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Baogang Xu (baogxu@njnu.edu.cn), School of Math. & Computer Science, Nanjing Normal Unversity, Nanjing, 210097 China, Nanjing, Jiangsu 210097, Peoples Rep of China, and Xingxing Yu* (yu@math.gatech.edu), School of Mathematics, Georgia Institue of Technoology, Atlanta, GA 30332. On judicious partitions of graphs.

Judicious partition problems ask for partitions of the vertex set of graphs so that several quantities are optimized simultaneously. We answer the following judicious partition question of Bollobás and Scott in the affirmative: For any positive integer k and for any graph G of size m, does there exist a partition of V(G) into V_1, \ldots, V_k such that the total number of edges joining different V_i is at least

$$\frac{k-1}{k}m + \frac{1}{2k}\left(\sqrt{2m + \frac{1}{4}} - \frac{1}{2}\right),$$

and for each $i \in \{1, 2, ..., k\}$ the total number of edges with both ends in V_i is at most

$$\frac{m}{k^2} + \frac{k-1}{2k^2} \left(\sqrt{2m + \frac{1}{4}} - \frac{1}{2} \right)$$
?

We also point out a connection between our result and another judicious partition problem of Bollobás and Scott. (Received August 31, 2007)