Reifying the Research: Mathematics Education in Taiwan

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A bronze Confucius sits on a bronze pillow, surrounded by his bronze disciples. The bas-relief decorates the entrance to the Chien-Kuo Senior High School, a prestigious public high school for boys in Taipei.

I am visiting the school in Taipei as a guest of the Nine Nine Cultural and Educational Foundation, to help them set up a mathematics contest modeled after that of the American Regions Mathematics League, an American competition with which I am associated. As a high school classroom teacher, I have asked my hosts to show me what goes on in Taiwan's secondary schools.

We have heard much recently about how students in Pacific Rim countries excel in their study of mathematics. But what is actually happening in these places? How do the numbers translate into practices from which we can learn? My visits to Taiwan presented a picture that reifies and extends what we learn from the research.

The Chien-Kuo School has recently celebrated its one-hundredth birthday, having been founded shortly after the Japanese occupied the island in 1895. After the customary glass of tea with the school's principal, I am ushered into a large room where one hundred young men have gathered to hear my talk about the American Regions Mathematics League competition. One of the examples I give uses Ptolemy's theorem about the diagonals and sides of a cyclic quadrilateral. I speak through an interpreter. My host, Yang-Ming Ho, has worked extensively with these students but had not been told in advance about the content of my talk. When I mention Ptolemy's theorem at one point, he interrupts to give a quick explanation in Chinese. The students do not know this theorem but are intrigued. He challenges them: "Can you prove it?" Then he writes some preliminary formulas on the chalkboard. "Now can you prove it?" His eyes burn with intensity, and the students are mesmerized. I am confident that many will go home and investigate the theorem.

I also wonder how many American high school teachers would have a working knowledge of Ptolemy's theorem at the ready.

The girls in Taipei Wesley High School are dressed in identical uniforms, but each has altered small details of her dress in some personal way. The mathematics class I am visiting consists of forty-five girls, a bit smaller than most high school classes in Taiwan. They are reviewing word problems that lead to quadratic equations. About three-quarters of the way through the lesson, the teacher reminds them that they know three ways of solving quadratic equations: factoring, completing the square, and applying the formula. She presents a problem, then invites the students to use whatever method of solution they find convenient. One student, chosen at random, is called to the board. Here she completes the square, although this method had not been reviewed in the day's lesson.

These students are in the eighth grade. American students usually learn to solve quadratics by factoring in the ninth grade and learn the

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quadratic formula in grade ten or eleven. Many of them will not find the method of completing the square either in their courses or in their textbooks.

In Tung-Shan High School, a private school pressed against the mountains that surround Taipei, sixty students are crowded into a large classroom. These are high school juniors studying precalculus. The teacher stands in front of the room with a microphone, while the students work at tiny desks bolted to the floor in a rectangular array. Their workspace is about half what an American classroom would offer. When I squeeze into the back of the room with my retinue of translator, assistant principal, and four other dignitaries, some of the students offer us their desks and double up in the tiny spaces of others.

The room is quiet. Faint noises float in from outside. (Taiwan's subtropical climate allows for open doors and windows in January and requires them by April or May.) Virtually all the students are on task. I find this particularly remarkable, since their task is merely to listen to the teacher. Only twice does he stop, once for a question ("Shall I do the derivation over for the second case?"), which he answers himself ("Well, I like it, so I will do it again"). The second pause is for a student's question. The student rises to ask it, the teacher replies, and the lecture continues.

This lesson is standard fare in traditional texts worldwide, a derivation of the equation of a hyperbola from its definition as a locus. It is essentially the same as the derivation in the text, which is used by all students in Taiwan. While I cannot read Chinese, I can read mathematics and found an interesting point in the text that I had not known about before. If we choose any point on a hyperbola and drop perpendiculars to the two asymptotes, the product of these perpendiculars is constant (this is easy to prove algebraically). The text uses this fact to prove that the distance from a point on the hyperbola to the nearest asymptote vanishes as we go up the hyperbola. The hour-long lesson I observed did not get this far.

The students in the class do not fidget, do not pass notes, do not gossip. They attend to the teacher or look down at a notebook or text. But the real surprise comes at the end of the period, marked by a gentle chime. The teacher finishes his thought, while the students remain attentive. When the teacher has finished, one student stands and issues a command to the others. They all rise and bow to the teacher, chanting *hsieh-hsieh, lao-tse* (thank you, teacher). Then they file out.

Lunch is no different from recess in an American high school. Kids rush down the corridors (which are open to the subtropical weather) and up the stairs, weaving around the staid visitors, teasing and calling to each other in organized chaos.

So Taiwanese students are not all that different from American students. Outside the classroom they seem to have the same energy and intensity as my own. It is the nature of the social contract that differs. Taiwanese teachers can prepare lessons assuming that the students in front of them will be interested and work hard. American teachers face a greater responsibility: they must create in the classroom an atmosphere in which hard work and intellectual curiosity are standard. I think to myself that if I could master the language (no mean feat!), I could teach in Taiwan. But a Taiwanese colleague would have to learn much more about American life than the English language to succeed in my classroom.

The food in Taiwan is delicious but rarely familiar. It is only after tasting it that one recognizes the fish or beef that has been presented in an exotic way. Over an array of such dishes I speak with Chi-Lin Yen, a professor at Taiwan Normal University. He tells me about the education of teachers. The university entrance examination, taken by most students, is quite rigorous, and the Normal University gets some of the most successful candidates.

He outlines the encyclopedic course of study in mathematics for prospective high school teachers, amounting to some 80 credits in subject matter alone (no wonder Yang-Ming Ho knew so much about Ptolemy's theorem!). Since the Normal University was free for a long time, many people who did not intend to be teachers got their degrees there. Some ended up in teaching. Others ended up as research mathematicians and had the background to pursue this career.

The evening air in downtown Taiwan is full of the aroma of frying, baking, and broiling. People on their way home stop to enjoy a quick snack from the food vendors on every corner. Many of these people are wearing high school uniforms. High school students come downtown one or more nights a week to attend evening classes. These are enrichment classes that virtually all students enroll in. The prices are low, and the parents eager to pay. Yang-Ming Ho runs an after-school center and is its most popular teacher. I visit his class of four hundred (not a misprint!) students. They are assembled in a large room with microphone and TV monitors. It is not like a lecture at a large state university, but more like a television show with a live audience. Ho works the audience, telling jokes and directing remarks at those students he knows well.

Here is a problem he gave, one of a series on division of polynomials with remainder: What is the remainder when the polynomial

$$x^{33} + x^{22} + x^{11} + x + 2$$

is divided by the polynomial $x^2 + x + 1$? Ho explained that we can write

$$P(x) = Q(x)(x^2 + x + 1) + (ax + b)$$

and that we can find the constants a and b by plugging in the roots of the equation $x^2 + x + 1 = 0$. Luckily these roots are the complex cube roots of unity, and luckily most of the 33rd degree polynomial drops out after substitution. The students were all on task. They were all in the tenth grade. Could they all do the problem? We do not really know, since there are no examinations in the enrichment program: students come to get ahead in their regular studies. But I could see that at least 90 percent of the students were interested and were following the discourse closely.

On a return visit eight months later, I see the results of this work. The first competition of the Taiwan Regions Mathematics League attracts 1,200 students and their teachers to a two-day celebration of mathematics. It is held during the students' summer vacation. Its American prototype, now twenty-four years old, serves 1,800 students out of a much larger population and is held (for most of this population) during the school year. It has taken years for this event to become popular. In Taiwan its success was much quicker.

The National Palace Museum of Taipei holds the world's most comprehensive collection of Chinese art. From neolithic ceramics to contemporary scroll paintings, its exhibits display the bottomless cultural wealth of Chinese civilization. In one of the galleries I find a rendering of the god Kuei-Hsing, whose special charge was the success of candidates in the official examinations of the Mandarin system. The students I talk with all know about Kuei-Hsing, and incense burns before his image in local temples. While there are no more Mandarin examinations, Kuei-Hsing has now taken responsibility for success on various local tests. Success in school here seems to be more than a point of personal pride or of self-improvement.

In the entrance to another building there is another copy of the bas-relief of Confucius. This copy is larger and a bit finer, for the building is the National Ministry of Education. I am talking here with Chao-Hsien Lin, the deputy minister of education, trying to get insights into the remarkable classrooms I have visited. His story is interesting but does not always answer the questions I have. Taiwan was a relatively backward part of China when the Japanese annexed it in 1895. By and large, education under the Japanese involved leaving the island for Japan at a certain point in the student's life. When the Nationalist government came to Taiwan, this changed. Particularly in the last few years, an enlightened leadership has emphasized mathematics and the sciences as the keys to economic prosperity. And the key has turned. Taiwan is indeed prosperous.

What changes are coming to Taiwanese education? Lin mentions that the ministry is exploring a variety of alternatives for college admission. The current system relies on a single examination, and the Taiwanese are looking to the American system to develop some alternatives. What if, asks Lin, a good student is ill on the day of the examination?

Tuan Tuan Lee, my hostess as the vice president of the Nine Nine Foundation, nods. This was exactly what happened to her daughter, who had to go to America for her undergraduate work. This proved a silver lining, she explained, as her daughter likes America and has learned a lot, both about business administration and about other cultures.

Lin tells me that many Taiwanese leave the island for graduate education, and those that study mathematics go mostly to the United States. Here is an echo of one of the paradoxes of American education. While our graduate mathematics programs are the envy of the world, our precollege education suffers in comparison to that of other developed countries.

Do the graduate students return to Taiwan? Sometimes, says Lin, and they are returning more and more often. The economic conditions at home are attractive, and they would rather live and work in their own country. While I do not tell him this, Lin has touched on another problem of American education: our best minds have recently been immigrants. Currently, fewer than half our graduate students in mathematics are native-born Americans. From the point of view of other countries, this brain drain is destabilizing. From the American point of view, it ties the success of our educational efforts to the success of our economy. If we slip, we will have very far to fall. In the case of Taiwanese students, this has happened already. If the trend that Lin points out continues, we may get fewer Taiwanese immigrants, and, in particular, fewer mathematicians, in years to come.

One Taiwanese mathematician who has stayed in the U.S. is my guide and host, Peter Shiue of the University of Nevada at Las Vegas. Shiue grew up on a poor fishing island off the coast of Taiwan. His talent was spotted early by an attentive junior high school teacher, and Shiue was sent to school on Taiwan, then in the U.S. He still keeps in touch with his junior high school teacher, who has herself moved to America. What would have become of his talent if she had not been able to spot it? How many American teachers of middle school mathematics would be able to recognize, in a classroom context, a real mathematical talent and not just a diligent and obedient student?

On another visit I was privileged to give a talk at the high school of the Pescadore Islands, where

Peter Shiue had spent his childhood. Thirty students and teachers came to the school, which was open for the talk despite the summer vacation. The students wore their uniforms. All engaged readily in the problems I posed for them.

Later, K. C. Shiue, principal of the school, told me that 80 percent of his students go on to college in Taiwan. Most do not return. The local economy, built on fishing and tourism, provides only a limited number of jobs. But even those who stay work on significant mathematics in high school. And those who leave are well prepared, despite their having attended school in this remote part of the country. How would this compare with a rural American school?

Most of the schools I visited in Taiwan were among the most successful on the island. As private or church schools, they serve students who can afford to pay tuition or who earn scholarships. But this observation does not lessen the achievement. It would be difficult to find an American middle school where eighth-grade students can complete the square, and I know of no after-school enrichment program in mathematics that draws anywhere near the number of students who attend those in Taipei. And few American enrichment programs are as advanced. Even if we compare the best American students and institutions to those in Taiwan, there is still much to learn.

What indeed is to be learned from this extraordinary system? There are those who look longingly at the educational results of Asian countries and urge us to copy their methods. This argument underestimates the enormous influence of culture. It is difficult to envision American students sitting at tiny desks. They have no special god to watch over their success in examinations. They will not flock to after-school programs, nor sit in a room with four hundred others to learn how to do complicated mathematics problems.

Likewise, American teachers cannot be expected to use a microphone to address classes of sixty. Even in large universities, where this form of instruction has been institutionalized, there is a move away from it. And few American school districts will vote for budgets based on their teachers working with only two classes each day, as their Taiwanese colleagues do.

We cannot plug Taiwanese practices directly into American schools. Taiwanese teaching is fashioned for Taiwanese students. But neither can we ignore what their experiences tell us about variations in teaching and learning. Like the cooking, the teaching in Taiwan takes some effort to appreciate. But in both cases the effort is quickly repaid.