



Letters to a Young Mathematician

Reviewed by Emma Carberry

Letters to a Young Mathematician

Ian Stewart

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“So,” my grandmother would say, fixing me with the expectant look of the schoolteacher she once was, “What is it that you do, exactly?”. Often this enquiry is interpreted on a purely practical level: what are the problems that mathematicians solve, and why are these important? The outcomes of our research are what justify its funding. But what my grandmother really wanted to understand is the more human question of what it is like to do mathematics. We may feel that we have some idea of what is involved in building a house or performing hand surgery, despite a lack of experience with these tasks. However few nonmathematicians would assert an understanding of the human experience of mathematical research.

For students considering embarking upon a mathematical career, the questions of whether they find mathematics to be a worthwhile endeavor and what being a mathematician is like are of fundamental importance. The main strength of Ian Stewart’s book *Letters To a Young Mathematician* is the way in which it addresses these questions in an entertaining and accessible manner. It is the first scientific entry in Basic Books’ *Art of Mentoring* series, whose volumes are each structured in homage to the collection of Rainer Maria Rilke’s letters to the young poet Franz Xaver Kappus. Stewart’s book takes the form of letters to his fictitious niece Meg as she progresses from high school through to an assistant professor of mathematics. It provides an intriguing glimpse into the intellectual life of her mathematical uncle: a personal window into the process of jousting with mathematical problems and how being a mathematician has affected the way in which he views the world. Meg ages around fifteen years through

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the life of the book, and the issues of relevance to her change significantly. Stewart handles this by avoiding in-depth discussions of practical matters, instead keeping his explicit advice witty and brief. The result is an enjoyable collection of essays whose chief mentoring value lies in the insight they provide into the life of a mathematician. Some of the issues discussed in the latter part of the book, such as teaching and the tenure process, do not lend themselves well to such light treatment, and I feel that the book would have been stronger had it ended earlier in Meg’s life. However, it contains some beautiful pieces of mathematical exposition and will be a valuable resource for those wishing to know more about the world of mathematics and of mathematicians.

Stewart characterises *Letters* as his attempt to update parts of Hardy’s *A Mathematician’s Apology*. Hardy’s view on the public’s perception of mathematics is rosy indeed. “There are now few studies more generally recognised, for good reasons or bad, as profitable and praiseworthy.” “The mass of mathematical truth is obvious and imposing; its practical applications...obtrude themselves on the dullest imagination. The public does not need to be convinced that there is something in mathematics.” Sixty-six years after Hardy penned those words, their veracity is no longer so clear. There is by no means universal public awareness of the role that mathematics plays in daily life, and Stewart begins by undertaking the task that Hardy once deemed unnecessary. Stewart has the expositor’s flair for invoking visual images to make his point, and I enjoyed his rueful wish for red stickers emblazoned with the words “Math inside” to appear on everything that uses mathematics, to increase awareness of the many roles that the subject plays. In his eyes, mathematics is indeed ubiquitous; soon everything from airplane tickets to the vegetables in the local grocery store is wearing a red sticker. The spacing of birds perching on phone lines prompts a discussion of crystal lattices, a passing dog becomes the subject of gait analysis with applications to robotics and orthopaedic rehabilitation, and he tells how his

understanding of the rainbow enhances its physical beauty with an intellectual one.

It is natural for high school students to assume that the type of thinking involved in higher mathematics is similar in nature, if not in scope, to that they have encountered in mathematics classes. It is also far from the truth, and to describe to Meg the type of thinking in which mathematicians engage, Stewart first dispels this notion. “Calling [these classes] ‘mathematics’ debases the currency of mathematical thought; it’s a bit like using the term ‘composing’ to describe routine exercises in playing musical scales.” An indication of what the currency of mathematical thought might be comes later, in letters written whilst Meg is an undergraduate, on the nature and necessity of proof. Unsurprisingly, he does an excellent job of discussing both elementary puzzles and famous theorems, distilling the ideas down to their essence in order to provide an accessible account. A word puzzle (the SHIP-DOCK theorem) is used here to demonstrate the difference between a proof and less rigorous reasoning; a chessboard puzzle introduces the idea (unfortunately not always part of the high-school curriculum) that not all mathematics problems can be solved; and there is a discussion of the impossibility of trisecting the angle that is eminently readable, if somewhat lacking in detail. Gauss’s logarithmic integral approximation for the number of primes less than a specified number is invoked to demonstrate that proofs really are necessary. Gauss’s approximation exceeds the correct value for all numbers that have ever been tested, yet as Littlewood has shown, the approximation and the correct value swap places infinitely often. The aesthetics of proof is also discussed; he characterises a proof as a story, and invokes Erdős’s notion of God’s Book of exquisite proofs. Indeed, there are a number of forays into the philosophy of mathematics, as Stewart ponders whether the mathematics of aliens would agree with ours (he is not convinced) and how exactly to define mathematics and mathematicians. As with the rest of the book, the overarching goal of keeping the material reasonably undemanding does make the level of discussion somewhat superficial, but the book is filled with references for those whose appetite has been sufficiently whetted that they are hungry for more.

Central to the question of what it is like to be a mathematician is the process of tussling with a problem. Stewart provides readers with a glimpse into the inner sanctum of mathematical creativity; those cherished moments when the confusion clears and all is simple and elegant. These moments do not appear by magic of course, and my favourite part of the book is Stewart’s elucidation of this process. He invokes Poincaré’s three stages: “preparation”, “incubation followed by illumination”, and “verification”, and follows up Poincaré’s

beautiful account of this process with a story of his own that conveys a strong sense of the inner world into which Meg is entering.

Towards the end of the book Stewart touches on subjects of relevance to young postdocs and faculty: teaching, giving talks at conferences, the pleasures and perils of collaboration, the mores of the mathematics community, and tenure. These are matters that will not be of immediate relevance to most readers of this volume, and so the advice is perfunctory; these letters serve as an indication of the kind of things a young mathematician needs to think about. I feel that the inclusion of these chapters detracts somewhat from the impact of the book. They offer less for the general reader than do the earlier parts, and for those currently dealing with these issues, the advice is too simplistic. Sometimes this is because he focuses on fairly shallow aspects of the matters under discussion; for example his advice on teaching includes a long description of the things that can go wrong with the equipment in a lecture theatre, but very little about how to teach in a way that maximises student learning. At other times he directly addresses more complex issues, but his treatment is too shallow. The most helpful advice is often that which enables the recipient to avoid or navigate difficulties, and Stewart several times glosses over situations that offer exactly this opportunity for the passing on of wisdom. For example, he gives a rosy description of the current landscape for women in mathematics, and assures Meg that “The idea that math is not a suitable subject for women is stone-cold dead”. Meg could easily conclude from his discussion that gender issues in mathematics are a thing of the past. Unfortunately this is not yet true. Good advice is certainly encouraging, but not because it pretends that complications may not exist, but rather because it helps one to see how to deal with them.

Letters to a Young Mathematician succeeds well in opening a door into the world of mathematics and enticing the reader inside. I would recommend it to those curious about mathematics and life as a mathematician, particularly high school students and undergraduates considering further mathematical study. Readers of the *Notices* may well find it enjoyable reading, or wish to pass it on to their students. Reading it is rather as I imagine a conversation with Uncle Ian-the-mathematician could be; one does not so much feel the benefit of a ream of practical advice, but rather of exposure to the inner realm of mathematics, and the enlightenment that that provides. It goes a good way to answering the question of what it is that we do; I suspect that my grandmother would have been satisfied with Stewart’s response to her question.