2003 Birkhoff Prize

The 2003 George David Birkhoff Prize in Applied Mathematics was awarded at the 109th Annual Meeting of the AMS in Baltimore in January 2003.

The Birkhoff Prize recognizes outstanding contributions to applied mathematics in the highest and broadest sense and is awarded every three years (until 2001 it was awarded usually every five years). Established in 1967, the prize was endowed by the family of George David Birkhoff (1884–1944), who served as AMS president during 1925–26. The prize is given jointly by the AMS and the Society for Industrial and Applied Mathematics (SIAM). The recipient must be a member of one of these societies and a resident of the United States, Canada, or Mexico. The prize carries a cash award of $5,000.

The recipients of the Birkhoff Prize are chosen by a joint AMS-SIAM selection committee. For the 2003 prize the members of the selection committee were: Douglas N. Arnold, Paul H. Rabinowitz, and Donald G. Saari (chair).

Previous recipients of the Birkhoff Prize are Jürgen K. Moser (1968), Fritz John (1973), James B. Serrin (1973), Garrett Birkhoff (1978), Mark Kac (1978), Clifford A. Truesdell (1978), Paul R. Garabedian (1983), Elliott H. Lieb (1988), Ivo Babuška (1994), S. R. S. Varadhan (1994), and Paul H. Rabinowitz (1998).

The 2003 Birkhoff Prize was awarded to John Mather and to Charles S. Peskin. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

John Mather

Citation

John Mather is a mathematician of exceptional depth, power, and originality.

His earliest work included contributions to foliation theory in topology and to the theory of singularities for smooth and analytic maps on R^n where he provided the rigorous foundations of this theory. Among his main contributions is a stability result. Here stability of a map means that any nearby map is equivalent to it up to diffeomorphisms of the domain and target manifolds. While this is very difficult to check directly, Mather proved that infinitesimal stability, a condition that can often be verified constructively, implies stability, and he developed an algorithm for describing the local forms of these stable mappings. These astonishing generalizations of the earlier work of Hassler Whitney have provided approaches to understand a variety of applied issues ranging from the structure of the Pareto set of the utility mapping in economics to phase transitions in physics.

Switching to the theory of dynamical systems, Mather has made several major contributions. An early highlight was his result with Richard McGehee proving that binary collisions in the Newtonian 4-body problem could accumulate in a manner that would force the system to expand to infinity in finite time. He was a co-founder of Aubry-Mather theory where, in particular, he proved that twist maps of an annulus possess so-called Aubry-Mather invariant sets for any irrational rotation number. These sets are Cantor sets, and the diffeomorphism on them is equivalent to a rigid rotation of a circle. Since KAM theory, which extends research going back to the work of Birkhoff, provides information about such situations when the rotation number is Diophantine, Mather found the missing circles in KAM theory.

Mather extended this work to multidimensional positive definite Lagrangian systems. He proved the invariant sets he found here—called Mather sets—are Lipschitz graphs over configuration space. He also developed a variational method for constructing shadowing trajectories first for twist maps and then for positive definite Lagrangian systems. In the twist map setting, he established the existence of heteroclinic orbits joining Aubry-Mather sets in the same Birkhoff instability region.

Currently he is doing seminal work on Arnold diffusion. In particular Mather proved the existence of Arnold diffusion for a generic perturbation of an a priori unstable integrable Hamiltonian system, solving the problem left standing from Arnold's famous 1964 paper.

Mather is a member of the U.S. and Brazilian National Academies of Sciences, a Guggenheim and

Sloan Fellow, and the winner of the 1978 John J. Carty Medal from the U.S. Academy.

Biographical Sketch

John N. Mather was born in Los Angeles, California, on June 9, 1942. He received a B.A. from Harvard University in 1964 and a Ph.D. from Princeton University in 1967. From 1967 to 1969 he was *professeur associé* (visiting professor) at the Institut des Hautes Études Scientifiques (IHÉS) in France. In 1969 he joined the faculty of Harvard University as associate professor and was promoted to professor in 1971. He was a visiting professor at Princeton University in 1974–75 and joined the faculty of Princeton University as professor in 1975. He was a visiting professor at IHÉS in 1982–83 and at the Eidgenössische Technische Hochschule in Zurich in 1989–90.

Mather was an editor of the *Annals of Mathematics* from 1990 to 2001 and has been an editor of the *Annals of Math. Studies* from 1990 to the present.

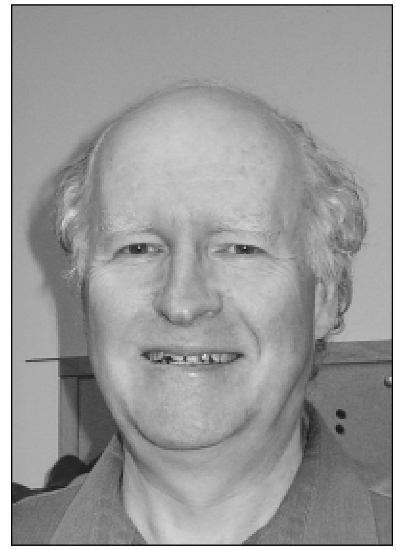
Mather was a Sloan Fellow in 1970–72 and a Guggenheim Fellow in 1989–90. He was elected a member of the National Academy of Sciences in 1988 and a member of the Brazilian Academy of Sciences in 2000. He received the John J. Carty Medal of the National Academy of Sciences in 1978 and the Ordem Nacional do Mérito Científico from the Brazilian Academy of Sciences in 2000.

Mather's current research is in the area of Hamiltonian dynamics. In the past he has worked in the theories of singularities of mappings and foliations.

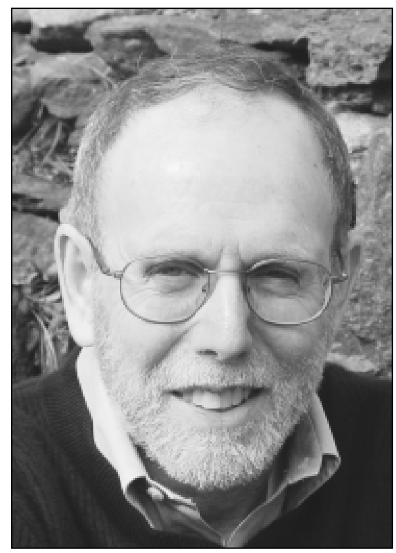
Response

It is a pleasure to accept the Birkhoff Prize for my work in singularities of mappings, the theory of foliations, and Hamiltonian dynamics. I greatly appreciate the generous citation of my achievements, as well as the honor of the prize. While I have not (yet) worked on applications of mathematics as such, I have always been fascinated by theoretical mathematical questions that originated in applications, for example, the n-body problem in Newtonian mechanics. Poincaré showed long ago that the study of the dynamics of area-preserving mappings of surfaces provides important insights into this problem. G. D. Birkhoff greatly extended Poincaré's work on area-preserving mappings, and his work was one of the inspirations for my contribution to Aubry-Mather theory.

I am grateful to my teachers at Harvard University and Princeton University, as well as colleagues and friends, from whom I have learned so much. I also wish to express my appreciation for the system of higher education, which makes a career of mathematical research possible.



John Mather



Charles S. Peskin

Charles S. Peskin

Citation

Charles Samuel Peskin has devoted much of his career to understanding the dynamics of the human heart. Blurring disciplinary boundaries, he has brought an extraordinarily broad range of expertise to bear on this problem: mathematical modeling, differential equations, numerical analysis, high performance computing, fluid dynamics, physiology, neuroscience, physics, and engineering. His primary tool for understanding the heart is computer simulation. In work spanning more than two decades, much of it with David McQueen, Peskin has developed a computer model that simulates blood circulation through the four chambers of the heart and in and out of the surrounding circulatory system along with the deformation of the cardiac muscle and the valves. This virtual heart enables experimentation in silico that would be impossible in vivo and is of tremendous value to the study of normal heart function and a variety of pathologies, to plan interventions, and to design prosthetic devices.

Peskin's computer simulations are based on the immersed boundary method, a unique numerical method he developed for the solution of dynamic fluid-structure interactions. This method, which is built on a novel approach to couple a fluid description in Eulerian coordinates to a solid description in Lagrangian coordinates, was originally designed to describe the flow of blood around cardiac valve surfaces. But it has found much wider use, allowing simulation of a variety of complex systems, such as the inner ear, swimming fish, locomoting microbes, flowing suspensions, and filaments flapping in soap films. The development and analysis of the immersed boundary method is an ongoing and active field of study.

While the heart is a large biological motor, much of Peskin's recent research concerns biological motors at the smallest scales. Here too he brings innovative mathematical modeling and computational simulation to bear, exploring and explaining the microscopic machinery inside cells which harness Brownian motion for transport and motility.

A former MacArthur fellow, Charles Peskin is a member of the American Academy of Arts and Sciences, the National Academy of Sciences, and the Institute of Medicine.

Biographical Sketch

Charles S. Peskin was born in New York City in 1946. His mathematical education began at the Ethical Culture School, where arithmetic was done with sticks tied together, when possible, in bundles of ten to explain the decimal system. His father, an electrical engineer, was another early mathematical influence, teaching him the elements of algebra from the simple yet mysterious example $x + y = 10, x - y = 2$. At Morristown (New Jersey) High School, Peskin had an inspiring mathematics teacher named Betty Wagner, who emphasized sketching graphs of functions and who was kind about undone homework. There is a picture of Peskin in his high school yearbook standing in front of these words written in chalk on the blackboard: "Resolved: That Homework Be Abolished".

Peskin studied engineering and applied physics at Harvard (A.B., 1968). "Engineering at Harvard? Isn't that MIT?" was a common comment he heard at the time. He then entered the M.D.-Ph.D. program at the Albert Einstein College of Medicine, Bronx, NY, but dropped out of the M.D. part of the program after completing a Ph.D. (1972) in physiology with a thesis entitled "Flow patterns around heart valves: A digital computer method for solving the equations of motion". This thesis was the beginning of the work that has now led to the Birkhoff Prize. Once he had decided not to go on to the M.D., Peskin nevertheless remained at the Albert Einstein College of Medicine for a year, studying pediatric cardiology and pulmonary medicine. During this time he developed an interest in fetal circulation and congenital heart disease, and he has since done mathematical modeling in these areas.

In 1973 Peskin joined the faculty of the Courant Institute of Mathematical Sciences, New York University, where he has been ever since. He became a professor of mathematics in 1981 and received the additional title professor of neural science in 1995. At NYU, Peskin teaches courses like Mathematical Aspects of Heart Physiology, Mathematical Aspects of Neurophysiology, Partial Differential Equations in Biology, Biomolecular Motors, and a freshman seminar on Computer Simulation. He is the coauthor (with Frank Hoppensteadt) of *Modeling and Simulation in Medicine and Biology,* Second Edition (Springer-Verlag, 2002). At New York University, Peskin has received the Sokol Faculty Award in the Sciences (1992) and the Great Teacher Award of the NYU Alumni Association (1999).

Peskin's other honors are the MacArthur Fellowship (1983–88), SIAM Prize in Numerical Analysis and Scientific Computing (1986), Gibbs Lecturer (1993), Cray Research Information Technology Leadership Award (joint with David M. McQueen, 1994), Sidney Fernbach Award (1994), Mayor's Award for Excellence in Science and Technology (1994), and von Neumann Lecturer (1999). He is a fellow of the American Institute for Medical and Biological Engineering (since 1992), fellow of the American Academy of Arts and Sciences (since 1994), member of the National Academy of Sciences (since 1995), fellow of the New York Academy of Sciences (since 1998), and member of the Institute of Medicine (since 2000).

Response

It is a pleasure to accept the George David Birkhoff Prize of the Society for Industrial and Applied Mathematics and the American Mathematical Society. I am awed to be placed in the company of former winners such as Jürgen Moser, Fritz John, Marc Kac, Paul Garabedian, and S. R. S. Varadhan, whom I also count as colleagues and friends. Although some of them are no longer with us, their influence, both mathematical and personal, surely lives on. Some of that influence is encapsulated in particularly memorable remarks. I especially remember when Mark Kac greeted me in his booming voice: "Ah, Peskin, the man with the two-dimensional heart!" I think he would be pleased to see that I have now won this great honor in large part for a three-dimensional heart model. Then there is the famous remark of Fritz John (that he claims never to have said) that the rewards of mathematics are the grudging admiration of a few friends. As the recipient of a reward of mathematics today, I would like to thank the mathematics community for welcoming me without proper credentials (my Ph.D. is in physiology) and (with no hint of grudging that I have ever detected) for honoring my research.

I would like to thank my father, Edward Peskin, and my thesis advisors, Edward Yellin and Alexandre Chorin, for starting me off on the road that has now led to the Birkhoff Prize. It was my father, an electrical engineer, who first suggested to me that it might be a good idea to apply mathematical methods to biological problems. It was Yellin, a mechanical engineer turned physiologist, who first introduced me to the fascinating dynamics of the heart and its valves. Around this time I had the incredible good luck to meet Alexandre Chorin, who invited me to his course on fluid mechanics at the Courant Institute. Chorin taught me his new projection method for incompressible flow; set me up with an office and an account on the CDC6600 (which we programmed with punch cards—I still recall the satisfying sounds of the keypunch and the relaxed mode of submitting a deck of cards to the computer and then going for a walk around Washington Square while awaiting the result); and introduced me to such inspiring characters as Peter Lax, Cathleen Morawetz, and Olof Widlund, each of whom has had a profound influence on my life and work.

My long-term colleagues in the research that is described in the citation for the Birkhoff Prize are David McQueen (in the case of the heart) and George Oster (in the case of biological motors). Both deserve a large share of the credit. McQueen handles all of the details of heart model construction, conducts our computer experiments, and visualizes the results with custom software of his own design. My role is to think about the methodology and suggest changes as needed. In the case of biomolecular motors, I am particularly grateful to George Oster for introducing me to this exciting field. Most of the concepts in our joint work have been his. I have been happy to help him reduce some of these concepts to specific mathematical models and computer simulation programs, which we can then use to see whether the concepts are capable of explaining the observed behavior of the biomolecular motor.

I would like to conclude with a few words of explanation about the immersed boundary method. This is a numerical method for fluid-structure interaction that I originally introduced to study the flow patterns around heart valves. Heart valve leaflets are thin membranes that move passively in the flow of blood and yet have a profound influence on the fluid dynamics. Examples of this influence are that they stop the flow when the valve is closed, and when the valve is open, the leaflets shear the flowing blood to create vortices that then participate in efficient valve closure, as was first described by Leonardo da Vinci.

The standard way to model this situation would be to treat the valve leaflet as an elastic membrane obeying Newton's laws of motion with forces calculated in part from the elasticity of the membrane and in part by evaluating the fluid stress tensor on both sides of the membrane. Then the fluid equations would have to be supplemented by the constraint that the velocity of the fluid on either side of the membrane must agree with the instantaneously known velocity of the elastic membrane itself. There are two difficulties with this standard approach to the problem. First, the valve leaflet is incredibly thin and light, with hardly any mass per unit area. (Indeed, if the mass per unit area were zero, the dynamics of the valve would not be noticeably different.) Because of its small mass, the valve leaflet is supersensitive to any imbalance in the forces acting upon it. The second challenge is the practical one of evaluating the fluid stress tensor on either side of the boundary. This seems difficult (or at least messy) to do numerically, unless the computational grid is aligned with the boundary. On the other hand, in a moving boundary problem, it is both expensive and complicated to recompute the grid at every time step in order to achieve alignment.

In the immersed boundary method, the mass of the heart valve leaflet is idealized as zero. (Recent work shows how to handle immersed boundaries of nonzero mass, but I won't discuss that here.) This means that the sum of the elastic force and the fluid force on any part of the immersed boundary has to be zero. Once we know this, it becomes unnecessary to evaluate the fluid stress tensor at the boundary at all! We can find the force of any part of the boundary on the fluid by evaluating the elastic force on that part of the boundary. (Note the use of Newton's third law: the force of boundary on fluid is minus the force of fluid on boundary.) All we need is a method for transferring the elastic force from the immersed boundary to the fluid. On a Cartesian grid, this may be done by spreading each element of the boundary force out over nearby grid points. The particular way that this is done in the immersed boundary method involves a carefully constructed approximation to the Dirac delta function. This force-spreading operation defines a field of force on the Cartesian lattice that is used for the fluid computation. Then the fluid velocity is updated under the influence of that force field. The Navier-Stokes solver that updates the fluid velocity does not know about the geometry of the heart valve leaflet; it just works with a force field that happens to be zero everywhere except in the immediate neighborhood of the leaflet. Note that there is no constraint on the fluid velocity coming from the state of motion of the leaflet. On the contrary, since the mass of the leaflet is zero, the leaflet velocity is not a state variable of the problem. Indeed, the no-slip condition has been turned on its head: it is now the equation of motion of the leaflet instead of a constraint on the fluid. The local fluid velocity at a point of the leaflet is evaluated by interpolation from the Cartesian grid. The same approximate delta function that was used to spread force can also be used to get an interpolation operator that is the adjoint (or transpose) of the force-spreading operator.

In summary, the immersed boundary method avoids many of the difficulties and pitfalls of the standard approach to fluid-structure interaction. By representing an immersed elastic boundary in terms of the forces applied by the immersed elastic boundary to the fluid, the immersed boundary method avoids any consideration of boundary geometry in the fluid computation; makes it unnecessary to evaluate the fluid stress tensor at the immersed elastic boundary; and makes it possible to simulate immersed elastic boundaries that are essentially massless, like the valve leaflets of the human heart.