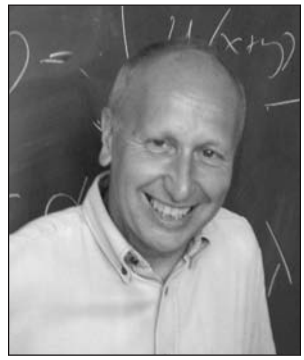
2009 Steele Prizes

I. G. Macdonald

Richard Hamilton



Luis Caffarelli

The 2009 AMS Leroy P. Steele Prizes were presented at the 115th Annual Meeting of the AMS in Washington, DC, in January 2009. The Steele Prizes were awarded to I. G. MACDONALD for Mathematical Exposition, to RICHARD HAMILTON for a Seminal Contribution to Research, and to LUIS CAFFARELLI for Lifetime Achievement.

Mathematical Exposition: I. G. Macdonald

Citation

Symmetric Functions and Hall Polynomials, Second edition. The Clarendon Press, Oxford University Press, New York, 1995.

I. G. Macdonald's book gathers a wealth of material related to symmetric functions into a beautifully organized exposition. Pioneering work of Frobenius, Schur, and others established important connections between symmetric functions and the representation theory of the symmetric group and complex general linear group. Since their work, many further connections were developed with representation theory, algebraic geometry, intersection theory, enumerative combinatorics, special functions, random matrix theory, and other areas. Until the first edition of Macdonald's book appeared in 1979, the theory of symmetric functions was scattered throughout the literature and very difficult to learn. This first edition collected, unified, and expanded such material as Schur functions, Hall polynomials, Hall-Littlewood symmetric functions, the characters of $GL_n(q)$, and the Hecke ring of GL_n over a local field, none of which had previously received an adequate exposition. The second edition of 1995 added a huge amount of new material, including Jack polynomials and the two-parameter Macdonald polynomials, which have subsequently arisen in many unexpected areas, such as in the Hilbert scheme of n points in the plane and in the representation theory of affine Hecke algebras and quantum affine algebras. An especially notable feature of Macdonald's book is the "examples" (really exercises with solutions) which include a vast variety of additional results, many of them original. The importance and popularity of Macdonald's book is evidenced by the more than 3,600 citations on Google Scholar. Macdonald's book has been and continues to be an invaluable resource to researchers throughout mathematics.

Biographical Sketch

Ian Macdonald was born in Middlesex, England, in 1928. After army service he went to Cambridge University in 1949 and graduated in 1952. He spent the next five years in the British Civil Service (government administration). Subsequently, he held teaching positions successively at the universities of Manchester, Exeter, Oxford, Manchester (again), and London. He was elected a Fellow of the Royal Society of London in 1979 and was awarded the Pólya Prize by the London Mathematical Society in 1991.

His research since the 1960s has been mainly in the general area of Lie theory, in particular the combinatorial infrastructure (root systems, Weyl groups) and associated objects such as orthogonal polynomials and power series identities.

Response

I am both honoured and delighted to be awarded a Steele Prize for Mathematical Exposition for my book, *Symmetric Functions and Hall Polynomials*.

The origins of that book go back to the beginning of my mathematical career at Manchester in the late 1950s. Whilst there, I learned about

Hall polynomials and what are now called Hall-Littlewood symmetric functions from Sandy Green, who had recently made crucial use of them in his determination of the character tables of the finite general linear groups. Some years later I was invited to write a survey article on Hall polynomials for the *Jahresbericht der DMV*. That article never got written, partly for the usual reasons but also partly because it became clear to me that such a survey would for the sake of clarity need to be prefaced by a self-contained account of the algebra of symmetric functions, which at that time was lacking in the mathematical literature. I hope my book may have been of service to students and others who need to know the basic facts about symmetric functions, even if their interest in Hall polynomials is minimal.

Seminal Contribution to Research: Richard Hamilton

Citation

The 2009 Leroy P. Steele Prize for Seminal Contribution to Research is awarded to Richard Hamilton for his paper "Three-manifolds with positive Ricci curvature", *J. Differential Geom.* **17** (1982), 255–306.

Differential geometry includes the study of Riemannian metrics and their associated geometric entities. These include the curvature tensor, geodesic distance function, natural differential operators on functions, forms, and tensors as well as many others. A given smooth manifold has an infinite-dimensional space of Riemannian metrics whose geometric behavior may vary dramatically. By its very nature geometry must be coordinate invariant, so two Riemannian metrics which are related by a diffeomorphism of the manifold must be considered equivalent. The question of choosing a natural metric from the infinite-dimensional family is nicely illustrated by the case of compact oriented two-dimensional surfaces. For surfaces of genus 0 there is a unique choice of equivalence class of metrics with curvature 1, while for genus 1 (respectively genus greater than 1) there is a finite-dimensional moduli space of inequivalent metrics with curvature 0 (respectively curvature -1).

The cited paper of Richard Hamilton introduced a profoundly original approach to the construction of natural metrics on manifolds. This approach is the Ricci flow, which is an evolution equation in the space of Riemannian metrics on a manifold. The stationary points (for the normalized flow) are the Einstein metrics (constant curvature in dimensions 2 and 3). The Ricci flow is a nonlinear diffusion equation which may be used to deform any chosen initial metric for a short time interval. In the cited paper Hamilton showed that, in dimension 3, if the initial metric has positive Ricci curvature, then the flow exists for all time and converges to a constant curvature metric. This implies the remarkable result that a three manifold of positive Ricci curvature is a spherical space form (a space of constant curvature). Over the next twenty years Hamilton laid the groundwork for understanding the long time evolution for an arbitrary initial metric on a three-manifold with an eye toward the topological classification problem. For this purpose he developed the idea of the Ricci flow with singularities in which the flow would be continued past singular times by performing surgeries in a controlled way. Finally, through the spectacular work of Grisha Perelman in 2002, the difficult issues remaining in the construction were resolved, and the program became successful.

In addition to the applications to the topology of three-manifolds, the Ricci flow has had, and continues to have, a wide range of applications to Riemannian and Kähler geometry. The cited paper truly fits the definition of a seminal contribution; that is, "containing or contributing the seeds of later development".

Biographical Sketch

Richard Streit Hamilton was born in Cincinnati, Ohio, in 1943. He graduated from Yale summa cum laude in 1963, and received his Ph.D. from Princeton in 1966, writing his thesis under Robert Gunning. He has taught at Cornell University, the University of California at San Diego, and UC Irvine, where he held a Bren Chair. He is currently Davies Professor of Mathematics at Columbia University in New York City, where he does research on geometric flows. In 1996 Richard Hamilton was awarded the Oswald Veblen Prize of the American Mathematical Society, and he is a Member of the American Academy of Arts and Sciences and the National Academy of Sciences.

Response

It is a great honor to receive the Steele Prize acknowledging the role of my 1982 paper in launching the Ricci flow, which has now succeeded even beyond my dreams. I am deeply grateful to the prize committee and the AMS.

When I first arrived at Cornell in 1966, James Eells Jr. introduced me to the idea of using a nonlinear parabolic partial differential equation to construct an ideal geometric object, lecturing on his brilliant 1964 paper with Joseph Sampson on harmonic maps, which was the origin of the field of geometric flows. This now encompasses the harmonic map flow, the mean curvature flow (used in physics to describe the motion of an interface, and also in image processing as well as isoperimetric estimates), the Gauss curvature flow (describing wear under random impact), the inverse mean curvature flow (used by Huisken and Ilmanen to prove the Penrose conjecture in relativity), and many others, including Ricci flow.

James Eells Jr. also first suggested I use analysis rather than topology to prove the Poincaré conjecture on the grounds that it is difficult for topologists to solve a problem where the hypothesis is the absence of topological invariants. And indeed as Lysander said, "Where the lion's skin will not reach, we must patch it out with the fox's." So I started thinking in the 1970s about how to use a parabolic flow to round out a general Riemannian metric to an Einstein metric by spreading the curvature evenly over the manifold. Now the Ricci curvature tensor is in a certain sense the Laplacian of the metric, so that zero Ricci curvature in the Riemannian case is really the elliptic equation for a harmonic metric, while in the Lorentzian case it is the hyperbolic wave equation for a metric, which is Einstein's theory of relativity. So it is only natural to guess that the parabolic heat equation for a metric is to evolve it by its Ricci curvature, which is the Ricci flow.

It is often the case that the credit for a discovery goes not to the first person to stumble upon a thing, but to the first who sees how to use it. So the significance of my 1982 paper was that it proves a very nice result in geometry, that a three-dimensional manifold with a metric of positive Ricci curvature is always a quotient of the sphere. To prove this I developed a number of new techniques and estimates that opened up the field, in particular using the maximum principle on systems to obtain pinching estimates on curvature. Right afterward Shing-Tung Yau pointed out to me that the Ricci flow would pinch necks, performing a connected sum decomposition. I was very fortunate that shortly after I moved to UCSD where I could collaborate with Shing-Tung Yau, Richard Schoen, and Gerhard Huisken, who coached me in the use of blow-ups to analyze singularities, making it possible to handle surgeries. It was also very important that Peter Li and Yau pointed out the fundamental importance of their seminal 1986 paper on Harnack inequalities, leading to my Harnack estimate for the Ricci flow, which is fundamental to the classification of singularities. And in 1997 I proved a surgical decomposition on four-manifolds with positive isotropic curvature. In 1999 I published a paper outlining a program for proving geometrization of three-manifolds by performing surgeries on singularities and identifying incompressible hyperbolic pieces as time goes to infinity, only as I was still lacking control of the injectivity radius, I had to assume a curvature bound. This was supplied four years later in 2003 by the brilliant work of Grigory Perelman in his noncollapsing estimate, which led to his remarkable pointwise derivative estimates, allowing him to complete the program.

But the importance of Ricci flow is not confined to three dimensions. For example, we can hope to prove results on four-manifold topology, which are far more difficult. The Ricci flow on canonical Kähler manifolds is well advanced, based on the work of Huai-Dong Cao and Grigory Perelman, which might lead to a theorem in algebra. Ricci flow also is closely connected to the renormalization group in string theory, and might be used to find stationary Lorentzian Einstein metrics in higher dimensions, giving applications to physics. And just recently we have the very lovely result of Richard Schoen and Simon Brendle using Ricci flow to prove the much stronger result in differential geometry of diffeomorphism rather than homeomorphism in the quarter-pinching theorem using the much weaker assumption of pointwise rather than global pinching. Now that many outstanding mathematicians are working on it, the story of the Ricci flow is just beginning.

Lifetime Achievement: Luis Caffarelli

Citation

Luis Caffarelli is one of the world's greatest mathematicians studying nonlinear partial differential equations (PDE). This is a difficult field: there are rarely exact formulas for solutions of nonlinear PDEs, and rarely will exact algebraic calculations yield useful expressions.

Instead researchers must typically invoke functional analysis to build "generalized" solutions for many important equations. What remains is the profound and profoundly technical problem of proving regularity for these weak solutions and, by universal acclaim, the greatest authority on regularity theory is Luis Caffarelli.

His breakthroughs are so many, and yet so technical, that they defy any simple recounting here. But it was certainly Caffarelli's work on "free boundary" problems that first showed his deep insights. Free boundary problems entail finding not only the solution of some PDEs, but also the very region within which the equation holds. Luis Caffarelli's vast work totally dominates this field, starting with his early papers on the obstacle problem. In estimate after estimate, lemma after lemma, he shows that the generalized solution and the free boundary have a bit more regularity than is obvious, then a bit more, and then more; until finally he proves under a mild geometric condition that the solution is smooth and the free boundary is a smooth hypersurface. The arguments are intricate, but completely elementary.

Later papers introduce countless technical innovations that broaden the analysis to PDE free boundary problems of all sorts. Caffarelli has likewise revolutionized the study of fully nonlinear elliptic PDEs, and particularly the Monge-Ampère equation. His breakthroughs here include boundary second derivative estimates, classifications of possible degeneracies for solutions, regularity theory for optimal mass transfer schemes, etc. In all this work Caffarelli is an endlessly inventive technical magician, for instance using the maximum principle in one paper to derive L_p estimates for second derivatives of solutions.

During his years at the University of Minnesota, the University of Chicago, New York University, the Institute for Advanced Study, and now the University of Texas, Luis Caffarelli has collaborated widely and directed many Ph.D. students. He is extraordinarily generous, in both his personal and professional lives. One of his coauthors at a conference once described extending to a fully nonlinear equation some previous research on a linear PDE. He reported that the earlier workers on the linear equation used a formula for the solution, but that "we had something better than an exact formula. We had Luis."

Biographical Sketch

Luis A. Caffarelli was born in Buenos Aires, Argentina, in December of 1948. He completed his Ph.D. in mathematics at the Universidad de Buenos Aires in 1972 under the direction of Calixto Calderón. In 1973, he came to the United States with a postdoctoral fellowship to the University of Minnesota, where by 1979 he attained a professorship.

He has been a professor at the University of Chicago (1983–1986), the Courant Institute (1980–1982 and 1994–1997), and the Institute for Advanced Study (1986–1996).

Since 1997, Luis Caffarelli has been a professor in the Department of Mathematics and the Institute for Computational Engineering and Science at the University of Texas at Austin, holding the Sid Richardson Chair 1.

He is a member of several academies, including the National Academy of Sciences, holds several honorary degrees and professorships, and has been awarded several distinguished prizes, including the Bôcher Prize of the AMS and the Rolf Schock Prize of the Royal Swedish Academy of Sciences. Finally, he has delivered the AMS Colloquium Lectures; AMS Invited Addresses; and a plenary lecture at the International Congress of Mathematics in Beijing, 2002; and the International Congress of Industrial and Applied Mathematics in Zurich, 2007.

His main mathematical interests are in nonlinear analysis and partial differential equations. He has made contributions in areas concerning phase transitions, free boundary problems, the Landau-Ginzburg equation; fluid dynamics, Navier-Stokes and quasi-geostrophic flows; fully nonlinear equations from optimal control, the Monge-Ampère equation and optimal transportation; and more recently nonlinear homogenization in periodic and random media and nonlinear problems involving nonlocal diffusion processes.

Response

On this very happy occasion, I would like to thank the Selection Committee for having awarded me this great distinction.

I would also like to thank my parents, my wife Irene, and my children Alejandro, Nicolas, and Mauro, for accompanying me through the years and sharing with me their love and their encouragement.

I came to the United States to the University of Minnesota in January of 1973. There was no email, no fax, and even the telephone was very expensive. But I found at Minnesota and in the midwest an extraordinary group of people. My colleagues were extremely generous, dedicated, and friendly, and they taught me much of what I know. They shared their ideas and gave me guidance as I began my research program.

Through the years, I have had the opportunity to belong to wonderful institutions and to befriend and collaborate with extraordinary scientists all over the world. This led to further opportunities to mentor very talented young people who have invigorated my research with new ideas.

The Steele Prize, which so much honors me, should also serve to recognize the many mathematicians who have contributed in so many ways to make nonlinear analysis and applied mathematics a central part of science today.

About the Prize

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository-research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. Each Steele Prize carries a cash award of US$5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2009 prizes, the members of the selection committee were: Enrico Bombieri, Russel Caflisch, Lawrence C. Evans, Lisa C. Jeffrey, Nicholas M. Katz, Gregory F. Lawler, Richard M. Schoen, Julius L. Shaneson (chair), and Richard P. Stanley.

The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.