**New Publications Offered by the AMS**

**Algebra and Algebraic Geometry**

**Sub-Laplacians with Drift on Lie Groups of Polynomial Volume Growth**
Georgios K. Alexopoulos, University of Paris, Orsay, France

This item will also be of interest to those working in analysis.

**Contents:** Introduction and statement of the results; The control distance and the local Harnack inequality; The proof of the Harnack inequality from Varopoulos's theorem and propositions 1.6.3 and 1.6.4; Hölder continuity; Nilpotent Lie groups; Sub-Laplacians on nilpotent Lie groups; A function which grows linearly; Proof of propositions 1.6.3 and 1.6.4 in the case of nilpotent Lie groups; Proof of the Gaussian estimate in the case of nilpotent Lie groups; Polynomials on nilpotent Lie groups; A Taylor formula for the heat functions on nilpotent Lie groups; Harnack inequalities for the derivatives of the heat functions on nilpotent Lie groups; Harmonic functions of polynomial growth on nilpotent Lie groups; Proof of the Berry-Eness estimate in the case of nilpotent Lie groups; The nil-shadow of a simply connected solvable Lie group; Connected Lie groups of polynomial volume growth; Proof of propositions 1.6.3 and 1.6.4 in the general case; A Berry-Eness estimate for the heat kernels on connected Lie groups of polynomial volume growth; Polynomials on connected Lie groups of polynomial volume growth; A Taylor formula for the heat functions on connected Lie groups of polynomial volume growth; Harnack inequalities for the derivatives of the heat functions; Harmonic functions of polynomial growth; Berry-Eness type of estimates for the derivatives of the heat kernel; Riesz transforms; Bibliography.

**Memoirs of the American Mathematical Society**, Volume 155, Number 739

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**Recommended Text**

**Introduction to Quantum Groups and Crystal Bases**
Jin Hong and Seok-Jin Kang, Korea Institute for Advanced Study, Seoul, Korea

The notion of a “quantum group” was introduced by V.G. Dinfeld and M. Jimbo, independently, in their study of the quantum Yang-Baxter equation arising from 2-dimensional solvable lattice models. Quantum groups are certain families of Hopf algebras that are deformations of universal enveloping algebras of Kac-Moody algebras. And over the past 20 years, they have turned out to be the fundamental algebraic structure behind many branches of mathematics and mathematical physics, such as solvable lattice models in statistical mechanics, topological invariant theory of links and knots, representation theory of Kac-Moody algebras, representation theory of algebraic structures, topological quantum field theory, geometric representation theory, and C*-algebras.

In particular, the theory of “crystal bases” or “canonical bases” developed independently by M. Kashiwara and G. Lusztig provides a powerful combinatorial and geometric tool to study the representations of quantum groups. The purpose of this book is to provide an elementary introduction to the theory of quantum groups and crystal bases, focusing on the combinatorial aspects of the theory.

The authors start with the basic theory of quantum groups and their representations, and then give a detailed exposition of the fundamental features of crystal basis theory. They also discuss its applications to the representation theory of classical Lie algebras and quantum affine algebras, solvable lattice model theory, and combinatorics of Young walls.

**Contents:** Lie algebras and Hopf algebras; Kac-Moody algebras; Quantum groups; Crystal bases; Existence and uniqueness of crystal bases; Global bases; Young tableaux and crystals; Crystal graphs for classical Lie algebras; Solvable lattice models; Perfect crystals; Combinatorics of Young walls; Bibliography; Index of symbols; Index.

**Graduate Studies in Mathematics**


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**December 2001**

**Notices of the AMS**

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Analysis

Sur Les Inégalités de Sobolev Logarithmiques
S Blanchere, D Chaïff, P Fougeres, I Gentil, F Malrieu, C Roberto, and G Scheffer

A publication of the Société Mathématique de France.

This book is an overview of logarithmic Sobolev inequalities. These inequalities have been the subject of intense activity in recent years, from analysis and geometry in finite and infinite dimensions to probability theory and statistical mechanics. And many developments are still to come.

The book is a "pedestrian approach" to logarithmic Sobolev inequalities, accessible to a wide audience. It is divided into several chapters of independent interest. The fundamental example of the Bernoulli and Gaussian distributions is the starting point for logarithmic Sobolev inequalities, as they were defined by Gross in the mid-seventies. Hypercontractivity and tensorisation form two main aspects of these inequalities, which are actually part of the larger family of classical Sobolev inequalities in functional analysis.

A chapter is devoted to the curvature-dimension criterion, which is an efficient tool for establishing functional inequalities. Another chapter describes a characterization of measures which satisfy logarithmic Sobolev or Poincaré inequalities on the real line, using Hardy’s inequalities.

Interactions with various domains in analysis and probability are developed. A first study deals with the concentration of measure phenomenon, which is useful in statistics as well as geometry. The relationships between logarithmic Sobolev inequalities and the transportation of measures are considered, in particular through their approach to concentration. A control of the speed of convergence to equilibrium of finite state Markov chains is described in terms of the spectral gap and the logarithmic Sobolev constants. The last part is a modern reading of the notion of entropy in information theory and of the several links between information theory and the Euclidean form of the Gaussian logarithmic Sobolev inequality. The genesis of these inequalities can be traced back to the early contributions of Shannon and Stam.

This book focuses on the specific methods and the characteristics of particular topics, rather than the most general fields of study. Chapters are mostly self-contained. The bibliography, without being encyclopedic, tries to give a rather complete state of the art on the topic, including some very recent references.

Contents: Preface; Avant-propos; L’exemple des lois de Bernoulli de Gauss; Sobolev logarithmique et hypercontractivité; Tensorisation et perturbation; Familles d’inégalités fonctionnelles; Le critère de courbure-dimension; Inégalités sur la droite réelle; Concentration de la mesure; Inégalités de Sobolev logarithmique et de transport; Sobolev logarithmique et chaînes de Markov finies; Inégalités entropiques en théorie de l’information; Bibliographie; Index.

Panoramas et Synthèses, Number 10
Mathematics Subject Classification: 60J60, 26D10, 58D25, 39B72, 58J65, 47D07, 60J10, 94A15, 94A17, Individual member $40, List $44, Order code PASY/10N

Rotation C*-Algebras and Almost Mathieu Operators
Florin-Petre Boca, Cardiff University, UK

A publication of the Theta Foundation.

This book delivers a swift, yet concise, introduction to some aspects of rotation C*-algebras and almost Mathieu operators. The two topics come from different areas of analysis: operator algebras and the spectral theory of Schrödinger operators, but can be approached in a unified way. The book does not try to be the definitive treatise on the subject, but rather presents a survey highlighting the important results and demonstrating this unified approach.

For each real number $\alpha$, the rotation C*-algebra $A_{\alpha}$ can be abstractly defined as the universal C*-algebra generated by two elements $U$ and $V$ subject to the relation $UV = e^{2\pi i \alpha} VU$. When $\alpha$ is an integer, $A_{\alpha}$ is isomorphic to the commutative C*-algebra of continuous functions on a two-dimensional torus. When $\alpha$ is not an integer, the algebra is sometimes called a non-commutative 2-torus. In this respect, some of the methods you will find here can be regarded as a sort of non-commutative Fourier analysis. An almost Mathieu operator is a type of self-adjoint operator on the Hilbert space $L^2 = L^2(\mathbb{Z})$.

The exposition is geared toward a wide audience of mathematicians: researchers and advanced students interested in operator algebras, operator theory and mathematical physics. Readers are assumed to be acquainted with some functional analysis, such as definitions and basic properties of C*-algebras and von Neumann algebras, some general results from ergodic theory, as well as the Fourier transform (harmonic analysis) on elementary abelian locally compact groups of the form $\mathbb{R}^d \times \mathbb{Z}^k \times \mathbb{Z}^1 \times F$, where $F$ is a finite group.

Much progress has been made on these topics in the last twenty years. This present book will introduce you to the subjects and to the significant results.

Distributed worldwide, except in Romania, by the AMS.

Contents: Prerequisites on rotation C*-algebras; Almost Mathieu operators and automorphisms of $A_{\alpha}$; Perturbations of the spectrum of $H_{\alpha,\lambda}$; The spectrum of almost Mathieu operators for rational $\alpha$; The absence of isolated points in the spectrum of $H_{\alpha,\lambda}$; Lyapunov exponents and pure point spectrum; The Lebesgue measure of $\text{spec}(H_{\alpha,\lambda})$; Some estimates for the Lebesgue measure of $\text{spec}(H_{\alpha,\lambda})$; Spectral computations for certain non-self-adjoint operators; Projections in rotation C*-algebras; The approximation of irrational rotation C*-algebras; The approximation of irrational non-commutative spheres; Subject index; Notation.

International Book Series of Mathematical Texts
Mathematics Subject Classification: 46L35, 81Q15, 47B39; 46L85, 81Q10, 47B36, All AMS members $52, List $58, Order code THETA/2N
Dynamical, Spectral, and Arithmetic Zeta Functions
Michel L. Lapidus, University of California, Riverside, and Machiel van Frankenhuyzen, Rutgers University, Piscataway, NJ, Editors

The original zeta function was studied by Riemann as part of his investigation of the distribution of prime numbers. Other sorts of zeta functions were defined for number-theoretic purposes, such as the study of primes in arithmetic progressions. This led to the development of L-functions, which now have several guises. It eventually became clear that the basic construction used for number-theoretic zeta functions can also be used in other settings, such as dynamics, geometry, and spectral theory, with remarkable results.

This volume grew out of the special session on dynamical, spectral, and arithmetic zeta functions held at the annual meeting of the American Mathematical Society in San Antonio, but also includes four articles that were invited to be part of the collection. The purpose of the meeting was to bring together leading researchers, to find links and analogies between their fields, and to explore new methods. The papers discuss dynamical systems, spectral geometry on hyperbolic manifolds, trace formulas in geometry and in arithmetic, as well as computational work on the Riemann zeta function.

Each article employs techniques of zeta functions. The book unifies the application of these techniques in spectral geometry, fractal geometry, and number theory. It is a comprehensive volume, offering up-to-date research. It should be useful to both graduate students and confirmed researchers.

This item will also be of interest to those working in number theory and geometry and topology.


Contemporary Mathematics, Volume 290


Operators, Functions, and Systems: An Easy Reading
Volume 1: Hardy, Hankel, and Toeplitz
Nikolai K. Nikolskii, University of Bordeaux I, Talence, France, and Steklov Institute of Mathematics, St. Petersburg, Russia

This unique book combines together four distinct topics of modern analysis and its applications:

A. Hardy classes of holomorphic functions
B. Spectral theory of Hankel and Toeplitz operators
C. Function models for linear operators and free interpolations,
D. Infinite-dimensional system theory and signal processing

This volume, Volume I, contains Parts A and B; Volume II will contain Parts C and D.

Hardy classes of holomorphic functions: This topic is known to be the most powerful tool of complex analysis for a variety of applications, starting with Fourier series, through the Riemann ζ-function, all the way to Wiener’s theory of signal processing.

Spectral theory of Hankel and Toeplitz operators: These now become the supporting pillars for a large part of harmonic and complex analysis and for many of their applications. In this book, moment problems, Nevanlinna-Pick and Caratheodory interpolation, and the best rational approximations are considered to illustrate the power of Hankel and Toeplitz operators.

Function models for linear operators and free interpolations: This is a universal topic and, indeed, is the most influential operator theory technique in the post-spectral-theorem era. In this book, its capacity is tested by solving generalized Carleson-type interpolation problems.

Infinite-dimensional system theory and signal processing: This topic is the touchstone of the three previously developed techniques. The presence of this applied topic in a pure mathematics environment reflects important changes in the mathematical landscape of the last 20 years, in that the role of the main consumer and customer of harmonic, complex, and operator analysis has more and more passed from differential equations, scattering theory, and probability, to control theory and signal processing.

The book is geared toward a wide audience of readers, from graduate students to professional mathematicians. It develops an elementary approach while retaining an expert level that can be applied in advanced analysis and selected applications.

Contents: An invitation to Hardy classes/Contents: Invariant subspaces of L^2(µ); First applications; H^p classes. Canonical factorization; Szegö infimum, and generalized Phragmén-Lindelöf principle; Harmonic analysis in L^2(T,µ); Transfer to the half-plane; Time-invariant filtering; Distance formulae and zeros of the Riemann ζ-function; Hankel and Toeplitz operators/Contents: Foreword; Hankel operators and their symbols; Compact Hankel operators; Applications to Nevanlinna-Pick interpolation; Essential spectrum. The first step: Elements of Toeplitz operators; Essential spectrum. The second step: The
New Publications Offered by the AMS

Hilbert matrix and other Hankel operators; Hankel and Toeplitz operators associated with moment problems; Singular numbers of Hankel operators; Trace class Hankel operators; Inverse spectral problems, stochastic processes, and one-sided invertibility; Subject index; Author index; Symbol index; Bibliography.

Mathematical Surveys and Monographs, Volume 92


Editors

$69, Institutional member $55, Order code CONM-289N

N. L. Vasilevski

8218-2708-1, LC 2001053556, 2000

Contemporary Mathematics

Theorem with applications to nonlinear elliptic equations.

35S30, 81Q50, 35A15, 35J65,

Classification

1372 NOTICES OF THE AMS VOLUME 48, NUMBER 11

Second Summer School in Analysis and Mathematical Physics

Topics in Analysis: Harmonic, Complex, Nonlinear and Quantization

Salvador Pérez-Esteva,

Università Nacional Autònoma de México, Cuernavaca, Morelos, México, and Carlos Villegas-Blas, Universidad Nacional Autònoma de México, Editors

For the second time, a Summer School in Analysis and Mathematical Physics took place at the Universidad Nacional Autònoma de México in Cuernavaca. The purpose of the schools is to provide a bridge from standard graduate courses in mathematics to current research topics, particularly in analysis. The lectures are given by internationally recognized specialists in the fields. The topics covered in this Second Summer School include harmonic analysis, complex analysis, pseudodifferential operators, the mathematics of quantum chaos, and non-linear analysis.

This item will also be of interest to those working in mathematical physics.

This volume is a joint publication of the American Mathematical Society and the Sociedad Matemática Mexicana. Members of the SMM may order directly from the AMS at the AMS member price.


Contemporary Mathematics


Differential Equations

Lyapunov Exponents and Smooth Ergodic Theory

Luis Barreira, Instituto Superior Técnico, Lisbon, Portugal, and Yakov B. Pesin, Pennsylvania State University, University Park

This book is a systematic introduction to smooth ergodic theory. The topics discussed include the general (abstract) theory of Lyapunov exponents and its applications to the stability theory of differential equations, stable manifold theory, absolute continuity, and the ergodic theory of dynamical systems with nonzero Lyapunov exponents (including geodesic flows).

The authors consider several nontrivial examples of dynamical systems with nonzero Lyapunov exponents to illustrate some basic methods and ideas of the theory.

Contents: Introduction; Lyapunov stability theory of differential equations; Elements of nonuniform hyperbolic theory; Examples of nonuniformly hyperbolic systems; Local manifold theory; Ergodic properties of smooth hyperbolic measures; Bibliography; Index.

University Lecture Series, Volume 23


A Stability Index Analysis of 1-D Patterns of the Gray-Scott Model

Arjen Doelman, University of Amsterdam, Netherlands, Robert A. Gardner, University of Massachusetts, Amherst, MA, and Tasso J. Kaper, Boston University

Contents: Introduction; The Evans function and the stability index; Tracking the fast subbundle; The slow subbundle; Calculation of the stability index; Concluding remarks; Bibliography.
**Introduction to the Theory of Differential Inclusions**

Georgi V. Smirnov, University of Porto, Portugal

A differential inclusion is a relation of the form $x \in F(x)$, where $F$ is a set-valued map associating any point $x \in \mathbb{R}^n$ with a set $F(x) \subseteq \mathbb{R}^n$. As such, the notion of a differential inclusion generalizes the notion of an ordinary differential equation of the form $\dot{x} = f(x)$. Therefore, all problems usually studied in the theory of ordinary differential equations (existence and continuation of solutions, dependence on initial conditions and parameters, etc.) can be studied for differential inclusions as well. Since a differential inclusion usually has many solutions starting at a given point, new types of problems arise, such as investigation of topological properties of the set of solutions, selection of solutions with given properties, and many others.

Differential inclusions play an important role as a tool in the study of various dynamical processes described by equations with a discontinuous or multivalued right-hand side, occurring, in particular, in the study of dynamics of economical, social, and biological macrosystems. They also are very useful in proving existence theorems in control theory.

This text provides an introductory treatment to the theory of differential inclusions. The reader is only required to know ordinary differential equations, theory of functions, and functional analysis on the elementary level.

Chapter 1 contains a brief introduction to convex analysis. Chapter 2 considers set-valued maps. Chapter 3 is devoted to the Mordukhovich version of nonsmooth analysis. Chapter 4 contains the main existence theorems and gives an idea of the approximation techniques used throughout the text. Chapter 5 is devoted to the viability problem, i.e., the problem of selection of a solution to a differential inclusion that is contained in a given set. Chapter 6 considers the controllability problem. Chapter 7 discusses extremal problems for differential inclusions. Chapter 8 presents stability theory, and Chapter 9 deals with the stabilization problem.

This item will also be of interest to those working in applications.

**Contents:**
- Foundations: Convex analysis; Set-valued analysis; Nonsmooth analysis; Differential inclusions: Existence theorems; Viability and invariance; Controllability; Optimality; Stability; Stabilization; Comments; Bibliography; Index.

**Graduate Studies in Mathematics**, Volume 41N

General and Interdisciplinary

Assistantships and Graduate Fellowships 2001

Review of a previous edition:
This directory is a tool for undergraduates in mathematics majors seeking information about graduate programs in mathematics. Although most of the information can be gleaned from the Internet, the usefulness of this directory for the prospective graduate student is the consistent format for comparing different mathematics graduate programs without the hype. Published annually, the information is up-to-date, which is more than can be said of some Websites. Support for graduate students in mathematics is a high priority of the American Mathematical Society, which also provides information for fellowships and grants they offer as well as support from other societies and foundations. The book is highly recommended for academic and public libraries.

—American Reference Books Annual

This publication is an indispensable source of information for students seeking support for graduate study in the mathematical sciences. Providing data from a broad range of academic institutions, it is also a valuable resource for mathematical sciences departments and faculty.

Assistantships and Graduate Fellowships brings together a wealth of information about resources available for graduate study in mathematical sciences departments in the U.S. and Canada. Information on the number of faculty, graduate students, and degrees awarded (bachelor’s, master’s, and doctoral) is listed for each department when available. Stipend amounts and the number of awards available are given, as well as information about foreign language requirements. Numerous display advertisements from mathematical sciences departments throughout the country provide additional information.

Also listed are sources of support for graduate study and travel, summer internships, and graduate study in the U.S. for foreign nationals. Finally, a list of reference publications for fellowship information makes Assistantships and Graduate Fellowships a centralized and comprehensive resource.


Mathematical Sciences Professional Directory, 2002

This annual directory provides a handy reference to various organizations in the mathematical sciences community. Listed in the directory are the following: officers and committee members of over thirty professional mathematical organizations (terms of office and other pertinent information are also provided in some cases); key mathematical sciences personnel of selected government agencies; academic departments in the mathematical sciences; mathematical units in nonacademic organizations; and alphabetic listings of colleges and universities. Current addresses, telephone numbers, and electronic addresses for individuals when provided are listed in the directory.


Geometry and Topology

The Submanifold Geometries Associated to Grassmannian Systems

Martina Brück and Xi Du,
Joonsang Park, Dongguk University, Seoul, Korea, and
Chuu-Lian Terng,
Northeastern University, Boston

Contents: Introduction; The $U/K$-system; $G_{m,n}$-systems; $G_{m,n}^+$-systems; Moving frame method for submanifolds; Submanifolds associated to $G_{m,n}$-systems; Submanifolds asso-
Homotopy Theory of Diagrams

Wojciech Chachólski, Yale University, New Haven, and Jérôme Scherer, Université de Lausanne, Switzerland

Contents: Introduction; Model approximations and bounded diagrams; Homotopy theory of diagrams; Properties of homotopy colimits; Appendix B. Categorical preliminaries; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 155, Number 736

Global Differential Geometry: The Mathematical Legacy of Alfred Gray

Marisa Fernández, University of the Basque Country, Bilbao, Spain, and Joseph A. Wolf, University of California at Berkeley, Editors

This book represents the state of the art in modern differential geometry, with some general expositions of some of the more active areas: special Riemannian manifolds, Lie groups and homogeneous spaces, complex structures, symplectic manifolds, geometry of geodesic spheres and tubes and related problems, geometry of surfaces, and computer graphics in differential geometry.

Contents: I. K. Babenko and I. A. Taimanov, On the formality problem for symplectic manifolds; T. F. Banchoff, Osculating tubes and self-linking for curves on the three-sphere; E. Boeckx and L. Vanhecke, Isoparametric functions and harmonic and minimal unit vector fields; S. K. Donaldson, The Seiberg-Witten equations and almost-Hermitian geometry; H. Ferguson, Sculpture inspired by work with Alfred Gray: Kepler elliptic curves and minimal surface sculptures of the planets; M. Gromov, Mesoscopic curvature and hyperbolicity; N. Hitchin, Stable forms and special metrics; A. Huckleberry and M. Völlner, A CR-momentum Ansatz for nilpotent groups; O. Kowalski, M. Sekizawa, and Z. Vlašek, Can tangent sphere bundles over Riemannian manifolds have strictly positive sectional curvature?; V. Miquel, Volumes of geodesic balls and spheres associated to a metric connection with torsion; E. Musso and L. Nicolodi, Special isothermic surfaces and solitons; A. Ros, The isoperimetric and Willmore problems; S. Salamon, Almost parallel structures; P. Wellington, Technical computing with Mathematica; J. A. Wolf, Complex geometry and non-holonomic mechanics; M. Völler, Morse-Novikov integrals on nilmanifolds; J. Wolf, On a new class of contact Riemannian manifolds; N. Cohen, C. J. C. Negreiros and I. A. Taimanov, Description of (1,2)-symplectic metrics on flag manifolds; G. Dloussky, Complex surfaces with Betti numbers $b_1 = 1$, $b_2 > 0$ and finite quotients; L. Bruna and J. Girbau, Is it admissible to linearize the Einstein equation in the presence of matter?; J. Bures, Solutions of some conformally invariant equations of the first order; R. Caddeo, S. Montaldo, and P. Piu, On biharmonic maps; M. A. Cañadas-Pinedo and C. Ruiz, Characterizations of Pfaffian systems. Consequences in dimension five; J. T. Cho, On new classes of almost contact Riemannian manifolds; N. Cohen, C. J. C. Negreiros and I. A. T. Martin, Description of (1,2)-symplectic metrics on flag manifolds; G. Dloussky, Complex surfaces with Betti numbers $b_1 = 1$, $b_2 > 0$ and finite quotients; L. G. Dotti and A. Fino, Hypercomplex nilpotent Lie groups; M. J. Druetta, E-spaces of Iwasawa type and Damek-Ricci spaces; M. E. E. L. M. Gálvão and C. Goes, Deformations of constant mean curvature surfaces in half space models; P. Gilkey and R. Ivanova, The geometry of the skew-symmetric curvature operator in the complex setting; H. Gollek, Representing minimal surfaces in $C^3$ by differential operators; L. Guijarro, Isometric immersions without positive Ricci curvature; J. Janyska, Natural Poisson and Jacobi structures on the tangent bundle of a pseudo-Riemannian manifold; H. Jiménez and S. López de León, On Alfred Gray’s elliptical catenoid; J. Koiller, P. R. Rodrigues, and P. Pitanga, Sub-riemannian geometry and non-holonomic mechanics; M. Kures, Weil algebras of generalized higher order velocities bundles; I. A. Taimanov, The good Lie algebras from certain Riemannian viewpoint; R. López, How to use Mathematica to find cyclic surfaces of constant curvature in Lorentz-Minkowski space; E. Loubeau, The Fuglede-Ishihara and Baird-Eells theorems for $p > 1$; D. V. Millionschikov, Cohomology of nilmanifolds and...
New Publications Offered by the AMS

Gontcharova’s theorem; V. Muñoz, F. Presas, and I. Sols, Asymptotically holomorphic embeddings of contact manifolds in projective spaces; H. Omori, Y. Maeda, N. Miyazaki, and A. Yoshioka, Convergent star products on Fréchet linear Poisson algebras of Heisenberg type; A. Onischenko, D. Repovš, and A. Skopenkov, Resolutions of 2-polyhedra by fake surfaces and embeddings into $\mathbb{R}^2$; L. Ornea and P. Piccinni, Cayley 4-frames and a quaternion Kähler reduction related to Spin(7); M. Parton, Old and new structures on products of spheres; L. Del Riego, 1 homogeneous sprays in Finsler manifolds; J. I. R. Prieto, The Gysin sequence for riemannian flows; T. Rybicki, On contact groupoids and Legendre bisections; M. Salvai, Affine maximal tori intersecting a fixed one; A. Savo, On the asymptotic series of the heat content; Y. Tazawa, Visualization of flat slant surfaces in $\mathbb{C}^2$; A. Talle, On solvable Lie groups without lattices; J.-P. Bourguignon, E. Calabi, J. Eells, O. García-Prada, and M. Gromov, Where does geometry go? A research and education perspective.

Contemporary Mathematics, Volume 288

Our first knowledge of differential geometry usually comes from the study of the curves and surfaces in $\mathbb{R}^3$ that arise in calculus. Here we learn about line and surface integrals, divergence and curl, and the various forms of Stokes’ Theorem. If we are fortunate, we may encounter curvature and such things as the Serret-Frenet formulas.

With just the basic tools from multi-variable calculus, plus a little knowledge of linear algebra, it is possible to begin a much richer and rewarding study of differential geometry, which is what is presented in this book. It starts with an introduction to the classical differential geometry of curves and surfaces in Euclidean space, then leads to an introduction to the Riemannian geometry of more general manifolds, including a look at Einstein spaces. An important bridge from the low-dimensional theory to the general case is provided by a chapter on the intrinsic geometry of surfaces.

The first half of the book, covering the geometry of curves and surfaces, would be suitable for a one-semester undergraduate course. The local and global theories of curves and surfaces are presented, including detailed discussions of surfaces of rotation, ruled surfaces and minimal surfaces.

The second half of the book, which could be used for a more advanced course, begins with an introduction to differentiable manifolds, Riemannian structures and the curvature tensor. Two special topics are treated in detail: spaces of constant curvature and Einstein spaces.

The main goal of the book is to get started in a fairly elementary way, then to guide the reader toward more sophisticated concepts and more advanced topics. There are many exercises and exercises to help along the way. Numerous figures help the reader visualize key concepts and examples, especially in lower dimensions.

Contents: Notations and prerequisites from analysis; Curves in $\mathbb{R}^n$; The local theory of surfaces; The intrinsic geometry of surfaces; Riemannian manifolds; The curvature tensor; Spaces of constant curvature; Einstein spaces; Bibliography; Index.

Student Mathematical Library

A Tour of Subriemannian Geometries, Their Geodesics and Applications
Richard Montgomery, University of California, Santa Cruz

Subriemannian geometries, also known as Carnot-Caratheodory geometries, can be viewed as limits of Riemannian geometries. They also arise in physical phenomenon involving “geometric phases” or holonomy. Very roughly speaking, a subriemannian geometry consists of a manifold endowed with a distribution (meaning a $k$-plane field, or subbundle of the tangent bundle), called horizontal together with an inner product on that distribution. If $k = n$, the dimension of the manifold, we get the usual Riemannian geometry. Given a subriemannian geometry, we can define the distance between two points just as in the Riemannian case, except we are only allowed to travel along the horizontal lines between two points.

The book is devoted to the study of subriemannian geometries, their geodesics, and their applications. It starts with the simplest nontrivial example of a subriemannian geometry: the two-dimensional isoperimetric problem reformulated as a problem of finding subriemannian geodesics. Among topics discussed in other chapters of the first part of the book we mention an elementary exposition of Gromov’s surprising idea to use subriemannian geometry for proving a theorem in discrete group theory and Cartan’s method of equivalence applied to the problem of understanding invariants (diffeomorphism types) of distributions. There is also a chapter devoted to open problems.

The second part of the book is devoted to applications of subriemannian geometry. In particular, the author describes in detail the following four physical problems: Berry’s phase in quantum mechanics, the problem of a falling cat righting herself, that of a microorganism swimming, and a phase...
problem arising in the N-body problem. He shows that all
these problems can be studied using the same underlying type
of subriemannian geometry; that of a principal bundle
endowed with $G$-invariant metrics.

Reading the book requires introductory knowledge of differential
geometry, and it can serve as a good introduction to this
new exciting area of mathematics.

**Contents:** Geodesics in subriemannian manifolds: Dido meets Heisenberg; Chow’s theorem: Getting from A to B; A remark-
able horizontal curve; Curvature and nilpotentization; Singular
curves and geodesics; A zoo of distributions; Cartan’s
approach; The tangent cone and Carnot groups; Discrete
groups tending to Carnot geometries; Open problems;
Mechanics and geometry of bundles: Metrics on bundles; Classical
particles in Yang-Mills fields; Quantum phases; Falling,
swimming, and orbiting; Appendices: Geometric mechanics;
Bundles and the Hopf fibration; The Sussmann and Ambrose-Singer
theorems; Calculus of the endpoint map and existence
of geodesics; Bibliography; Index.

**Mathematical Surveys and Monographs,** Volume 91

2001053538, 2000 Mathematics Subject Classification: 58E10,
53C17, 53C23, 49Q20, 58A30, 53C22, 58A15, 58D15, 58E30,
**Individual member $41,** List $69, Institutional member $55,
Order code SURV/91N

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**Convex Polyhedra with Regularity Conditions and Hilbert’s Third Problem**

A. R. Rajwade, Panjab University, Chandigarh, India

A publication of the Hindustan Book Agency.

Since antiquity, people knew that there are only five regular
solids, i.e. polyhedra whose all faces are regular polygons and
all solid angles are also regular. These solids are, of course, the
tetrahedron, the octahedron, the cube, the icosahedron, and
the dodecahedron. Later, much attention was drawn to the
question of how to describe polyhedra with other types of
regularity conditions. The author puts together many facts
known in this direction. He formulates four regularity condi-
tions (two for faces and two for solid angles) and for any
combination of their conditions lists all the corresponding
polyhedra. In this way, he obtains such very interesting classes
of solids as 13 semiregular solids, or 8 deltahedra, or 92 regu-
larly faces polyhedra, etc. In later chapters the author presents
some related topics of geometry of solids, like star polyhedra
and plane tessellations. In the concluding chapter, a complete
solution of the Hilbert 3rd problem is given.

Supplied with many figures, the book can be easily read by
anyone interested in this beautiful classical geometry.

This item will also be of interest to those working in general
and interdisciplinary areas.

**Contents:** Introduction; Definitions and notations; Theorems of Euler and Descartes; The regularity restrictions and the five

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**Lectures on Algebraic Model Theory**

Bradd Hart and Matthew Valeriote, McMaster University,
Hamilton, ON, Canada, Editors

In recent years, model theory has had
remarkable success in solving impor-
tant problems as well as in shedding
new light on our understanding of
them. The three lectures collected here
present recent developments in three
such areas: Anand Pillay on differential
fields, Patrick Speissegger on o-minimality and Matthias
Clasen and Matthew Valeriote on tame congruence theory.

**Contents:** Differential fields: Lectures on o-
minimality: Lectures on o-minimality; Tame congruence theory:
The structure of finite algebras; Varieties; Bibliography.

**Fields Institute Monographs,** Volume 15

2001053718, 2000 Mathematics Subject Classification: 03C64;
12L12, 03C50, **Individual member $18,** List $30, Institutional
member $24, Order code FIM/15N

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**Mathematical Physics**

**Multiparticle Quantum Scattering in Constant Magnetic Fields**

Christian Gérard, Ecole Polytechnique, Paris, France,
and Izabella Laba, University of British Columbia,
Vancouver, BC, Canada

This monograph offers a rigorous mathematical treatment of the
scattering theory of quantum N-particle systems in an external
constant magnetic field. In particular, it addresses the question of
**asymptotic completeness,** a classification of all possible trajec-

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**New Publications Offered by the AMS**
Mathematical Physics in Mathematics and Physics
Quantum and Operator Algebraic Aspects

Roberto Longo, University of Rome II, Italy, Editor

The beauty and the mystery surrounding the interplay between mathematics and physics is captured by E. Wigner’s famous expression, “The unreasonable effectiveness of mathematics”. We don’t know why, but physical laws are described by mathematics and good mathematics sooner or later finds applications in physics, often in a surprising way.

In this sense, mathematical physics is a very old subject—as Egyptian, Phoenician, or Greek history tells us. But mathematical physics is a very modern subject as any working mathematician or physicist can witness. It is a challenging discipline that has to provide results of interest for both mathematics and physics. Ideas and motivations from both these sciences give it a vitality and freshness that is difficult to find anywhere else.

One of the big physical revolutions in the twenty-first century, quantum physics, opened a new magnificent era for this interplay. With the appearance of noncommutative analysis, the role of classical calculus has been taken by commutation relations, a subject still growing in an astonishing way.

A good example where mathematical physics showed its power, beauty, and interdisciplinary character is the Doplicher-Haag-Roberts analysis of superselection sectors in the late 1960s. Not only did this theory explain the origin of statistics and classify it, but year after year, new connections have emerged, for example with Tomita-Takesaki modular theory, Jones theory of subfactors, and Doplicher-Roberts abstract duality for compact groups.

This volume contains the proceedings of the conference, “Mathematical Physics in Mathematics and Physics”, dedicated to Sergio Doplicher and John E. Roberts held in Siena (Tuscany, Italy). The articles offer current research in various fields of mathematical physics, primarily concerning quantum aspects of operator algebras.


Fields Institute Communications, Volume 30

New Publications Offered by the AMS
Winter School on Mirror Symmetry, Vector Bundles and Lagrangian Submanifolds

Cumrun Vafa and S.-T. Yau,
Harvard University,
Cambridge, MA, Editors

The collection of articles in this volume are based on lectures presented during the Winter School on Mirror Symmetry held at Harvard University. There are many new directions suggested by mirror symmetry which could potentially have very rich connections in physics and mathematics.

This book brings together the latest research in a major area of mathematical physics, including the recent progress in mirror manifolds and Lagrangian submanifolds. In particular, several articles describing homological approach and related topics are included.


This item will also be of interest to those working in algebra and algebraic geometry.


**AMS/IP Studies in Advanced Mathematics**

December 2001, approximately 377 pages, Softcover, ISBN 0-8218-2159-8, LC 2001043765, 2000 Mathematics Subject Classification 14-06, 32-06, 81-06, 53D12, 14F05, 14J32, All AMS members $34, List $42, Order code AMSIP/23N

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Number Theory

**Generalized Whittaker Functions on SU(2, 2) with Respect to the Siegel Parabolic Subgroup**

Yasuro Gon, Saitama University, Japan

**Contents:** Introduction; Generalized Whittaker functions and representation theory of SU(2, 2); Generalized Whittaker functions for P[subscript 1] principal series representations; Generalized Whittaker functions for the discrete series representations; Bibliography.

**Memoirs of the American Mathematical Society,** Volume 155, Number 738


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Probability

**Lectures on Monte Carlo Methods**

Neal Madras, York University, Toronto, ON, Canada

Monte Carlo methods form an experimental branch of mathematics that employs simulations driven by random number generators. These methods are often used when others fail, since they are much less sensitive to the “curse of dimensionality”, which plagues deterministic methods in problems with a large number of variables. Monte Carlo methods are used in many fields: mathematics, statistics, physics, chemistry, finance, computer science, and biology, for instance.

This book is an introduction to Monte Carlo methods for anyone who would like to use these methods to study various kinds of mathematical models that arise in diverse areas of application. The book is based on lectures in a graduate course given by the author. It examines theoretical properties of Monte Carlo methods as well as practical issues concerning their computer implementation and statistical analysis. The only formal prerequisite is an undergraduate course in probability.

The book is intended to be accessible to students from a wide range of scientific backgrounds. Rather than being a detailed treatise, it covers the key topics of Monte Carlo methods to the depth necessary for a researcher to design, implement, and analyze a full Monte Carlo study of a mathematical or scientific problem. The ideas are illustrated with diverse running examples. There are exercises sprinkled throughout the text. The topics covered include computer generation of random variables, techniques and examples for variance reduction of

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**Recommended Text**

Neal Madras, Lectures on Monte Carlo Methods, York University, Toronto, ON, Canada

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**AMERICAN MATHEMATICAL SOCIETY**

**International Press**

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**DECEMBER 2001 NOTICES OF THE AMS 1379**
Monte Carlo estimates, Markov chain Monte Carlo, and statistical analysis of Monte Carlo output.

**Contents:** Introduction; Generating random numbers; Variance reduction techniques; Markov chain Monte Carlo; Statistical analysis of simulation output; The Ising model and related examples; Bibliography.

**Fields Institute Monographs, Volume 16**

January 2002, 103 pages, Hardcover, ISBN 0-8218-2978-5, LC 2001053551, 2000 Mathematics Subject Classification: 65C05, 60-01; 60F10, 62C10, 82B80, All AMS members $24, List $30, Order code FIM/16N

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**Algebraic Methods in Statistics and Probability**

Marlos A. G. Viana, University of Illinois at Chicago, and Donald St. P. Richards, University of Virginia, Charlottesville, Editors

Algebraic methods and arguments in statistics and probability are well known, from Gauss's least squares principle through Fisher's method of variance decomposition. The relevance of group-theoretic arguments, for example, became evident in the 1980s. Such techniques continue to be of interest today, along with other developments, such as the use of graph theory in modelling complex stochastic systems.

This volume is based on lectures presented at the AMS Special Session on Algebraic Methods and Statistics held at the University of Notre Dame (Indiana) and on contributed articles solicited for this volume. The articles are intended to foster communication between representatives of the diverse scientific areas in which these functions are utilized and to further the trend of utilizing algebraic methods in the areas of statistics and probability.

This is one of few volumes devoted to the subject of algebraic methods in statistics and probability. The wide range of topics covered in this volume demonstrates the vigorous level of research and opportunities ongoing in these areas.


**Contemporary Mathematics, Volume 287**


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**Previously Announced Publications**

**Ramanujan: Essays and Surveys**

Bruce C. Berndt, University of Illinois, Urbana-Champaign, IL, and Robert A. Rankin, University of Glasgow, Scotland, Editors

This book contains essays on Ramanujan and his work that were written especially for this volume. It also includes important survey articles in areas influenced by Ramanujan’s mathematics. Most of the articles in the book are nontechnical, but even those that are more technical contain substantial sections that will engage the general reader.

The book opens with the only four existing photographs of Ramanujan, presenting historical accounts of them and information about other people in the photos. This section includes an account of a cryptic family history written by his younger brother, S. Lakshmi Narasimhan. Following are articles on Ramanujan’s illness by R. A. Rankin, the British physician D. A. B. Young, and Nobel laureate S. Chandrasekhar. They present a study of his symptoms, a convincing diagnosis of the cause of his death, and a thorough exposition of Ramanujan’s life as a patient in English sanitariums and nursing homes.

Following this are biographies of S. Janaki (Mrs. Ramanujan) and N. Narayana Iyer, Chief Accountant of the Madras Port Trust Office, who first communicated Ramanujan’s work to the Journal of the Indian Mathematical Society. The last half of the book begins with a section on “Ramanujan’s Manuscripts and Notebooks”. Included is an important article by G. E. Andrews on Ramanujan’s lost notebook.

The final two sections feature both nontechnical articles, such as Jonathan and Peter Borwein’s “Ramanujan and pi”, and more technical articles by Freeman Dyson, Atle Selberg, Richard Askey, and G. N. Watson.

This volume complements the book Ramanujan: Letters and Commentary, Volume 9, in the AMS series, History of Mathematics. For more on Ramanujan, see these AMS publications Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work, Volume 136, and Collected Papers of Srinivasa Ramanujan, Volume 159, in the AMS Chelsea Publishing series.
Classical Groups and Geometric Algebra

Larry C. Grove, *University of Arizona, Tucson, AZ*

"Classical groups", named so by Hermann Weyl, are groups of matrices or quotients of matrix groups by small normal subgroups.

Thus the story begins, as Weyl suggested, with "Her All-embracing Majesty", the general linear group $GL_n(V)$ of all invertible linear transformations of a vector space $V$ over a field $F$. All further groups discussed are either subgroups of $GL_n(V)$ or closely related quotient groups.

Most of the classical groups consist of invertible linear transformations that respect a bilinear form having some geometric significance, e.g., a quadratic form, a symplectic form, etc. Accordingly, the author develops the required geometric notions, albeit from an algebraic point of view, as the end results should apply to vector spaces over more-or-less arbitrary fields, finite or infinite.

The classical groups have proved to be important in a wide variety of venues, ranging from physics to geometry and far beyond. In recent years, they have played a prominent role in the classification of the finite simple groups.

This text provides a single source for the basic facts about the classical groups and also includes the required geometrical background information from the first principles. It is intended for graduate students who have completed standard courses in linear algebra and abstract algebra. The author, L. C. Grove, is a well-known expert who has published extensively in the subject area.