A Tribute to Boris Weisfeiler

In 1985 the mathematician Boris Weisfeiler disappeared while hiking alone in a remote area of Chile. At the time, he was a professor of mathematics at Pennsylvania State University and was widely recognized for his work in algebraic groups. What happened to him remains a mystery, and to this day it is not known whether he is still alive.

Born in the Soviet Union, Weisfeiler received his Ph.D. in 1970 from the Leningrad branch of the Steklov Institute, where his adviser was E. B. Vinberg. Weisfeiler emigrated to the United States in 1975 and worked with Armand Borel at the Institute for Advanced Study. The next year he joined the faculty at Pennsylvania State University. In 1981 he became an American citizen.

Weisfeiler's disappearance has been the subject of several newspaper articles (see, for example, "Chilean Mystery: Clues to Vanished American", by Larry Rohter, *New York Times*, May 19, 2002; and "Tracing a Mystery of the Missing in Chile", by Pascale Bonnefoy, *Washington Post*, January 18, 2003). Further information about media coverage, as well as the present status of the investigation into his disappearance, may be found at `http://weisfeiler.com/boris`.

On the occasion of the publication in Chile of a book about Weisfeiler's disappearance, the *Notices* decided to present a brief tribute to his life and work. What follows is a short summary of his mathematical work and a review of the book. This is not an obituary, as hope remains that Weisfeiler is still alive. Nevertheless, it seems appropriate to commemorate this lost member of the mathematical community, whose absence is keenly felt.

—Allyn Jackson

The Mathematics of Boris Weisfeiler

Alexander Lubotzky

Boris Weisfeiler's mathematical activity extended over more than twenty years of research in the USSR and the USA, during which time he published three dozen research papers. All this ended abruptly in early 1985 with his disappearance—right after he had published his most influential papers.

Weisfeiler's area was algebraic groups in all their aspects and directions. During his years of activity the theory of algebraic groups over algebraically closed fields had already been well understood, and Weisfeiler was part of the trend of studying the more difficult questions concerning the case when the field is not algebraically closed and the groups do not split or—even worse—are nonisotropic. His specialty was the positive characteristic case. In these cases the connection between the algebraic group and its Lie algebra is more subtle, and a great deal of effort was needed to study questions whose solutions in characteristic zero were pretty standard.

A popular subject of that period was the study of "abstract homomorphisms" between algebraic groups; i.e., assume k and k' are fields and G and G' are algebraic groups defined over k and k', respectively, and let $\varphi : G(k) \to G'(k')$ be a homomorphism of groups. The standard "wishful" result in this theory is the claim that, under suitable conditions, such a homomorphism is algebraic; i.e. there is a monomorphism of fields $k \to k'$ and after identifying k as a subfield of k', the homomorphism φ is a homomorphism in the category of algebraic groups, i.e., given by polynomial maps. Various methods have developed to tackle such problems. Usually, one associates some geometry with the algebraic groups in the spirit of Klein's Erlangen program, as projective geometry is associated with $GL_n(k)$, and hence the map φ induces a map between the associated geometries, which turns out to be a useful way to study the original φ. The works of Borel, Tits, O'Meara, and others settled most questions for isotropic groups, but the

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Photographs courtesy of Olga Weisfeiler.

Boris Weisfeiler

nonisotropic cases were always of special difficulty. This was a direction in which Weisfeiler made some important contributions, but unfortunately they are too technical to be elaborated upon here.

Weisfeiler is mainly remembered and quoted for the two contributions he made in the last year (1984) just before his disappearance. The first one is truly spectacular, although it took some time to realize that: In [W1] (following a partial result in [MVW]) Weisfeiler proved a strong approximation theorem for general linear groups. To get a feeling of what it says, let us take a very special case: Let $SL_n(\mathbb{Z})$ be the group of $n \times n$ integral matrices of determinant one, and let $\Gamma \leq SL_n(\mathbb{Z})$ be a Zariski-dense subgroup. Then, Weisfeiler's theorem says (in fact, this case is already covered by [MVW]) that Γ is almost dense in $SL_n(\mathbb{Z})$ in the congruence topology of $SL_n(\mathbb{Z})$; i.e., its closure there is of finite index. The congruence topology of $SL_n(\mathbb{Z})$ is the one for which the groups

$$\{\Gamma(m) = Ker(SL_n(\mathbb{Z}) \to SL_n(\mathbb{Z}/m\mathbb{Z}) | m \in \mathbb{Z}\}$$

serve as a system of neighborhoods of the identity.

On the surface, this is a technical theorem comparing two topologies of $SL_n(\mathbb{Z})$, but, in fact, this result has many extremely useful consequences. Being Zariski dense is a very weak condition, but having the quality of being congruence dense gives a lot of corollaries on the finite quotients of Γ. It says, for example, that for almost every prime number p, the finite simple group $PSL_n(p)$ is a quotient of Γ. For a general, finitely generated linear group Γ, it implies, for example, that either Γ is almost solvable or it has a finite index subgroup with infinitely many different finite simple quotients. It is not surprising that Weisfeiler's strong approximation theorem has since played an important role in the study of infinite linear groups, in asymptotic group theory in general and subgroup growth in particular. (The reader is referred to [LS] and especially to Window 9, "Strong Approximation for Linear Groups", which describes some of the applications). Interestingly enough Weisfeiler used the (at that time brand new) classification of finite simple groups (CFSG) for his proof. He was probably the first to see how CFSG could be used for infinite linear groups. (By now, there are proofs which do not require the CFSG; see [LS] and the references therein.)

In [W2], Weisfeiler made another remarkable use of the CFSG for linear groups. He announced a sharp bound on the index of the abelian normal subgroup of finite linear subgroups of GL_n. The existence of such a bound was proved by Jordan, with extensions and better bounds proven by many including Brauer and Feit. Weisfeiler's result is still, twenty years later, the best known result. Unfortunately, a detailed proof has never appeared.

Boris Weisfeiler's mathematical work was suddenly and tragically cut—but his name is still remembered by all those who knew him or who work in related areas.

References

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El Ultimo Secreto de Colonia Dignidad— A Book Review

Reviewed by Neal Koblitz

El Ultimo Secreto de Colonia Dignidad
(The Last Secret of Colonia Dignidad)
Carlos Basso
Santiago, Chile
Mare Nostrum Publishers, 2002

Boris Weisfeiler disappeared on January 5, 1985, while hiking in a remote, mountainous area of south-central Chile near the border with Argentina. His body has never been found, and his disappearance remains a mystery. The book *El Ultimo Secreto de Colonia Dignidad* (*The Last Secret of Colonia Dignidad*) is a detailed account of the case

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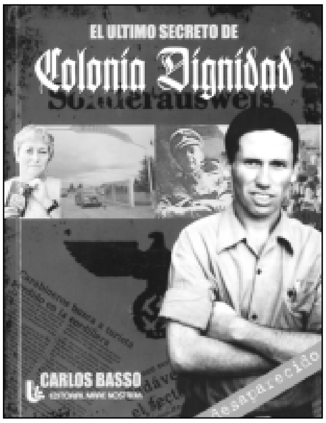

by Chilean journalist Carlos Basso. Basso bases his investigation on recently declassified U.S. Embassy cables and memos, as well as earlier published sources.

Three theories have emerged about what happened: (1) Weisfeiler drowned accidentally while trying to cross a river. This was the official version promulgated by the Chilean authorities at the time. (2) He was shot by local police or a military patrol, who thought that he was a subversive infiltrator from Argentina and who covered up the incident when they realized their mistake. (3) A military patrol arrested him and handed him over to nearby Colonia Dignidad, an enclave of ultrarightist German immigrants founded and at the time still led by ex-Nazi Paul Schäfer. Thinking that Weisfeiler was a "Jewish spy" working for Nazi-hunters, they imprisoned and eventually killed him.

Sifting through the conflicting reports of witnesses and informants, Basso argues persuasively that the third theory is most likely correct. Ever since the U.S.-backed coup d'état on September 11, 1973, which ousted the democratically elected President Salvador Allende, the military dictatorship had a close relationship with Colonia Dignidad and used the colony as a place for the detention and torture of dissidents and political opponents. Even after Chile's transition to a democratic government in 1990, the colony continued to be protected by its allies in the military and intelligence services. In 1996 the government unsuccessfully tried to search the colony and to arrest Paul Schäfer on child-rape charges. To this day none of the leaders of the cult have been prosecuted for their crimes.

Basso's book is not without flaws. He has a tendency toward sensationalism. For example, he asks why the U.S. Embassy delayed for over a year before starting to press the Chileans for a full investigation of Weisfeiler's disappearance. He correctly observes that the Reagan administration had a friendly relationship with the Pinochet regime, which in turn had close ties to Colonia Dignidad. This—along with the usual bureaucratic inertia—would have been enough to explain the U.S. government's procrastination. However, Basso goes much further in his speculations. He recounts the now well-known story of how in the early years of the Cold War the U.S. relied on ex-Nazis to set up spy networks in Eastern Europe and help with other anti-Soviet operations. He also recalls President Reagan's controversial visit in 1985 to Bitburg, Germany, where the President laid a wreath at a cemetery where SS soldiers were buried. Basso suggests that as late as 1985 factions of the U.S. government had secret ties with and sympathy for German Nazis and that this explains the reluctance to investigate the role of Colonia Dignidad in Weisfeiler's disappearance. His argument here is unconvincing and far-fetched.

Despite the weaknesses in the book, Basso has performed a valuable service. One can hope that the publication of his book in Chile will increase pressure on the government there to pursue the Weisfeiler case, determine who was responsible for his disappearance, and bring them to justice.