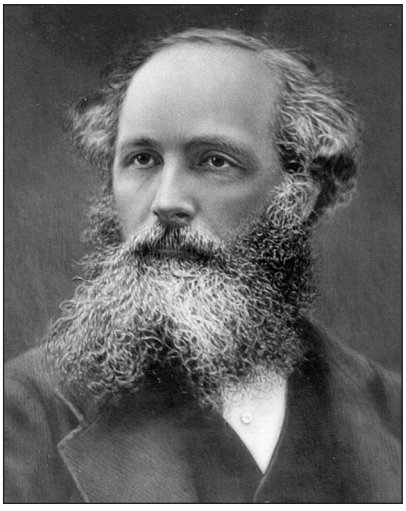
The Last Poem of James Clerk Maxwell

Daniel S. Silver



James Clerk Maxwell

"My soul's an amphicheiral knot," proclaimed the greatest mathematical physicist of the nineteenth century. Many are familiar with James Clerk Maxwell for his elegant equations describing the electromagnetic field or for his profound ideas about the kinetic theory of gases and thermodynamics. Fewer know that Maxwell was a compulsive poet. This is the story of Maxwell's last poem, written during his final days in 1878. An enigmatic piece, expressed with odd references to topology, cosmology, and evolution, *Paradoxical Ode* in fact reflects Maxwell's private thoughts about the relationship between science and religion, choice and chance, death and eternity.

Paradoxical Ode was composed for a close friend, the Scottish physicist Peter Guthrie Tait. The two had known each other since their school days at Edinburgh Academy. In 1867 Tait had demonstrated the mutual interaction of smoke-rings in his laboratory for the benefit of William Thomson (the recently knighted Lord Kelvin). Thomson rashly proposed a "vortex atom theory", asserting that knotted vortices in the ether comprise all chemical elements. Tait was now laboring hard to classify knots.

The original version of *Paradoxical Ode* was written on three sheets of plain paper. It is preserved in a scrapbook that was recently donated to the James Clerk Maxwell Foundation, Edinburgh, by a relative of Tait. The poem was published in *The Life of James Clerk Maxwell*, an extensive and definitive biography written in 1882 by Lewis Campell and William Garnett. It has since been reproduced many times, often altered and with little or no explanation. The poem appears below in its original form including Maxwell's indentation.

Superscripts in the poem and text refer to endnotes. Some words and phrases in the poem reappear later in boldface as they are explained.

To Hermann Stoffkraft, Ph.D.
A Paradoxical Ode
After Shelley

I

My soul's an amphicheiral knot
Upon a liquid vortex wrought
By Intellect in the Unseen residing,
While thou dost like a convict sit
With marlinspike[1] untwisting it
Only to find my knottiness abiding;[2]
Since all the tools for my untying
In four-dimensioned space are lying,
Where playful fancy intersperses
Whole avenues of universes;
Where Klein and Clifford fill the void
With one unbounded, finite homaloid,
Whereby the Infinite[3] is hopelessly destroyed.

II

But when thy Science lifts her pinions
In Speculation's wild dominions,
I treasure every dictum thou emittest;
While down the stream[4] of Evolution
We drift, and look for no solution
But that of the survival of the fittest.[5]
Till in that twilight of the gods

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When earth and sun are frozen clods,
When, all its energy degraded,[6]
Matter in æther shall have faded,
We, that is, all the work we've done,
As waves in æther, shall for ever run
In swift-expanding spheres, through heavens beyond the sun.

III

Great Principle of all we see,
Thou endless Continuity!
By thee are all our angles gently rounded;[7]
Our misfits are by thee adjusted,
And as I still[8] in thee have trusted,
So let my methods never be confounded!
O never may direct Creation
Break in upon my contemplation,
Still may the causal chain, ascending,
Appear unbroken and unending,
And, where that chain is lost to sight
Let viewless fancies[9] guide my darkling[10] flight
Through Æon[11]-haunted worlds, in order infinite.

$$\frac{\partial p}{\partial t}\,^{12}$$

Besides wordplay, Maxwell's nonscientific passions included literature and philosophy. *Paradoxical Ode* is a pastiche of a passage from the lyrical drama, *Prometheus Unbound*, by the English romantic poet Percy Shelley.

The first verse of *Paradoxical Ode* accurately suggests that Maxwell was familiar with new mathematical ideas of the day. He understood a knot to be a simple closed curve in 3-dimensional space, just as we do today. An **amphicheiral knot** is one that can be deformed to its mirror image, while **liquid vortex** is a reference to Thomson's vortex atom theory.

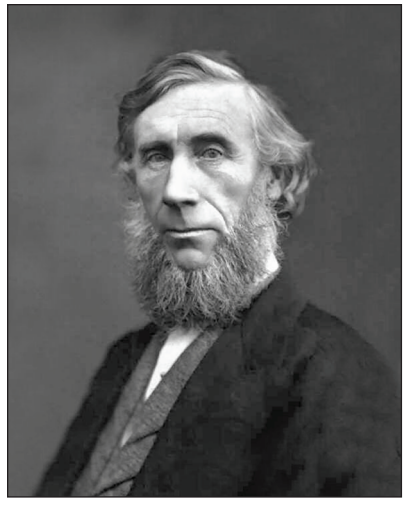
From correspondence between Tait and Maxwell, we know that Maxwell was directly involved in the nascent subject of knot theory, although he did not publish any knot theoretical results of his own.

Maxwell's interest in knots, and topology in general, had a serious side. It was Maxwell who informed Tait about the work of Johann Benedict Listing, a student of Gauss, who coined the word "topology" in 1848 and had begun his own study of knots. Maxwell's *Treatise on Magnetism and Electricity* made early and profound use of Listing's homological notions. In it, we find a new interpretation of Gauss's linking integral for two disjoint, simple closed curves as the work done by a charged particle moving along a path described by one knot against the magnetic field induced by an electrical current running though the other. Maxwell was fascinated by the idea that two knots can have zero algebraic linking number and yet be inseparable.

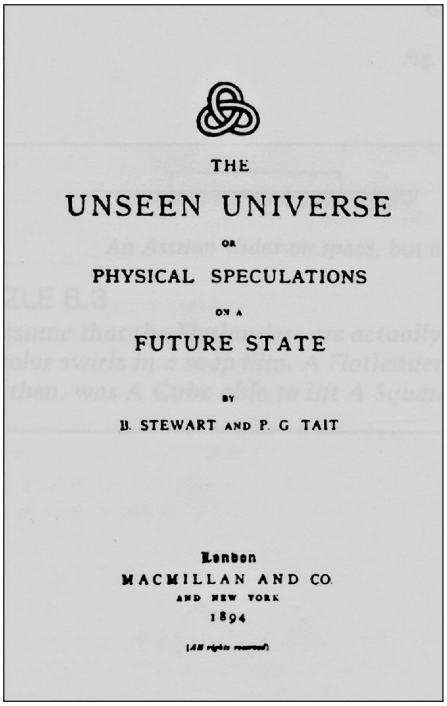
Intellect in the Unseen is the first of several references to *The Unseen Universe or Physical Speculations on a Future State*, a book that Tait co-authored with (yet another) Scottish physicist, Balfour Stewart, in 1875. In it they argued that religious miracles and the immortality of the soul are compatible with modern science. God is hidden from us because all human thought is "conditioned", an idea that traces back to Kant. Nevertheless, the "principle of Continuity", which *Unseen Universe* announced, comforts all with the gentle reassurance that nature will never do anything to confound us permanently:

> [W]hat the principle of Continuity demands is an endless development of the conditioned. We claim it as the heritage of intelligence that there shall be an endless vista, reaching from eternity to eternity, in each link of which we shall be led only from one form of the conditioned to another, never from the conditioned to the unconditioned or absolute, which would be to us no better than an impenetrable intellectual barrier. … Finally our argument has led us to regard the production of the visible universe as brought about by an intelligent agency residing in the unseen.

Stewart and Tait wrote *Unseen Universe* in response to John Tyndall's Presidential Address to the British Association for the Advancement of Science during the previous year. As controversy raged over the teaching of science at the Catholic University in Ireland, Tyndall, an Irish physicist and successful popularizer of science, had felt the need to speak out. "All religious theories, schemes and systems, which embrace notions of cosmogony, or which otherwise reach into the domain of science," Tyndall contended, "must, in so far as they do this, submit to the control of science, and relinquish all thought of controlling it." Tyndall's call for reason over revelation was seen by some, including Stewart and Tait, as an attack on religion. *Unseen Universe* was hastily written and was a huge success. The authors published anonymously until the fourth edition appeared. A trefoil knot adorned the spine and title page.



John Tyndall



THE

UNSEEN UNIVERSE

OR

PHYSICAL SPECULATIONS

ON A

FUTURE STATE

BY

B. STEWART AND P. G TAIT

London
MACMILLAN AND CO.
AND NEW YORK
1894

Title page of *Unseen Universe.*

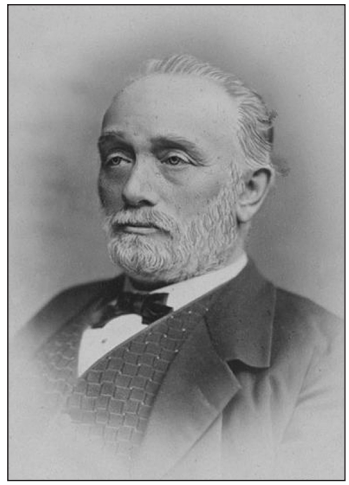
Just a few years before *Paradoxical Ode* was composed, Felix **Klein** had published a proof that any knot can be undone in **four-dimensioned space**. By the time that the poem was sent to Tait, the American magician and medium Henry Slade had incorporated the idea into his act, claiming to unknot ropes in the fourth dimension. Edwin Abbott's multidimensional exploration, *Flatland*, would appear five years later.

Like many, Maxwell was interested in speculations about a fourth dimension. In an amusing letter to C. J. Monroe, dated 1871, he doubted that we live in a universe of more than 3 spatial dimensions: "If you have 4 dimensions this becomes a puzzle—for first, if three of them are in our space, then which three?"

In a multidimensional world, many things such as **whole avenues of universes** are possible. Stewart and Tait were not timid about transporting a novel scientific theory to unfamiliar surroundings. Readers of *Unseen Universe* learned that their every thought and action broadcasts eternal vibrations throughout the perfect ether. Inconveniently, nothing in our visible world, including fluids, is perfect. The authors were undeterred. Particles of good ether were next to particles of better ether, readers were told, while particles of better ether next to even better, ad infinitum. In effect, *Unseen Universe* postulated parallel avenues of universes. More importantly, it described a world in which miracles and life after death were scientifically possible.

At the age of thirty-three, William Kingdon **Clifford** had already established himself as one of the most brilliant and iconoclastic mathematicians. An atheist, Clifford had little use for *Unseen Universe*. His review in the *Fortnightly Review* was biting and entertaining.[13]

Like his slightly younger German colleague Klein, Clifford was fascinated by the possibility that the universe is non-Euclidean. He even speculated that gravity might be the effect of "a variation in the curvature of space". Two years before Maxwell penned *Paradoxical Ode*, Clifford had coined the term **homaloid** and had used it in the title of an article in the *Proceedings of the London Mathematical Society*. By the term, Clifford meant a "flat" space of any number of dimensions, that is, a space with curvature zero. Maxwell's use of the term is perplexing.[14] However, his intention becomes clearer when we understand that most scientists of his day believed that the universe is flat and that a flat universe is necessarily infinite. Likely, Maxwell understood that while the first assertion might be true, the second is not. He attended the1873 Bradford meeting of the British Association for the Advancement of Science at which Clifford showed how to construct flat tori as quotients of Euclidean space.



Ludwig Büchner

At Northwestern University in 1893, Klein summarized the situation:

> It is evident from this point of view many assertions concerning space made by previous writers are no longer correct (e.g., that infinity of space is a consequence of zero curvature), so that we are forced to the opinion that our geometrical demonstrations have no absolute objective truth, but are true only for the present state of our knowledge.

Tyndall and Clifford were just two of many Victorians who hoped that modern science would sweep away Christian dogma. The second verse of *Paradoxical Ode* enters a noisy debate about science and religion that continues today.

Maxwell declared more than once that science had nothing to say on matters of religion.[15] Maxwell's biographer, Campbell, wrote that while his subject was devout, he distrusted the "practical applications or the popular dissemination of what appeared to him as crude and half-baked theories about the highest subjects."

In response to a rumor that Stewart and Tait were writing a sequel, Maxwell wrote to Tait, using characteristic humor to cloak annoyance with his friend's public religious pronouncements.

> It is said in *Nature* that U. U. [*Unseen Universe*] is germinating into some higher form. If you think of extending the collection of hymns given in the

original work, do not forget to insert "How happy could I be with Ether."

Paradoxical Philosophy, the sequel to *Unseen Universe*, appeared in 1878. It takes the form of a debate between scientifically-minded Christians and a fictitious philosopher, **Hermann Stoffkraft**. A blunt polemic eventually convinces Stoffkraft to trade his erroneous convictions for belief in the doctrines of *Unseen Universe*.

Stoffkraft's convictions were paraphrased from *Kraft und Stoff (Force and Matter)*, an influential work published by Ludwig Büchner in 1855. An atheist, Büchner had found reassurance for his "scientific materialism" in Charles Darwin's writings. Natural selection provided a mechanism for survival and reproduction without an appeal to divine power. Büchner summarized his views in the following way.

> No force without matter—no matter without force!

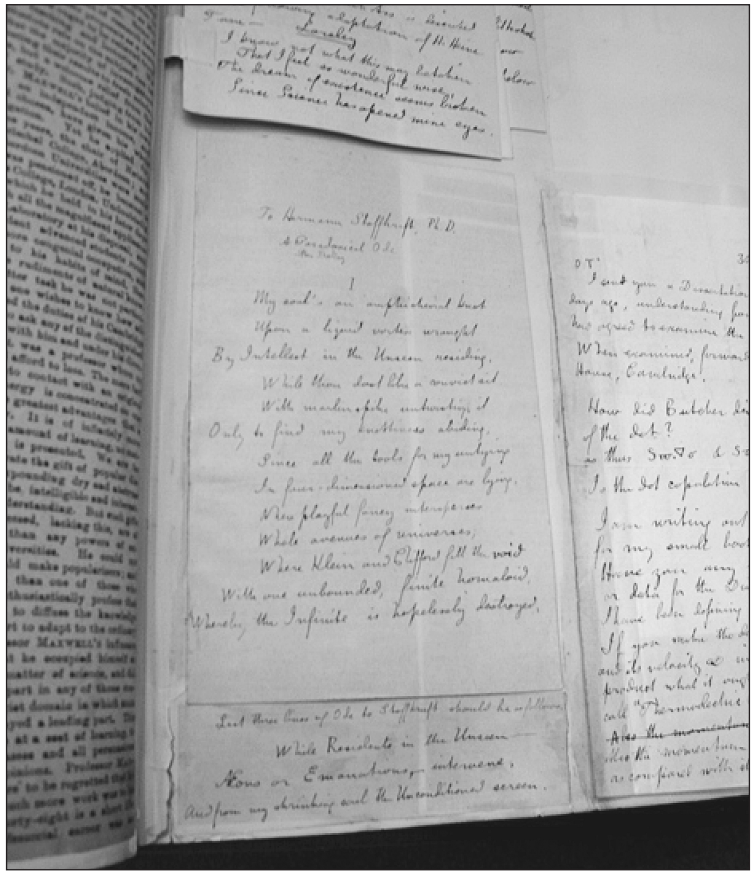
Hermann Stoffkraft, Ph.D., is both the hero of *Paradoxical Philosophy* and the dedicatee of *Paradoxical Ode*.[16] In fact, Maxwell seems to be speaking directly to him.

Maxwell's public attitudes about *Paradoxical Philosophy* were dismissive. In his review, published in the 1878 volume of *Nature*, he concluded:

> The progress of science, therefore, so far as we have been able to follow it, has added nothing of importance to what has always been known about the physical consequences of death, but has rather tended to deepen the distinction between the visible part, which perishes before our eyes, and that which we are ourselves, and to shew that this personality, with respect to its nature as well as to its destiny, lies quite beyond the range of science.

Questions about the soul's immortality were no longer merely academic for Maxwell. He was dying, and possibly knew it by now. For months he had been suffering from stomach pains but he had consulted no doctors. By the time that Maxwell wrote his review, he was having difficulty swallowing.[17] He would soon learn that he had the same cancer that took his mother at the very age that he was now. He would die within a year.

Maxwell wrote his last poem borrowing from Percy Shelley's popular *Prometheus Unbound*. Shelley had been ejected from Oxford for publishing an atheistic tract. Prometheus, the hero of Shelley's four-act play, resembled Jesus in the sense that he was not afraid to speak the truth to his oppressor. *Paradoxical Ode* is addressed to Stoffkraft, who like Prometheus attempted to liberate man from the ancient gods.



Original version of *Paradoxical Ode*.

Taped to the bottom of the page of Tait's scrapbook is an addendum, sent by Maxwell some time afterwards:[18]

> Last three lines of Ode to Stoffkraft should be as follows.
>
> While Residents in the Unseen—
> Æons or Emanations—intervene,
> And from my shrinking soul the Unconditioned screen.[19]

Was Maxwell teasing Tait by pointing out an unfortunate consequence of his philosophy—that the Principle of Continuity should prevent us from ever encountering the Unconditioned, even after death?

Maxwell's thoughts about dying remain hidden from us. Whatever they were, they must have been certain and comforting to him. According to an attending physician, "No man ever met death more consciously or more calmly."

Acknowledgments

I am grateful for the insights of my student Adam Brown, who investigated aspects of Maxwell's *Paradoxical Ode* as a course assignment. I also thank Susan Williams for numerous valuable

suggestions, and the James Clerk Maxwell Foundation for making its materials available.

Notes

1. Convicts in Victorian England were punished with the unpleasant task of recovering hemp from worn, tarred rope. Sailors employed a pointed tool called a **marlinspike** to unravel the rope; convicts had to use their fingers.
2. Professor Keith Moffatt observes: "The invariance of *helicity* is embedded in this verse. Perhaps Maxwell really was a hunderd years ahead of his time!"
3. The Infinite was often identified with God.
4. In Shelley's poem, Asia also drifted down a stream.
5. Inspired by Darwin's *On the Origin of Species*, the English philosopher and economist Herbert Spencer introduced the popular phrase "survival of the fittest" in his 1851 work *Social Statics*.
6. Dr. Stoffkraft asserts that all energy degrades. With the end of humanity, collective consciousness will disappear.
7. Maxwell appears to be assuming a Principle of Differentiability.
8. always.
9. invisible spirits.
10. in the dark.
11. divine power.
12. Maxwell often signed his letters to Tait with $\partial p/\partial t$, $\partial p/\partial t$ = JCM being short-hand for one of the laws of thermodynamics.
13. Clifford wrote: Let us contemplate the reposeful picture of the universal divan, where these intelligent beings whiled away the tedium of eternity by blowing smoke-rings from sixty-three kinds of mouths…How fertile of resource is the theologic method, while it once has clay for its wheel!
14. A review of *Life of James Clerk Maxwell* that appeared in the journal *Nature* in 1882 called Maxwell's use here of the term "poetic license".
15. It appears that Maxwell regarded religious ideas much like the analogies and mechanical devices that guided his scientific intuition. The attitude is suggested in the draft of a letter (never sent) to the Victoria Institute, a society that attempted to reconcile Christianity with science:

> Sir - I do not think it my duty to become a candidate for admission into the Victoria Institute. Among the objects of the Society are some of which I think very highly. I think men of science as well as other men need to learn from Christ, and I think Christians whose minds are scientific are bound to study science that their view of the glory of God may be as extensive as their being is capable. But I think that the results which each man arrives at in his attempts to harmonize his science with his Christianity ought not to be regarded as having any significance except to the man himself, and to him only for a time, and should not receive the stamp of a society… .

16. The appended title "Ph.D." would have seemed exotic to English ears. Although the Ph.D. degree had been awarded in Europe since the twelfth century, its significance as the highest possible degree had begun in German universities during the early nineteenth century. The Ph.D. degree was not awarded in Britain until 1917.
17. According to Campbell, Maxwell's illness had begun by the spring of 1877.
18. The addendum bears no date.
19. hide from view.

References

[1] A. Brown, *Maxwell's Paradoxical Ode*, research paper, University of South Alabama, Fall Term 2006.
[2] L. Campell and W. Garnett, *Life of James Clerk Maxwell*, Macmillan and Co., London, 1882.
[3] C. W. F. Everitt, *James Clerk Maxwell, Physicist and Natural Philosopher*, Charles Scribner's Sons, New York, 1975.
[4] F. Klein, *Lectures on Mathematics*, Macmillan and Co., New York, 1884.
[5] C. G. Knott, *Life and Scientific Work of P. G. Tait*, Cambridge University Press, 1911.
[6] P. M. Harman (ed.), *The Scientific Letters and Papers of James Clerk Maxwell*, 3 volumes, Cambridge University Press, 1990, 1995, 2002.
[7] W. D. Niven (ed.), *The Scientific Papers of James Clerk Maxwell*, Dover, 1965.
[8] K. Moffatt, Homage to James Clerk Maxwell, in *James Clerk Maxwell Commemorative Booklet*, The James Clerk Maxwell Foundation, 1999.
[9] B. Stewart and P. G. Tait, *The Unseen Universe or Physical Speculations on a Future State,* Macmillan and Co., London, 1875.
[10] B. Stewart and P. G. Tait, *Paradoxical Philosophy: A Sequel to The Unseen Universe*, Macmillan and Co., London, 1878.