2005 Bôcher Prize

The 2005 Maxime Bôcher Memorial Prize was awarded at the 111th Annual Meeting of the AMS in Atlanta in January 2005.

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–10. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the American Mathematical Society or the paper should have been published in a recognized North American journal. Currently, this prize is awarded every three years. The prize carries a cash award of $5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2005 prize the members of the selection committee were: Charles L. Fefferman, Leon Simon (chair), and Daniel I. Tataru.

Previous recipients of the Bôcher Prize are: G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), Thomas Wolff (1999), Daniel Tataru (2002), Terence Tao (2002), and Fanghua Lin (2002).

The 2005 Bôcher Prize was awarded to Frank Merle. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Frank Merle

Citation

The Bôcher Prize is awarded to Frank Merle (Cergy-Pontoise, France) for his fundamental work in the analysis of nonlinear dispersive equations, represented most recently by his joint work "Stability of blow-up profile and lower bounds for blow-up rate for the critical generalized KdV equation" (with Y. Martel), *Annals of Math.* 155 (2002), 235–280; "Blow up in finite time and dynamics of blow up solutions for the L^2-critical generalized KdV equation" (with Y. Martel), *J. Amer. Math. Soc.* 15 (2002), 617–664; and "On universality of blow-up profile for L^2 critical nonlinear Schrödinger equation" (with P. Raphael), *Invent. Math.* 156 (2004), no. 3, 565–672.

Biographical Sketch

Frank Merle was born November 22, 1962, in Marseille, France. He received his Ph.D. at the École Normale Supérieure in 1987 and held a Centre National de la Recherche Scientifique (CNRS) research position there from 1988 to 1991. From 1989 to 1990 he was assistant professor at the Courant Institute. Since 1991 he has been professor of mathematics at the Université de Cergy-Pontoise. From 1998 to 2003 he was a member of the Institut Universitaire de France and in 1996 and from 2003 to 2004, a member of the Institute for Advanced Study in Princeton.

Over the years he has held various visiting positions at the University of Chicago, Rutgers University, Stanford University, the Courant Institute, the Institute for Advanced Study in Princeton, the Mathematical Sciences Research Institute in Berkeley, the University of Tokyo, the CNRS, and Leiden University.

Merle's awards and honors include the Institut Poincaré Prize in Theoretical Physics (1997), the Charles-Louis de Saulse de Freycinet Prize of the Académie des Sciences de Paris (2000), and an invitation to speak at the International Congress of Mathematicians in 1998.

Response

It is a great honor to be awarded the Bôcher Memorial Prize. I am grateful to the prize committee and to the American Mathematical Society for their recognition of this research. I am also deeply grateful to Jean Bourgain and Carlos Kenig for their constant support and early recognition of this work, and to George Papanicolaou, who introduced me to these problems and supported me. I would like to thank people who influenced me early in my career and over the years, such as Henri Berestycki; Haim Brezis; Louis Nirenberg (who was a role model); Hiroshi Matano; Robert V. Kohn; Abbas Bahri; Jean Ginibre; my close collaborators Yvan Martel, Pierre Raphael, and Hatem Zaag; and family and friends.

The cited work is concerned with the Critical Generalized Korteweg-de Vries (CGKdV) and Critical Schrödinger (CNLS) equations. We considered the existence and description of solutions which break down (or blow up) in finite time and related qualitative properties of the equations such as long-time behavior of global solutions. Such problems were proposed as models for understanding breakdown in the Hamiltonian context. A number of people, including Ya. G. Sinai and V. E. Zakharov, first investigated these problems in the 1970s using formal asymptotics combined with numerical methods. Initial work led to less-than-clear results for CGKdV and to a controversial blow-up rate for CNLS. In 1988, for the generic behavior of the breakdown, Papanicolaou and coauthors suggested a rate equal to the scaling rate corrected by $\sqrt{Loglog(t)}$, but this rate is different from that of the explicit blow-up solution.

In the last decade, from Bourgain's seminal work; from the work of Kenig, Gustavo Ponce, Luis Vega; and now from the work of a large mathematical community, a huge breakthrough has arisen out of analytical methods based on frequency localization properties of the solution of dispersive equations. This approach extends linear-type behavior (in particular global existence results) to various nonlinear contexts. For nonlinear-type behavior (in particular for the qualitative study of breakdown), little is known apart from stability results of the 1980s based on global energy arguments by P.-L. Lions and M. Weinstein.

The approach we took for these problems was not to justify possible formal asymptotics and construct one solution with a given behavior; instead, we looked for properties of these equations that were rigid enough to classify different blow-up and dynamical behaviors. Since the 1980s this geometrical approach has had great success in elliptic theory and in geometry. Earlier research on the nonlinear heat equation (Merle and Zaag) suggested that this approach might also be successful for evolution equations. Using the Hamiltonian structure, we were able to localize in physical space dispersive effects which occur naturally at infinity. By their local nature, these effects give a new set of estimates and provide a dynamical rigidity for the asymptotic behavior of solutions (by way of a monotonicity formula, or by local quantities which do not oscillate in time or which satisfy a maximum principle).

For the CGKdV problem, a mechanism of balance between local dispersive effects and Hamiltonian constraints on the solutions allows us to prove and describe blow-up. In the process, we also eliminate the formally expected candidate. Nevertheless, getting a sharp lower bound for the blow-up rate remains an open problem. In the subcritical case, these techniques give asymptotic stability in the energy space of a soliton or finite sum of solitons.

For the CNLS problem, an exact description of blow-up is given (at least for solutions with a single blow-up point). It confirms that the remarkable conjecture of Papanicolaou (along with M. Landman, C. Sulem, and P.-L. Sulem) is the only generic behavior. Additional rigidities for the global behavior of solutions are also exhibited.

In the future, I think three directions should be investigated. The first is to extend this approach to other dispersive problems. Bearing in mind the qualitative elliptic theory of the 1980s and 1990s, the second direction is to carry out a similar program in the context of oscillatory integral problems. In particular, I think questions from the dynamical systems viewpoint should be considered, such as classification of connections between critical points. The last direction is to develop techniques using localization in both space and frequency to investigate a new set of questions.

Again, I thank the prize committee for honoring these lines of research, and I look forward to continued work on them.