state our main theorems. Section 2 is devoted to proving the following result.

Theorem 1. *Let $m < n$ and $m \equiv n \mod p$. Let δ be the homomorphism of vector spaces defined by*

$$\delta : Z^{S_n}_{p'} \longrightarrow Z^{S_m}_{p'},\ \delta(C^+_\mu) := \begin{cases} C^+_\mu|_{S_m}, & \text{if } W(\mu) \leq m, \\ 0, & \text{if } W(\mu) > m. \end{cases}$$

Then δ is a homomorphism of algebras.

Let $e_1, ..., e_r$ denote the primitive central idempotents of $\mathbb{F}_pS_n$. Then δ has the following properties:

1) *If $\delta(e_i) \neq 0$, then $\delta(e_i)$ is a primitive central idempotent of $\mathbb{F}_pS_m$.*
2) *For every primitive central idempotent f of $\mathbb{F}_pS_m$ there is an $i \in \{1, ..., r\}$ such that $f = \delta(e_i)$.*

We remark that our Theorem 1 is related to Theorem 1.6 of [16].

In Section 3 we prove the following two theorems:

Theorem 2. *$Z^{S_n}_{p'} \cap Z^{S_n}_{p-nreg}$ is an algebra.*

Theorem 3. *The primitive central idempotents of $\mathbb{F}_pS_n$ are contained in $Z^{S_n}_{p'} \cap Z^{S_n}_{p-nreg}$, i.e. the support $\operatorname{supp} e_i$ of a primitive central idempotent e_i of $\mathbb{F}_pS_n$ consists of p-near-regular p'-conjugacy classes.*

These theorems help to speed up the program, so we were able to compute the primitive central idempotents of $\mathbb{F}_2S_n$ and $\mathbb{F}_4A_n$ for $n \leq 50$ using the computer algebra package GAP [2] and a program written in SYMMETRICA [10] provided by A. Kohnert. The Appendix contains the computational results.

We note that there is a theoretical result which provides one of the primitive central idempotents of the group rings $\mathbb{F}_2S_n$, if n is of the form $n = \frac{m(m+1)}{2}$ with an integer m. R. Gow proves in Theorem 3 of [3] that in this case one of the primitive central idempotents of $\mathbb{F}_2S_n$ has the form $e = C^+$, where C is the conjugacy class of elements corresponding to the partition $(2m-1, 2m-5, 2m-9, ...)$ of n. We don't use this result for our computations for symmetric groups, but we use it to determine the primitive central idempotents of $\mathbb{F}_4A_n$, because Gow also proves that this idempotent is the only idempotent of $\mathbb{F}_2S_n$ which splits in $\mathbb{F}_4A_n$.

1. Computation of primitive central idempotents of finite group rings

Let F be a splitting field for G of characteristic $p > 0$. We already mentioned that the usual method for computing the primitive central idempotents of a group ring FG uses the character table of G over $\mathbb{C}$. But what if we do not know the character table? Can we compute the idempotents within FG? The first observation is that we can assume F to be finite because every group has a finite splitting field of characteristic p according to a theorem of Brauer ([6], Theorem VII.2.6). Now if

FG is finite and $B \in Z(FG)$, then the sequence $(B^n)_{n\in\mathbb{N}}$ has to be periodic, i.e. there exist $r, m \in \mathbb{N}$ such that $B^r = B^{r+m}$. This idea can be used to construct central idempotents and leads to Algorithm 7, which is already described in [12]. The following theorem ([12], Satz 2.1) is the foundation of the algorithm. As we will need it in section 2, we prove it here again. Now let F be an arbitrary finite field of characteristic $p > 0$ and let $\overline{F}$ denote the algebraic closure of F. We will mention it explicitly if we assume F to be a splitting field for G or for $Z(FG)$.

Theorem 4. *Let $B \in Z(FG)$ and let $m \in \mathbb{N}$ be such that $B^r = B^{r+m}$, for all $r \in \mathbb{N}$ suitable large. Choose $r = l \cdot m$, where $l \neq 0$ and suppose that $m = p^s \cdot d$, where $s \geq 0$ and $p \nmid d$. Let $\zeta \in \overline{F}$ be a primitive d-th root of unity and put*

$$f_k := d^{-1} \sum_{i=0}^{d-1} (\zeta^k)^i B^{r+p^s \cdot i}$$

for $0 \leq k \leq d-1$. Then $f_k = 0$ or f_k is a central idempotent of $F(\zeta^k)G$, the group ring of G over the field $F(\zeta^k)$. For $k \neq n$ we have $f_k f_n = 0$ in $F(\zeta)G$. If $B^r \neq 0$, then there is a k such that $f_k \neq 0$ and B^r is a central idempotent itself.

Proof. For $0 \leq k \leq d-1$ we define

$$D_k := \sum_{i=0}^{d-1} (\zeta^k)^i \cdot B^{r+p^s \cdot i}.$$

Now let $w := \zeta^k$. Then we have $w^d = 1$ and get

$$w^j B^{r+p^s \cdot j} \cdot D_k = \sum_{i=0}^{d-1} w^{i+j} B^{r+lm+p^s \cdot (i+j)} = D_k$$

using the periodicity of the sequence $(B^n)_{n\in\mathbb{N}}$. Hence we obtain $D_k^2 = d \cdot D_k$, so $f_k = d^{-1} D_k$ is 0 or a central idempotent in $F(\zeta^k)G$.

If $0 \leq n \leq d-1$ and $n \neq k$, then

$$D_k D_n = \sum_{i=0}^{d-1} (\zeta^k)^i (\zeta^n)^{-i} \left[(\zeta^n)^i B^{r+p^s \cdot i} D_n \right] = D_n \cdot \sum_{i=0}^{d-1} (\zeta^{k-n})^i = 0.$$

Therefore the f_k with $f_k \neq 0$ are orthogonal idempotents.

Finally, if $B^r \neq 0$, then

$$\sum_{k=0}^{d-1} D_k = dB^r \neq 0,$$

i.e. there is an $f_k \neq 0$. As the f_k are orthogonal we obtain

$$(B^r)^2 = \left(\sum_{k=0}^{d-1} f_k \right)^2 = \sum_{k=0}^{d-1} f_k^2 = \sum_{k=0}^{d-1} f_k = B^r.$$

A similar computation leads to the following corollary ([12], 2.3):

Corollary 5. *Let $e_1, ..., e_r$ be the primitive central idempotents of FG. Then the span*

$$\langle e_1, ..., e_r \rangle_F = \{ B \in Z(FG) \mid B^{|F|} = B \}.$$

The following theorem (see [12], Satz 2.17) is the foundation for our algorithm:

Theorem 6. *Let F be a splitting field for $Z(FG)$. Let $C_1, ..., C_k$ be the p'-conjugacy classes of G and let $e_1, ..., e_r$ be the primitive central idempotents of FG. Then there is an $n_0 \in \mathbb{N}$ such that*

$$\langle e_1, ..., e_r \rangle_F = \left\langle (C_1^+)^{p^n}, ..., (C_k^+)^{p^n} \right\rangle_F$$

for all $n \geq n_0$.

Proof. Let $C_{k+1}, ..., C_c$ be the p-singular conjugacy classes of G, i.e. the classes of elements whose order is divisible by p. As the class sums $C_1^+, ..., C_c^+$ form a basis of $Z(FG)$, we get

$$\langle e_1, ..., e_r \rangle_F = \left\langle (C_1^+)^{p^n}, ..., (C_c^+)^{p^n} \right\rangle_F$$

for all $n \in \mathbb{N}$ suitable large by [11], p. 434 (an elementary proof can be found in [12], Satz 2.11). According to the theorem of Osima ([8], Theorem 23.6) we know $\langle e_1, ..., e_r \rangle_F \subset \langle C_1^+, ..., C_k^+ \rangle_F$. Now let n_0 be a multiple of $|F|$, which is suitably large. Then

$$\varphi : Z(FG) \longrightarrow Z(FG), B \longmapsto B^{p^{n_0}}$$

is a homomorphism of vector spaces, hence we obtain

$$\begin{aligned} \langle e_1, ..., e_r \rangle_F &= \varphi(\langle e_1, ..., e_r \rangle_F) \subset \left\langle (C_1^+)^{p^{n_0}}, ..., (C_k^+)^{p^{n_0}} \right\rangle_F \\ &\subset \left\langle (C_1^+)^{p^{n_0}}, ..., (C_c^+)^{p^{n_0}} \right\rangle_F = \langle e_1, ..., e_r \rangle. \end{aligned}$$

Thus we get our statement for all $n \geq n_0$.

This leads to the following algorithm for computing primitive central idempotents of finite group rings, which can be found in [12], 2.21. But the algorithm in [12] contains some minor gaps, which we fill here.

Algorithm 7. *Let G be a finite group and p a prime. Let F be a splitting field for $Z(FG)$ with $\operatorname{char} F = p$. The computation of the primitve central idempotents of FG can be accomplished by the following steps:*

1) *Compute the p'-conjugacy classes $C_1, ..., C_b$ of G.*
2) *For $i = 1, ..., b$, do the following: By computing successive powers of the class sum C_i^+, determine the minimal integers $r_i, m_i \geq 1$ such that $(C_i^+)^{r_i} = (C_i^+)^{r_i+m_i}$, and r_i is a multiple of m_i. Write $m_i = p^{s_i} \cdot d_i$, where $s_i \geq 0$ and $p \nmid d_i$.*
3) *Compute the idempotents*

$$f_k^{(i)} := d_i^{-1} \sum_{j=0}^{d_i-1} (\zeta_i^k)^j (C_i^+)^{r_i+p^{s_i} \cdot j}$$

for $1 \leq i < b$ and $0 \leq k \leq d_i - 1$, where ζ_i denotes a primitive d_i-th root of unity in a suitable extension of the field $\mathbb{F}_p$.

4) *Choose a basis $f_1, ..., f_r$ of the mostly linear dependent set*

$$\left\{ f_k^{(i)} \mid 1 \leq i \leq b,\ 0 \leq k \leq d_i - 1 \right\}.$$

If $r = 1$, then $f_1 = 1$ is the only central idempotent of FG and the computation is finished. For $r > 1$ we have to accomplish one more step:

5) *For $i = 1, ..., r-1$ do:*
   { *For $j = i+1, ..., r$ do:*

{ *Set* $w := f_i$.
If $f_i \cdot f_j = f_j$ *then*
{ *set* $f_i := f_j,\ f_j := w,\ w := f_i$. }
else
{ *If* $f_i \cdot f_j \neq 0$ *then* { *replace* f_i *with* $f_i \cdot f_j$. }
If $\dim\langle f_1, ..., f_r\rangle < r$ *then*
{ $k := j$.
While $\dim\langle f_1, ..., f_{k-1}, w, f_{k+1}, ..., f_r\rangle < r$ *do*
{ $k := k + 1$. }
$f_k := w$.
}
}
}
For $j = i + 1, ..., r$ *do:*
{ *If* $f_i \cdot f_j = f_i$ *then:*
{ *If* $1-\left(\sum_{k=1}^{i-1} f_k\right) - f_j \neq 0$ *then replace* f_j *with* $1-\left(\sum_{k=1}^{i-1} f_k\right) - f_j$
else replace f_j *with* $1 - \sum_{k=1}^{i} f_k$. }
}
}

The elements $f_1, ..., f_r$ *are the primitive central idempotents of* FG.

Remark. 1) It is not necessary to compute a splitting field for $Z(FG)$ in advance, as the ζ_i arise during the computation. In general, $\mathbb{F}_p(\zeta_1, ..., \zeta_k)$ is not a splitting field for G, but it is a splitting field for the centre $Z(FG)$ of the group ring FG: Indeed it is the minimal splitting field of characteristic p for $Z(FG)$. A proof can be found in [12], Satz 3.16.

2) Further knowledge about the group ring FG can be used to speed up the program. For example an automorphism α of $Z(FG)$ can be applied in step 2 of the algorithm to deduce the powers of $\alpha(C^+)$ very fast, if the powers of C^+ are already computed.

This remark together with a previous lemma and the decomposition

$$Z(FG) = \langle f_1, ..., f_r\rangle_F \oplus Rad(Z(FG))$$

of $Z(FG)$ as a vector space ([8], Lemma 25.1) allows us to compute generators of $Rad(Z(FG))$ as a vector space:

Theorem 8. *Let* F *be a finite splitting field of characteristic* p *for* $Z(FG)$ *with* $|F| = p^n$. *Let* $C_1, ..., C_c$ *be the conjugacy classes of* G *(here we need them all). Then*

$$Rad(Z(FG)) = \left\langle C_1^+ - (C_1^+)^{p^n}, ..., C_c^+ - (C_c^+)^{p^n} \right\rangle \text{ (as a vector space).}$$

Proof. Let $e_1, ..., e_r$ be the primitive central idempotents of FG. Let $i \in \{1, ..., c\}$. According to [11], p. 434, there is an $m \in \mathbb{N}$ with $(C_i^+)^{p^m} \in \langle e_1, ..., e_r\rangle$. Using Corollary 5 we obtain

$$\left(C_i^+ - (C_i^+)^{p^n}\right)^{p^m} = (C_i^+)^{p^m} - \left((C_i^+)^{p^m}\right)^{p^n} = (C_i^+)^{p^m} - (C_i^+)^{p^m} = 0$$

and thus $C_i^+ - (C_i^+)^{p^n} \in Rad(Z(FG))$.

Now we consider the map

$$\varphi : Z(FG) \longrightarrow Z(FG),\ \varphi(x) := x - x^{p^n}.$$

φ is a homomorphism of vector spaces. According to Corollary 5 the kernel is $\langle e_1, ..., e_r\rangle$. The image is a subset of $Rad(Z(FG))$. By [8], Lemma 25.1, we obtain our statement.

Remark 9. We keep the notation of the last theorem. The proof of the preceding theorem also provides a method to compute the projection of an arbitrary element $B \in Z(FG)$ to the vector spaces $\langle e_1, ..., e_r\rangle_F$ and $Rad(Z(FG))$: Choose $m = k \cdot n$, such that $B^{p^m} \in \langle e_1, ..., e_r\rangle$ (this is true if $p^m > c^{p^n}$). Then $B - B^{p^m} \in Rad(Z(FG))$ and B^{p^m} and $B - B^{p^m}$ are the projections of B to $\langle e_1, ..., e_r\rangle_F$ and $Rad(Z(FG))$, respectively.

In the following sections we will see how we can improve the algorithm for symmetric groups.

2. Connections between primitive central idempotents of $\mathbb{F}_p S_n$ for different n

The methods developed in the first section work for all finite groups and fields of characteristic p, even in the case where $p \nmid |G|$. But to apply the algorithm successfully to bigger symmetric groups we have to speed up the algorithm. In the algorithm we have to multiply sums of class sums. How can we do that quickly? If $C_1, ..., C_m$ are the conjugacy classes of S_n, then we get

$$C_i^+ C_j^+ = \sum_{k=1}^{m} c_{ijk} C_k^+$$

with coefficients $c_{ijk} \in \mathbb{F}_p$. According to [4], Theorem 4.6, we know that

$$c_{ijk} \equiv \frac{|C_i||C_j|}{|S_n|} \sum_{\chi \in \operatorname{Irr} S_n} \chi(g_i)\chi(g_j)\frac{\chi(g_k^{-1})}{\chi(1)} \mod p,$$

where $g_i \in C_i$ for $i \in \{1, ..., m\}$. So we can compute the class multiplication coefficients c_{ijk}, if we know some parts of the character table of S_n. Character values for symmetric groups can be computed very fast with the Murnaghan-Nakayama formula, see [7], 2.4.7, even for S_{50}. But the number of class multiplication coefficients we have to compute must not be too big, if we want to get results for a group like S_{50}. So we need theoretical results to reduce the work. A first result in this direction is a theorem of Murray ([14], Corollary 5): $Z_{p'}^{S_n}$ is an algebra. So if we multiply class sums of p'-conjugacy classes we only need to compute the coefficients of p'-class sums. Due to Algorithm 7 we only need to consider p'-conjugacy classes, so we can use this result for all our computations. Here and in the following sections we will prove some more theorems which allow us to reduce the number of coefficients we need to compute.

The computations are simplified by the fact that $\mathbb{F}_p$ is a splitting field for S_n according to [7], 2.1.12, so we can choose $F = \mathbb{F}_p$.

Lemma 10. *Let $n > p$ and let δ_n be the homomorphism of vector spaces defined by*

$$\delta_n : Z_{p'}^{S_n} \longrightarrow Z_{p'}^{S_{n-p}}, C_\mu^+ \longmapsto \begin{cases} C_\mu^+|_{S_{n-p}}, & \text{if } W(\mu) \leq n-p, \\ 0, & \text{if } W(\mu) > n-p. \end{cases}$$

Then δ_n is a homomorphism of algebras.

Proof. We write S_{n-p} for the subgroup of S_n containing the permutations, which fix the numbers $n-p+1, ..., n$. Let $\tau := (n-p+1, ..., n) \in S_n$. Then for the centralizer we get $\mathrm{C}_{S_n}(\tau) = S_{n-p} \times \langle\tau\rangle$. Now let

$$\mathrm{Br}_{\langle\tau\rangle} : Z(\mathbb{F}_p S_n) \longrightarrow Z(\mathbb{F}_p \,\mathrm{C}_{S_n}(\tau))$$

be the Brauer-homomorphism (see [15], Theorem 4.9). $\mathrm{Br}_{\langle\tau\rangle}$ is a homomorphism of algebras with $\mathrm{Br}_{\langle\tau\rangle}(C_\mu^+) = (C_\mu \cap \mathrm{C}_{S_n}(\tau))^+$. Thus δ_n is the restriction of $\mathrm{Br}_{\langle\tau\rangle}$ to $Z_{p'}^{S_n}$, and is therefore a homomorphism of algebras, by [14, Corollary 5].

Remark 11. If we define the homomorphism δ_n of the preceding lemma for $Z(\mathbb{F}_p S_n)$ we usually do not get a homomorphism of algebras: If C_τ denotes the conjugacy class of τ in S_n, then $C_\tau \cap \mathrm{C}_{S_n}(\tau)$ is not contained in the subgroup S_{n-p} of $\mathrm{C}_{S_n}(\tau)$ and the same problem occurs for conjugacy classes of partitions containing p. For example for the conjugacy class C_2 of transpositions it is easy to see that $(C_2^+)^2 = C_1^+ + C_3^+$ for $n \equiv 2, 3 \mod 4$ and $(C_2^+)^2 = C_3^+$ for $n \equiv 0, 1 \mod 4$ in $\mathbb{F}_2 S_n, n \geq 3$.

Now we can prove our Theorem 1.

Proof of Theorem 1. We keep the notation of Lemma 10. If $m = n - k \cdot p$, then δ is the composition of $\delta_n, \delta_{n-p}, ..., \delta_{n-(k-1)p}$ and thus is a homomorphism of algebras. Therefore $\{\delta(e_1), ..., \delta(e_r)\} \backslash \{0\}$ is a set of central orthogonal idempotents. We prove the remaining statements in several steps:

1) For every central idempotent $f \in Z(\mathbb{F}_p S_m)$ there is a central idempotent $e \in Z(\mathbb{F}_p S_n)$ with $\delta(e) = f$:

   We define a homomorphism Δ of vector spaces by

$$\Delta : Z(\mathbb{F}_p S_m) \longrightarrow Z(\mathbb{F}_p S_n),\ C_\mu^+|_{S_m} \mapsto C_\mu^+|_{S_n}$$

   and put $F := \Delta(f)$. Now let $r, m \in \mathbb{N}$ such that $F^r = F^{r+m}$ and $r = l \cdot m$ for an $l \in \mathbb{N}$. Let $m = p^s \cdot d$ with $p \nmid d$. We set

$$e := d^{-1} \sum_{i=0}^{d-1} F^{r+p^s \cdot i}.$$

   Then e is 0 or a central idempotent of $\mathbb{F}_p S_n$ by Theorem 4. We obtain

$$\delta(e) = d^{-1} \sum_{i=0}^{d-1} \delta(F)^{r+p^s \cdot i} = d^{-1} \sum_{i=0}^{d-1} f = f \neq 0,$$

   so $e \neq 0$ and $\delta(e) = f$.

2) For every primitive central idempotent $f \in Z(\mathbb{F}_p S_m)$ there is $i \in \{1, ..., r\}$ with $f = \delta(e_i)$:

   By 1) there exists a central idempotent e of $Z(\mathbb{F}_p S_n)$ with $\delta(e) = f$. Now let $e = \sum_{i=1}^{k} e_i$ be the decomposition of e in primitive central idempotents of

$Z(\mathbb{F}_p S_n)$. Then we have

$$f = \delta(e) = \sum_{i=1}^{k} \delta(e_i),$$

and as δ is a homomorphism of algebras, the $\delta(e_i)$ are 0 or central orthogonal idempotents of $Z(\mathbb{F}_p S_m)$. But f is primitive, therefore there is an $i_0 \in \{1, ..., k\}$ such that $\delta(e_{i_0}) \neq 0$, $\delta(e_j) = 0$ for $j \neq i_0$ and we obtain $f = \delta(e_{i_0})$.

3) If $e \in \{e_1, ..., e_r\}$ and $\delta(e) = f \neq 0$, then f is a primitive central idempotent of $\mathbb{F}_p S_m$:

We assume that f is not primitive as a central idempotent. Then let $f = \sum_{i=1}^{k} f_i$ be a decomposition of f in primitive central idempotents. By 2) there are primitive central idempotents $e_1, ..., e_k$ of $\mathbb{F}_p S_n$ with $\delta(e_i) = f_i$. As $\delta(e) = f \neq f_1 = \delta(e_1)$ we have $e \neq e_1$. Now both e and e_1 are different primitive central idempotents of $\mathbb{F}_p S_n$, so they are orthogonal. Hence we obtain

$$0 = \delta(e \cdot e_1) = \delta(e) \cdot \delta(e_1) = f \cdot f_1 = f_1,$$

a contradiction.

Theorem 1 allows us to write down the primitive central idempotents of $\mathbb{F}_p S_n$ in a very compact way: For example, it will do to write down the primitive central idempotents of $\mathbb{F}_2 S_{50}$ and $\mathbb{F}_2 S_{49}$ to know the primitive central idempotents of $\mathbb{F}_2 S_n$ for all $n \leq 50$. Furthermore our computations in connection with Theorem 1 lead to statements like the following:

$$C_{11} \notin \operatorname{supp} e \text{ for all primitive central idempotents } e \text{ of } \mathbb{F}_2 S_n, n \in \mathbb{N},$$

because C_{11} is neither included in the support of the primitive central idempotents of $\mathbb{F}_2 S_{11}$ nor in the according support in $\mathbb{F}_2 S_{12}$.

For the computation of idempotents the most important part of Theorem 1 is the fact that δ is a homomorphism of algebras because this provides the following corollary:

Corollary 12. *Let λ, μ, ν be p'-partitions of n with $W(\mu) \geq W(\lambda)$. Let $m \equiv n \bmod p$ be minimal with $W(\nu) \leq m$. Then for the class multiplication coefficient $c_{\lambda\mu\nu}|_{S_n}$ we have*

$$c_{\lambda\mu\nu}|_{S_n} = \begin{cases} 0, & \text{if } W(\nu) \leq W(\mu) - p \text{ or } W(\nu) > W(\mu) + W(\lambda), \\ c_{\lambda\mu\nu}|_{S_m}, & \text{if } W(\mu) - p < W(\nu) \leq W(\mu) + W(\lambda). \end{cases}$$

Proof. If $\pi \in C_\lambda$ and $\sigma \in C_\mu$, then π moves $W(\lambda)$ points and σ moves $W(\mu)$ points, so $\pi\sigma$ moves at most $W(\lambda) + W(\mu)$ points. Thus if $W(\nu) > W(\lambda) + W(\mu)$, then obviously $c_{\lambda\mu\nu}|_{S_n} = 0$. Now let $W(\nu) \leq W(\mu) - p$ and let δ_n be as in Lemma 10. As δ_n is a homomorphism and $\delta_n(C_\mu^+) = 0$, but $\delta_n(C_\nu^+) \neq 0$, it follows that $c_{\lambda\mu\nu}|_{S_n} = 0$, in this case. The remaining statement is clear by applying δ_n.

3. A further subalgebra of $\mathbb{F}_p S_n$ containing the idempotents

To prove Theorem 2 we need the following version of a lemma proved in [1], which is just a formulation of the statement that the Brauer homomorphism is an algebra homomorphism. Additionally it is a formulation of another theorem of Osima ([15, Theorem 4.1]): If C is a conjugacy class of G and $g \in C$, then a

Sylow-p-subgroup of $\mathrm{C}_G(g)$ is called a defect group of C. If $Z_D(FG)$ denotes the F-span of all class sums C^+ such that the defect groups of C are contained in a G-conjugate of a certain p-subgroup D of G, then $Z_D(FG)$ is an ideal of $Z(FG)$. But Lemma 13 is more detailed specifying elements and making a statement about the Sylow-p-subgroups of these elements. In the proof of Theorem 2 we apply this more detailed version of the theorem of Osima.

Lemma 13. *Let G be a finite group. Let $C_1, ..., C_s$ be the conjugacy classes of G and let $C_1^+, ..., C_s^+$ be the corresponding class sums in $\mathbb{Z}G$. Now let*

$$C_i^+ C_j^+ = \sum_{k=1}^{s} a_{ijk} C_k^+.$$

If $a_{ijk} \not\equiv 0 \mod p$, then for every element $z \in C_k$ and every Sylow-p-subgroup P_z of $\mathrm{C}_G(z)$ there are elements $x \in C_i$ and $y \in C_j$ as well as Sylow-p-subgroups P_x of $\mathrm{C}_G(x)$ and P_y of $\mathrm{C}_G(y)$, such that $xy = z$ and $P_z \leq P_x \cap P_y$.

Proof. See the proof of [1], Lemma 87.9.

Proof of Theorem 2. $Z_{p'}^{S_n}$ is an algebra by Corollary 5 of [14]. Now let $(a_1, ..., a_n)$ and $(b_1, ..., b_n)$ be cycle types of p-near-regular p'-partitions σ and τ. We have to show that $C_\pi \not\subset \operatorname{supp} C_\sigma^+ C_\tau^+$ for p'-classes C_π, which are not p-near-regular.

Let $(c_1, ..., c_n)$ be the cycle type of π. If $c_1 \geq p$ we can apply the Brauer-homomorphism and get

$$\mathrm{Br}_{\langle (1,...,p)\rangle}(C_\pi^+) = C_{\pi'}^+ \neq 0,$$

where π' has cycle type $(c_1 - p, c_2, ..., c_{n-p})$. Thus we get $c_{\sigma\tau\pi}|_{S_n} = c_{\sigma'\tau'\pi'}|_{S_{n-p}}$, where $C_{\sigma'}^+ = \mathrm{Br}_{\langle (1,...,p)\rangle}(C_\sigma^+)$ and analogously for $C_{\tau'}^+$. Moreover, $C_{\sigma'}$ and $C_{\tau'}$ are p-near-regular p'-classes. Therefore we can assume that $c_1 < p$.

Now let $z \in C_\pi$ and let $x \in C_\sigma$, $y \in C_\tau$ such that $xy = z$. It is well known that

$$(*) \qquad \mathrm{C}_{S_n}(x) \cong S_{a_1} \times (\mathbb{Z}_2 \wr S_{a_2}) \times ... \times (\mathbb{Z}_n \wr S_{a_n})$$

and $|\mathbb{Z}_k \wr S_k| = a_k! \cdot k^{a_k}$ ([17], 3.2.13). As $a_k < p$ for $k \geq 2$ and $a_k = 0$ for $p \mid k$ we obtain that p divides $|\mathrm{C}_{S_n}(x)|$ if and only if $a_1 \geq p$.

Hence the Sylow-p-subgroup of $\mathrm{C}_{S_n}(x)$ is a subgroup of the subgroup isomorphic to S_{a_1}. This subgroup is the symmetric group on the set of numbers which are fixed by x. The same is true for y. Now let P_x, P_y be Sylow-p-subgroups of $\mathrm{C}_{S_n}(x)$, $\mathrm{C}_{S_n}(y)$, respectively and let $P := P_x \cap P_y$. Then all numbers which are moved by an element of P are fixed by x and y and therefore are fixed by z. But $c_1 < p$, so z fixes less than p numbers, i.e. no permutation of the fixed points of z has order p. Thus we obtain $P_x \cap P_y = \langle 1 \rangle$.

If P_z is a Sylow-p-subgroup of $\mathrm{C}_{S_n}(z)$, then $|P_z| > 1$ because there is an $i \geq 2$ such that $c_i \geq p$. Thus we obtain $P_z \not\subset P_x \cap P_y$ for all $x \in C_\sigma$, $y \in C_\tau$ with $xy = z$. By Lemma 13 we get $C_\pi \not\subset \operatorname{supp}(C_\sigma^+ C_\tau^+)$.

Table 1 shows the dimension of $U_p := Z_{p'}^{S_n} \cap Z_{p-nreg}^{S_n}$ for $p = 2, 3$ in comparison with the dimensions of $Z(\mathbb{F}_p S_n)$ and $Z_{p'}^{S_n}$.

The following remark is a special case of a more general theorem of Osima already mentioned above ([15, Theorem 4.1]).

Remark 14. $Z_{p'}^{S_n} \cap Z_{p-reg}^{S_n}$ is an algebra.

For the rest of this section we fix the following notation:

n	$\dim Z(\mathbb{F}_p S_n)$	$\dim Z_{2'}^{S_n}$	$\dim U_2$	$\dim Z_{3'}^{S_n}$	$\dim U_3$
10	42	10	7	22	18
20	627	64	30	202	129
30	5604	296	95	1225	622
40	37338	1113	260	5834	2405
50	204226	3658	632	23603	8008

Table 1.

Notation 15. By $\sigma = (a_1, ..., a_n)$ we denote the cycle type of a p'-partition with $a_i \geq p$ for an $i \geq 2$, $a_j < p$ for $j < i$. C_σ denotes the conjugacy class of elements of cycle type σ in S_n. Thus the p'-partition σ is not p-regular, but elements in C_σ fix at most $p-1$ symbols in $\{1, ..., n\}$.

The idea of the proof of Theorem 3 is the following: If the support of an idempotent e contains C_σ, then the support also contains $\operatorname{supp}(C_\sigma^+)^p$. The support of $(C_\sigma^+)^p$ can contain classes C_μ, where μ is not p-regular. But if $(b_1, ..., b_n)$ is the cycle type of such a class C_μ and if $b_k \geq p$, then $k > i$, i.e. the position where the 'irregularity' occurs, grows. So a class C_σ in the support of e with a 'minimal position of irregularity' is not contained in the support of $e^p = e$, a contradiction.

For the proof of Theorem 3 we first collect the necessary information about C_σ in several lemmas. We start with a lemma about the centralizers of an element of cycle type σ.

Lemma 16. *Let $x \in C_\sigma$ and let $y \in \mathrm{C}_{S_n}(x)$ have order p. Then y is a product of i or more commuting p-cycles.*

Proof. As y centralizes x, it permutes the orbits of x on $\{1, ..., n\}$. Let O be one orbit of x which is not fixed by y. Then $O, yO, ..., y^{p-1}O$ are distinct orbits of x. Thus the cycle decomposition of x consists of at least p cycles of length $|O|$. Our choice of x gives $|O| \geq i$, and y is a product of at least $|O|$ commuting p-cycles.

Corollary 17. *Let σ be as in Notation 15, let $C_\mu \subset S_n$ be a p'-conjugacy class and let $\lambda = (c_1, ..., c_n)$ be the cycle type of a p'-partition with $C_\lambda \subset \operatorname{supp}(C_\mu^+ C_\sigma^+)$. Then $c_j < p$ for $2 \leq j < i$.*

Proof. Let $x \in C_\mu$, $y \in C_\sigma$ and $z \in C_\lambda$. Suppose that z contains at least p commuting j_0-cycles in its cycle decomposition, where $j_0 < i$. Then an element permuting p of the orbits of z on $\{1, ..., n\}$ of length j_0 is an element in $C_{S_n}(z)$, and its cycle decomposition consists of j_0 commuting p-cycles. Thus every Sylow-p-subgroup P_z of $\mathrm{C}_{S_n}(z)$ contains an element of this cycle type. But $\mathrm{C}_{S_n}(x)$ does not contain an element of such a cycle type according to Lemma 16. Hence if P_x, P_y are Sylow-p-subgroups of $\mathrm{C}_{S_n}(x)$, $\mathrm{C}_{S_n}(y)$, respectively, then $P_z \not\subset P_x \cap P_y$ and by Lemma 13 we obtain $C_\lambda \not\subset \operatorname{supp}(C_\mu^+ C_\lambda^+)$.

We remark that we just proved that a defect group of C_λ is not contained in a defect group of C_μ up to conjugacy, so Corollary 17 also follows by the theorem of Osima mentioned above ([15, Theorem 4.1]).

Lemma 18. *Let π be a product of i commuting p-cycles, where i is coprime to p. Then all p'-elements of the same cycle type contained in $\mathrm{C}_{S_{p \cdot i}}(\pi)$ are conjugate in $\mathrm{C}_{S_{p \cdot i}}(\pi)$.*

Proof. $\mathrm{C}_{S_{p\cdot i}}(\pi) \cong \mathbb{Z}_p \wr S_i$ (see $(*)$) is a semidirect product of a p-group (generated by the cycles of π) and a copy of S_i. Using the explicit description of the conjugacy classes of $\mathbb{Z}_p \wr S_i$ in [9, 3.2, 3.13] we can deduce that every p'-element of $\mathrm{C}_{S_{p\cdot i}}(\pi)$ is conjugate in $\mathrm{C}_{S_{p\cdot i}}(\pi)$ to an element of the copy of S_i. Therefore the projection $\mathrm{C}_{S_{p\cdot i}}(\pi) \longrightarrow S_i$ induces a bijection between the p'-classes of the two groups. Due to [7, 4.1.18, 4.2.17] this bijection maps a p'-element of cycle type $(p\lambda_1, ..., p\lambda_i)$ to an element of cycle type $(\lambda_1, ..., \lambda_i)$. Thus we obtain our result.

Corollary 19. *We keep the notation of Lemma* 18. *Let $g \in \mathrm{C}_{S_{p\cdot i}}(\pi)$ be a p'-element and let $\chi \in \mathrm{Irr}\,\mathrm{C}_{S_{p\cdot i}}(\pi)$. Then $\chi(g) \in \mathbb{Z}$.*

Proof. Let $m \in \mathbb{N}$ with $\gcd(m, \mathrm{ord}\, g) = 1$. By Lemma 18 the elements g and g^m are conjugate in $\mathrm{C}_{S_{p\cdot i}}(\pi)$, thus we obtain $\chi(g) \in \mathbb{Z}$ for $\chi \in \mathrm{Irr}\,\mathrm{C}_{S_{p\cdot i}}(\pi)$ using [5, V.13.7.b)] and [4, 6.3b), 6.4a)].

Remark 20. With the notation of Lemma 18 the vector space $Z_{p'}^{\mathrm{C}_{S_{p\cdot i}}(\pi)}$ is an algebra.

Proof. $Z_{p'}^{S_{p\cdot i}}$ is an algebra by Corollary 5 of [14], and the Brauer homomorphism

$$\mathrm{Br}_{\langle\pi\rangle} : Z(\mathbb{F}_p S_{p\cdot i}) \longrightarrow Z(\mathbb{F}_p\, \mathrm{C}_{S_{p\cdot i}}(\pi))$$

is a projection. By Lemma 18 this projection is surjective: If C is a p'-class of $S_{p\cdot i}$, then $C \cap C_{S_{p\cdot i}}(\pi) = \emptyset$ or $C \cap C_{S_{p\cdot i}}(\pi)$ is a p'-class of $C_{S_{p\cdot i}}(\pi)$.

Lemma 21. *Let π and i be as in Lemma* 18. *Let $C \subset \mathrm{C}_{S_{p\cdot i}}(\pi)$ be the conjugacy class of elements of cycle type $(c_1, ..., c_{pi})$, where $c_i = p$ and $c_j = 0$ for $j \neq i$. Then $(C^+)^2 = 0$ in $\mathbb{F}_p\, \mathrm{C}_{S_{p\cdot i}}(\pi)$.*

Proof. $\mathrm{supp}(C^+)^2$ consists of p'-elements according to Remark 20. Now let $D \subset \mathrm{C}_{S_{p\cdot i}}(\pi)$ be a p'-conjugacy class. According to [4], 4.6, the coefficient a_{CCD} of D^+ in $(C^+)^2 \in \mathbb{Z}\,\mathrm{C}_{S_{p\cdot i}}(\pi)$ is

$$a_{CCD} = \frac{|C|^2}{|\,\mathrm{C}_{S_{p\cdot i}}(\pi)|} \sum_{\chi \in \mathrm{Irr}\,\mathrm{C}_{S_{p\cdot i}}(\pi)} \chi^2(g) \cdot \frac{\chi(h^{-1})}{\chi(1)},$$

where $g \in C$ and $h \in D$. By Corollary 19 we know that $\chi(g)$ and $\chi(h^{-1})$ are integers. Using [9], 3.9, we obtain

$$\frac{|C|^2}{|\,\mathrm{C}_{S_{p\cdot i}}(\pi)|} = \frac{(p^{i-1} \cdot (i-1)!)^2}{p^i \cdot i!} = \frac{p^{i-2}(i-1)!}{i}.$$

The group $\mathrm{C}_{S_{p\cdot i}}(\pi) \cong (\mathbb{Z}_p)^i \rtimes S_i$ contains an abelian normal subgroup of order p^i, so by a theorem of Ito ([4], 19.9) we get

$$\chi(1) \mid [\mathrm{C}_{S_{p\cdot i}}(\pi) : (\mathbb{Z}_p)^i] = \frac{p^i \cdot i!}{p^i} = i!.$$

As $p \nmid i$ this provides $a_{CCD} \equiv 0 \mod p$ for $i > 2$. For $i = 2$ the centralizer of π is a group of order $2p^2$. Therefore the product of two elements of C is an element of the Sylow-p-subgroup of $\mathrm{C}_{S_{p\cdot i}}(\pi)$, i.e. $C \not\subset \mathrm{supp}(C^+)^2$. As $p \mid |C|$ provides $1 \notin \mathrm{supp}(C^+)^2$, we obtain $a_{CCD} \equiv 0 \mod p$ for the case $i = 2$, because $\{1\}$ and C are the only p'-classes of $\mathrm{C}_{S_{p\cdot i}}(\pi)$ in this case.

We remark that the fact that C^+ is nilpotent can already be deduced from [15, 4.7].

Lemma 22. *Let σ, i be as in Notation 15, and let $\mu = (b_1, ..., b_n)$ be the cycle type of a p'-partition with $b_i \geq p$. Then $C_\mu \not\subset \mathrm{supp}(C_\sigma^+)^p$.*

Proof. There are elements $x \in C_\sigma$ and $y \in C_\mu$ such that

$$\pi = (1, 2, ..., p)(p+1, ..., 2p)...((i-1)p+1, ..., ip)$$

is an element of $\mathrm{C}_{S_n}(x)$ and of $\mathrm{C}_{S_n}(y)$. We apply the Brauer-homomorphism

$$\mathrm{Br}_{\langle\pi\rangle} : Z(\mathbb{F}_p S_n) \longrightarrow Z(\mathbb{F}_p \, \mathrm{C}_{S_n}(\pi))$$

and get

$$C_\mu \subset \mathrm{supp}(C_\sigma^+)^p \Longleftrightarrow \mathrm{supp}\,\mathrm{Br}_{\langle\pi\rangle}(C_\mu)^+ \subset \mathrm{supp}(\mathrm{Br}_{\langle\pi\rangle}(C_\sigma^+))^p,$$

as $\mathrm{Br}_{\langle\pi\rangle}(C_\mu)^+ \neq 0$.

The coefficient a_μ of C_μ^+ in $(C_\sigma^+)^p \in \mathbb{Z}S_n$ is the number of solutions $(g_1, ..., g_p)$ of the equation

$$g_1 \cdot ... \cdot g_p = y,$$

where $g_i \in C_\sigma \cap \mathrm{C}_{S_n}(\pi)$. Using the decomposition $\mathrm{C}_{S_n}(\pi) \cong \mathrm{C}_{S_{p\cdot i}}(\pi) \times S_{n-ip}$ we can write $g_t = g_{t,1}g_{t,2}$ and $y = y_1y_2$ with $g_{t,1}, y_1 \in \mathrm{C}_{S_{p\cdot i}}(\pi)$ and $g_{t,2}, y_2 \in S_{n-ip}$ and obtain equations

$$g_{1,1}g_{2,1}...g_{p,1} = y_1, \qquad g_{1,2}...g_{p,2} = y_2.$$

If r_1, r_2 denote the number of solutions of the first and second equation, respectively, then $a_\mu = r_1 \cdot r_2$. We show that $r_1 \equiv 0 \mod p$.

According to Lemma 18 there is exactly one conjugacy class $C \subset \mathrm{C}_{S_{pi}}(\pi)$ of elements of cycle type $(c_1, ..., c_{pi})$ with $c_i \neq 0$ and $c_j = 0$ for $j \neq i$, and in fact $c_i = p$ for this class. Let $g_t = g_{t,1}g_{t,2}$ be a decomposition of an element $g_t \in C_\sigma \cap \mathrm{C}_{S_n}(\pi)$. We prove that $g_{t,1} \in C$ and that for every element $g_{t,1} \in C$ there exists an element $g_{t,2} \in S_{n-ip}$ such that $g_{t,1}g_{t,2} \in C_\sigma \cap \mathrm{C}_{S_n}(\pi)$. Thus we have to count the number of solutions $(g_{1,1}, ..., g_{p,1})$ of the first equation, where $g_{t,1} \in C$ for all t. Then we prove that $y_1 \in C$ as well and that therefore r_1 is the coefficient of C^+ in $(C^+)^p$.

All p'-elements in $\mathrm{C}_{S_{pi}}(\pi)\backslash\{(1)\}$ have a cycle type of the form $(pd_1, ..., pd_i)$. As $a_j < p$ for $1 \leq j < i$ we obtain $g_{t,1} \in C$ for all $g_t = g_{t,1}g_{t,2} \in \mathrm{C}_{S_n}(\pi) \cap C_\sigma$ and for every $g_{t,1} \in C$ there exists a $g_{t,2} \in S_{n-ip}$ such that $g_t = g_{t,1}g_{t,2} \in \mathrm{C}_{S_n}(\pi) \cap C_\sigma$.

Now let $\lambda = (b_1, ..., b_n)$ be the cycle type of a p'-partition with $C_\lambda \subset \mathrm{supp}(C_\sigma^+)^k$. Using induction and Corollary 17 we see that $b_j < p$ for all $j < i$. This also provides $y_1 \in C$ for all $y = y_1y_2 \in \mathrm{C}_{S_n}(\pi) \cap C_\mu$. Therefore r_1 is the coefficient of C^+ in $(C^+)^p \in Z(\mathbb{F}_p \, \mathrm{C}_{S_{p\cdot i}}(\pi))$. But by Lemma 21 we have $(C^+)^2 = 0$ and thus $(C^+)^p = 0$, i.e. $r_1 \equiv 0 \mod p$.

Corollary 23. *Let σ, i be as in Notation 15, and let $\mu = (b_1, ..., b_n)$ be the cycle type of a p'-partition with $C_\mu \subset \mathrm{supp}(C_\sigma^+)^p$. Then $b_j < p$ for $1 \leq j \leq i$.*

Proof. For $1 \leq j < i$ the statement $b_j < p$ follows by induction and Corollary 17. Lemma 22 provides $b_i < p$.

Proof of Theorem 3. We consider a minimal counterexample in the following sense: For a given prime p let n be minimal such that a primitive central idempotent $e \in \mathbb{F}_p S_n$ exists with $e \notin Z_{p'}^{S_n} \cap Z_{p-nreg}^{S_n}$. According to a theorem of Osima ([8], 7.4) we know that $e \in Z_{p'}^{S_n}$, therefore there is a p'-partition σ of cycle type $(a_1, ..., a_n)$

with $a_i \geq p$ for some $i \geq 2$, such that $C_\sigma \subset \operatorname{supp} e$. Now we choose σ to be the partition, where $i \geq 2$ is minimal with $a_i \geq p$ under all partitions with $C_\sigma \subset \operatorname{supp} e$.

Let $e = \sum_{C_\tau \subset \operatorname{supp} e} a_\tau C_\tau^+$. Then

$$e = e^p = \sum_{C_\tau \subset \operatorname{supp} e} a_\tau^p (C_\tau^+)^p = \sum_{C_\tau \subset \operatorname{supp} e} a_\tau (C_\tau^+)^p.$$

For conjugacy class sums $C_\tau^+ \in Z_{p'}^{S_n} \cap Z_{p-nreg}^{S_n}$ we know $(C_\tau^+)^p \in Z_{p'}^{S_n} \cap Z_{p-nreg}^{S_n}$ by Theorem 2, thus $C_\sigma \not\subset \operatorname{supp}(C_\tau^+)^p$ for these τ. Now let $(b_1, ..., b_n)$ be the cycle type of a partition τ with $C_\tau \subset \operatorname{supp} e$ and $b_j \geq p$ for some $j \geq 2$. Using Theorem 1 and the minimality of n we get that $b_1 < p$ for all these τ. Now let j be minimal with $b_j \geq p$ for the given τ. If $j \geq i$, then $C_\sigma \not\subset \operatorname{supp}(C_\tau^+)^p$ according to Corollary 23. As we chose i to be minimal there is no τ such that $C_\tau \subset \operatorname{supp} e$ and $b_j \geq p$ for a j with $2 \leq j < i$. Hence we obtain

$$C_\sigma \not\subset \operatorname{supp} e^p = \operatorname{supp} e,$$

a contradiction.

Now we want to use the theorems we proved to speed up the algorithm described in Algorithm 7 for the computation of the primitive central idempotents of finite group rings of symmetric and alternating groups. For symmetric groups the number r of primitive central idempotents of $\mathbb{F}_p S_n$ can be computed using Nakayamas Conjecture, [7], 6.1.21. To compute the primitive central idempotents of $\mathbb{F}_p S_n$ we use Algorithm 7 with the following changes:

- We compute the p-near-regular p'-conjguacy classes in step 1) of Algorithm 7.
- We subsume steps 2), 3) and 4) of Algorithm 7 in a loop and check if the basis computed in 4) already has r elements. If this is the case we do not need to compute further idempotents and we can continue with step 5.
- For step 2) and step 5) of Algorithm 7 it is necessary to compute the product of class sums in $\mathbb{F}_p S_n$. We compute the class multiplication coefficients over the field $\mathbb{C}$ of complex numbers according to [4], Theorem 4.6, using a program written in SYMMETRICA ([10]) provided by A. Kohnert and then reduce them modulo p. As we are multiplying class sums of p-near-regular p'-classes we only have to compute coefficients c_{ijk} of p-near regular p'-classes C_k according to Theorem 2. We also use Corollary 12 to reduce the number of coefficients we have to compute.
- We store the products of class sums because they usually occur several times during the computation.

Considering the changes described above we see that the best situation occurs for $p = 2$, because in this case the numbers of p'-classes and of p-near-regular classes both are much smaller than for all other primes (see Table 1). But there are even more possibilities to speed up the program for $p = 2$. The step of Algorithm 7 consuming the most time is step 5), because the idempotents f_i are sums of many class sums and it takes a long time to compute a product $f_i \cdot f_j$. The philosophy for $p = 2$ is to compute only squares of class sums if possible. So for $p = 2$ we also made the following changes of Algorithm 7:

- It turned out that we get the right number of idempotents using 1 and the powers of the class sums $C_3^+, C_7^+, ..., C_{4\cdot(r-2)+3}^+$. We didn't prove that the powers of these class sums always generate the vector space spanned by the primitive central idempotents, but the program will stop if it doesn't find enough linear independent idempotents.
- In step 2) we compute $(C_i^+)^{2^j}$ until $(C_i^+)^{2^k} = (C_i^+)^{2^{k+1}}$ building only squares of class sums. Then $f_i := (C_i^+)^{2^k}$ is the idempotent occuring in step 3). We keep in mind that f_i "comes from" C_i for step 5).
- We store all class multiplication coefficients, because we can use them for bigger symmetric groups according to Corollary 12.
- The products $f_i \cdot f_j$ occuring in step 5) are computed in the following way: We kept in mind that f_i came from C_i and f_j came from C_j. So instead of multiplying f_i and f_j directly—both are usually large sums of class sums—we multiply C_i^+ and C_j^+ and compute $(C_i^+C_j^+)^{2^j}$ until $(C_i^+C_j^+)^{2^k} = (C_i^+C_j^+)^{2^{k+1}}$. Then $f_i \cdot f_j = (C_i^+C_j^+)^{2^k}$. The advantage of this procedure is that we only have to compute squares of class sums and no mixed products except $C_i^+C_j^+$. We store all squares we compute, because they usually occur several times.

  If we replace an idempotent f_i by $f_i \cdot f_j$ or by a sum of idempotents, then we have to keep in mind that our new idempotent comes from $C_i^+ \cdot C_j^+$ or from a sum of class sums. Here we get a delicate problem: After several steps of the loop in step 5) our idempotents are powers of expressions like $(C_i^+)^2 \cdot C_j^+ + (C_k^+)^3 \cdot C_m^+ + C_k^+ \cdot (C_m^+)^2 + ...$ In every step the expressions become longer and the multiplying of two such expressions takes more and more time. Therefore we have to keep these expressions simple: The idempotent generated by $(C_i^+)^2 \cdot C_j$ is also generated by $C_i^+ \cdot C_j$ so we have to filter out powers in our expressions. If we do that we will see that the idempotents generated by the summands $(C_k^+)^3 \cdot C_m^+$ and $C_k^+ \cdot (C_m^+)^2$ in the expression above are the same, so we can delete those summands from our expression. Thus for every idempotent f_i we store such an expression, and this expression has to be updated and simplified whenever we replace an idempotent f_i by a product or a sum of idempotents.

These changes allowed us to compute the primitive central idempotents of $\mathbb{F}_2S_n$ for $n \leq 50$. The results can be seen in the Appendix. For our computations we used a dual core computer with two Opteron 265 1.8GHz processors. Approximately the time needed to compute the idempotents of S_{n+2} using the results of the computation for S_n is twice the time needed for the computation of the idempotents for S_n. So it takes about 7s to carry out the computation for S_{20}, 4m for S_{30}, 3h31m for S_{40} and 10d7h14m for S_{50} using the results for S_{28}, S_{38} and S_{48}, respectively.

The necessary information about the primitive central idempotents of the group rings $\mathbb{F}_2A_n$ can be found in corollaries 4 and 5 of [3]: If n is not of the form $n = \frac{m(m+1)}{2}$ with an integer m, then the primitive central idempotents of $\mathbb{F}_2S_n$ are also the primitive central idempotents of $\mathbb{F}_{2^k}A_n$. If $n = \frac{m(m+1)}{2}$, then this is also true for all idempotents of $\mathbb{F}_2S_n$ except for one. In this case $e = C^+$ is a primitive central idempotent of $\mathbb{F}_2S_n$ according to Theorem 3 of [3], where C is the conjugacy class of elements corresponding to the partition $(2m-1, 2m-5, 2m-9, ...)$ of n.

This idempotent e splits in a sum of two primitive central idempotents of $\mathbb{F}_{2^k}A_n$. As the class C splits in two conjugacy classes C_- and C_+ of A_n we can compute the remaining primitive central idempotents of $\mathbb{F}_{2^k}A_n$ by computing the powers of the class sums C_-^+ and C_+^+. For alternating groups the field $\mathbb{F}_2$ is not always a splitting field, but at least $\mathbb{F}_4$ is a splitting field for A_n according to Corollary B of [13]. Thus it may happen that the two idempotents are not elements of $\mathbb{F}_2A_n$, but they are elements of $\mathbb{F}_4A_n$. The results show that our Theorems 1, 2, 3 are not true for alternating groups.

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Appendix

Here our notation is slightly different to the notation in the rest of the article: If $\mu = 2^{\alpha_2}, ..., n^{\alpha_n}$ is a partition we write $\overline{2^{\alpha_2}, ..., n^{\alpha_n}}$ for the class sum $C_\mu^+ \in \mathbb{F}_2S_m$, where $m \geq W(\mu)$. According to Theorem 1 one can easily deduce the primitive central idempotents of $\mathbb{F}_2S_n$ for $n < 49$ from the primitive central idempotents of $\mathbb{F}_2S_{50}$ and $\mathbb{F}_2S_{49}$. To simplify that task we added tokens of the form $|_{16}$ to indicate where the primitive central idempotent of $\mathbb{F}_2S_{16}$ ends.

Primitive central idempotents of $\mathbb{F}_2S_n$ for n odd and $n \leq 49$:

$$
\begin{aligned}
e_1 \quad = \quad & \overline{1}|_1 + \overline{3}|_3 + \overline{5}|_5 + \overline{3,5} + \overline{9}|_9 + \overline{7,9} + \overline{5,11} + \overline{3,13} + \overline{17}|_{17} + \overline{3,7,11}|_{21} + \overline{5,7,11} + \overline{3,9,11} + \\
& \overline{3,7,13}|_{23} + \overline{5,9,11} + \overline{5,7,13} + \overline{3,9,13}|_{25} + \overline{3,5,7,11} + \overline{7,9,11} + \overline{5,9,13} + \overline{3,11,13} + \\
& \overline{3,7,17}|_{27} + \overline{3,5,9,11} + \overline{3,5,7,13} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,7,17} + \overline{3,9,17}|_{29} + \overline{3,5,9,13} + \\
& \overline{5,9,17}|_{31} + \overline{5,7,9,11} + \overline{3,5,11,13} + \overline{3,5,7,17} + \overline{15,17} + \overline{13,19} + \overline{11,21} + \overline{9,23} + \overline{7,25} + \\
& \overline{5,27} + \overline{3,29} + \overline{9,11,13} + \overline{7,9,17} + \overline{3,13,17} + \overline{33}|_{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,5,9,17} + \\
& \overline{3,5,7,9,11} + \overline{7,11,17} + \overline{5,13,17} + \overline{3,11,21} + \overline{3,7,25}|_{35} + \overline{5,7,11,13} + \overline{3,9,11,13} + \\
& \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{3,15,19} + \overline{5,11,21} + \overline{3,13,21} + \overline{3,11,23} + \overline{5,7,25} + \\
& \overline{3,9,25} + \overline{3,7,27}|_{37} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,15,19} + \overline{3,17,19} + \overline{5,13,21} + \overline{3,15,21} + \\
& \overline{5,11,23} + \overline{3,13,23} + \overline{5,9,25} + \overline{3,11,25} + \overline{5,7,27} + \overline{3,9,27} + \overline{3,7,29}|_{39} + \overline{7,9,11,13} + \\
& \overline{5,7,11,17} + \overline{3,7,13,17} + \overline{3,5,11,21} + \overline{3,5,7,25} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \\
& \overline{11,13,17} + \overline{5,17,19} + \overline{9,11,21} + \overline{5,15,21} + \overline{5,13,23} + \overline{7,9,25} + \overline{5,11,25} + \overline{5,9,27} + \\
& \overline{5,7,29} + \overline{3,9,29}|_{41} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,5,15,19} + \overline{3,5,13,21} + \overline{3,5,11,23} + \\
& \overline{3,5,9,25} + \overline{3,5,7,27} + \overline{9,15,19} + \overline{9,13,21} + \overline{3,19,21} + \overline{9,11,23} + \overline{3,17,23} + \overline{3,15,25} + \\
& \overline{7,9,27} + \overline{3,13,27} + \overline{5,9,29} + \overline{3,11,29} + \overline{3,7,33}|_{43} + \overline{3,5,17,19} + \overline{3,5,15,21} + \\
& \overline{3,5,13,23} + \overline{3,5,11,25} + \overline{3,5,9,27} + \overline{3,5,7,29} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \\
& \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,7,33} + \\
& \overline{3,9,33}|_{45} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29} + \overline{5,9,33}|_{47} + \\
& \overline{3,5,7,9,11,13} + \overline{7,9,15,17} + \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{5,11,13,19} + \overline{3,11,13,21} + \\
& \overline{3,5,19,21} + \overline{5,9,11,23} + \overline{3,5,17,23} + \overline{3,7,13,25} + \overline{3,5,15,25} + \overline{5,7,9,27} + \overline{3,5,13,27} + \\
& \overline{3,5,11,29} + \overline{3,5,7,33} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \\
& \overline{3,5,9,13,19} + \overline{3,5,9,11,21} + \overline{11,17,21} + \overline{9,19,21} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{9,15,25} + \\
& \overline{7,17,25} + \overline{9,13,27} + \overline{9,11,29} + \overline{3,17,29} + \overline{7,9,33} + \overline{3,13,33}
\end{aligned}
$$

$$
\begin{aligned}
e_2 = {} & \overline{3}|_{3} + \overline{5}|_{5} + \overline{3,5} + \overline{9}|_{9} + \overline{5,9}|_{15} + \overline{3,13} + \overline{3,5,9} + \overline{17}|_{17} + \overline{7,11} + \overline{5,13} + \overline{3,7,9} + \\
& \overline{3,5,11}|_{19} + \overline{9,11} + \overline{7,13} + \overline{5,7,9} + \overline{3,7,11} + \overline{3,5,13}|_{21} + \overline{5,17} + \overline{5,7,11} + \overline{3,9,11} + \\
& \overline{3,7,13}|_{23} + \overline{3,5,7,9} + \overline{11,13} + \overline{7,17} + \overline{5,7,13} + \overline{3,5,17}|_{25} + \overline{3,5,7,11} + \overline{9,17} + \overline{7,9,11} + \\
& \overline{5,9,13} + \overline{3,11,13} + \overline{3,7,17}|_{27} + \overline{3,5,9,11} + \overline{3,5,7,13} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,7,17} + \\
& \overline{3,9,17}|_{29} + \overline{3,5,9,13} + \overline{5,9,17}|_{31} + \overline{3,7,9,13} + \overline{3,5,7,17} + \overline{11,21} + \overline{7,25} + \overline{3,29} + \\
& \overline{9,11,13} + \overline{5,11,17} + \overline{33}|_{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,5,9,17} + \overline{15,19} + \overline{13,21} + \\
& \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,5,7,9,11} + \overline{7,11,17} + \overline{5,13,17} + \overline{3,15,17} + \overline{3,13,19} + \\
& \overline{3,9,23} + \overline{3,5,27}|_{35} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{17,19} + \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \\
& \overline{7,29} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,15,17} + \overline{5,13,19} + \overline{3,15,19} + \overline{3,13,21} + \\
& \overline{5,9,23} + \overline{3,11,23} + \overline{3,9,25} + \overline{3,7,27} + \overline{3,5,29}|_{37} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,33} + \\
& \overline{5,15,19} + \overline{3,17,19} + \overline{5,13,21} + \overline{3,15,21} + \overline{5,11,23} + \overline{3,13,23} + \overline{5,9,25} + \overline{3,11,25} + \\
& \overline{5,7,27} + \overline{3,9,27} + \overline{3,7,29}|_{39} + \overline{7,9,11,13} + \overline{5,7,11,17} + \overline{3,7,13,17} + \overline{3,5,15,17} + \\
& \overline{3,5,13,19} + \overline{19,21} + \overline{3,5,9,23} + \overline{17,23} + \overline{15,25} + \overline{13,27} + \overline{11,29} + \overline{7,33} + \overline{3,5,9,11,13} + \\
& \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{9,13,19} + \overline{5,17,19} + \overline{5,15,21} + \overline{5,13,23} + \overline{5,11,25} + \\
& \overline{5,7,29} + \overline{3,5,33}|_{41} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,5,15,19} + \overline{3,5,13,21} + \overline{3,5,11,23} + \\
& \overline{3,5,9,25} + \overline{3,5,7,27} + \overline{9,33} + \overline{9,15,19} + \overline{9,13,21} + \overline{3,19,21} + \overline{9,11,23} + \overline{3,17,23} + \\
& \overline{3,15,25} + \overline{7,9,27} + \overline{3,13,27} + \overline{5,9,29} + \overline{3,11,29} + \overline{3,7,33}|_{43} + \overline{3,5,17,19} + \overline{3,5,15,21} + \\
& \overline{3,5,13,23} + \overline{3,5,11,25} + \overline{3,5,9,27} + \overline{3,5,7,29} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \\
& \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,7,33} + \\
& \overline{3,9,33}|_{45} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29} + \overline{5,9,33}|_{47} + \\
& \overline{3,5,7,9,11,13} + \overline{3,13,15,17} + \overline{7,9,11,21} + \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \\
& \overline{5,7,11,25} + \overline{3,5,15,25} + \overline{23,25} + \overline{21,27} + \overline{3,7,9,29} + \overline{19,29} + \overline{3,5,7,33} + \overline{15,33} + \\
& \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{13,17,19} + \\
& \overline{3,5,9,11,21} + \overline{9,19,21} + \overline{3,5,7,9,25} + \overline{9,15,25} + \overline{9,13,27} + \overline{5,17,27} + \overline{9,11,29} + \overline{5,11,33}
\end{aligned}
$$

$$
\begin{aligned}
e_3 = {} & \overline{5,9}|_{15} + \overline{7,9} + \overline{5,11} + \overline{3,5,9}|_{17} + \overline{7,11} + \overline{5,13} + \overline{3,7,9} + \overline{3,5,11}|_{19} + \overline{9,11} + \overline{7,13} + \\
& \overline{5,7,9} + \overline{3,7,11} + \overline{3,5,13}|_{21} + \overline{5,17} + \overline{5,7,11} + \overline{3,9,11} + \overline{3,7,13}|_{23} + \overline{3,5,7,9} + \overline{11,13} + \\
& \overline{7,17} + \overline{5,7,13} + \overline{3,5,17}|_{25} + \overline{3,5,7,11} + \overline{9,17} + \overline{7,9,11} + \overline{5,9,13} + \overline{3,11,13} + \overline{3,7,17}|_{27} + \\
& \overline{3,5,9,11} + \overline{3,5,7,13} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,7,17} + \overline{3,9,17}|_{29} + \overline{3,5,9,13} + \overline{5,9,17}|_{31} + \\
& \overline{3,7,9,13} + \overline{3,5,7,17} + \overline{15,17} + \overline{13,19} + \overline{9,23} + \overline{5,27} + \overline{9,11,13} + \overline{5,11,17}|_{33} + \overline{5,7,9,13} + \\
& \overline{3,7,11,13} + \overline{3,5,9,17} + \overline{15,19} + \overline{13,21} + \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,5,7,9,11} + \\
& \overline{7,11,17} + \overline{5,13,17} + \overline{3,15,17} + \overline{3,13,19} + \overline{3,9,23} + \overline{3,5,27}|_{35} + \overline{5,7,11,13} + \overline{3,9,11,13} + \\
& \overline{17,19} + \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \overline{7,29} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \\
& \overline{5,15,17} + \overline{5,13,19} + \overline{3,15,19} + \overline{3,13,21} + \overline{5,9,23} + \overline{3,11,23} + \overline{3,9,25} + \overline{3,7,27} + \\
& \overline{3,5,29}|_{37} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,33} + \overline{5,15,19} + \overline{3,17,19} + \overline{5,13,21} + \overline{3,15,21} + \\
& \overline{5,11,23} + \overline{3,13,23} + \overline{5,9,25} + \overline{3,11,25} + \overline{5,7,27} + \overline{3,9,27} + \overline{3,7,29}|_{39} + \overline{7,9,11,13} + \\
& \overline{5,7,11,17} + \overline{3,7,13,17} + \overline{3,5,15,17} + \overline{3,5,13,19} + \overline{19,21} + \overline{3,5,9,23} + \overline{17,23} + \overline{15,25} + \\
& \overline{13,27} + \overline{11,29} + \overline{7,33} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{9,13,19} + \\
& \overline{5,17,19} + \overline{5,15,21} + \overline{5,13,23} + \overline{5,11,25} + \overline{5,7,29} + \overline{3,5,33}|_{41} + \overline{5,9,11,17} + \overline{3,9,13,17} + \\
& \overline{3,5,15,19} + \overline{3,5,13,21} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{3,5,7,27} + \overline{9,33} + \overline{9,15,19} + \overline{9,13,21} + \\
& \overline{3,19,21} + \overline{9,11,23} + \overline{3,17,23} + \overline{3,15,25} + \overline{7,9,27} + \overline{3,13,27} + \overline{5,9,29} + \overline{3,11,29} + \\
& \overline{3,7,33}|_{43} + \overline{3,5,17,19} + \overline{3,5,15,21} + \overline{3,5,13,23} + \overline{3,5,11,25} + \overline{3,5,9,27} + \overline{3,5,7,29} + \\
& \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \\
& \overline{7,9,29} + \overline{5,11,29} + \overline{5,7,33} + \overline{3,9,33}|_{45} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \\
& \overline{5,7,9,25} + \overline{3,5,9,29} + \overline{5,9,33}|_{47} + \overline{3,5,7,9,11,13} + \overline{3,13,15,17} + \overline{7,9,11,21} + \\
& \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,5,15,25} + \overline{23,25} + \overline{21,27} + \\
& \overline{3,7,9,29} + \overline{19,29} + \overline{3,5,7,33} + \overline{15,33} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \\
& \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{13,17,19} + \overline{3,5,9,11,21} + \overline{9,19,21} + \overline{3,5,7,9,25} + \\
& \overline{9,15,25} + \overline{9,13,27} + \overline{5,17,27} + \overline{9,11,29} + \overline{5,11,33}
\end{aligned}
$$

$$
\begin{aligned}
e_4 = {} & \overline{3,7,11}|_{21} + \overline{5,7,11} + \overline{3,9,11} + \overline{3,7,13}|_{23} + \overline{5,9,11} + \overline{5,7,13} + \overline{3,9,13}|_{25} + \overline{3,5,7,11} + \\
& \overline{7,9,11} + \overline{5,9,13} + \overline{3,11,13} + \overline{3,7,17}|_{27} + \overline{3,5,9,11} + \overline{3,5,7,13} + \overline{7,9,13} + \overline{5,11,13} + \\
& \overline{5,7,17} + \overline{3,9,17}|_{29} + \overline{3,5,9,13} + \overline{5,9,17}|_{31} + \overline{5,7,9,11} + \overline{3,5,11,13} + \overline{3,5,7,17} + \\
& \overline{9,11,13} + \overline{7,9,17} + \overline{3,13,17}|_{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,5,9,17} + \overline{3,5,7,9,11} + \\
& \overline{7,11,17} + \overline{5,13,17} + \overline{3,11,21} + \overline{3,7,25}|_{35} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{3,5,7,9,13} + \\
& \overline{9,11,17} + \overline{7,13,17} + \overline{3,15,19} + \overline{5,11,21} + \overline{3,13,21} + \overline{3,11,23} + \overline{5,7,25} + \overline{3,9,25} + \\
& \overline{3,7,27}|_{37} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,15,19} + \overline{3,17,19} + \overline{5,13,21} + \overline{3,15,21} + \overline{5,11,23} + \\
& \overline{3,13,23} + \overline{5,9,25} + \overline{3,11,25} + \overline{5,7,27} + \overline{3,9,27} + \overline{3,7,29}|_{39} + \overline{7,9,11,13} + \overline{5,7,11,17} + \\
& \overline{3,7,13,17} + \overline{3,5,11,21} + \overline{3,5,7,25} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{5,17,19} + \\
& \overline{9,11,21} + \overline{5,15,21} + \overline{5,13,23} + \overline{7,9,25} + \overline{5,11,25} + \overline{5,9,27} + \overline{5,7,29} + \overline{3,9,29}|_{41} + \\
& \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,5,15,19} + \overline{3,5,13,21} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{3,5,7,27} + \\
& \overline{9,15,19} + \overline{9,13,21} + \overline{3,19,21} + \overline{9,11,23} + \overline{3,17,23} + \overline{3,15,25} + \overline{7,9,27} + \overline{3,13,27} + \\
& \overline{5,9,29} + \overline{3,11,29} + \overline{3,7,33}|_{43} + \overline{5,9,13,17} + \overline{3,5,17,19} + \overline{3,5,15,21} + \overline{3,5,13,23} + \\
& \overline{3,5,11,25} + \overline{3,5,9,27} + \overline{3,5,7,29} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \\
& \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,7,33} + \overline{3,9,33}|_{45} + \\
& \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29} + \overline{3,5,9,13,17} + \\
& \overline{5,9,33}|_{47} + \overline{3,5,7,9,11,13} + \overline{7,11,13,17} + \overline{5,9,15,19} + \overline{5,9,13,21} + \overline{3,11,13,21} + \\
& \overline{3,5,19,21} + \overline{5,9,11,23} + \overline{3,5,17,23} + \overline{3,7,13,25} + \overline{3,5,15,25} + \overline{5,7,9,27} + \overline{3,5,13,27} + \\
& \overline{3,5,11,29} + \overline{3,5,7,33} + \overline{5,7,9,11,17} + \overline{3,5,9,11,21} + \overline{11,17,21} + \overline{9,19,21} + \overline{9,17,23} + \\
& \overline{3,5,7,9,25} + \overline{9,15,25} + \overline{7,17,25} + \overline{9,13,27} + \overline{9,11,29} + \overline{3,17,29} + \overline{7,9,33} + \overline{3,13,33}
\end{aligned}
$$

$$\begin{aligned}
e_5 \quad = \quad & \overline{5,9,13,17}|_{45} + \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{3,5,9,13,17}|_{47} + \\
& \overline{7,11,13,17} + \overline{7,9,15,17} + \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{5,11,13,19} + \overline{5,9,15,19} + \\
& \overline{5,9,13,21} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19}
\end{aligned}$$

Primitive central idempotents of $\mathbb{F}_2 S_n$ for n even and $n \leq 50$:

$$\begin{aligned}
e_1 \quad = \quad & \overline{1}|_2 + \overline{5}|_6 + \overline{7} + \overline{3,5}|_8 + \overline{9}|_{10} + \overline{15} + \overline{7,9} + \overline{5,11} + \overline{3,13}|_{16} + \overline{17}|_{18} + \overline{5,9,13}|_{28} + \overline{7,9,13} + \\
& \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{7,9,15} + \overline{5,11,15} + \overline{5,9,17} + \overline{31} + \overline{3,7,9,13} + \\
& \overline{3,5,11,13} + \overline{3,5,9,15} + \overline{15,17} + \overline{13,19} + \overline{11,21} + \overline{9,23} + \overline{7,25} + \overline{5,27} + \overline{3,29}|_{32} + \overline{9,11,13} + \\
& \overline{7,11,15} + \overline{5,13,15} + \overline{7,9,17} + \overline{5,11,17} + \overline{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \\
& \overline{3,5,11,15} + \overline{3,5,9,17}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \overline{5,7,11,13} + \\
& \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \\
& \overline{7,13,17} + \overline{5,13,19} + \overline{5,9,23} + \overline{5,7,9,17} + \overline{3,5,13,17}|_{38} + \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \\
& \overline{11,13,15} + \overline{7,13,19} + \overline{5,15,19} + \overline{5,13,21} + \overline{7,9,23} + \overline{5,11,23} + \overline{5,9,25} + \overline{7,9,11,13} + \\
& \overline{5,9,11,15} + \overline{3,9,13,15} + \overline{3,5,13,19} + \overline{3,5,9,23}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \\
& \overline{11,13,17} + \overline{9,13,19} + \overline{7,15,19} + \overline{5,17,19} + \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \\
& \overline{7,9,25} + \overline{5,11,25} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \overline{3,7,13,19} + \overline{3,5,15,19} + \\
& \overline{3,7,11,21} + \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \\
& \overline{9,13,21} + \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \\
& \overline{5,7,13,19} + \overline{3,9,13,19} + \overline{3,5,17,19} + \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \\
& \overline{3,5,13,23} + \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \\
& \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \\
& \overline{5,9,13,19} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \\
& \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \overline{3,5,7,13,19} + \overline{13,15,19} + \overline{3,5,7,11,21} + \overline{7,19,21} + \\
& \overline{3,5,7,9,23} + \overline{9,15,23} + \overline{7,17,23} + \overline{7,15,25} + \overline{7,13,27} + \overline{5,15,27} + \overline{7,11,29} + \\
& \overline{7,9,31} + \overline{5,11,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{7,9,15,17} + \overline{3,13,15,17} + \overline{5,11,13,19} + \\
& \overline{7,9,11,21} + \overline{3,11,13,21} + \overline{3,5,19,21} + \overline{5,9,11,23} + \overline{3,5,17,23} + \overline{3,7,13,25} + \\
& \overline{3,5,15,25} + \overline{3,5,13,27} + \overline{3,7,9,29} + \overline{3,5,11,29} + \overline{3,5,9,31}|_{48} + \overline{5,7,9,11,17} + \\
& \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{13,17,19} + \overline{3,5,9,11,21} + \\
& \overline{13,15,21} + \overline{9,19,21} + \overline{11,15,23} + \overline{3,5,7,9,25} + \overline{9,13,27} + \overline{7,15,27} + \overline{5,17,27} + \overline{9,11,29} + \\
& \overline{5,15,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{7,9,33} + \overline{5,11,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \\
& \overline{7,11,13,19} + \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \\
& \overline{3,11,13,23} + \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \overline{3,7,15,25} + \\
& \overline{3,7,13,27} + \overline{3,5,15,27} + \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33}
\end{aligned}$$

$$\begin{aligned}
e_2 \quad = \quad & \overline{5}|_6 + \overline{7} + \overline{3,5}|_8 + \overline{9} + \overline{3,7}|_{10} + \overline{5,7} + \overline{3,9}|_{12} + \overline{5,9}|_{14} + \overline{3,5,7} + \overline{15} + \overline{5,11}|_{16} + \overline{3,5,9} + \\
& \overline{17} + \overline{7,11} + \overline{5,13} + \overline{3,15}|_{18} + \overline{9,11} + \overline{7,13} + \overline{5,15} + \overline{3,17}|_{20} + \overline{5,7,9} + \overline{3,5,13} + \overline{5,17}|_{22} + \\
& \overline{5,7,11} + \overline{3,7,13} + \overline{3,5,15} + \overline{3,5,7,9} + \overline{11,13} + \overline{9,15}|_{24} + \overline{5,9,11} + \overline{3,9,13} + \overline{3,5,17} + \\
& \overline{9,17}|_{26} + \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{3,13,15} + \\
& \overline{5,9,17} + \overline{31} + \overline{5,7,9,11} + \overline{3,5,9,15} + \overline{13,19} + \overline{9,23} + \overline{5,27}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \\
& \overline{5,13,15} + \overline{3,13,17} + \overline{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,5,9,17} + \\
& \overline{15,19} + \overline{13,21} + \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,31}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \\
& \overline{5,13,17} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17} + \\
& \overline{17,19} + \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \overline{7,29} + \overline{5,31} + \overline{3,33}|_{36} + \overline{3,5,7,9,13} + \\
& \overline{9,11,17} + \overline{7,13,17} + \overline{5,15,17} + \overline{5,11,21} + \overline{5,7,25} + \overline{3,5,29} + \overline{5,7,9,17} + \overline{3,5,13,17} + \\
& \overline{5,33}|_{38} + \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,15,17} + \overline{5,15,19} + \overline{7,11,21} + \\
& \overline{5,13,21} + \overline{5,11,23} + \overline{5,9,25} + \overline{5,7,27} + \overline{3,7,29} + \overline{3,5,31} + \overline{7,9,11,13} + \overline{5,9,11,15} + \\
& \overline{3,9,13,15} + \overline{3,5,15,17} + \overline{3,5,11,21} + \overline{19,21} + \overline{17,23} + \overline{3,5,7,25} + \overline{15,25} + \overline{13,27} + \\
& \overline{11,29} + \overline{9,31}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{7,15,19} + \overline{5,17,19} + \\
& \overline{9,11,21} + \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{5,11,25} + \overline{5,9,27} + \overline{3,9,29} + \\
& \overline{3,5,33} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \overline{3,7,13,19} + \overline{3,5,15,19} + \overline{3,7,11,21} + \\
& \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{9,33}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \\
& \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \overline{5,7,13,19} + \\
& \overline{3,9,13,19} + \overline{3,5,17,19} + \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,5,13,23} + \\
& \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \\
& \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \\
& \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \\
& \overline{3,5,7,13,19} + \overline{3,5,7,11,21} + \overline{11,15,21} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{7,17,23} + \overline{7,13,27} + \\
& \overline{7,11,29} + \overline{3,15,29} + \overline{3,13,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{5,11,15,17} + \overline{7,9,13,19} + \\
& \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,5,15,25} + \overline{23,25} + \overline{5,7,9,27} + \\
& \overline{21,27} + \overline{19,29} + \overline{3,5,9,31} + \overline{17,31}|_{48} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \\
& \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{3,5,9,11,21} + \overline{13,15,21} + \overline{11,17,21} + \overline{9,19,21} + \\
& \overline{11,15,23} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{7,17,25} + \overline{9,13,27} + \overline{7,15,27} + \overline{9,11,29} + \overline{5,15,29} + \\
& \overline{3,17,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{3,13,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \\
& \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \overline{3,11,13,23} + \\
& \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \\
& \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33} + \overline{17,33}
\end{aligned}$$

$$
\begin{aligned}
e_3 \quad = \quad & \overline{3,7}|_{10} + \overline{5,7} + \overline{3,9}|_{12} + \overline{5,9}|_{14} + \overline{3,5,7} + \overline{7,9} + \overline{3,13}|_{16} + \overline{3,5,9} + \overline{7,11} + \overline{5,13} + \\
& \overline{3,15}|_{18} + \overline{9,11} + \overline{7,13} + \overline{5,15} + \overline{3,17}|_{20} + \overline{5,7,9} + \overline{3,5,13} + \overline{5,17}|_{22} + \overline{5,7,11} + \\
& \overline{3,7,13} + \overline{3,5,15} + \overline{3,5,7,9} + \overline{11,13} + \overline{9,15}|_{24} + \overline{5,9,11} + \overline{3,9,13} + \overline{3,5,17} + \overline{9,17}|_{26} + \\
& \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{3,13,15} + \overline{5,9,17} + \\
& \overline{5,7,9,11} + \overline{3,5,9,15} + \overline{15,17} + \overline{11,21} + \overline{7,25} + \overline{3,29}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \\
& \overline{3,13,17} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,5,9,17} + \overline{15,19} + \overline{13,21} + \\
& \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,31}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \\
& \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17} + \overline{17,19} + \\
& \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \overline{7,29} + \overline{5,31} + \overline{3,33}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \\
& \overline{7,13,17} + \overline{5,15,17} + \overline{5,11,21} + \overline{5,7,25} + \overline{3,5,29} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,33}|_{38} + \\
& \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,15,17} + \overline{5,15,19} + \overline{7,11,21} + \overline{5,13,21} + \\
& \overline{5,11,23} + \overline{5,9,25} + \overline{5,7,27} + \overline{3,7,29} + \overline{3,5,31} + \overline{7,9,11,13} + \overline{5,9,11,15} + \overline{3,9,13,15} + \\
& \overline{3,5,15,17} + \overline{3,5,11,21} + \overline{19,21} + \overline{17,23} + \overline{3,5,7,25} + \overline{15,25} + \overline{13,27} + \overline{11,29} + \overline{9,31}|_{40} + \\
& \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{7,15,19} + \overline{5,17,19} + \overline{9,11,21} + \\
& \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{5,11,25} + \overline{5,9,27} + \overline{3,9,29} + \overline{3,5,33} + \\
& \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \overline{3,7,13,19} + \overline{3,5,15,19} + \overline{3,7,11,21} + \overline{3,5,13,21} + \\
& \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{9,33}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \overline{7,15,21} + \\
& \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \overline{5,7,13,19} + \\
& \overline{3,9,13,19} + \overline{3,5,17,19} + \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,5,13,23} + \\
& \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \\
& \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \\
& \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \\
& \overline{3,5,7,13,19} + \overline{3,5,7,11,21} + \overline{11,15,21} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{7,17,23} + \overline{7,13,27} + \\
& \overline{7,11,29} + \overline{3,15,29} + \overline{3,13,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{5,11,15,17} + \overline{7,9,13,19} + \\
& \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,5,15,25} + \overline{23,25} + \overline{5,7,9,27} + \\
& \overline{21,27} + \overline{19,29} + \overline{3,5,9,31} + \overline{17,31}|_{48} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \\
& \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{3,5,9,11,21} + \overline{13,15,21} + \overline{11,17,21} + \overline{9,19,21} + \\
& \overline{11,15,23} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{7,17,25} + \overline{9,13,27} + \overline{7,15,27} + \overline{9,11,29} + \overline{5,15,29} + \\
& \overline{3,17,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{3,13,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \\
& \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \overline{3,11,13,23} + \\
& \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \\
& \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33} + \overline{17,33}
\end{aligned}
$$

$$
\begin{aligned}
e_4 \quad = \quad & \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{7,9,15} + \overline{5,11,15} + \\
& \overline{5,9,17} + \overline{3,7,9,13} + \overline{3,5,11,13} + \overline{3,5,9,15}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \overline{7,9,17} + \\
& \overline{5,11,17} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,5,9,17}|_{34} + \overline{9,11,15} + \\
& \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,7,11,15} + \\
& \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,13,19} + \\
& \overline{5,9,23} + \overline{5,7,11,15} + \overline{3,9,11,15} + \overline{3,7,13,15} + \overline{5,7,9,17} + \overline{3,7,11,17} + \overline{3,5,13,17}|_{38} + \\
& \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,13,19} + \overline{5,15,19} + \overline{5,13,21} + \overline{7,9,23} + \\
& \overline{5,11,23} + \overline{5,9,25} + \overline{7,9,11,13} + \overline{5,7,13,15} + \overline{5,7,11,17} + \overline{3,9,11,17} + \overline{3,7,13,17} + \\
& \overline{3,5,13,19} + \overline{3,5,9,23}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,11,15} + \overline{3,5,7,9,17} + \overline{11,13,17} + \\
& \overline{9,13,19} + \overline{7,15,19} + \overline{5,17,19} + \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{7,9,25} + \\
& \overline{5,11,25} + \overline{7,9,11,15} + \overline{5,9,13,15} + \overline{3,11,13,15} + \overline{5,7,13,17} + \overline{3,7,13,19} + \overline{3,5,15,19} + \\
& \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25}|_{42} + \overline{3,5,9,11,15} + \overline{3,5,7,13,15} + \\
& \overline{3,5,7,11,17} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \\
& \overline{5,9,29} + \overline{7,9,13,15} + \overline{5,11,13,15} + \overline{7,9,11,17} + \overline{5,9,13,17} + \overline{3,11,13,17} + \overline{5,7,13,19} + \\
& \overline{3,9,13,19} + \overline{3,7,15,19} + \overline{3,5,17,19} + \overline{3,7,13,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,7,11,23} + \\
& \overline{3,5,13,23} + \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{3,5,9,13,15} + \overline{3,5,9,11,17} + \overline{3,5,7,13,17} + \\
& \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \\
& \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,13,19} + \overline{5,7,15,19} + \\
& \overline{3,9,15,19} + \overline{3,7,17,19} + \overline{5,7,13,21} + \overline{3,9,13,21} + \overline{3,7,15,21} + \overline{5,7,11,23} + \overline{3,9,11,23} + \\
& \overline{3,7,13,23} + \overline{5,7,9,25} + \overline{3,7,11,25} + \overline{3,5,9,29}|_{46} + \overline{3,7,9,13,15} + \overline{3,5,9,13,17} + \\
& \overline{3,5,7,13,19} + \overline{13,15,19} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{9,15,23} + \overline{7,17,23} + \overline{7,15,25} + \\
& \overline{7,13,27} + \overline{5,15,27} + \overline{7,11,29} + \overline{7,9,31} + \overline{5,11,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \\
& \overline{9,11,13,15} + \overline{5,11,13,19} + \overline{5,9,15,19} + \overline{5,7,17,19} + \overline{3,9,17,19} + \overline{5,9,13,21} + \\
& \overline{5,7,15,21} + \overline{3,9,15,21} + \overline{3,5,19,21} + \overline{5,7,13,23} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \\
& \overline{3,9,11,25} + \overline{3,5,15,25} + \overline{3,5,13,27} + \overline{3,7,9,29} + \overline{3,5,11,29} + \overline{3,5,9,31}|_{48} + \\
& \overline{5,7,9,13,15} + \overline{3,7,11,13,15} + \overline{3,7,9,13,17} + \overline{3,5,9,13,19} + \overline{3,5,7,15,19} + \overline{13,17,19} + \\
& \overline{3,5,7,13,21} + \overline{13,15,21} + \overline{9,19,21} + \overline{3,5,7,11,23} + \overline{11,15,23} + \overline{3,5,7,9,25} + \overline{9,13,27} + \\
& \overline{7,15,27} + \overline{5,17,27} + \overline{9,11,29} + \overline{5,15,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{7,9,33} + \overline{5,11,33} + \\
& \overline{3,5,7,9,11,15} + \overline{9,11,13,17} + \overline{7,11,13,19} + \overline{5,11,15,19} + \overline{3,13,15,19} + \overline{5,9,17,19} + \\
& \overline{5,11,13,21} + \overline{5,9,15,21} + \overline{3,7,19,21} + \overline{7,9,11,23} + \overline{3,9,15,23} + \overline{3,7,17,23} + \\
& \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \\
& \overline{3,5,9,33}
\end{aligned}
$$

$$
\begin{aligned}
e_5 \quad = \quad & \overline{3,7,11,15}|_{36} + \overline{5,7,11,15} + \overline{3,9,11,15} + \overline{3,7,13,15} + \overline{3,7,11,17}|_{38} + \overline{5,9,11,15} + \\
& \overline{5,7,13,15} + \overline{3,9,13,15} + \overline{5,7,11,17} + \overline{3,9,11,17} + \overline{3,7,13,17}|_{40} + \overline{3,5,7,11,15} + \\
& \overline{7,9,11,15} + \overline{5,9,13,15} + \overline{3,11,13,15} + \overline{5,9,11,17} + \overline{5,7,13,17} + \overline{3,9,13,17} + \\
& \overline{3,7,15,17} + \overline{3,7,11,21}|_{42} + \overline{3,5,9,11,15} + \overline{3,5,7,13,15} + \overline{3,5,7,11,17} + \overline{7,9,13,15} + \\
& \overline{5,11,13,15} + \overline{7,9,11,17} + \overline{5,9,13,17} + \overline{3,11,13,17} + \overline{5,7,15,17} + \overline{3,9,15,17} + \\
& \overline{3,7,15,19} + \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,7,13,21} + \overline{3,7,11,23}|_{44} + \overline{3,5,9,13,15} + \\
& \overline{3,5,9,11,17} + \overline{3,5,7,13,17} + \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,15,17} + \overline{5,7,15,19} + \\
& \overline{3,9,15,19} + \overline{3,7,17,19} + \overline{5,9,11,21} + \overline{5,7,13,21} + \overline{3,9,13,21} + \overline{3,7,15,21} + \overline{5,7,11,23} + \\
& \overline{3,9,11,23} + \overline{3,7,13,23} + \overline{3,7,11,25}|_{46} + \overline{5,7,9,11,15} + \overline{3,5,11,13,15} + \overline{3,5,9,13,17} + \\
& \overline{3,5,7,15,17} + \overline{3,5,7,11,21} + \overline{9,11,13,15} + \overline{7,9,15,17} + \overline{3,13,15,17} + \overline{5,9,15,19} + \\
& \overline{5,7,17,19} + \overline{3,9,17,19} + \overline{7,9,11,21} + \overline{5,9,13,21} + \overline{3,11,13,21} + \overline{5,7,15,21} + \\
& \overline{3,9,15,21} + \overline{5,9,11,23} + \overline{5,7,13,23} + \overline{3,9,13,23} + \overline{5,7,11,25} + \overline{3,9,11,25} + \\
& \overline{3,7,13,25}|_{48} + \overline{5,7,9,13,15} + \overline{3,7,11,13,15} + \overline{5,7,9,11,17} + \overline{3,5,11,13,17} + \\
& \overline{3,5,9,15,17} + \overline{3,5,7,15,19} + \overline{3,5,9,11,21} + \overline{3,5,7,13,21} + \overline{3,5,7,11,23} + \\
& \overline{3,5,7,9,11,15} + \overline{9,11,13,17} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,9,15,19} + \overline{3,13,15,19} + \\
& \overline{5,9,17,19} + \overline{7,9,13,21} + \overline{5,11,13,21} + \overline{5,9,15,21} + \overline{3,11,15,21} + \overline{3,7,19,21} + \\
& \overline{7,9,11,23} + \overline{5,9,13,23} + \overline{3,11,13,23} + \overline{3,7,17,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25}
\end{aligned}
$$

For alternating groups $\mathbb{F}_4$ is always a splitting field. The primitive central idempotents of $\mathbb{F}_4 A_n$ are the primitive central idempotents of $\mathbb{F}_2 S_n$ except for one case: If $n = \frac{m(m+1)}{2}$, then there is an idempotent $e = C^+$ of $\mathbb{F}_2 S_n$, where C is the conjugacy class corresponding to the partition $(2m-1, 2m-5, 2m-9, ...)$ of n. This idempotent splits in two primitive central idempotents f_1 and f_2 of $\mathbb{F}_4 A_n$. We computed these two idempotents. If a class C of S_n splits in two conjugacy classes of A_n, then we write C_- and C_+ for the A_n-classes. ζ denotes a generator of $\mathbb{F}_4$ over $\mathbb{F}_2$. To save space we only write f_1, the second idempotent f_2 can easily be computed via $f_2 = f_1 + \overline{2m-1, 2m-5, 2m-9, ..._+} + \overline{2m-1, 2m-5, 2m-9, ..._-}$.

n	f_1
3	$\overline{1} + \zeta^2 \cdot \overline{3_+} + \zeta \cdot \overline{3_-}$
6	$\overline{3} + \zeta^2 \cdot \overline{5_+} + \zeta \cdot \overline{5_-} + \overline{3^2}$
10	$\overline{7} + \overline{3,5} + \overline{3^3} + \overline{5^2} + \zeta^2 \cdot \overline{3,7_+} + \zeta \cdot \overline{3,7_-}$
15	$\overline{5,7} + \overline{3,9} + \overline{3^3,5} + \overline{7^2} + \zeta \cdot \overline{5,9_+} + \zeta^2 \cdot \overline{5,9_-} + \overline{5^3} + \overline{3,5,7_+} + \overline{3,5,7_-} + \overline{3^2,9}$
21	$\overline{7,11} + \overline{3,7,9} + \overline{3,5,11} + \overline{3,5^2,7} + \overline{3^3,11} + \overline{3^3,5,7} + \overline{7^3} + \overline{5,7,9_+} + \overline{5,7,9_-} + \overline{3,9^2} + \overline{5^2,11} + \overline{3,7,11_+}$
28	$\overline{5,9,11} + \overline{5,7,13} + \overline{3,9,13} + \overline{3,5^3,9} + \overline{3^2,5,7,9} + \overline{9^3} + \overline{7,9,11_+} + \overline{7,9,11_-} + \overline{5,11^2} + \overline{3^3,5,13} + \overline{7^2,13} + \overline{5,9,13_+} + \overline{5,7^2,9} + \overline{3,5,9,11_+} + \overline{3,5,9,11_-} + \overline{5^3,13} + \overline{3,5,7,13_+} + \overline{3,5,7,13_-} + \overline{3^2,9,13}$
36	$\overline{7,11,15} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,7^3,11} + \overline{3,5,7,9,11_+} + \overline{3,5,7,9,11_-} + \overline{3,5^2,7,15} + \overline{3^3,11,15} + \overline{3,5^3,7,11} + \overline{3^3,7,9,11} + \overline{7,9^2,11} + \overline{3,11^3} + \overline{5,7,11,13_+} + \overline{5,7,11,13_-} + \overline{3,9,11,13_+} + \overline{3,9,11,13_-} + \overline{3,7,13^2} + \overline{3^3,5,7,15} + \overline{7^3,15} + \overline{5,7,9,15_+} + \overline{5,7,9,15_-} + \overline{3,9^2,15} + \overline{5^2,11,15} + \overline{3,7,11,15_+}$
45	$\overline{5,9,13,15} + \overline{5,9,11,17} + \overline{5,7,13,17} + \overline{3,9,13,17} + \overline{5^3,7,9,13} + \overline{3,5,7^2,9,13} + \overline{3^2,5,9,11,13} + \overline{9,11^2,13} + \overline{5,13^3} + \overline{7,9,13,15_+} + \overline{7,9,13,15_-} + \overline{5,11,13,15_+} + \overline{5,11,13,15_-} + \overline{5,9,15^2} + \overline{3,5^3,9,17} + \overline{3^2,5,7,9,17} + \overline{9^3,17} + \overline{7,9,11,17_+} + \overline{7,9,11,17_-} + \overline{5,11^2,17} + \overline{3^3,5,13,17} + \overline{7^2,13,17} + \overline{5,9,13,17_+} + \overline{5,9^3,13} + \overline{5,7,9,11,13_+} + \overline{5,7,9,11,13_-} + \overline{3,5,9,13,15_+} + \overline{3,5,9,13,15_-} + \overline{5,7^2,9,17} + \overline{3,5,9,11,17_+} + \overline{3,5,9,11,17_-} + \overline{5^3,13,17} + \overline{3,5,7,13,17_+} + \overline{3,5,7,13,17_-} + \overline{3^2,9,13,17}$
55	$\overline{7,11,15,19} + \overline{3,7,11,15,17} + \overline{3,7,11,13,19} + \overline{3,7,9,15,19} + \overline{3,5,11,15,19} + \overline{3,7,9^2,11,15} + \overline{3,5,7,11,13,15_+} + \overline{3,5,7,11,13,15_-} + \overline{3,7^3,11,19} + \overline{3,5,7,9,11,19_+} + \overline{3,5,7,9,11,19_-} + \overline{3,5^2,7,15,19} + \overline{3^3,11,15,19} + \overline{3,5,7^3,11,15} + \overline{3,5^2,7,9,11,15} + \overline{7,11^3,15} + \overline{3^3,7,11,13,15} + \overline{7,9,11,13,15_+} + \overline{7,9,11,13,15_-} + \overline{3,11,13^2,15} + \overline{3,7,15^3} + \overline{5,7,11,15,17_+} + \overline{5,7,11,15,17_-} + \overline{3,9,11,15,17_+} + \overline{3,9,11,15,17_-} + \overline{3,7,13,15,17_+} + \overline{3,7,13,15,17_-} + \overline{3,7,11,17^2} + \overline{3,5^3,7,11,19} + \overline{3^3,7,9,11,19} + \overline{7,9^2,11,19} + \overline{3,11^3,19} + \overline{5,7,11,13,19_+} + \overline{5,7,11,13,19_-} + \overline{3,9,11,13,19_+} + \overline{3,9,11,13,19_-} + \overline{3,7,13^2,19} + \overline{3^3,5,7,15,19} + \overline{7^3,15,19} + \overline{5,7,9,15,19_+} + \overline{5,7,9,15,19_-} + \overline{3,9^2,15,19} + \overline{5^2,11,15,19} + \zeta^2 \cdot \overline{3,7,11,15,19_+} + \zeta \cdot \overline{3,7,11,15,19_-}$

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