We define a functor from the category of unital C*-algebras with compact quantum group actions to the category of comodule algebras by extending the notion of the algebra of regular functions (spanned by the matrix coefficients of the irreducible unitary corepresentations) from compact quantum groups to unital C*-algebras on which they act. We call it the Peter-Weyl functor. Combined with the Gelfand transform, it translates compact group actions on compact Hausdorff spaces into a general algebraic framework. On the other hand, the Galois condition for finite field extensions is also translated into this comodule-algebraic setting, and is the founding stone of noncommutative Hopf-Galois theory. The main result is that the Galois condition for compact quantum group actions is preserved under one-surjective pullbacks (fibre products) of comodule algebras. This talk will be focused on deducing from here the equivalence of the freeness of a classical compact group action on a compact Hausdorff space and the Galois condition for its Peter-Weyl comodule algebra. This result parallels the well-known equivalence of Galois coverings and discrete group principal bundles. (Based on joint work with P.F.Baum, U.Kraehmer, R.Matthes, E.Wagner and B.Zielinski.) (Received August 12, 2008)