About the Cover

Automata in Coxeter groups

Many interesting subsets of infinite Coxeter groups, and algorithms involved in computation within those groups, are related to automata or finite state machines, as the article by Paul Gunnells in this issue demonstrates. One automaton that arises is that which generates elements of the group one by one, the ShortLex automaton, as proven by Brigitte Brink and Bob Howlett in a classic paper many years ago. Others, more conjectural, are those which seem to describe the Kazhdan-Lusztig cells of an arbitrary Coxeter group.

The cover illustrates both of these types for the Coxeter group with Coxeter numbers 2, 3, 7. In the background the alcoves are colored in pastels according to the state of the ShortLex automaton they are associated to. In darker, primary colors are three left cells of the group, with the repetitiveness characterizing the structures determined by finite state machines singled out.

The relationship between subsets of Coxeter groups and finite state machines is just a small part of a theory not yet clearly perceived in which very complicated infinite patterns of all kinds are described by finite data structures related to the theory of languages. Here the languages are the regular languages, those recognized by finite state machines. Even in Coxeter groups, structures related to the level of complexity in language theory above regular languages, something like of context free languages, appear.

The explicit finite state machines required to draw the Kazhdan-Lusztig cells on the cover were supplied by Gunnells.

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