Book Review

Indiscrete Thoughts

Reviewed by Steven G. Krantz

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Gian-Carlo Rota
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The period beginning in 1946 and on into the early 1960s was a golden age for American mathematics. Postwar euphoria combined with an influx of stunning talent from Europe to create a maelstrom of mathematical activity. Conferences, travel, and collaboration proliferated as never before. Milestone theorems, such as the Nash isometric embedding theorem, the Calderón-Zygmund theorem, and the independence of the continuum hypothesis, were proved during this time. The notion of K-theory was developed. Perhaps the culmination and crowning achievement of the age was the Atiyah-Singer Index Theorem. Gian-Carlo Rota grew up in this milieu and was thoroughly immersed in the activity. This book tells some of his memories and many of his opinions.

But this is not a memoir, nor an autobiography in the usual sense. Rather, it is a collection of Rota's essays—many of them already published elsewhere—that are organized around certain central themes. Among these themes are: (i) the special nature of the Princeton and Yale mathematics departments, (ii) the contributions of Jacob Schwartz to modern mathematics, (iii) phenomenology, (iv) the role of finite and combinatorial mathematics in the current scientific infrastructure, and (v) the nature of algebra.

Rota certainly paints a very personal and loving picture of his undergraduate days in the Princeton mathematics department. He notes that Alonzo Church would never say, "It is raining," because such a statement taken in isolation makes no sense. Instead, Church would say, "I must postpone my departure for Nassau Street, inasmuch as it is raining, a fact which I can verify by looking out the window."

Rota speaks dolefully of bigotry in the American mathematical establishment, both the prejudice that prevented Princeton from ever giving Kodaira a regular appointment as well as the intolerance that Lefschetz and others exhibited toward Church, a mere logician. Rota points out, with detailed examples, that great mathematicians are not necessarily nice guys. Tell me about it.

Rota describes Feller's "loud and entertaining" lectures and how Kac characterized Feller's technique as "proof by intimidation." Feller's lectures contained little tirades and asides with titles such as "Ghandi was a phony" or "Velikovsky is not as wrong as you think" or "ESP is a sinister plot against civilization".

The imposing figure of Emil Artin receives dutiful treatment: Artin dressed like a Luftwaffe pilot, had strong opinions about algebra and algebraists, and sat in a special chair each day at tea in the Princeton "Common Room" dispensing aphorisms and witticisms in the style of Wittgenstein.

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Rota characterizes Lefschetz as "rude to everyone" and a self-styled anti-Semite (though only after his arrival in Princeton) in spite of being Jewish himself. Lefschetz's lectures "came close to incoherence," and "it was said of him that he had never given a completely correct proof, but had never made a wrong guess either." Notable is the story of Lefschetz attending an E. H. Moore lecture. Moore began, "Let a be a point, and let b be a point." "But why don't you just say, 'Let a and b be points'?" asked Lefschetz. "Because a may equal b," answered Moore. Lefschetz got up and left the room.

Rota's account of Yale is more desultory. There are anecdotes, but to my taste they have less universal appeal than the Princeton stories. He pays due homage to Jacob Schwartz (of whom Rota was the first and only Ph.D. student at Yale), to Hille and Ore, to Kakutani and Rickart and Dunford. I was tickled to learn that Dunford wrote his first paper while working as an independent scholar in the St. Louis Public Library. Rota gives a suitably fulsome and detailed description of the genesis and writing of the monumental work *Linear Operators* [1]. Rota was very much involved with proofreading chapters and solving problems for that book; he recalls the embarrassment of being unable to solve problem 20 of Section 9, Chapter III, Volume 1. His pain was somewhat alleviated when he found that Dunford could not solve the problem either; he felt better still when he learned that Schwartz could not do it. The problem was solved three years later by a first-year graduate student named Robert Langlands. The chapter on Yale ends with a story of the bright undergraduate who spent a summer checking all the special functions problems in [1]. Rota notes that "Section 1 of Chapter XIII is now spotless." The undergraduate was (later to be Fields Medalist) John Thompson.

Rota's stories of Princeton and Yale convey a lovely sense of history, of the early lives of many now venerable and distinguished mathematicians, and of the development of some of the important American mathematics departments. Though surely colored by Rota's personal memories and opinions, these are nonetheless charming and informative vignettes.

While I am exceedingly fond of this little book, I do not accord equal love to every word in it. For personal reasons I truly enjoy Rota's reminiscences about Princeton and Yale, I appreciate his enthusiasm about Jack Schwartz, I nod bemusedly at his commentary about graph theory, and I chuckle at his personal jibes at pomp and arrogance. It is perhaps my own inadequacies that prevent me from appreciating Rota's discussions of phenomenology. Like deconstructionism, phenomenology professes to separate what an object is and does from what it purports to be. But the considerations often degenerate into a disquisition on, for instance, the significance of the roller ball in the end of your pen when you are writing a love letter to your betrothed. I don't get it.

One of the themes of the book is a defensive one: Rota feels that graph theory (and lattice theory), à la Rodney Dangerfield, "doesn't get any respect." I have written only one paper on graph theory—with Paul Erdős, no less—so can hardly speak as an authority in the matter. But I doubt that the primacy of graph or lattice theory will be altered even one iota by such wistful thinking. Surely none of us feels adequately appreciated. Graph theory and lattice theory do not have the structural coherence and depth of a subject like algebraic geometry; given the values of late twentieth-century mathematics, it is no surprise that graph theory (and perhaps lattice theory) is generally not viewed as being at the heart of things.

Some of Rota's most scintillating commentary concerns the development of parts of modern algebra (see Rota's chapter entitled "Combinatorics, Representation Theory, and Invariant Theory: The Story of a Ménage à Trois"). This seems to be a playing field on which emotions run high: witness for instance the Mordell review of Lang's book *Diophantine Geometry*, the letter of C. L. Siegel to Mordell about the book, and the subsequent discussions in the *Notices* of the AMS (see [5, 4]). Rota's remarks about what is important in modern algebra and what is not are bound to raise hackles; they bring into focus some of the differences between the modern, abstract French/German school and the more traditional and concrete British/Italian school of thought.

Rota's remarks on algebra center around a perceived rivalry between Alfred Young and Ferdinand G. Frobenius over the application of representation theory to the calculation of invariants. Young was a combinatorialist: he counted things and proved identities. Frobenius was a theorist: he developed concepts and machinery, such as group characters, to attack problems. Rota abstracts from this competition a notion of "the problem solvers vs. the theorists." Quoth Rota,

> To the problem solver, the supreme achievement in mathematics is the solution to a problem that had been given up as hopeless. It matters little that the solution may be clumsy; all that counts is that it should be the first and that the proof be correct. Once the problem solver finds the solution, he will permanently lose interest in it, and will listen to new and

> simplified proofs with an air of condescension suffused with boredom.

By contrast:

> To the theorizer, the supreme achievement of mathematics is a theory that sheds sudden light on some incomprehensible phenomenon. Success in mathematics does not lie in solving problems but in their trivialization. The moment of glory comes with the discovery of a new theory that does not solve any of the old problems but renders them irrelevant.

For Rota "the problem solver is a conservative at heart," while "the theorizer is a revolutionary at heart." He goes on to say,

> If I were a space engineer looking for a mathematician to help me send a rocket into space, I would choose a problem solver. But if I were looking for a mathematician to give a good education to my child, I would unhesitatingly prefer a theorizer.

Rota's essay "Ten Lessons I Wish I Had Been Taught" is a delight. Though I may not endorse every opinion expressed here, I am certainly grateful that an intelligent and sensitive person has pondered these matters and shared with us his conclusions. Here are some examples:

- When you give a colloquium lecture, talk about just one thing. In Rota's words,

  > Every lecture should make only one main point. The German philosopher G. W. F. Hegel wrote that any philosopher who uses the word "and" too often cannot be a good philosopher. I think he was right, at least insofar as lecturing goes. Every lecture should state one main point and repeat it over and over, like a theme with variations. An audience is like a herd of cows, moving slowly in the direction that they are being beaten into. If we make one point, we have a good chance that our audience will take the beaten direction; if we make several points, then the cows will scatter all over the field. The audience will lose interest and go back to the thoughts they interrupted in order to come to our lecture.

- In your written work keep repeating yourself. He particularly recalls F. Riesz, who would publish a tentative version of his ideas in an obscure Hungarian journal, a more polished version in *Comptes Rendus*, and a final version (often in English) some years later.
- Every mathematician has only a few tricks, which he must cultivate and learn to use with skill.
- Every mathematician passes through a professional period when he is the new kid on the block and everyone refers to him as "the youngster". At a later stage the mathematician is the "old-timer". But, observes Rota, there is no in-between stage: one passes from tyro to sage with no intermediate period of passage.
- Nobody ever gives you proper credit for your ideas. If you prove a great theorem, they will either say that you got lucky or that this follows naturally from what you had already been thinking about. [What Rota describes here may sound like self-serving doubletalk, but it is certainly consistent with my own observation.] To compensate, we should be generous in giving credit to others.

An essay with a title cognate to the last one is "Ten Lessons for the Survival of a Mathematics Department". These include:

- Do not criticize other areas of mathematics. For example, if you are an applied mathematician, don't trash the logicians.
- Never break rank and go above the head of the department. In other words, don't rat your chairman out to your dean.
- Do not look down on good teachers. The point being that teaching is also an important part of what we do. [Contrast with Paul Halmos's statement[1] that in the 1950s research was not just the most important thing, it was the *only* thing.]
- Write expository papers. Here Rota attacks the intransigence and unwillingness of American mathematicians to engage in expository work. I wonder whether anyone will listen.
- Attack flakiness. "Flakiness is nowadays creeping into the sciences like a virus through a computer, and it may be the present threat to our civilization…mathematics is not and never will be flaky."

Well, you get the idea.

The "Ten Lessons for Survival" essay is more lugubrious and less playful than the "Ten Lessons I Wish I Had Been Taught" essay. But it contains valuable advice that we should all heed. The late 1990s are a time of trial for mathematics. Not only is the job market dreadful, but funding support is in danger, and the public is suspicious of scientists. We need strong chair-

[1] *Private communication.*

men and a well-informed professoriate to protect what is valuable and important about mathematics.

A few years ago Rota published the book *Discrete Thoughts* (see [3]). Though similar in title to the one now under review, the book [3] is more of a scholarly tract. Less concerned with personal thoughts and reminiscences (though there is a nice piece devoted to the memory of Stan Ulam, and the same piece appears in the book under review), the authors Kac, Rota, and Schwartz instead treat us to different views of discrete mathematics and its place in the mathematical infrastructure. They mount a spirited case for the importance of lattice theory, graph theory, Ramsey theory, statistics, and just plain counting. I believe that [3] will appeal primarily to those whose drum the authors are beating. The book under review, by contrast, will appeal to everyone.

For reasons about which one can only speculate, American mathematicians have traditionally been loth to publicly discuss their personal feelings about the discipline and its denizens. Notable exceptions are Halmos's memoir *I Want to Be a Mathematician* [2] and the book under review. In my opinion we can learn much about ourselves and about our subject by reading these books.

References

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[3] M. KAC, G.-C. ROTA, and J. SCHWARTZ, *Discrete thoughts*, Birkhäuser, Boston, 1992.

[4] S. LANG, Mordell's review, *Siegel's letter to Mordell, diophantine geometry, and 20th century mathematics*, Notices Amer. Math. Soc. **42** (1995), 339–350.

[5] L. MORDELL, Review of *Diophantine geometry* by S. Lang, Bulletin Amer. Math. Soc. **70** (1962), 491–498.