Book Review

Is Mathematics Misapplied to the Environment?

Review by Christopher K. R. T. Jones

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
Orrin H. Pilkey and Linda Pilkey-Jarvis
Columbia University Press, 2006
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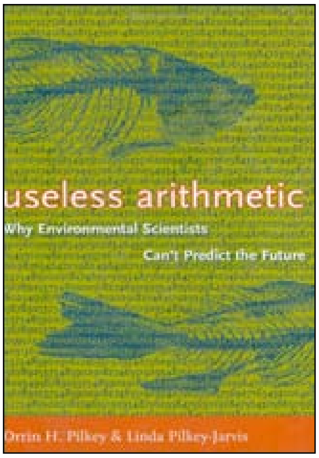
This book is about disastrous decisions made on critical environmental issues. The authors document multiple cases ranging from nuclear waste disposal to plant invasion. Between the initial acceptance of a particular environmental issue and the eventual decision for action (or inaction), they show that something all too often goes very wrong. The cases they cite are unarguable, and they are right, at least about the outcome, in each instance: extremely bad decisions have been made that have led to situations that should outrage all of us. These have included the almost complete depletion of cod stocks on George's Bank and the engineering of beaches that has had the reverse effect of what was purportedly intended, the saving of those beaches.

Having been living recently in North Carolina, I am aware of the role of one of the authors, Orrin Pilkey, in combating beach development. He has stood up to the rampant and unthinking commercial exploitation of the coastline and exposed the obfuscation of developers and politicians as well as the miscalculations of coastal engineers. He is a hero of the environmental movement in North Carolina and deserves a hearing, even with a book having a title as provocative as this one.



One of the most compelling examples is the authors' analysis of pit lake contamination. This is in particular reference to the Berkeley pit in Montana. The problem is to predict the environmental impact of open-pit mining many years down the road when the various contaminating by-products of the mining operation have been allowed to seep away from the original pit and potentially contaminate groundwater. The various parties, either governmental or with direct business interests, have used models to make predictions. In spite of the underlying physics and chemistry of the pit lake pollution being only crudely estimated and modeled, the resulting predictions are presented with an unwarranted level of specificity and certainty. The authors claim that the predictions have not been borne out by later facts. Although they, unfortunately, do not make a detailed comparison, they do make a strong case that the original modeling is misguided and that little later verification and assessment was carried out despite many key decisions being based on the earlier predictions.

I was convinced by their arguments that something goes awry in the process of environmental decision-making and that absurdly optimistic predictions, based on the use of mathematical models, are involved. The important question is what

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exactly does go wrong and how can it be fixed? The answer offered by the authors is unequivocal: the real culprit is the very use of mathematical models in environmental science, and the fix is to rid the subject of such models altogether.

This is a fairly extreme position. Models, after all, are neutral in themselves, and if they are at the root of the problem then it is surely because they are being misused. Such misuse can often be traced to people seeking to bolster their self-interest by appealing to supposedly objective results. The authors make this point themselves, but they also hammer on the inappropriateness of mathematical models and advocate their near total banishment from environmental sciences. Although I ultimately disagree with their position, it is worthwhile to give it full consideration as there are serious issues raised in the book and important messages for mathematics as a discipline.

The authors make a distinction between quantitative and qualitative models. In their lexicon, quantitative models are designed to give numbers representing a specific prediction, while qualitative models are used for understanding underlying physical processes. This distinction is based on the uses of the models rather than any of their intrinsic properties and is, in my opinion, misleading.

A clearer view of modeling is as a continuum, with one end being computational models, which involve intensive use of numerical solutions of some complex system of equations, and, at the other end, models that are more like rules-of-thumb. The authors use their distinction inappropriately at times as they classify almost anything they disagree with as quantitative. For example, they devote a considerable portion of the book to the discussion of beach erosion in which the Bruun Rule, which describes the amount of horizontal retreat of the shore-face as a response to sea-level rise, plays a starring role. In my view, this rule is far on the "rule-of-thumb" end of the modeling spectrum, and yet they group it with quantitative modeling. It should surely be viewed as qualitative modeling because it makes a broad brush stroke of the physics. They disagree with its use in practice and even its applicability, but that would make it bad qualitative—not quantitative—modeling.

A more straightforward distinction to make would be between the uses of models for prediction as opposed to understanding. Models all along the spectrum may be used for either objective. It is the use of models for prediction that the authors question. I will argue below that to understand the limits to the predictive capacity of models, we must view them as working in concert with data. But, first, I will look in more detail at their arguments.

Models of environmental situations involve the setting of a large number of parameters. The proper values of these parameters are largely unknown and are often estimated in a subjective process with settings that are based, at best, on convenience or, at worst, with the intention of producing a particular outcome. Moreover, the complexity of the models means that many subprocesses are either included only very approximately or omitted altogether. The result is a crude model whose predictive output may have little to do with the true outcome. This inadequacy of the models is compounded by their intrinsic limits to predictability due to internal dynamics; for instance they may well be chaotic.

Nevertheless, the models are used extensively by (coastal) engineers, government planners, and others for precise and specific predictions. Given this state of affairs, it is not surprising that the predictions have often proved to be inaccurate and have led to disastrous decisions. A modest and reasonable response at this point would be to propose caution and advocate the restricted use of models and then only with considerable validation and verification. This, however, is not the authors' approach, as they take the extreme position of advocating the wholesale rejection of mathematical models in environmental sciences. Their case for such a drastic proposal takes us into the psycho-sociological realm.

The authors have no fond feelings for the politicians, planners, and developers who ultimately make the decisions about policies affecting the environment. My guess is that they would agree these decision-makers would use other convenient justifications for their decisions were the output of models not available or not well regarded. Their point, however, is that mathematical models carry an inherent susceptibility to manipulation in the hands of not-so-well-intentioned.

They use terms like "priesthood" for the practitioners of mathematical modeling and quote a number of times a colleague whom they report as having said, "I stress that the problem was not mathematics per se but the place of idolatry we have given it. And it is idolatry" [1]. Perhaps we should be relieved that mathematics itself is freed of blame in their view, but, as an applied mathematician, these statements give me considerable cause for concern.

Their point is that mathematical modeling is being applied in a domain to which it is not relevant, namely studies of the environment, and yet is held in such awe by the public that even ludicrous decisions can hold sway if backed by mathematical predictions. They seem to be suggesting that a form of "math anxiety" is at play here, and I cannot help feeling that they are betraying a certain degree of such a condition themselves. We are all very aware in the mathematics community of the reaction of the public to our erudite subject, and it is sometimes all too easy to hide behind its mystique. It is, however, in our interest to demystify mathematics and its uses as much as possible. If they are correct that bad decisions are being made in our name, then that should certainly concern us. But is this an argument for the complete rejection of mathematical models in environmental sciences?

A closer look at their scientific case for the inappropriateness of mathematical models to the environment is warranted. They argue that this use of mathematical models is extending mathematics beyond its proper home into a setting that is just too complex to model. Its success in concrete physical situations is well-known; they call this the land of bricks and mortar. In a recent review of this book in *Nature* by Roger Pielke Jr. [2], the reviewer points out that mathematics has met with extraordinary success in two areas of great relevance to flying in an airplane, which is what he was doing while writing the review, namely weather prediction and the successful flight of the plane itself. He writes that these uses of mathematical modeling are very different from those discussed in the book. Unfortunately, he does not fully explore what makes them different. The prediction of weather, even on short time scales, involves an extremely complex system that suffers from all the issues present in the kind of environmental systems addressed in the book: unknown parameters, unresolved processes, and underlying chaotic dynamics. Airplane flight is arguably more straightforward to model, but is still a highly complex system given that the ambient air is undergoing all the dynamic effects that make weather prediction so complex.

What then is the key difference? I would contend that it is not the physical basis of these models that makes them so different but rather the data that are being incorporated into the predictive process. The assimilation of data into the process of forecasting weather is critical in its success. We know that weather models will fail to predict accurately after not too many hours without the incorporation of available data. The data serves to correct the inaccuracies due to the inadequacies of the model as well as its intrinsic instabilities. The mechanics of compromising between model output and data is emerging as both an important and fascinating area of science: data assimilation. It involves a blend of statistical and dynamical thinking that in itself offers many mathematical challenges; see [3].

The flying of an airplane is a similar story. There is continual incorporation of information on the response of the plane to its ambient environment, and the plane is then controlled to keep its flight on track. It is an extraordinary feat of engineering that flights run so smoothly given the complexity of the physics. In both of these cases it is then the mutual support that data and model give each other that leads to successful predictions. Neither can live without the other: models will go wrong fairly quickly without corrections in light of data, and the data are insufficient to provide a description from which a prediction might be extrapolated without the use of a model to fill in away from the data points.

A serious weakness of this book is the lack of discussion of data: how data can and indeed must be incorporated into models. There is, of course, a vicious circle here in that the topic is prediction and we do not have data from future events. The point is that once we realize the significance of data, we can start to circumscribe the valid use of models in making predictions. It is determined by not going too far from where data are available. In the case of weather prediction, it means forecasting a few days out and understanding that longer time predictions are to be taken much more lightly.

The authors argue that mathematical models are being taken out of their domain of applicability by moving from the safe ground of physically based modeling, such as for bridges and buildings, to complex situations like those that occur in the environment. I would argue that they are correct in saying that models are being taken to a place they do not belong, but it is not the complexity of the situation that is the problem, but its disconnection from data. Once the point is understood properly that data and models go hand-in-hand, a new focus can be adopted that places data assimilation in a central position.

Reading this book, one realizes that the scientific research community has a very different view of mathematical models from those who typically put them into use, and the authors point this out repeatedly. The scientific community sees models as a testing ground for ideas. New phenomena can be discovered in models and relationships between physical effects exposed and understood; in other words, hypotheses about cause and effect can be tested.

One of my favorite examples is the work of Hodgkin and Huxley on nerve impulses [4]. They formulated a model for the propagation of a voltage action potential along a nerve axon. It involved all kinds of approximations concerning the chemical concentration differences across the membrane. The biochemical processes are not even physically modeled, only their effect on the membrane is accounted for. Nevertheless, they showed, using a primitive computational device which was little more than an adding machine, that this modeling of the mechanisms for chemical passage through the membrane leads to a propagating wave (the nerve impulse) in the equation for the voltage. They thus showed that the postulated chemical concentration differences and their changes in response to electrical excitation could explain why nerve axons support propagating impulses. This groundbreaking piece of work won them the Nobel Prize and would not have been possible without the computations they performed on the mathematical model. This example is not of environmental modeling, but neither is it from the land of bricks-and-mortar.

Another variant of the use of mathematical models is to test "what-if scenarios". Information is fed into a model that reflects a particular set of choices or decisions. The output of computational runs of the model can then give tremendous insight into the potential consequences of the original choices. The importance of this use of modeling is emphasized by Naomi Oreskes in a series of very insightful and interesting articles about prediction and models [5].

A fascinating example of this type, which is in the context of an environmental issue, was pointed out to me by Margaret Beck [6]. Loggerhead sea turtles are a species that has been in danger of extinction. Crouse et al. [7] tested various conservation management strategies in a population dynamics model. They concluded that the management practices of the time, with their focus on eggs on nesting beaches, were not the most effective as this is the least responsive life stage. Based on runs of the model, they proposed specific protection efforts for juvenile loggerheads, for instance using turtle excluder devices that prevent turtles from getting caught in nets. There is evidence that this approach has been successful; see the recent articles [8].

These examples are typical of the great successes of mathematics applied through models. Different physical effects are shown to be connected through their being modeled and the resulting equations solved computationally. The suggestions of a replacement for mathematical models by the authors are rather vague. Their recommendation appears to amount to putting trust in the environmental experts who understand the underlying physical processes. Without belittling their own expertise and contributions, I would suggest that depending solely on expert advice would constitute a system more vulnerable to abuse than one based on mathematical models.

It is almost certain that mathematical models are here to stay. Moreover, we should be happy about this as they can be used to expose an enormous amount about the underlying physical mechanisms. It is critical to understand cause and effect in environmental situations and to be able to test the possible outcomes of different decision strategies. Mathematical models play the key role in this enterprise, and they allow us to cover cases that are far more complex than can be handled by well-informed expertise. We do need to be circumspect, however, about very specific predictions and reserve our faith in such predictions for cases where reasonable data have been available and assimilated into the model. The issue of delineating the validity of predictions and their presentation is fascinating and critically important. This is essentially a mathematical subject but also has sociological and philosophical dimensions. It has received much recent attention because of its significance in addressing climate change; see [9] for an interesting and current discussion.

It should be of concern to us as a community that mathematical models have been abused in environmental engineering as described in this book. But this is reason to get rid of the abuse, not the models.

References

[1] The authors attribute this to Jim O'Malley, who is a representative of the fishing industry and is referred to extensively in Chapter 1. This quote is on p. xiii in the Preface, see also p. 156.

[2] Roger Pielke Jr., When the numbers don't add up, *Nature* **447** (2007), 35–6.

[3] The recent special issue of *Physica D* gives a good overview of the state-of-the-art in data assimilation: Data Assimilation, (Kayo Ide and Christopher K. R. T. Jones, eds.), *Physica D* **230** (2007). This volume is based on a special semester of activity held at SAMSI (Statistical and Applied Mathematical Sciences Institute), an NSF funded mathematics institute in Research Triangle Park, North Carolina.

[4] A. Hodgkin and A. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve, *J. Physio.* **117** (1952), 500–544.

[5] N. Oreskes, Why believe a computer? Models, measures, and meaning in the natural world, *The Earth Around Us: Maintaining a Livable Planet*, (Jill S. Schneiderman, ed.), W. H. Freeman and Co., San Francisco, 2000, pp. 70–82; The role of quantitative models in science, *Models in Ecosystem Science*, (Charles D. Canham, Jonathan J. Cole, and William K. Lauenroth, eds.), Princeton University Press, Princeton, 2003, pp. 13–31.

[6] Margaret Beck is currently an NSF Mathematical Sciences Postdoctoral Fellow, and she dates her interest in applied mathematics to the time she read this paper as an undergraduate.

[7] Deborah T. Crouse, Larry B. Crowder, and Hal Caswell, A stage-based population model for loggerhead sea turtles and implications for conservation, *Ecology* **68** (1987), 1412–1423; and Rebecca L. Lewison, Sloan A. Freeman, and Larry B. Crowder, Quantifying the effects of fisheries on threatened species: the impact of pelagic longlines on loggerhead and leatherback sea turtles, *Ecology Letters* **7** (2004), 221–231.

[8] See: `http://www.ens-newswire.com/ens/dec2002/2002-12-18-06.asp`; `http://www.dukeresearch.duke.edu/database/pagemaker.cgi?992632903`; `http://www.commondreams.org/news2004/1129-06.htm`; `http://moray.ml.duke.edu/faculty/crowder/`.

[9] D. A. Stainforth, M. R. Allen, E. R. Tredger, and L. A. Smith, Confidence, uncertainty and decision-support relevance in climate predictions, *Phil. Trans. Roy. Soc. A* **365** (2007), 2145–2161.