An Interview with Vladimir Arnol′d

by S. H. Lui

Utilius scandalum nasci permittur quam veritas relinquatur.
(One should speak the truth even at risk of provoking a scandal.)

—Decretalium V of Pope Gregory IX, 1227–1241

Vladimir Arnol′d is currently professor of mathematics at both the Steklov Mathematical Institute, Moscow, and Ceremade, Université de Paris–Dauphine. Professor Arnol′d obtained his Ph.D. from the Moscow State University in 1961. He has made fundamental contributions in dynamical systems, singularity theory, stability theory, topology, algebraic geometry, magneto-hydrodynamics, partial differential equations, and other areas. Professor Arnol′d has won numerous honors and awards, including the Lenin Prize, the Crafoord Prize, and the Harvey Prize.

This interview took place on November 11, 1995. The following articles may be of interest to the reader:

1) *Conversation with Vladimir Igorevich Arnol′d,* by S. Zdravkovska, Mathematical Intelligencer **9**:4 (1987).

2) *A mathematical trivium,* by V. I. Arnol′d, Russian Math. Surveys **46**:1 (1991).

3) *Will Russian mathematics survive?*, by V. I. Arnol′d, Notices of the AMS **40**:2 (1993).

4) *Why Mathematics?*, by V. I. Arnol′d, Quantum, 1994.

5) *Will mathematics survive? Report on the Zurich Congress*, by V. I. Arnol′d, Mathematical Intelligencer **17**:3 (1995).

Lui: *Please tell us a little bit about your early education. Were you already interested in mathematics as a child?*

Arnol′d: The Russian mathematical tradition goes back to the old merchant problems. Very young children start thinking about such problems even before they have any knowledge of numbers. Children five to six years old like them very much and are able to solve them, but they may be too difficult for university graduates, who are spoiled by formal mathematical training. A typical example is:

You take a spoon of wine from a barrel of wine, and you put it into your cup of tea. Then you return a spoon of the (nonuniform!) mixture of tea from your cup to the barrel. Now you have some foreign substance (wine) in the cup and some foreign substance (tea) in the barrel. Which is larger: the quantity of wine in the cup or the quantity of tea in the barrel at the end of your manipulations?

Slightly older children, knowing the first few numbers, like the following problem. Jane and John wish to buy a children's book. However, Jane needs seven more cents to buy the book, while John needs one more cent. They decide to buy only one book together but discover that they

S. H. Lui is an assistant professor at the Hong Kong University of Science and Technology. His e-mail address is shlui@uxmail.ust.hk.

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do not have enough money. What is the price of the book? (One should know that books in Russia are *very* cheap!)

Many Russian families have the tradition of giving hundreds of such problems to their children, and mine was no exception. The first real mathematical experience I had was when our schoolteacher I. V. Morozkin gave us the following problem: Two old women started at sunrise and each walked at a constant velocity. One went from A to B and the other from B to A. They met at noon and, continuing with no stop, arrived respectively at B at 4 p.m. and at A at 9 p.m. At what time was the sunrise on this day?

I spent a whole day thinking on this oldie, and the solution (based on what is now called scaling arguments, dimensional analysis, or toric variety theory, depending on your taste) came as a revelation. The feeling of discovery that I had then (1949) was exactly the same as in all the subsequent much more serious problems—be it the discovery of the relation between algebraic geometry of real plane curves and four-dimensional topology (1970) or between singularities of caustics and of wave fronts and simple Lie algebra and Coxeter groups (1972). It is the greed to experience such a wonderful feeling more and more times that was, and still is, my main motivation in mathematics.

Lui: *What was it like studying at Moscow State University? Can you tell us something about the professors (Petrovskii, Kolmogorov, Pontriagin, Rokhlin,...)?*

Arnol′d: The atmosphere of the Mechmat (Moscow State University Mechanics and Mathematics Faculty) in the fifties when I was a student is described in detail in the book *Golden Years of Moscow Mathematics*, edited by S. Zdravkovska and P. L. Duren and published jointly by the AMS and LMS in 1993. It contains reminiscences of many people. In particular, my article was on A. N. Kolmogorov, who was my supervisor.

The constellation of great mathematicians in the same department when I was studying at the Mechmat was really exceptional, and I have never seen anything like it at any other place. Kolmogorov, Gelfand, Petrovskii, Pontriagin, P. Novikov, Markov, Gelfond, Lusternik, Khinchin, and P. S. Alexandrov were teaching students like Manin, Sinai, S. Novikov, V. M. Alexeev, Anosov, A. A. Kirillov, and me.

All these mathematicians were so different! It was almost impossible to understand Kolmogorov's lectures, but they were full of ideas and were really rewarding! I recall his explanation of his theory of the size of the minimal cube into which you can embed every graph having N vertices (balls of fixed size), each connected with at most K others by wires of fixed thickness. He explained that when N is very large (while K is fixed), the diameter of the cube grows like $\sqrt{N}$ by the following argument: the grey matter (the body of the neurons) is on the surface of the human brain, while the white matter (the connections) fills the interior part. Since the brain is embedded into the head as economically as possible, a sufficiently complicated brain of N neurons can only be embedded in a cube of size $\sqrt{N}$ (while a trivial brain, like that of a worm, needs only the size $\sqrt[3]{N}$).

Kolmogorov's work on what is now called KAM theory of Hamiltonian systems was a by-product of compulsory exercises that he gave to all second-year undergraduate students. One of the problems was the study of some nontrivial completely integrable systems (like the motion of a heavy particle along the surface of a horizontal torus of revolution). No computers were available then! He observed that the motion in all such classical examples was quasiperiodic and tried to find examples of more complicated motion ("mixing", or in today's language, "chaos") in the case of nonintegrable perturbed systems.

His attempts were unsuccessful. The problem which motivated his study is still open—no one has been able to find an invariant torus carrying mixing flows in generically perturbed systems. However, the by-products of this investigation are far more important than the initial technical problem on mixing. They include the discovery of the persistent nonresonant tori, the "accelerated convergence" method and the related implicit function theorems in function spaces, the proof of stability of motion in many Hamiltonian systems (e.g., gyroscopes and planetary orbits), and the proof of the existence of magnetic surfaces in the Tokamak geometry, which is used in the study of plasma containment for controlled thermonuclear fusion.

That consequences of an investigation are more important than the original question is a general phenomenon. The initial goal of Columbus was to find a new way to India. The discovery of the New World was just a by-product.

Pontriagin was already very weak when I was a student at Mechmat, but he was perhaps the best of the lecturers. He had just turned from topology to control theory, and his personality had also changed a lot. He later explained his reasons for switching to applied mathematics and his antisemitic ideas in his autobiography published in the *Russian Mathematical Surveys*. When he submitted this paper to the Editorial Board, the KGB representative suggested that the article should not be published as it was because of its extreme openness. I would prefer to see the original text published—what you now find is rather softened. Some people claim that his antisemitism might be simply a manifestation of his fear that some part of his blood might be Jewish and that this might be discovered.

However, Pontriagin was not always like this! During the war his best student, V. A. Rokhlin, was wounded and imprisoned by the Germans. Later, Rokhlin was liberated by the Americans, returned to Russia, and continued to serve in the Russian army, which was still fighting. One day, while he was transporting a captured German officer to his superior, he met a drunk KGB officer, who wanted to shoot the German officer immediately. Rokhlin objected. Fortunately, Rokhlin was saved by his superior, who immediately sent him to a different regiment. However, in the end Rokhlin was, as were all the Russians who were saved from the German camps by the Allies, sent to the gulag (Russian concentration camp) in the north of Russia.

Some months later, someone who was liberated from this camp came to Moscow and told Pontriagin that Rokhlin was still alive but dying from starvation in the camp. Pontriagin, with the help of Kolmogorov, Alexandrov, and others, wrote a letter to Beria, the KGB chief, claiming that Rokhlin should be immediately released because he was the most talented mathematician of his generation. Beria signed the order to liberate Rokhlin, who was subsequently given a machine gun and continued his military service as a guard at the same camp where he had been held prisoner. Pontriagin and others wrote a second letter to Beria, and Rokhlin finally was able to return to Moscow.

Rokhlin had no right to *propiska* in Moscow since returning from the gulag. [Propiska is Russian, meaning the right to live in a specified area—one is not free to live elsewhere. Propiska is applied to everybody!] Pontriagin was completely blind and had a right to hire a personal secretary at the Moscow Steklov Institute. He was brave enough to give this position to Rokhlin, who later became one of the leading Soviet mathematicians in topology and dynamical systems. Rokhlin had a lot of influence on the younger generation of mathematicians (like S. Novikov, Sinai, Anosov, and me) and later created a very important mathematical school at St. Petersburg. Some of his illustrious students include Vershik, Gromov, Eliashberg, Viro, Shustin, Turaev, and Kharlamov. I met him in the sixties when he held a seminar in Moscow. He came to Moscow from one hundred miles away, where his propiska allowed him to live.

Rokhlin was of Jewish origin and survived the German prisoner camp by pretending to be a Muslim. Indeed, he was born in Baku, Azerbaijan. It was really dangerous for Pontriagin to help him and to approach Beria. Pontriagin preserved his high opinion of Rokhlin even after he became an active antisemite. My personal relation with Pontriagin was rather good. He invited me to his house and to his seminar and showed genuine interest in my work, especially on singularity theory. This was partially due to our common interests in differential topology and control and game theory. The main reason, however, was that he wanted to say something against me at an international meeting. Pontriagin was then the Russian representative in the International Mathematical Union (IMU) and had done a lot to prevent any vote for dissident Russians. (I was blacklisted because I, along with 99 other mathematicians, had signed a letter protesting the imprisonment of a perfectly healthy Soviet mathematician in a psychiatric hospital. This was the standard method of eliminating dissidents.) The IMU had always been very political, and he succeeded. In his reminiscences Pontriagin revealed that quite a few of the IMU officers shared his cannibalistic views. I hope we shall know their names. Curiously enough, I am now in his former position, representing Russia in the IMU.

Petrovskii, who was then the rector of the university, usually met Rokhlin in the elevator just before the seminar. I think it was dangerous for him to be seen in the company of Rokhlin. Petrovskii was no longer active in mathematics. However, he was extremely important for the Moscow mathematical community, always trying to support genuine mathematicians in difficult fights with the Communist Party.

His mathematical taste was rather classical, based on the Italian school of algebraic geometry more than the set-theoretic conceptions. Sir Michael Atiyah once told me that he was always delighted by the way Petrovskii dealt with algebraic geometry in his works on PDEs. One of these, the paper on the lacunas of hyperbolic PDEs, was later rewritten by Atiyah, Bott, and Gårding in modern terminology in two long papers in *Acta Mathematica*. It is a far-reaching generalization of the well-known fact of the impossibility of acoustic communication in the even-dimensional spaces (for instance, in the "plane" world), while in our three-dimensional world we communicate easily. It is interesting that in this paper, Petrovskii proved that the cohomology classes of the complement of an algebraic variety are representable by rational differential forms—a result which is usually attributed to Grothendieck.

The works of Petrovskii (1933 and 1938) on real algebraic geometry (related to the 16th Hilbert problem on the shape of real plane algebraic curves) started an important branch of modern mathematics—the topology of real algebraic varieties. Results of this theory (for example, a bound on the Betti numbers in terms of the degrees of the equations) are very useful in many branches of mathematics, including complexity theory. For instance, they were used by Khovanskii in his fewnomial theory, by Smale in his study of the "real P-NP" problem, and so on. In the West these results are usually attributed to Thom and to Milnor (1965), while the papers by Petrovskii and his student Oleinik, published in the forties, contained better estimates (and were, by the way, quoted by Thom and by Milnor). This is, however, a very standard situation—it is too easy to omit quoting Russian fundamental papers in the modern world of the job hunters.

Petrovskii had never been a party member. This was unknown to most Communists. He was highly influential, partially because of his personal relation to his former students, who had attained very high positions in the Soviet hierarchical system. Petrovskii was made a member of the Presidium of the Supreme Soviet, which was the "collective president" of the Soviet Union. He died at the door of the Party Central Committee building in Moscow of a heart attack after a long fight at a meeting for the support of fundamental science. His last words were "I won."

After his death the party and the KGB worked for twenty years to destroy the mathematical center at Mechmat created by him. They had stopped the appointment of talented people to the faculty, and they have by now almost succeeded in killing the center.

Lui: *Can you tell us your philosophy of teaching undergraduates and of supervising graduate students and how many you have had in Russia and France?*

Arnol'd: The number of Ph.D. theses defended under my supervision is something like forty. I cannot give the exact number for several reasons. In the "stagnation" period, I was not allowed to supervise foreign graduate students at Moscow University because I was not a party member. They still were studying with me, but the official supervisor was some friendly party member who also got paid for it. Some graduate students had other supervisors but wrote their theses on topics discussed in my seminars and were practically my students. Three examples are S. M. Gusein-Zade, Yu. Iliashenko, and A. I. Neistadt. At present, I'm working with two undergraduates and three graduates in Moscow and with four graduates in Paris. Two or three more are supposed to start in January.

I learn a lot from my students, especially undergraduates. I never assign a thesis topic to my students. This is like assigning them a spouse. I merely show them what is known and unknown.

My Moscow seminar, working even when I am abroad, consists of about thirty mathematicians, mostly my former graduate students, but there are always others. The seminar has existed for about thirty years, and among the participants in different years were Ya. Sinai, V. Alexeev, S. Novikov, M. Kontsevich, A. Goncharov, D. B. Fuchs, G. Tjurina, A. Tjurin.…

Life in Moscow is so difficult that most students have to earn their living independently of their scientific work. Some, for instance, start their own businesses. The rate of crime is so high, however, that in starting a business, one risks being killed. One of my graduate students in Moscow, who has just finished his thesis but has not defended it, disappeared a few weeks ago. We have doubts about whether he is alive or not.

Lui: *Do you have any mathematical heroes?*

Arnol'd: I would mention Barrow, Newton (who was, however, a very unpleasant person—see my book *Huygens and Barrow, Newton and Hooke* published by Birkhäuser, 1990), Riemann, Poincaré, Minkowski, Weyl, Kolmogorov, Whitney, Thom, Smale, and Milnor. One-half of the mathematics I know comes from the book of F. Klein *Lectures on the Development of Mathematics in the 19th Century*. I have also learned a lot from many mathematicians like Gelfand, Rokhlin, S. Novikov, P. Deligne, Fuchs, and from my own students like Khovanskii, Nekhoroshev, Varchenko, Zakaljukin, Vassiliev, Givental, Goryunov, O. Scherbak, Chekanov, and Kazarian.

I am deeply indebted to Thom, whose singularity seminar at the Institut des Hautes Études Scientifiques, which I frequented throughout the year 1965, profoundly changed my mathematical universe. I was always delighted by the way in which Thom discussed mathematics, using sentences obviously having no strict logical meaning at all. While I was never able to completely free myself from the straitjacket of logic, I was forever poisoned by the dream of the irresponsible mathematical speculation with no exact meaning. "One can always find imbeciles to prove theorems" was, according to Thom's students, his principle.

Milnor's talks at Leningrad in 1961 on the differential structures on the sphere made such a profound impression on my supervisor, Kolmogorov, that he suggested that I put this in my graduate curriculum. This forced me to study differential topology from Novikov, Fuchs, and Rokhlin. This came in handy because, a year later, I was on the jury for Novikov's thesis defense on the differential structures on the products of spheres.

Smale was one of the first foreign mathematicians I met when he came to Moscow in

1961. His influence on Russian works in dynamical systems and on me was enormous.

Lui: *Do you notice any differences in the way people from different cultures do mathematics?*

Arnol'd: I was unaware of these differences for many years, but they do exist. A few years ago, I was participating in an International Science Foundation (ISF) meeting in Washington, DC. This organization distributes grants to Russian scientists. One American participant suggested support for some Russian mathematician because "he is working in a good American style." I was puzzled and asked for an explanation. "Well," the American answered, "it means that he is traveling a lot to present all his latest results at all our conferences and is personally known to all experts in the field." My opinion is that ISF should better support those who are working in the good Russian style, which is to sit at home working hard to prove fundamental theorems which will remain the cornerstones of mathematics forever!

Russian salaries are (and were) so small, that if someone is doing mathematics, it means that for him it is *the* goal and not a means to earn money. It is still possible to attain a high reputation in the Western mathematical community by simply rewriting (or modernizing) classical Russian achievements and ideas unknown to the West.

The Russian attitude toward knowledge, science, and mathematics always conforms to the old traditions of the Russian *intelligentsiya*. This word does not exist in other languages, since no other country has a similar caste of scholars, medical doctors, artists, teachers, etc., who find more reward from their contributions to society than from personal or monetary gains.

My friend Vershik recently tried to obtain an American visa in Paris. "What is your salary in St. Petersburg?" asked the staff at the American consulate. After hearing his honest reply, the staff asked, "Do you wish to persuade us that you intend to return to St. Petersburg at such a salary?" Vershik answered, "Of course. Money is not all!" The staff was so shocked that Vershik was given the visa immediately.

I was applying for a visa a week earlier, and they put me on a waiting list for three weeks. Their reasoning was that my papers must be checked in Washington since I am a "donkey". I asked for an explanation. "Well," they replied, "we have such names for every crime: dog, cat, tiger, camel, and so on." They showed me the list, and "donkey" is a pseudonym for a Russian scientist.

One other characteristic of the Russian mathematical tradition is the tendency to regard all of mathematics as one living organism. In the West it is quite possible to be an expert in mathematics modulo 5, knowing nothing about mathematics modulo 7. One's breadth is regarded as negative in the West to the same extent as one's narrowness is regarded as unacceptable in Russia.

The French mathematical school was brilliant for several centuries, up to the penetrating works of Leray, H. Cartan, Serre, Thom, and Cerf. The Bourbakists claimed that all the great mathematicians were, using the words of Dirichlet, replacing blind calculations by clear ideas. The Bourbaki manifesto containing these words was translated into Russian as "all clear ideas were replaced by blind calculations." The editor of the translation was Kolmogorov. His French was excellent. I was shocked to find such a mistake in the translation and discussed it with Kolmogorov. His answer was: I had not realized that something was wrong in the translation since the translator described the Bourbaki style much better than the Bourbakists did. Unfortunately, Poincaré left no school in France.

A typical example of the French narrow-mindedness is the recent discussion at the Academy of Sciences. Gromov was a foreign associate for many years, but he recently chose the French nationality and hence could no longer remain a foreign associate. The problem was to transfer him to be an ordinary fellow of the Academy. The French mathematicians, however, were opposed to this, saying that "those places are for the really French people!" In my opinion, all the "really French" candidates were incomparably below the level of Gromov, who is one of the world's leading mathematicians. In the end, Gromov is still not a fellow.

To teach in France is very difficult because of the formalized Bourbaki training the students have. For example, at a written examination in dynamical systems for fourth-year students at Paris-Dauphine, one problem was to find the limit of the solution of a system of Hamiltonian equations on the phase plane starting with some given initial point when time goes to infinity. The idea was to choose the initial point on a separatrix of a saddle, with the limit being the saddle point.

Preparing the examination problem, I made an arithmetical error, and the phase curve (the energy-level curve containing the initial point) was a closed oval instead of the separatrix. The students discovered this and concluded that there exists a finite time T at which the solution returns to the initial point. Using the unicity theorem, they were able to deduce that for any integer n the value of the solution at time nT is still the initial point. Then came the conclusion: since the limit at infinite time coincides with the limit for any subsequence of times going to infinity, the limit is equal to the initial point! This solution was invented *independently* by several *good* students sitting at different places in the examination hall. In all this reasoning, there are *no* logical mistakes. It is a *correct* deduction which one may also generate by a computer. It is apparent that the authors understood *nothing*. It is awful to think what kind of pressure the Bourbakists put on (evidently nonsilly) students to reduce them to formal machines! This kind of formalized education is completely useless for any practical problem and even dangerous, leading to Chernobyl-type events. Unfortunately, this plague of formal deduction is propagating in many countries, and the future of the mathematics infected by it is rather bleak.

The United States has a different danger. No Russian professor is able to solve correctly the problem they give in the Graduate Record Examination, the official entrance examination for graduate studies: find the closest pair to (angle, degree) among the pairs: (time, hour), (area, square inch), and (milk, quart). Every American immediately solves it correctly. The official explanation for the correct response (area, square inch) is: one degree is the minimal measure of angle, one square inch is the minimal measure of area, while an hour contains minutes and a quart contains two pints. I always wondered how it is possible for so many Americans to overcome such difficulties and become great mathematicians. One physicist in New York who solved the problem successfully told me that he had the correct model of the degree of stupidity of the authors of such problems.

H. Whitney told me that the problem (intended for fourteen-year-old American school children) of whether 120% of the number 80 is a number greater than, smaller than, or equal to 80 was correctly solved (in a nationwide test) by 30% of the students. People making the test thought that 30% of the school children understood percentages. Whitney explained to me, however, that the number of those who really understood was negligible with respect to the whole sample. Since there were three possible answers, the statistical prediction for a correct random choice was 33%, with a 5% uncertainty.

Recently, even the National Academy of Sciences decided that scientific education in America should be enhanced. What they propose is to eliminate from the curriculum unnecessary scientific facts too difficult for American children and replace them by really *fundamental, basic* knowledge, such as all objects have properties and all organisms have nature! (See *Nature* **372**:5606 December 8, 1994.) Undoubtedly, they will go far with this! Two years ago, I read in *USA Today* that American parents have formed a list of really necessary knowledge for children in each age category. At ten they have to know that water has two phases, and at fifteen that the moon has phases and rotates around the earth. In Russia we still teach children in primary school that water has three phases, but the new Americanized culture will undoubtedly win in the near future. There are, however, some remarkable advantages in the free American system, where a high school student may take, say, a course on the history of jazz instead of algebra.

A few months before his death, Whitney, who was still very active at the Institute for Advanced Study in Princeton, told me the story of his mathematical studies. He was an undergraduate in violin at Yale, and after the second year he was sent to one of the best centers in Europe for music. Unfortunately, I have forgotten which city it was, but in any case it was not far from the Alps, since he already was a mountain climber. There, a student had to pass an exam in a subject different from his own studies. Whitney asked his fellow students which subject was the most fashionable then, and they told him quantum mechanics. After his first class in quantum mechanics, Whitney approached the famous lecturer (Pauli? Schrödinger? Sommerfeld?) with the following words: "Dear Professor, it seems to me that something is wrong with your lectures. I'm the best student from Yale, and still I am unable to understand a word in your lecture." The lecturer, after being informed that Whitney was studying music, answered quite politely, "This is because you need some background, such as calculus and linear algebra." "Well," Whitney replied, "I hope these are not so brand new as your subject and someone has already written textbooks on these subjects." The lecturer agreed and mentioned the titles of some textbooks. (If someone knows about this story, I would like to know the name of the city, lecturer, and titles.) "In three weeks," Whitney continued, "I was understanding his lectures, and at the end of the semester I switched from music to mathematics."

Kolmogorov also started as a nonmathematician—he was studying history. His first paper, written when he was seventeen, was reported at a seminar given by Bakhrushin at Moscow University. Kolmogorov came to some conclusion based on an analysis of medieval tax records in Novgorod. After his talk, Kolmogorov asked Bakhrushin whether he agreed with the conclusions. "Young man," the professor said, "in history, we need at least five proofs for any conclusion." Next day, Kolmogorov switched to mathematics. The paper was rediscovered in his archive after his death and is now published and approved by the historians.

Lui: *Any comments on the relation between pure and applied mathematics?*

Arnol'd: According to Louis Pasteur, there exist no applied sciences—what do exist are the APPLICATIONS of sciences. The common opinion of both pure mathematicians and theoretical physicists on the applied mathematics community is that it consists of weak thinkers unable to produce something scientifically important and of those who are more interested in money than in mathematics. I do *not* think that this characteristic is *fully* deserved by the applied mathematics community. See my article "Apology of applied mathematics" in *Russian Mathematical Surveys*, 1996. It summarizes my talk at the opening of the Hamburg International Congress of Industrial and Applied Mathematics, July 1995. I think that the difference between pure and applied mathematics is social rather than scientific. A pure mathematician is paid for making mathematical discoveries. An applied mathematician is paid for the solution of given problems.

When Columbus set sail, he was like an applied mathematician, paid for the search of the solution of a concrete problem: find a way to India. His discovery of the New World was similar to the work of a pure mathematician. I do not think that the discoveries of Galileo (who was immediately exploiting them in a businesslike American style) are less important than, say, those of the pure philosopher Pascal. The real danger is not the applied mafia itself, but the divorce between pure mathematics and the sciences created by the (I would say criminal) formalization of mathematics and of mathematical education. The axiomatical-deductive Hilbert-Bourbaki style of exposition of mathematics, dominant in the first half of this century, is now fortunately giving place to the unifying trends of the Poincaré style geometrical mathematics, combining deep theoretical insight with real-world applications.

By the way, I read in a recent American book that geometry is the art of making no mistakes in long calculations. I think that this is an underestimation of geometry.

Our brain has two halves: one is responsible for the multiplication of polynomials and languages, and the other half is responsible for orientation of figures in space and all the things important in real life. Mathematics is geometry when you have to use both halves. See, for instance, "The geometry of formulae" by A. G. Khovanskii in the *Soviet Sci. Rev. Sect. C: Math. Phys. Rev.* V4 (1984).