1033-05-54

Matthias Beck* (beck@math.sfsu.edu), 1600 Holloway Ave, Dept. of Mathematics, SFSU, San Francisco, CA 94132, and Asia Matthews, Department of Mathematics & Statistics, Queen's University, Kingston, ON K7L 3N6, Canada. Generating Functions of Rational Polyhedra and Dedekind-Carlitz Polynomials. Preliminary report.

We study higher-dimensional analogs of the Dedekind-Carlitz polynomials,

$$c(u, v; a, b) := \sum_{k=1}^{a-1} u^{k-1} v^{\lfloor kb/a \rfloor},$$

where u and v are indeterminates and a and b are positive integers. These polynomials satisfy the reciprocity law

$$(u-1)c(u,v;a,b) + (v-1)c(v,u;b,a) = u^{a-1}v^{b-1} - 1,$$

from which one easily deduces many classical reciprocity theorems for the Dedekind sum and its generalizations, most notably by Hardy and Berndt–Dieter.

Dedekind–Carlitz polynomials appear naturally in generating functions of rational cones. We use this fact to give geometric proofs of the Carlitz reciprocity law. Our approach gives rise to new reciprocity theorems and a multivariate generalization of the Mordell–Pommersheim theorem on the appearance of Dedekind sums in Ehrhart polynomials of 3-dimensional lattice polytopes. (Received August 27, 2007)