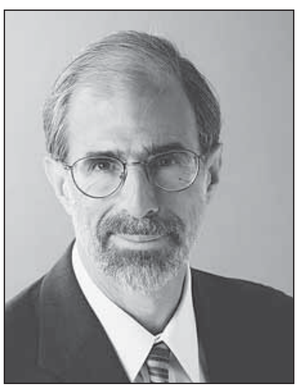
# 2010 Steele Prizes

The 2010 AMS Leroy P. Steele Prizes were presented at the 116th Annual Meeting of the AMS in San Francisco in January 2010. The Steele Prizes were awarded to David Eisenbud for Mathematical Exposition, to Robert Griess for a Seminal Contribution to Research, and to William Fulton for Lifetime Achievement.

## Mathematical Exposition: David Eisenbud

### Citation

The Leroy P. Steele Prize for Mathematical Exposition for 2010 is awarded to David Eisenbud in recognition of his book, *Commutative Algebra: With a View Toward Algebraic Geometry* (Graduate Texts in Mathematics, 150, Springer-Verlag, New York, 1995. xvi+785 pp.)



David Eisenbud

Commutative algebra has grown continuously over the last half-century. For many years, the classic book of Atiyah and MacDonald, *Introduction to Commutative Algebra,* which was first published in 1969, served as students' first glimpse of the field. But the subject has long since moved beyond the material in this book. Periods of strong growth, while enriching, sometimes contributed to distancing new researchers in the subject from one of its main reasons for being: algebraic geometry.

Published in 1995 by Springer, Eisenbud's book was designed with several purposes in mind. One was to provide an updated text on basic commutative algebra reflecting the intense activity in the field during the author's life. Another was to provide algebraic geometers, commutative algebraists, computational geometers, and other users of commutative algebra with a book where they could find results needed in their fields, especially those pertaining to algebraic geometry. But even more, Eisenbud felt that there was a great need for a book which did not present pure commutative algebra leaving the underlying geometry behind. In his introduction he writes, "It has seemed to me for a long time that commutative algebra is best practiced with knowledge of the geometric ideas that played a great role in its formation: in short, with a view toward algebraic geometry."

It is this view which permeates the book and makes it unique. Eisenbud distills from the pure beauty of the subject a "true meaning": he tries, and usually succeeds, in making clear to the reader what is going on behind the scenes—the "why", not only the "what", "who", and "how".

*Commutative Algebra: With a View Toward Algebraic Geometry* presents a wide range of topics, many not typically found in other texts. It gives serious attention to the all-important technique of Gröbner bases. Though there are other good books that use and explain this topic (such as the book by Cox, Little, and O'Shea), Eisenbud's book goes into depth concerning what is called "Gröbner deformation" and gives a full treatment of the critically important fact that generic initial ideals are Borel fixed. Computer and computational algebra is in full swing, a fact not lost in this book. Eisenbud has been on the forefront in relating geometry, syzygies, and regularity, and these topics are given much attention in the book. Likewise, his book has by far the most serious treatment of complexes arising from multilinear algebra. Numerous exercises are given, not just as individual problems but exhibiting the author's broad interests and experience, which further clarify underlying principles. Indeed, the book serves as far more than an introduction to the field; it is used by researchers around the world.

Throughout the text, Eisenbud sprinkles his own commentary, giving the book the strong sense of his own viewpoint. As the reviewer wrote in the *Mathematical Reviews*, "This text has 'personality'—those familiar with Eisenbud's own research will recognize its traces in his choice of topics and manner of approach. The book conveys infectious enthusiasm and the conviction that research in the field is active and yet accessible."

It is this personality, which conveys Eisenbud's broad vision of the field and insistence on conveying basic understanding, that makes *Commutative Algebra: With a View Toward Algebraic Geometry* so special and enduring.

**Biographical Sketch**

David Eisenbud was born on April 8, 1947, in New York City. He attended the University of Chicago for both his undergraduate and graduate education, receiving his Ph.D. in 1970 under Saunders Mac Lane and J. C. Robson. After his degree he joined the faculty at Brandeis University, where he remained until 1997, when he became director of the Mathematical Sciences Research Institute in Berkeley and joined the faculty at the University of California Berkeley. He stepped down as director in 2007 and assumed full-time responsibilities at UC Berkeley. In 2010 he will begin a part-time position as Vice President for Mathematics and the Physical Sciences at the Simons Foundation while continuing his activities at Berkeley.

In 2003–2004 Eisenbud served as president of the AMS. He has held numerous visiting positions at institutes and universities around the world. He has been a Sloan Foundation Fellow (1973–1975), was an invited speaker at the International Congress of Mathematicians in Vancouver in 1974, and was elected to the American Academy of Arts and Sciences in 2006. He has supervised twenty-six Ph.D. students and numerous postdocs. He has contributed to a wide variety of areas, including commutative algebra, algebraic geometry, and computational algebra, with fifty-six coauthors.

**Response**

While I was a graduate student at the University of Chicago (1967–1970), I listened at every chance I got to the beautiful lectures of Irving Kaplansky. He was then just finishing his book *Commutative Rings*, and lectured from it. I admired him and it a great deal, but—in the style of a rebellious adolescent—I was quite ready to proclaim that a lot was left out. In the fall of 1971, visiting at the University of Leeds in England, I had a chance to lecture on one of the things that I felt was missing: Noether normalization (in a version borrowed from Nagata's book, *Local Rings*).

This was the germ from which my own book grew…and grew and grew, by fits and starts, over more than twenty years. Kaplansky's book is a lapidary work, focused, polished, concentrated, like a fine short story. By contrast, mine seems a sprawling novel, trying somehow to include all of mathematics within its borders. I had a lot of fun writing it, though it seemed to take forever. I've been immensely gratified, in the nearly fifteen years since its publication, that people have found it useful and that at least some of them seem to have fun reading it (though that, too, might seem to take forever).

I feel truly privileged to have had such great teachers in commutative algebra and algebraic geometry: Kaplansky, David Buchsbaum, David Mumford, Antonius van de Ven, and Joe Harris in the ten years or so when I felt like a beginner; and, later on, a wonderful sequence of collaborators and students from whom I also learned a great deal. I wanted my book to make some of what I received from them accessible to the whole community. I'm honored to think that this prize recognizes some measure of success in that attempt.

## Seminal Contribution to Research: Robert Griess

**Citation**

The Leroy P. Steele Prize for Seminal Contribution to Research for 2010 is awarded to Robert L. Griess Jr. for his construction of the "Monster" sporadic finite simple group, which he first announced in "A construction of $F_1$ as automorphisms of a 196,883-dimensional algebra" (*Proc. Nat. Acad. Sci. U.S.A.* **78** (1981), no. 2, part 1, 686–691) with details published in "The friendly giant" (*Invent. Math.* **69** (1982), no. 1, 1–102).

Griess and, independently, Bernd Fischer of the University of Bielefeld had earlier suggested the existence of this group, whose order (number of elements) is a 54-digit number. The construction was accomplished by Griess, not only for the first time but also entirely by hand without the aid of a computer. It was a tour de force. We now know, with the completion of the classification of all finite simple groups, that this is the largest "sporadic" finite simple group—that is, the largest finite simple group not fitting into the patterns established by the continuous Lie groups, broadly viewed.

**Robert L. Griess**

But beyond the sheer magnitude of the numbers involved, the discovery of this group has touched science and mathematics very deeply. Connections have emerged with areas as diverse as string theory in physics and, within mathematics itself, in very sophisticated number theory; see, for instance, the papers by Richard E. Borcherds, "Sporadic groups and string theory" (*First European Congress of Mathematics*, Vol. I (Paris, 1992), 411–421, *Progr. Math.* **119**, Birkhäuser, Basel, 1994) and "Monstrous moonshine and monstrous Lie superalgebras" (*Invent. Math.* **109** (1992), 405–444). (Here "Monstrous Moonshine" is a term coined by John Conway in reaction to the surprising relationships, first observed empirically by John McKay, of character degrees of the Monster and modular function theory. See the Wikipedia article http://en.wikipedia.org/wiki/Monstrous_moonshine. Also on the Web are lectures by Edward Witten suggesting a role for the Monster in 3-dimensional quantum gravity—e.g., http://www.nonequilibrium.net/81-edward-wittens-talk-3d-gravity/.) In addition, the group and its construction by Griess have stimulated the development of the important new subject of vertex operator algebras, cf. Igor Frenkel, James Lepowsky, Arne Meurman, "Vertex operator algebras and the Monster" (*Pure and Applied Mathematics* **134**, Academic Press, Boston, MA, 1988). There are even philosophical implications, in that these discoveries, though certainly related to topics investigated from the point of view of continuous Lie group theory, were not at all found from that perspective but were revealed when one pushed hard enough in the world of finite structures. The group is the "jewel in the crown" for those mathematicians who worked so hard to understand all the finite simple groups.

**Biographical Sketch**

Robert L. Griess Jr. was born in Savannah, Georgia, in 1945. Shortly afterward, his family returned to Pittsburgh, Pennsylvania, where he attended public schools. He received his undergraduate and graduate degrees at the University of Chicago, studying with adviser John Thompson, and wrote a thesis on central extensions of simple groups. In 1971 he became a Hildebrandt Instructor at the University of Michigan, where he is currently a professor.

His honors include a Guggenheim Fellowship, an invited address at the International Congress of Mathematicians in 1983, the Harold Johnson Diversity Award at the University of Michigan, and membership in the American Academy of Arts and Sciences. He has held visiting positions at Rutgers University, the Institute for Advanced Study, Yale University, Ecole Normale Supérieure in Paris, University of California Santa Cruz, National Cheng Kung University in Taiwan, and Zhejiang University in China. His current research interests are finite groups, finite aspects of Lie theory, vertex operator algebras, and rational lattices.

**Response**

My sincerest thanks go to the individuals who chose me for this great honor and those who helped me during my career. My construction of the Monster took place during my first sabbatical at the Institute for Advanced Study in 1979–1980. It was the result of intense mental and physical passion over several months.

Within the finite group theory community in the early 1970s, a feeling grew that classification of finite simple groups might be possible. The majority view remained skeptical for years. By 1973, when Bernd Fischer and I quite independently found evidence for the Monster, new sporadic groups had been appearing steadily for about eight years. There was no obvious reason why the flow should stop. Much larger groups than the Monster could have been out there, waiting to be discovered.

By the late 1970s the optimism about classifying finite simple groups had increased. A new school of thought, Moonshine, showed us that the Monster group was connected to some classical number theory on the upper half complex plane. Other amazing coincidences involving sporadic groups were proposed. Suddenly, there were new contexts in which to regard the finite simple groups. I felt inspired to try a construction. After some trial explorations came full involvement in late 1979. I built a nonassociative, commutative algebra of dimension 196883 and gave enough automorphisms to generate a finite simple group with the right properties. On 14 January 1980 I mailed an announcement about my existence proof to group theorists. What a wonderful coincidence that I would be awarded the Steele Prize on 14 January 2010, exactly thirty years later!

The construction of the Monster resolved an existence question in the classification of finite simple groups and, moreover, as corollaries, there followed new existence proofs of several smaller sporadic groups which had been constructed earlier by combinatorial methods or computer. This brought some unity to the world of sporadic groups. The Monster involves twenty of the twenty-six sporadic groups. Uniqueness of the Monster was proved in the late 1980s by Meierfrankenfeld, Segev, and me. (Like the construction, this uniqueness result was relatively hard and had been an open problem for years.)

As the years went by, we saw the sporadic groups play roles in emerging vertex algebra theory, in theoretical physics and algebraic topology. The nonassociative algebra which I defined is part of the Moonshine vertex operator algebra of Frenkel, Lepowsky, and Meurman, constructed in the mid-1980s. This vertex operator algebra has the Monster as its full group of automorphisms. Borcherd's proof of the Moonshine conjectures made use of vertex algebras and infinite-dimensional Lie theory. With coauthors, I have studied vertex operator algebras and their automorphism groups. About 2005, Chongying Dong, Ching Hung Lam, and I proved the first partial uniqueness theorem for that Moonshine vertex operator algebra. I expect that thirty years from today—when I hope to see all of you again!—finite simple groups will be more fully integrated into other parts of mathematics.

About the citation, I have two comments. First, while I was in graduate school, I met Bernd Fischer and corresponded with him for years. He taught me so much about his beautiful and original ideas. Secondly, the citation mentions that most finite simple groups are analogues of continuous groups over finite fields, while also remarking that the Monster led to mathematical insights not obtained from the continuous (Lie-theoretic) point of view. I add that sporadic group theory, in general, involves significant finite mathematics which I still do not see as contained in or suggested by Lie theory. There is no theory for sporadic groups like BN-pairs for groups of Lie type. While there are many theorems about sporadic groups, how they should be placed within mathematics remains an open question.

## Lifetime Achievement: William Fulton

### Citation

The 2010 Steele Prize for Lifetime Achievement is awarded to William Fulton. Through his research, his writing, and his intellectual leadership, Fulton has played a pivotal role in shaping the direction of algebraic geometry and in forging and strengthening ties between algebraic geometry and adjacent fields. His teaching and mentoring have nurtured several generations of younger mathematicians. In short, he is a giant of the mathematical profession.

Fulton has made important contributions to many topics in algebraic geometry, but he is probably best known for his transformational work in intersection theory. This theory has stood at the center of algebraic geometry since the beginnings of the field, but the foundational revolutions led by Weil, Zariski, Serre, and Grothendieck in the 1950s and 1960s largely passed it by. As late as the mid-1970s, intersection theory remained a collection of ad hoc tools without an overarching organizational principle, and it was not clear in what generality the tools applied. Working in part with MacPherson, in the late 1970s Fulton created a completely new approach that revolutionized the subject and greatly extended its applicability.

In the classical approach to intersection theory, one started by proving a moving lemma to reduce to the situation in which cycles met properly, i.e., in the expected dimension. The fundamental innovation of Fulton and MacPherson was to develop a theory that worked directly with possibly excess intersection and so avoided the necessity of perturbing the original data. This led to a vast strengthening and clarification of the machine, and the basic method they used—the so-called deformation to the normal cone—has had many important ramifications throughout algebaic geometry and beyond. For instance, it suggested the construction of virtual fundamental classes, which in turn opened the door to the explosive development of Gromov–Witten theory in the last fifteen years. Fulton's research in this area appears in his *Ergebnisse* volume *Intersection Theory*. Universally recognized as a classic, this beautifully written book has become an essential part of the library of every mathematician working anywhere near algebraic geometry. It was awarded the Steele Prize for Exposition in 1996.

In many respects, Fulton's writing and scientific leadership have been as important as his research. He has repeatedly shown an uncanny ability to recognize areas that are ripe for development and to create the framework making this development possible. The theory of toric varieties provides a good case in point. Although there had been important applications of the machine to combinatorics by Stanley, until the late 1980s the theory of toric varieties was a relatively quiet area. Fulton realized that this was in fact a very fertile subject having contact with many parts of mathematics, and in 1993 he published lectures giving a clean and concise working out of the theory. His book was in large part responsible for a dramatic growth of activity in the field, and nowadays toric varieties are a central feature of the algebro-geometric landscape. Something similar happened more recently when Fulton was one of the first to recognize the importance of a circle of questions linking Hermitian matrices, invariant theory, and Schubert calculus following work of Klyachko and others. Through lectures and an influential article in the *Bulletin of the AMS*, Fulton vigorously promoted what subsequently became one of the most exciting topics on the boundary between algebraic geometry and representation theory. One should also mention Fulton's behind-the-scenes influence at the beginning of quantum cohomology: his notes with Pandharipande in 1997 fulfilled a critical foundational need just as the area was starting to take off.

**William Fulton**

Fulton is also famous for his magic touch in mentoring postdocs and graduate students. He is extremely generous both with mathematical guidance and with the sort of practical advice that is so important at the start of a career. Many of the younger leaders of contemporary algebraic geometry were in Fulton's orbit as postdocs at Brown, Chicago, or Michigan, and he has been equally successful as a Ph.D. advisor. Under his leadership, all of the institutions where he was employed became international centers of the field. Finally, Fulton has a remarkable gift for recognizing and encouraging budding mathematical talent. For years, he has worked tirelessly to promote the careers of promising young mathematicians all over the world.

### Biographical Sketch

William Fulton, born in 1939, grew up in Naugatuck, Connecticut, where he spent more time on music and sports than mathematics. As an undergraduate at Brown, the inspiration of John

Wermer and Herbert Federer led to a concentration on mathematics. He attended graduate school at Princeton, where John Milnor, John Moore, and Goro Shimura were particularly influential teachers; his thesis with Gerald Washnitzer was on tame fundamental groups.

During a postdoc at Brandeis, he taught a course on algebraic curves, which led to a text still in use (and available free on the Internet). He spent seventeen years at Brown, with Bob MacPherson and Paul Baum, joined later by Joe Harris, Dick Gross, and Jean-Luc Brylinski, where a remarkable center in algebraic geometry, topology, and number theory flourished. His book *Intersection Theory* appeared in 1984. This was followed by eleven years at the University of Chicago, where he became the Charles L Hutchinson Distinguished Service Professor. There he had the chance to teach splendid graduate students in advanced courses, leading to several texts. At Chicago he had the opportunity to interact with many stimulating postdocs, of whom Burt Totaro and Rahul Pandharipande were particularly influential.

In 1998 he moved to the University of Michigan, where he has held the Miner Keeler Chair in Mathematics and recently became the Oscar Zariski Distinguished University Professor. At Michigan he is enjoying the stimulating atmosphere provided by his colleagues, postdocs, and graduate students in algebraic geometry and surrounding areas. He has held visiting positions at the University of Genoa, Aarhus University, IHES, IAS, MSRI, Mittag-Leffler Institute as the Erlander Professor, and Columbia University as the Eilenberg Visiting Professor.

Fulton is a member of the National Academy of Sciences and the American Academy of Arts and Sciences, and he is a foreign member of the Royal Swedish Academy of Sciences. He was the managing editor of the *Journal of the AMS* from 1995 to 1998.

**Response**

Most pleasures in a mathematician's life, at least those related to mathematics, come from discovering something new or finding a proof of something one has worked on, alone or with others, for a long time. Other pleasures come from seeing colleagues or students solve problems one has thought about; indeed, the increase in these pleasures as one gets older compensates for fewer of the former. Being awarded a prize like this from one's peers is yet another fine pleasure for a mathematician, and I am most grateful to the committee and the AMS for this award.

I have indeed been fortunate in my career, with the inspiring teachers I had as an undergraduate and graduate student and the many splendid colleagues, postdocs, and students with whom it has been a joy to work. In particular, my career would be nothing like it has been without the collaboration with Bob MacPherson on intersection theory and the work with Rob Lazarsfeld on positivity. I am grateful to them and my many other collaborators for making this award possible.

## About the Prize

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository-research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. Each Steele Prize carries a cash award of US$5,000.

Beginning with the 1994 prize, there has been a five-year cycle of fields for the Seminal Contribution to Research Award. For the 2010 prize, the field was algebra. The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2010 prizes, the members of the selection committee were: Enrico Bombieri, Russel Caflisch (chair), Peter S. Constantin, Lisa C. Jeffrey, Gregory F. Lawler, Richard M. Schoen, Joel A. Smoller, Richard P. Stanley, and Terence C. Tao.

The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.