2005 Book Prize

The AMS Book Prize was awarded at the Joint Mathematics Meetings in Atlanta in January 2005.

The Book Prize was established in 2003 to recognize a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize amount is $5,000. This is the first time the prize has been given.

The Book Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2005 prize the members of the selection committee were: Rodrigo Banuelos, Steven G. Krantz (chair), H. W. Lenstra, Dale P. Rolfsen, and Bhama Srinivasan.

The 2005 Book Prize was awarded to WILLIAM P. THURSTON. The text that follows presents the selection committee's citation and a brief biographical sketch.

Citation

Three-Dimensional Geometry and Topology by William P. Thurston, edited by Silvio Levy.

William P. Thurston's "Geometrization Program" is one of the big events of modern mathematics. The main thrust of the program is to prove a classification of all 3-manifolds by showing that each such manifold can be broken up into pieces, each of which admits a geometric structure which is hyperbolic, Euclidean, spherical, or one of five other model 3-dimensional geometries. A corollary of the program would be the Poincaré conjecture.

More than twenty years ago, Thurston wrote an extensive set of notes explaining the key ideas of his program. These notes were circulated informally by the Princeton Mathematics Department—a copy could be had for the cost of the photocopying—and today the book is in most mathematics libraries. The contents of these notes cannot be considered to be a proof of the geometrization conjecture. They are instead a manifest, laying out all the key ideas and explaining how things fit together. The book, *Three-Dimensional Geometry and Topology*, is the first volume of a multi-volume work projected to provide all the details of the proof of Thurston's program. It begins at a quite elementary level, but takes the reader to a rather sophisticated stage of classifying the uniformizing geometries of a compact 3-manifold. This result is a major step of the geometrization program. Even though the geometrization program remains unproved, this is exciting and vital mathematics.

William P. Thurston

Thurston's book is nearly unique in the intuitive grasp of subtle geometric ideas that it provides. It has been enormously influential, both for graduate students and seasoned researchers alike. Certainly the army of people who are working on the geometrization program regard this book as "the touchstone" for their work. A book that has played such an important and dynamic role in modern mathematics is eminently deserving of the AMS Book Prize.

Biographical Sketch

William P. Thurston was born October 30, 1946, in Washington, D.C. He received his Ph.D. in mathematics from the University of California at Berkeley in 1972. He held positions at the Institute for Advanced Study in Princeton (1972–73) and the Massachusetts Institute of Technology (1973–74) before joining the faculty of Princeton University in 1974. Thurston returned to Berkeley, this time as a faculty member, in 1991, and became director of the Mathematical Sciences Research Institute in 1993. He moved to UC Davis in 1996 and in

2003 accepted his current position at Cornell University, where he holds joint appointments in the Department of Mathematics and the Faculty of Computing and Information Science.

Thurston held an Alfred P. Sloan Foundation Fellowship from 1974 to 1975. In 1976 he was awarded the AMS Oswald Veblen Geometry Prize for his work on foliations. In 1979 he became the second mathematician ever to receive the Alan T. Waterman Award, and in 1982 he was awarded the Fields Medal. He is a member of the American Academy of Arts and Sciences and the National Academy of Sciences.

Response

I feel especially honored to receive the AMS Book Prize for *Three-Dimensional Geometry and Topology*, because I invested so much of myself in it.

This book grew from a portion of lecture notes that I distributed to a large mailing list in the olden days before the Web, now available electronically at `http://www.msri.org/publications/books/gt3m/`. At the time, I had grown dissatisfied with the usual vehicles for recording and communicating mathematics. One issue has been that the informal drawings and diagrams that mathematicians often use when talking with each other are quite often missing from papers and books. When I was an undergraduate and graduate student, I enjoyed learning to study mathematics in a slow, laborious, step-by-step process, but as my study of mathematics progressed, I went through many experiences of struggling to digest laborious sequential arguments, finally catching on to whole-brain, instantaneous ways to understand the concepts and saying to myself, "Oh, is *that* what they were talking about? Why didn't they say so?" I started to realize that written mathematics is usually a highly denatured rendition of what sits inside mathematicians' heads. It's a hard task that often never happens to translate a detailed step-by-step proof or description into a conceptual understanding, far harder than to translate from concept to details. We've all also noticed that in seminar talks and even in informal one-on-one explanations of mathematics, it's common for one person to talk completely past another. Why is it so hard to communicate mathematics effectively?

I had the ambition to try to communicate on a more conceptual level, paying attention not only to the logical aspects of what is correct but also to the psychological aspects of how we can hold it in our heads and understand it. The geometric modules of our brains are the parts most severely neglected in most mathematical writing. Many papers, even about geometry and topology, lack the figures, or they have figures that are poor or mistaken. I was determined to include all the figures that would be natural to sketch if I were explaining things one-on-one to someone interested in understanding.

This was all much harder than I anticipated. The process of writing lecture notes flowed reasonably smoothly: I just wrote what I thought were good conceptual explanations, later filling in pencil sketches in spaces left by the typist. But as I started to hear of the arduous efforts people went through to digest the material, I realized that it wasn't as easy as I had thought to communicate mathematics more directly on a conceptual plane. Concepts that sit easily in the brain are often surprisingly hard to build up or to communicate. There is a big difference between proofs that are easy to follow but are hard to hold in mind vs. proofs that may be hard to get straight but that you see in a glance once you have them.

In writing the book, my goal was to make the ideas more accessible by filling in more of the unspoken assumptions. In doing so, I found it hard to avoid switching over to different brain modes that are centered more on language and symbols. At the time I started, the hardest problem was to find a workable system for reasonable renditions of the many figures. None of the artists or graphics professionals I tried seemed to be able to get the mathematical relationships correct, and I had neither the time nor the skill to draw professional-looking versions myself. Now, of course, there are computer drawing programs that make the job much easier although still laborious.

Personally, my mind always used to turn to jelly when it came time to do homework in school, and I have similar difficulties when I try to correct and edit something left over from past efforts. I would have given up in despair except for the support and hard work of many people, most notably the editing by Silvio Levy, who devoted many hours of his multifaceted talents and in particular solved the figure issue by developing good ways to do almost all the figures using Mathematica, in addition to working out the language and fixing up many mathematical issues. The other really remarkable contribution was through Al Marden, who organized several bookwriting workshops involving many colleagues to help with a task much larger than I had foreseen. I owe a huge debt to all these people.

In the end, I haven't discovered a solution to the problem of how to communicate mathematics effectively, but I have a better grasp of the problem than when I started. In any case, I am tremendously gratified to receive this recognition for my efforts and the efforts of all the people who helped.