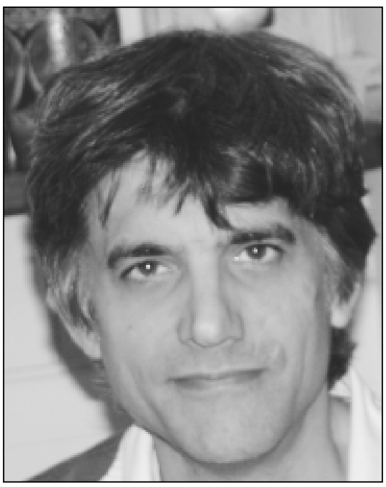
2004 Veblen Prize



David Gabai

The 2004 Oswald Veblen Prize in Geometry was awarded at the 110th Annual Meeting of the AMS in Phoenix in January 2004.

The Veblen Prize is awarded every three years for a notable research memoir in geometry or topology that has appeared during the previous five years in a recognized North American journal (until 2001 the prize was usually awarded every five years). Established in 1964, the prize honors the memory of Oswald Veblen (1880–1960), who served as president of the AMS during 1923–24. It carries a cash award of $5,000.

The Veblen Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2004 prize the members of the selection committee were: Andrew J. Casson, Yakov Eliashberg, and John W. Morgan (chair).

Previous recipients of the Veblen Prize are: Christos D. Papakyriakopoulos (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), Shing-Tung Yau (1981), Michael H. Freedman (1986), Andrew J. Casson (1991), Clifford H. Taubes (1991), Richard Hamilton (1996), Gang Tian (1996), Jeff Cheeger (2001), Yakov Eliashberg (2001), and Michael J. Hopkins (2001).

The 2004 Veblen Prize was awarded to David Gabai. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Citation

The 2004 Veblen Prize in Geometry is awarded to David Gabai of Princeton University in recognition of his work in geometric topology, in particular, the topology of 3-dimensional manifolds.

Since its beginnings in the early twentieth century, 3-dimensional topology has occupied a central role in geometric topology. It is tantalizingly close to what we can directly visualize, yet with its knotting phenomena it is an incredibly complex and difficult subject. For the last twenty years, Gabai has been one of the leading figures in this field. He has led many of the main avenues of development, developing tools in order to solve some of its most important problems himself, tools that have turned out to be central to the further development of the subject.

One aspect of 3-dimensional topology greatly influenced by Gabai is the study of surfaces inside a 3-manifold and the intersection patterns of two or more of these. His introduction of the notion of thin position, which he used to resolve the question known as "Property R" about when surgery on a knot in the 3-sphere can yield a 3-manifold homeomorphic to the product of the 2-sphere and a circle, has found application far beyond Gabai's original use, for example, in the proof that knots are determined by their complements. Gabai's study of surfaces is achieved in large part through the study of more general objects, codimension-one laminations, in a 3-manifold. In a sequence of papers beginning in the 1980s and summarized in his talk at the 1990 International Congress of Mathematicians in Kyoto entitled "Foliations and 3-manifolds", Gabai developed the theory of these objects. In Gabai's hands, they have served to help unlock some of the topological mysteries of 3-dimensional topology. This theory of these laminations has now grown to the extent that it is a subdomain of 3-dimensional topology in its own right.

More recently, Gabai has investigated 3-dimensional hyperbolic manifolds. Hyperbolic 3-manifolds are a rich and much studied class. Conjecturally at least, they are by far the richest and most interesting class. One of the central problems in 3-manifold topology is how to tell when a 3-manifold is hyperbolic, i.e., has a hyperbolic structure. By strikingly original arguments in a series of papers [listed below], Gabai has answered this question in a special case, by showing that every irreducible 3-manifold with the homotopy type of a hyperbolic manifold has a hyperbolic structure. Further developments of his methods led to a proof by Gabai of the Smale Conjecture for hyperbolic 3-manifolds—describing the homotopy type of the space of self-diffeomorphisms—and also to new estimates for the volumes of hyperbolic 3-manifolds.

"Homotopy hyperbolic 3-manifolds are virtually hyperbolic", *J. Amer. Math. Soc.* **7** (1994), no. 1, 193–8.

"On the geometric and topological rigidity of hyperbolic 3-manifolds", *J. Amer. Math. Soc.* **10** (1997), no. 1, 37–74.

(jointly with Robert Meyerhoff and Nathaniel Thurston) "Homotopy hyperbolic 3-manifolds are hyperbolic", *Ann. of Math.* (2) **157** (2003), no. 2, 335–431.

Biographical Sketch

David Gabai received his B.S. from the Massachusetts Institute of Technology (1976), his M.A. from Princeton University (1977), and his Ph.D. from Princeton (1980) under the direction of William Thurston. After positions at Harvard and the University of Pennsylvania, he spent most of the years 1986–2001 at the California Institute of Technology and has been at Princeton University since 2001. He held visiting positions at the Institute for Advanced Study, Princeton (1982–83, fall 1989); the Mathematical Sciences Research Institute, Berkeley (1984–85, 1996–97); and the Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France (1985–86). He has received a National Science Foundation Postdoctoral Fellowship, a Sloan Fellowship, and an AMS Centennial Fellowship. He gave a 45-minute invited talk at the 1990 International Congress of Mathematicians in Kyoto and an hour invited talk at the 1995 joint meeting of the AMS and the Mexican Mathematical Society in Guanajuato, Mexico. He also gave the 1996 Porter Lectures (Rice University), the 2001 Marston Morse memorial lectures (IAS), and the 2002 Unni Namboodiri memorial lectures (University of Chicago).

Response

I have been incredibly lucky all my life. As a graduate student at Princeton, Bill Thurston suggested an area of mathematics, foliations on 3-manifolds, which matched my talents. Being in a field then considered by many experts to be either finished or peripheral, I could slowly and happily build my intuition and technical skills without the distraction of noise or the danger of being trampled. It turned out my constructions of foliations were useful and of contemporary interest, so I got a great job at Caltech. In 1991 I attempted to teach a topics course on the hyperbolic geometry that I should have learned as a graduate student. I started with Chapter 1 of Thurston's 1977–78 lecture notes but could not get past Chapter 5. There he discussed why Mostow's rigidity theorem implies that a manifold finitely covered by a hyperbolic 3-manifold is homotopy equivalent to a hyperbolic 3-manifold. I got stuck trying to prove the converse. Ultimately, the converse became the start of work cited here.

It is a great pleasure to thank Ulrich Oertel, with whom I introduced essential laminations in 1986, and Rob Meyerhoff and Nathaniel Thurston, who worked with me on the homotopy hyperbolic project. I also thank my long-time collaborator Will Kazez and my collaborators Peter Milley and Valentin Poenaru. Victor Guillemin's beautiful differential topology course during my last semester at MIT kept me from opting out of mathematics for medical school. Bill Thurston's influence has been immense. I am in debt to my teachers, and I count my students among them. I very much appreciate the many mathematicians who have encouraged me over the years. Finally, I thank my many hosts in China, England, France, Israel, and Japan for providing quiet environments during short visits so that I could hide out and nurture my thoughts.

Much of what I know was done or inspired by former prizewinners. It is humbling to be the recipient of the 2004 Veblen Prize.