Intersections in Mathematics and Education: Learning, Teaching, Creating, and Using

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Mathematicians must be constructively involved at the interface of mathematics and education at all levels. This statement holds true regardless of the modifier attached to "mathematics" or "mathematician": applied, statistical, pure, educator, researcher, academic, industrial, administrator, or any other I might have missed. My purpose is to discuss this constructive involvement, what it might mean in practice at various levels of education, and how the National Science Foundation (NSF) is working to facilitate it.

Looking to the Past

There is, of course, a long history of action at the interface of mathematics and education. Many researchers in mathematics were involved in the development of mathematics curricula during the 1960s, which, to the extent they were actually implemented in our schools, influenced the directions many of us in this room took in getting where we are now.

After a fallow period, the mid-1980s saw a resurgence of interest in mathematics and science education. In an effort to break the cycle I might describe as "crisis, response, tepid implementation, criticism, falling back", formal and informal organizations developed new approaches to dealing with the recognized issues of mathematics in education. We saw the creation of the Mathematical Sciences Education Board at the National Academy of Sciences and the less-structured Mathematicians and Education Reform network, for example.

In higher education we heard calls from the mathematics community for reform of the calculus curriculum. The result was a first-time cooperative effort that linked the Division of Mathematical Sciences (DMS) with NSF's education arm to provide support for rethinking the curriculum. In K–12 education the development of standards by the National Council of Teachers of Mathematics (NCTM)[1] proved a defining moment for mathematics education, stimulating the development of state standards across the country, of comprehensive curricula for their implementation, and of programs to facilitate implementation of reform in mathematics and science education.

The mathematics and education communities responded with many different ideas and a variety of creative programs. Implementation of these programs has raised the level of attention to the intersection of mathematics and education both inside and outside the communities most directly involved.

Higher Education—Graduate and Postdoctoral

Let me turn first to the interface of mathematics with higher education, where most of us have daily experience. At the graduate and postdoctoral levels, learning, teaching, creating, and using merge. Mathematicians are the experts on mathematics education. The graduate education enterprise is highly oriented to specific disciplines and is

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[1]Curriculum and Evaluation Standards for School Mathematics, *NCTM, March 1989.*

comparatively underdeveloped as a field of expertise in itself.

Mathematics has much to be proud of at the graduate level of education, but reports on the status of the U.S. mathematics enterprise regularly point to signs of trouble even here. Dropping enrollments in graduate programs, the extremely high proportion of graduate students coming from other countries, and the low rate of participation by women and minorities are all areas for concern.

This is not a new story. It was told in the David Report[2] of the early 1980s and reiterated in numerous reports of the Board on Mathematical Sciences and in the Annual Surveys. Three of five recommendations for action made by the 1998 *Report of the Senior Assessment Panel of the International Assessment of the U.S. Mathematical Sciences*[3] had to do with graduate and postdoctoral education.

Discussions of how to address these issues generally take place within the mathematical sciences, with little interference from educators or from other disciplines. We find a similar distinction within NSF—the DMS addresses the specifics of mathematics graduate work, while the directorate for Education and Human Resources (EHR) supports activities that cover all areas of science and engineering.

So DMS developed the VIGRE program—Vertically Integrated Grants for Research and Education—to address the many issues raised in the mathematical community. In contrast, in EHR we find the Integrative Graduate Education and Research Training program. IGERT permits development of graduate programs in an interdisciplinary environment to broaden and strengthen our usual conception of graduate education. EHR's new program for Graduate Teaching Fellows in K–12 Education, or GK–12, as it is called, provides opportunities for graduate students to participate in K–12 education activities. One might regard GK–12 as developing a new intersection for mathematics and education at the graduate level.

These programs help, but we need the mathematics community continually involved in thinking about questions such as:

- How do we educate effectively at the doctoral level for careers other than academe—for the different things that "doing mathematics" can mean?
- How do we encourage more U.S. citizens, particularly women and minorities, to enter and pursue careers in mathematics? How do we retain them in programs that do not currently attract them?
- How do we involve graduate students in mathematics research from the moment they enter a graduate program?

Higher Education—Undergraduate

Many issues at the graduate level are related to ones at the undergraduate level, where the merging of mathematics with education is less complete. While mathematicians set directions for education at this level, there are others who are looking at undergraduate education more broadly and who may want and need to play a role as well.

The January 7, 2000, issue of the *Chronicle of Higher Education* carried the article "The Remaking of Math". The focus was not on the mathematics major—the primary interest of most mathematicians—but on the many issues impinging on the teaching of mathematics at the undergraduate level.

The article noted that most students taking undergraduate mathematics do not major in the subject. Many do not even major in the physical sciences or engineering, disciplines that helped shape the standard introductory sequences in mathematics. Those focused on undergraduate education ask what steps mathematics departments need to take to address these changes in student population. At the same time, advances in information technology make new activities possible both inside and outside the classroom. Both course content and modes of teaching may need to change as what students are trying to do with mathematics and how they will do it influences their approach to learning the subject.

The dichotomy between focused attention to mathematics majors and broader attention to undergraduate education is shared at NSF. For example, the article discussed the calculus curriculum effort as a predecessor to a broader effort in Mathematical Sciences and Their Applications throughout the Curriculum, a continuing example of cooperation between EHR and the DMS. These shared activities reflect a desire for substantive mathematical input on curricula that can affect many undergraduate students. The Research Experiences for Undergraduates, or REU, program is coordinated through EHR but managed and funded by the disciplines. To my mind, it is the REU program that has had the most significant impact on undergraduate majors in mathematics. In REU the learning, teaching, creating, and using merge, giving students a taste of what it means to be a mathematician.

Most active curriculum development activities are aimed not at mathematics majors but at broad undergraduate preparation in mathematics for many possible future directions. In this context, here are some questions to consider:

[2]Renewing U.S. Mathematical Sciences: Critical Resource for the Future, *National Academy of Sciences, 1984. Reprinted in the* Notices, *August 1984, pages 435–466.*

[3]*Published by the NSF. The report may be found at* `http://www.nsf.gov/pubs/1998/nsf9895/nsf9895.htm`.

- What are the implications for mathematics majors?
- How can we use different introductory courses to attract people to the discipline?
- How must we change our assumptions about the backgrounds of students and the subsequent coursework?
- Should this change how we think about preparation for graduate work?
- Where there are insufficient resources to run parallel tracks for majors and nonmajors, how do we make sure there are opportunities in mathematics for all?
- Are there advantages for mathematics majors to have different types of beginnings in the discipline?
- And finally—leading me into the K–12 arena—what are the implications for training teachers of mathematics? Where will they get the content they need to be effective teachers in the future?

K–12 Education

At the graduate and undergraduate levels, we come to grips regularly with issues at the interface of mathematics and education. While we have some choice in the issues we as individuals choose to confront, some level of involvement is part of our lives. However, at the K–12 level the intersections of the world of mathematics and the world of education are much more limited, in part because the education universe becomes so much larger.

Distinctions are made between mathematicians (meaning those who practice mathematics for a living), mathematics educators (meaning those who train teachers in mathematics, develop K–12 curricula, or do research on mathematics education), and teachers of mathematics (with an additional distinction between mathematics teachers and elementary teachers who teach mathematics along with everything else). And those are just the education groups directly involved with mathematics. There is also the broader education establishment to consider.

Over time the separateness of communities and the differing bodies of professional knowledge with which they work have created differing perspectives leading to friction, occasional stalemate, and now controversy that some would call "war" in K–12 mathematics education. We cannot afford war—as mathematicians, as educators, or as a nation. The stakes are too high. We must have a population that is well equipped to use mathematics in an increasingly technological environment. Their success as individuals and our economic success as a nation depend upon it. We need all our forces engaged in productive work rather than in battle.

While not personally directly involved with establishing standards, developing curricula, or training teachers, I have followed these activities and the issues that have arisen from them on behalf of the NSF director before I moved to EHR. In some respects I was blessed by being in a position to look at the issues from outside all the participating communities while maintaining links to them. I began by looking at the facts, assumptions, and constraints involved in addressing K–12 mathematics education. Here are just a few that confronted the NCTM in developing its standards.

First, K–12 mathematics was failing large portions of our student population, something we had known for a very long time but seldom confronted. From this fact we have the decision to focus discussions of standards and educational change in areas that would impact the broad student population.

Second, research shows us that individuals learn differently. Thus, if we are to increase achievement for all students, we need to address a range of learning modes. Doing so can be difficult, as traditional patterns of teaching are deeply ingrained.

Next, uses of mathematics in the working world and in the daily decisions of our lives were and are changing significantly. Already it was clear that students lacked ability to deal with complex, multistep problems drawn from realistic or not-so-realistic situations. So new elements need to be added to an already packed curriculum.

Finally, new capabilities arising from the advances in information technologies put a high premium on knowing what mathematical operations to perform when and being able to estimate what the answer should be.

All these issues and more set up a situation where choices must be made and human judgment exerted. After broad consultation with all stakeholders, including the mathematicians (I can remember sitting in a few sessions at Joint Meetings like this back in the late 1980s where that kind of consultation was taking place), the NCTM wrote something down that became a starting point for discussion and action.

And, of course, that is when controversy reared its head. There is a big difference between commenting on fairly abstract standards and seeing the results of their implementation in the schools after the lengthy processes of adaptation at state and local levels, development of curricula and other instructional materials, and teacher education and development.

The NSF has been a focal point for some of that action, supporting development of curricula and programs in teacher professional development and systemic education reform, many of which draw on the NCTM standards or comparable state standards. We have continued to support research activities that help us understand the processes of learning—in general and specifically in mathematics. We have used the cooperation engendered at the graduate and undergraduate levels to bring mathematicians into the discussion in substantive ways. Some of NSF's actions—or what has resulted from them—are under fire in parts of the mathematics community. So be it—no substantive change takes place without some negative reaction.

To speak in mathematical terms, mathematics education is a complex, dynamical system. Our problems come when we try to view it as static and see documents like the NCTM standards as set in concrete. We need the corrective mechanisms of dialogue and discussion to keep the dynamical system in reasonable equilibrium. For most mathematicians, our contact with K–12 education comes through personal experience, either our own or that of our children. It is rational for us to reason from example to generality, but as mathematicians we know that can be dangerous when trying to address characteristics of a system. One of the real problems we face is making sure that we understand what influences the system.

It is here that I think NSF, and particularly EHR, can play its most effective role. K–12 education—its drivers, the state and local decisions that control it, the professional societies through which it works, etc.—are largely unknown quantities for the mathematics community. Even NSF is really just beginning to understand their workings in the broader sense, through systemic reform activities and our partnering with the Department of Education in several areas. On the other hand, for most educators NSF—not to mention the larger mathematics and science community—is a black box. Many really think in terms of education broadly rather than in the specifics of a particular field.

I regard it as NSF's and EHR's unique responsibility to ensure the engagement of all parties in serious discussion. I would have us be a conduit for discussions between the mathematics and education communities—listening to both groups, facilitating communication, and synthesizing the outcomes of discussions for future consideration and action. I view it as a privilege to head EHR at a time when this kind of ferment shows real promise of sustained attention to complex problems with the potential to make a difference for students. The NCTM's work on updating of the standards is a model for the level of consultation and involvement that is required. We need to maintain that kind of momentum.

I have many questions like those I posed at the graduate and undergraduate levels that I believe deserve our attention. But I will refrain from listing them and instead pose just one.

- How do we develop and maintain a constructive involvement of mathematicians in K–12 education?

Summing up

Education across levels is interconnected and dynamic. What we do in K–12 education affects the abilities of our undergraduate and graduate students. What we do at the undergraduate and graduate levels affects what we can do at the K–12 level. We cannot address the issues of K–12 mathematics education in isolation from mathematics, because content knowledge of teachers and substance of curricula are important factors in student achievement. In large part content knowledge is attained and curricula are developed within the confines of higher education.

To make the kind of connections across mathematics and education communities I mentioned earlier requires effort on the part of a much broader group of people than we are seeing to date. We need mathematicians who interact with the teacher in the classroom, the person teaching teachers in the local school of education, the mathematics supervisor in the school district, and the parents of students working through mathematics classes. To be constructive, we need to approach our involvement with the same degree of professionalism with which we ply our trade and to recognize the added professional requirements of work at the intersection.

At the graduate and undergraduate levels, where we exert our professional mathematics expertise most directly, we may need to find out what research can tell us about learning and teaching mathematics, perhaps investing our own time and effort to do the research where it is missing. That may be the only way to keep our discipline vibrant for both those in mathematics and those studying in other fields. At the same time it may help us better prepare those going on for advanced work in mathematics. At the K–12 level our involvement may be of a different character, but we must find ways to exert our professional expertise in mathematics in conjunction with those whose professional expertise is in education.

As mathematicians we know that finding counterexamples is usually much easier than finding proofs. When we find a flaw in a proof of an important result, we work with our colleagues to create a new proof, often finding unexpected results in the process. In K–12 mathematics education it may be easy to find flaws or to find examples of individuals for whom a particular curriculum or style of teaching does not work. Our job as mathematicians is to point out problems, understand the issues and constraints that led to the situation, and work with our colleagues in the education world to make things work in the best way possible for the largest number of people.

EHR stands ready to spur such interactions. We can use the help of each and every one of you in developing guiding principles and funded activities to make them happen.