

Book Review

Bourbaki, A Secret Society of Mathematicians and *The Artist and the Mathematician*

Reviewed by Michael Atiyah

Bourbaki, A Secret Society of Mathematicians

Maurice Mashaal

AMS, June 2006

US\$29.00, 260 pages

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The Artist and the Mathematician: The Story of Nicolas Bourbaki, the Genius Mathematician Who Never Existed

Amir D. Aczel

Thunder's Mouth Press, August 2006

US\$23.95, 272 pages

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All mathematicians of my generation, and even those of subsequent decades, were aware of Nicolas Bourbaki, the Napoleonic general whose reincarnation as a radical group of young French mathematicians was to make such a mark on the mathematical world. His memory may now have faded, the books are old and yellowed, but his influence lives on. Many of us were enthusiastic disciples of Bourbaki, believing that he had reinvigorated the mathematics of the twentieth century and given it direction. But others believed that Bourbaki's influence had been pernicious and narrow, confining mathematics behind walls of rigour, and cutting off its external sources of inspiration.

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Now that we are in the twenty-first century it is perhaps the right time to look back and try to assess the overall impact of Bourbaki, before all the principal players leave the scene. The basic historical facts are well known and are set out in both the books under review. France had lost a whole generation of intellectuals in the 1914-18 war, and the young mathematicians of Paris, in the inter-war period of the 1920s and 1930s, were looking for new guidance and inspiration. Only Hadamard and Élie Cartan of the older generation still commanded respect. Talented youth, unconstrained by higher authority, is a powerful force and, whatever one's views about Bourbaki, there is no doubt that the talent was quite exceptional. The list of the early members of Bourbaki is truly impressive: André Weil, Henri Cartan, Claude Chevalley, Jean Dieudonné, Laurent Schwartz... Later recruits were of similar calibre: Jean-Pierre Serre, Armand Borel, Alexandre Grothendieck... Harnessing the powers of such a formidable group was not an easy task. There were fierce debates, some serious quarrels, and much passion. The remarkable fact is that the group, by and large, stayed together and kept Bourbaki alive and active over several decades. This was a tribute to the idealistic vision that they shared, that of remoulding the shape of mathematics in the twentieth century.

Much of the atmosphere of the early days is brought vividly to life by the many informal photographs in the Mashaal book. It is fascinating to see pictures of the young André Weil, relaxing in a deck chair, though Henri Cartan was always

impeccably dressed in jacket and tie, resisting trendy fashion.

I myself attended a Bourbaki conference in my youth and can attest to the lively experience of debating vigorously (and usually critically) the latest version of the next book. Summer sunshine in the south of France and the friendly and casual atmosphere did much to prevent arguments developing into armed conflict. To paraphrase Winston Churchill, “never in the course of human argument has so much been spoken by so many on so little.” It appeared a miracle that books, many of them, actually emerged from this process, a result undoubtedly due to the diligence and energy of Dieudonné. If Weil was the prime inspiration behind Bourbaki, it was Dieudonné who carried it to fruition.

So what were the basic aims of Bourbaki, and how much was achieved? Perhaps one can pick out two central objectives. One was that mathematics needed new and broad foundations, embodied in a series of books that would replace the old-fashioned textbooks. The other was that the key idea of the new foundations lay in the notion of “structure”, illustrated by the now common word “isomorphism”.

There is no doubt that, with its clear emphasis on “structure”, Bourbaki produced the right idea at the right time and changed the way most of us thought. Of course it fitted in well with Hilbert’s approach to mathematics and the subsequent development of abstract algebra. But structure was not confined to algebra, and it was particularly fruitful in topology and associated areas of geometry, all of which were to see spectacular developments in the period following World War II. Here the impact of Bourbaki was decisive, and, in the hands of Serre and Grothendieck, algebraic geometry rose to incredible heights.

Laying universal foundations is another matter. Each time it is tried it inevitably gets bogged down by the sheer scale and ambition of the operation. The “ne plus ultra” in this direction was the *Éléments de Géométrie Algébrique* of Grothendieck and Dieudonné, which expanded voluminously both forward and backward and was in danger of sinking under its own weight.

Laying ambitious foundations is not only a dangerous delusion, it can also be a didactic disaster. Encyclopaedias are not textbooks, and much of the critique directed against Bourbaki is that it was used, or perhaps misused, to reform school education. This may be unfair, since many of the great mathematicians in Bourbaki were excellent lecturers and knew well the difference between formal exposition and the conveying of ideas. But, as so often happens, the disciples are more extreme and fanatical than their masters, and education in France and elsewhere suffered from a dogmatic and ill-informed attempt at reform. Jesus Christ

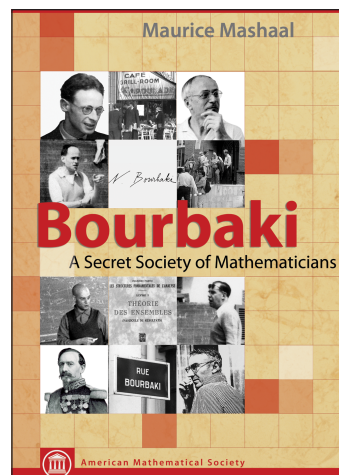
is not responsible for the excesses perpetrated in the name of Christianity.

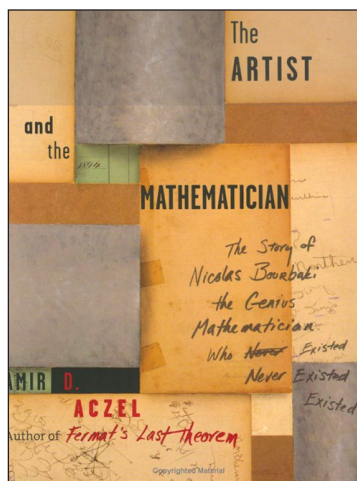
Bourbaki was to some extent the victim of its own success. The original aim had been the modest one of writing a modern replacement for Goursat’s *Cours d’Analyse* but, buoyed up by enthusiasm and the success of recruiting many of the leading mathematicians of the time, horizons broadened. All of mathematics was to be included, analysis, algebra, and geometry. For obvious reasons algebra lent itself best to the Bourbaki treatment. The volumes on commutative algebra and particularly on Lie groups were excellent and became standard references, due in large part to the personal contribution of Serre, whose influence and taste guided this whole area.

The formal aspects of analysis, as exemplified in functional analysis, also had success, though Bourbaki’s treatment of probability came in for severe criticism from the experts who argued that important parts of the theory were excluded by the restriction to locally compact spaces. A concern for elegance had led to too great a price being paid.

But this little battle over probability was a mere sideshow in the Bourbaki approach to analysis, a subject too varied, complex, and untidy to be taken over by Bourbaki. Glimmerings of these problems already appear in differential geometry, a subject at the interface between analysis and geometry, where structure, though present, is a less dominating concept. Though Riemann surface theory, after a century of active development, could conceivably be given a coherent Bourbaki treatment, the same could hardly be said for the current work of Thurston-Perelman in three dimensions. Another severe limitation of Bourbaki, no doubt conscious, was the restriction to pure mathematics. Applied mathematics is too messy and disparate to be included, and theoretical physics hovers on an uncertain borderline. One distinguishing feature of Bourbaki was the emphasis on clear and unambiguous definitions and on rigorous proofs. This was, as in algebraic geometry, a reaction against some sloppy treatments of the past, and it served a purpose in creating a firm platform for the future. Unfortunately, when taken to extremes, the requirement for total rigour excludes large areas of mathematics which are in their early creative stages. Had Euler worried too much about rigour, mathematics would have suffered.

Over the past thirty years, arguably in the declining years of Bourbaki, some of the most exciting developments in mathematics have arisen from the interface with physics and particularly quantum





field theory. New concepts and explicit results have emerged from this interaction, notably Donaldson's work on four-manifolds, mirror symmetry in algebraic geometry, and quantum cohomology. Much of this came directly from very heuristic work by physicists such as Edward Witten. Most of it, though by no means all, has now been given a cloak of respectability involving rigorous proofs.

Clarity and rigour have a vital place in mathematics but they must not be used as a barrier to new ideas from other fields.

Free trade is a benefit to us all and should not be inhibited by excessive attachment to national sovereignty.

Although Bourbaki recruited most of the famous French mathematicians of the time (and several from outside France), there were some notable exceptions, the most obvious being Jean Leray (who left very early) and René Thom. In retrospect it is clear that neither fitted the Bourbaki role. The fact that they were also two of the most original mathematicians of the time does perhaps suggest that such originality has difficulty flourishing in a constrained atmosphere. Both were also closer to applied mathematics than their colleagues.

Of the two books under review, the first by Maurice Mashaal might be described as "authorized". It has the sanction of the AMS and was first published several years ago in French. It seems clear that the author knew many of the French mathematicians personally and derived his information and in particular the photographs from this source. It is reliable on the history, the personalities, and the mathematics. It is also highly readable and noncontroversial.

The other book by Amir Aczel is totally different. It has a more ambitious aim, which is to examine the Bourbaki influence on "structure" in the social sciences. It is also highly controversial in its extensive treatment of the Grothendieck story. I was not convinced of the total reliability of its sources, nor of its philosophical credentials.

Although written in English this book is permeated by French intellectual ideas and will probably seem strange to those not part of that scene. A slightly tenuous link between André Weil and the sociologist Claude Levi-Strauss is used to claim that Bourbaki made a major impact on sociology and related fields such as psychology, anthropology, and linguistics. This grand aim is clearly set out by the title, and I have no expertise in any of these fields. It may be that the author is a polymath, an intellectual colossus, who straddles

the entire scene from mathematics to the social sciences. The only place where I can examine the evidence for this and make an informed comment, is in his treatment of mathematics and the people in it. Here I have profound misgivings, which relate mainly to Grothendieck, who occupies a central place in the author's pantheon.

There is no doubt that Grothendieck was an exceptional figure in the mathematical world and that he deserves a scholarly full-length biography, preferably written by a mathematician who knew him personally. I believe such a book is in preparation, and I look forward to reading it. Aczel's book does not measure up to the level of the subject, because of his uncritical acceptance of Grothendieck as the great prophet, spurned eventually by his people (including Bourbaki).

I knew Grothendieck well when he was in his prime. I greatly admired his mathematics, his prodigious energy and drive, and his generosity with ideas, which attracted a horde of disciples. But his main characteristic, both in his mathematics and in social life, was his uncompromising nature. This was, at the same time, the cause both of his success and of his downfall. No one but Grothendieck could have taken on algebraic geometry in the full generality he adopted and seen it through to success. It required courage, even daring, total self-confidence and immense powers of concentration and hard work. Grothendieck was a phenomenon.

But he had his weaknesses. He could navigate like no one else in the stratosphere, but he was not sure of his ground on earth—examples did not appeal to him and had to be supplied by his colleagues.

Aczel is right when he identifies Grothendieck as someone who took the new Bourbaki philosophy seriously and made a tremendous success of it. Where I part company with Aczel is in his assertion that Bourbaki made a fatal mistake in not taking Grothendieck's advice and rewriting its foundations in the new language of category theory. Aczel believes that Bourbaki had turned its face away from the future in not following Grothendieck. I doubt whether history will come to this verdict. Grothendieck's own *EGA*, as well as the general fate of over-confident universalists, might suggest otherwise. Moreover, given Grothendieck's uncompromising nature and supreme self-confidence, it is difficult to see how, with him at the helm, Bourbaki could have continued as a collegial enterprise.

Aczel's total endorsement of Grothendieck leads him to make such fatuous statements as: "Weil was a somewhat jealous person who clearly saw that Grothendieck was a far better mathematician than he was." Subtle balanced judgement is clearly not Aczel's forte, and it hardly encourages the reader to take seriously his confident and sweeping assertions in the social sciences.