Group Theory in the Bedroom, and Other Mathematical Diversions

Reviewed by David Austin

Group Theory in the Bedroom, and Other Mathematical Diversions
Brian Hayes
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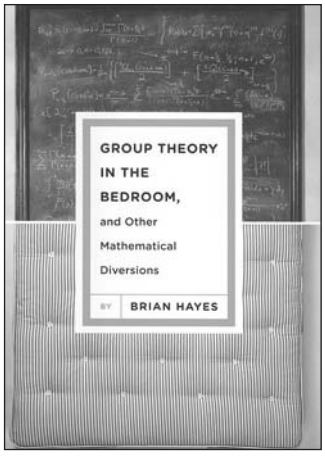
While driving across my home state of Michigan not so long ago, I noticed that the format for the state's license plates had changed. The old format had three letters followed by three digits; sometime recently, a fourth digit was added. Wondering why the change had been made, I figured that the old format had $26^3 \cdot 1000$ possible designations and quickly estimated that this gave around 16 million possibilities. I knew that the population of Michigan is about 10 million so I could see, assuming license plates are not reused, how we could be running low on plates either now or in the near future. Maybe this was a good time to add another digit.

Mathematicians do this kind of thinking frequently. There's no deep mathematics involved, but our comfort with computation can help explain observations we make as well as make a long drive a little less tedious.

In his wonderful new book, *Group Theory in the Bedroom, and Other Mathematical Diversions*, Brian Hayes shows how pleasurable this kind of thinking can be and the surprising places to which it can lead. The book collects and slightly reworks twelve of Hayes' essays, which originally appeared over the last decade in the *The Sciences* and *American Scientist*. Each essay also includes an "Afterthoughts" section in which Hayes reflects on the original work, scrupulously owns up to errors, and discusses responses received from readers.



Let's begin by looking at the essay, "Dividing the continent", appropriately placed in the middle of the collection. Hayes describes a recent driving trip, and the question that arose in his mind as he drove over the continental divide on the Idaho-Montana border: What is the mathematical nature of the continental divide and how would one determine its location? Written in the first person, the essay consists of his evolving thoughts as his journey continues ("Somewhere in North Dakota or Minnesota …I finally began to settle on an idea….").

Eventually, Hayes begins searching for an algorithm to detect the continental divide. Interestingly, his resources are limited; being on the road, he is away from a computer and a library. Thinking is his only tool. Continuing on, Hayes shows us several possible algorithms, which initially seem promising but are seen after further thought to be deficient, before arriving at an especially appealing, elegant solution.

Back at home, Hayes continues the project by acquiring a digital elevation map and implementing his algorithm. Sure enough, the continental divide shows up just where we expect. He also

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searches the literature on this subject, finding early papers by Arthur Cayley and James Clerk Maxwell and comparing their thoughts to his own. Hayes also finds that, in addition to the expected work in geography, this problem is also important in image analysis and artificial vision.

Relating these thoughts as part of a long trip serves Hayes' larger purpose. As he writes in the "Afterthoughts" section: "In telling the story, I wanted to focus not so much on the solution to the watershed problem as on the process by which people go about finding or inventing solutions. Where do the ideas come from? How do we evaluate alternatives? How do we know when to stop?" This story is about traveling rather than arriving, and we feel involved in it. At several points, I put the book down and developed an outline for my own algorithm.

While the other essays do not originate in a long car trip, many of them read like travel essays, as Hayes takes us on expeditions through landscapes surrounding interesting mathematical questions. Sometimes, such as when the continental divide appears before our eyes, we realize the joy has been in the journey as much as the destination. Other times, these investigations lead to surprising questions that reflect more broadly on who we are, what we value, and the nature of mathematics.

Another essay, "Statistics of deadly quarrels", presents a study of the number of human-caused casualties in recent history. Hayes begins by introducing us to Lewis Fry Richardson, a British meterologist who, after serving in the first world war, began to apply the mathematical techniques he had used in meterology to a study of war. Richardson painstakingly attempted to tally all deaths between 1820 and 1950 attributable to the actions of another person. This would, of course, include all war deaths as well as all murders and suicides. (It is easy to speculate that Richardson, perhaps haunted by the horrors he witnessed in the war, was hoping to explain what he had seen.)

Of course, this is a daunting task that is necessarily incomplete. However, a statistical analysis of the data is fascinating for what it does *not* show. Factors that might be thought to influence the outbreak of war—such as countries sharing a common language or engaging in an arms race—are seen to be statistically insignificant. Instead, the most striking observation is that the number of new wars per year closely follows a Poisson distribution, which would follow if the probability of a war breaking out at any given time is constant. This does not, of course, imply that individual wars occur randomly; rather, it appears that war is a perpetual feature of our human existence.

Topics of other essays include:

• a remarkable working clock, constructed with gears in the 1800s, built to be accurate for at least 10,000 years. Hayes deftly relates this to the crisis that was predicted to follow the Y2K phenomenon, a crisis thought to result from software engineers who failed to imagine their code would be running in a few decades;

• an investigation of randomness, whether it really exists, and what it means that mathematics has not found a way to create randomness;

• a survey of early attempts to explain the genetic code, the means by which a string of nucleotides on a DNA molecule represents a sequence of amino acids. Nature's code is much less elegant than the mathematical ones first proposed, but in the end nature seems to have gotten it right;

• an economic model of the distribution of wealth based on the kinetic theory of gases. Paradoxically, an economy based on theft leads to a more equitable distribution of wealth than one based on fair trading;

• the partitioning problem, and what it means for a problem to be hard;

• name spaces, such as radio station call letters and airport codes (and, yes, license plates), and how they fill up;

• ways in which the base three number system is superior to the more familiar binary and decimal systems;

• how it can be tricky to determine if two things are really the same;

• and night thoughts on how to flip a mattress periodically so that all possible orientations are used equally. This essay lends its title to the book as well.

As may be seen, the essays, for the most part, originate in "real world" issues. Hayes writes the "Computing Science" column in *American Scientist*, so it's no surprise that many of the topics have a computational component. Indeed, Hayes is quick to write some code to test an idea, and some readers, like this one, will no doubt be inspired to follow suit. It is not expected, however, that the audience shares this interest in computing. Instead, computation is used as a means to address other issues. In his blog, `http://bit-player.org/`, Hayes accurately describes some of his work as "inquisitive programming", which he defines as "computer programming [used] as a tool for exploring, experimenting, and problem-solving".

Most of the essays contain interesting bits of what could be called trivia, though they never seem superfluous. There are often interesting biographical sketches of little-known players as well as tidbits such as how random numbers are used in the Ethernet networking protocol; the origin of the equals sign; and why Brocot, a French watchmaker, was led to discover what we now call the

Stern-Brocot tree. The writing is uniformly good, exceptionally clear, and with abundant humor and humanity. Hayes writes with a refreshing openness; as part of his colloquial style, he is quick to admit errors and naive ideas.

Group Theory in the Bedroom should be accessible to a wide audience. Hayes uses almost no mathematical notation and assumes relatively little mathematical knowledge. For instance, when e is used at some point, Hayes writes, "e is the number 2.718 (known as Euler's number)." A formula for the Poisson distribution is given, but Hayes quickly gives a lucid interpretation of its meaning.

This is not a book that aims to teach grand mathematical theory or to follow in the footsteps of giants. So what is its aim? Asking the question almost feels wrong. Hayes seems to be enjoying himself so much, and the reader is so involved, that no greater goal seems required. These are, after all, "mathematical diversions". What I found, however, is that the essays, taken as a whole, demonstrate convincingly the joy of mathematical thinking and the real power available when it is applied to more general inquiries.

As much as any book I can name, *Group Theory in the Bedroom* conveys to a general audience the playfulness involved in doing mathematics: how questions arise as a form of play, how our first attempts at answering questions usually seem naive in hindsight but are crucial for finding eventual solutions, and how a good solution just feels right. As Hayes writes, "I'm not a mathematician, but I've been hanging out with some of them long enough to know how the game is played." In addition, Hayes' writing, with its openness, invites the reader to participate actively. I often felt I was having a conversation, and at times an argument, with the author.

We often hear it said that more students need to take more mathematics courses, even if they never subsequently use the content, for mathematics helps build general problem-solving and critical thinking skills. This argument seems dicey to me: why should mathematics, as least as traditionally taught, foster critical thinking any more than philosophy or literature or physics?

Without addressing the issue explicitly, *Group Theory in the Bedroom* presents a more compelling argument for the importance of mathematics. Whether it develops general skills or not, mathematics is a fundamental tool in an intellectual toolkit and is crucial for making sense of the world around us.

About the Cover

Long divide

This month's cover was produced by Brian Hayes, and is derived from images he included in the column "Dividing the Continent" that he wrote for the November 2000 issue of *American Scientist* (available at `http://www.americanscientist.org/issues/pub/dividing-the-continent/`). It illustrates how the algorithm he used proceeds to find the continental divide of North America, drawn as the thin red trail of pixels in the final image. Basically, it floods the ocean basins and keeps track of where they meet as the flood rises.

Brian Hayes has been writing the column "Computing Science" for *American Scientist* for many years, and it is a constant source of interesting mathematical and computational ideas. Some of his columns have been collected in the book *Group Theory in the Bedroom*, reviewed by David Austin in this issue of the *Notices*. The elevation data for Hayes's maps come from `http://www.ngdc.noaa.gov/mgg/global/etopo5.HTML`.

He writes: "The basic idea is to raise the level of both oceans in stages until the waters meet. The line along which they meet is the continental divide. This physical process is easy to describe, but that's because water performs an extraordinary parallel computation when it seeks its own level. Trying to emulate that process in a sequential, digital computer takes considerably more trouble." He also tells us that an animated version of the process can be found at `http://bit-player.org/bph-publications/AmSci-2000-11-Hayes-Cont-Divide/animate.html`.

—Bill Casselman, Graphics Editor
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