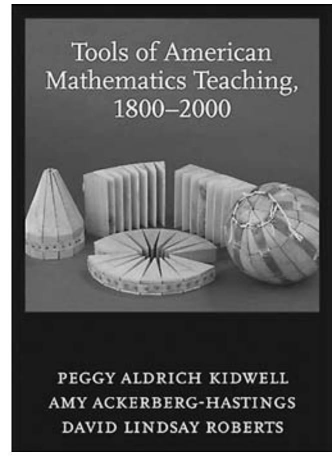
# Tools of American Mathematics Teaching, 1800–2000

*Reviewed by Andrew G. Bennett*




**Tools of American Mathematics Teaching, 1800–2000**
*Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts*
*Johns Hopkins University Press, July 2008*
*US$70.00, 440 pages*
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With the advent of the calculator and computer, questions have been raised about whether we should continue to teach mathematics the way we always have, or if we would be better served by adjusting instruction to take advantage of the power of these new devices. *Tools of American Mathematics Teaching* makes clear that this is not a new question. Indeed, the whole notion of teaching the way we always have makes little sense. As this volume makes clear, tools and instruction have changed throughout history.

This volume is a museum exhibit in book form, based in part on a temporary exhibit on "Slates, Slide Rules, and Software: Teaching Math in America", at the National Museum of American History (part of the Smithsonian Institution). Sections of the book focus on tools with a common purpose (presentation, calculation, measurement, etc.) while individual chapters focus on specific types of tools (blackboards, slide rules, graph paper, calculators, etc.). The individual chapters provide historical context not just for the particular devices but also for the pedagogical ideas that underlie their use in the classroom, and sometimes short biographies of the key figures in their development and adoption. There are many illustrations and the coverage is quite broad. Museums and books are different beasts, however. The focus on things rather than ideas can be rather limiting. In most cases, the chapters look at a limited time period based on when the tools changed form, rather than following ideas through history. For example, the chapter on standardized testing finishes up in the mid-1920s. From a museum perspective this makes sense, since the basic form of the exams is now set and a patron won't see much of anything new looking at additional college entrance exams in glass exhibit cases. However, leaving off in the mid-1920s hides the major changes in standardized testing and how it was used during the twentieth century, which would easily have fit into the text. Other times the focus on tools leads to overemphasizing or underemphasizing ideas based on whether they created tangible objects rather than their importance to mathematics or education. B. F. Skinner and his theories are discussed since he created a visible tool, the Skinner box, but Piaget's ideas get only a brief mention since they don't come with an object to display. This is unfortunate since Piaget's ideas are more important for understanding the current controversies in how to teach mathematics, including the appropriate use of tools.

As you might expect, the chapters can profitably be browsed in any order, though occasionally references are made to ideas introduced in other chapters, not always in order (for instance, "Pestalozzian ideals" are mentioned in chapter 1 but not defined until chapter 6, with additional discussion in chapters 8, 9, and 10). However, reading the book through you do get a sense of the broad themes of educational development over the last two centuries, both in general pedagogy and in specific approaches to mathematics.

An interesting aspect of the book is that it points out that the tools we take for granted—the textbook, the blackboard, graph paper, etc.—all have a history, and we used to do without them. Improvements in production and printing made it

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feasible for all children in a school to have their own textbooks. Along with the introduction of the large blackboard in place of the hand slate, this made possible new approaches where students worked problems from the book on the board for all to see. Soon this led to larger (and cheaper) classes mixing lecture with recitation, where students recited from the book. Some mathematics teachers recognized the need for a series of textbooks that students could use to build mathematical ideas over a series of classes. While America has never had an official national curriculum, the common textbook series have provided the basic outlines of a standard curriculum used across the country. In the uniquely American way, this curriculum has been shaped as much by entrepreneurs looking to profit from book sales as by pedagogical concerns, and the reader will recognize the names of some of the early publishers whose companies are still sending textbook representatives to visit us today.

Cheaper paper and printing also made it feasible to supply school children with prepared graph paper. While engineers made use of ruled paper for surveying and design in the nineteenth century, this was a specialty product and wasn't typically used in schools until early in the twentieth century. Prepared graph paper made it more reasonable to cover graphical ideas in algebra texts. Given the large role graphing plays in algebra classes today, it is striking that the first American algebra text to include graphical treatments was published in 1902, when graph paper was available.

Not all tools led to lasting changes. The overhead projector was developed to help nineteenth-century public lecturers share their slides with a large audience eager to hear about expeditions to faraway lands. While there were hopes that the ability to use prepared illustrations in class would greatly improve classroom attention, the biggest advantage the overhead actually delivered was to let the teacher face the class. The book devotes a chapter to cube root blocks, a dissected cube that could be used to illustrate the algebraic identities underlying the standard algorithm for the arithmetic extraction of cube roots. Despite these demonstrations, the students still found the process difficult and confusing. Since in practice one would use logarithms and/or tables if a cube root was needed, and from a theoretical standpoint the Newton-Raphson algorithm had more to offer, the tool and eventually the entire topic would disappear from the elementary curriculum.

Manipulatives for students to use to build conceptual ideas of numbers are often associated with recent "reform" curricula, but actually have a long history. The ideas of Pestalozzi and later Montessori on early childhood education supported the development of tools to help students visualize numbers and experiment with different ways of combining smaller numbers to make the same sum. These lead in a straight line to the modern Cuisenaire rods, unifix cubes, and base 10 blocks. As children progress, it is natural to move from counting specific objects to a number frame or abacus. Oddly, the early number frames usually featured twelve beads to a wire instead of ten. And, just as today, there were opponents to "object teaching". An 1835 pamphlet[1] "On the Dangerous Tendency to Innovation and Extremes in Education" has much that would fit easily into the current math wars, with complaints about textbooks being "adapted to please rather than to profit" and worries that the students would have difficulty later when studying more abstract topics.

Another situation where history repeats itself is in the discussion of the metric system. I have always found it odd that measurement is usually considered part of the mathematics curriculum, since mathematicians are not experimentalists and usually don't have a talent for measurement. While I presumed measurement fit into the mathematics curriculum because of rounding, it appears it actually entered as an attempt to get students to understand the metric system. Since mathematics has always been taught much more broadly than science, it made much more sense to teach the metric system as a mathematics unit if the goal was to prepare the whole community to adopt a new system. The text shows various demonstration materials that were intended to aid students in getting a sense of what a meter or liter or kilogram meant. However, as we know, the big push for the metric system in the 1970s was mostly a failure. What I didn't know is that there was an earlier push in the 1870s, which was also mostly a failure. The authors quote a Philadelphia editor in 1882: "Easy as it might be to furnish us with rules and sticks graduated off in metres and their parts and with measuring glasses and weights accurately made … it would be by no means so easy to furnish us with the mental standards of reference which our English feet, inches, and yards have become, after years of intimate knowledge of them." The same editorial could have run a century later. The main effect of this early effort to teach the metric system was to solidify measurement as part of the mathematical curriculum, and to ensure that students now all head off to school with a ruler, though for feet instead of centimeters. Perhaps in 2070 America will finally be ready to convert to the metric system.

The final section on electronic devices is by far the weakest of the book. The authors note that it is difficult to write a history of electronic

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[1] *Winslow Hubbard,* On the Dangerous Tendency to Innovations and Extremes in Education, *Tuttle & Weeks, Boston, MA, 1835. A copy of this work is in vol. 16 of the Benjamin Silliman Miscellaneous Pamphlets, Beinecke Library, Yale University, New Haven, CT.*

aids at this time because of the lack of historical perspective. With the focus on tools, there are too many examples covered too quickly to do justice to the big ideas. These tools also fit awkwardly in the framework the authors use. Since computers are used for both presentation and calculation, they don't fit neatly into either section. The authors have therefore decided to put electronic devices into their own section. This means early teaching machines and programmed instruction (such as the Skinner box) are discussed in chapter 5 while computer-aided instruction and other natural descendants aren't discussed until chapter 16. This section should be where the ideas introduced earlier come together to provide a context for understanding the current issues in how we should use tools (or not use tools) in teaching. Unfortunately, the focus on tools rather than ideas gets in the way of developing such a synthesis.

The writers have done a lot of research, with eighty-four pages of footnotes for 317 pages of text. Their research is historical and not mathematical, and the only equation in the text is one reference to $f(x, y) = 0$. There are a few places where I wish they had included a bit more technical detail. For example, in the section on the protractor there is a picture of a Ramsden dividing engine capable of marking a large protractor in divisions as small as five seconds. I was intrigued as to how the machine accomplished such a feat, but the text offered no discussion of this. However, there was enough historical detail given that it was easy to look up information online to find out how the engine worked. The indexing is well done, and when I did encounter references in one section that I vaguely recalled from another section, I didn't find it too hard to track them down.

The book is quite attractive with its many pictures, though they are all in black and white. It would be a nice addition to a department common room or undergraduate lounge, particularly given the format that provides a few pages of description for many different tools, making it a good book for browsing. For the serious student of the history of mathematics education or educational technology, or even the general history of mathematics, this volume provides much information unavailable elsewhere, and, because of that, I believe this is a book that belongs in your university library. However, I find myself unconvinced by the authors' claim that the history of mathematics education "is best understood by considering the stories of specific teaching tools." Such an approach is better suited to a museum than a book. This is an excellent book about the history of teaching tools, but just a good book on the history of mathematics education.