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Notices

of the American Mathematical Society

May 2010

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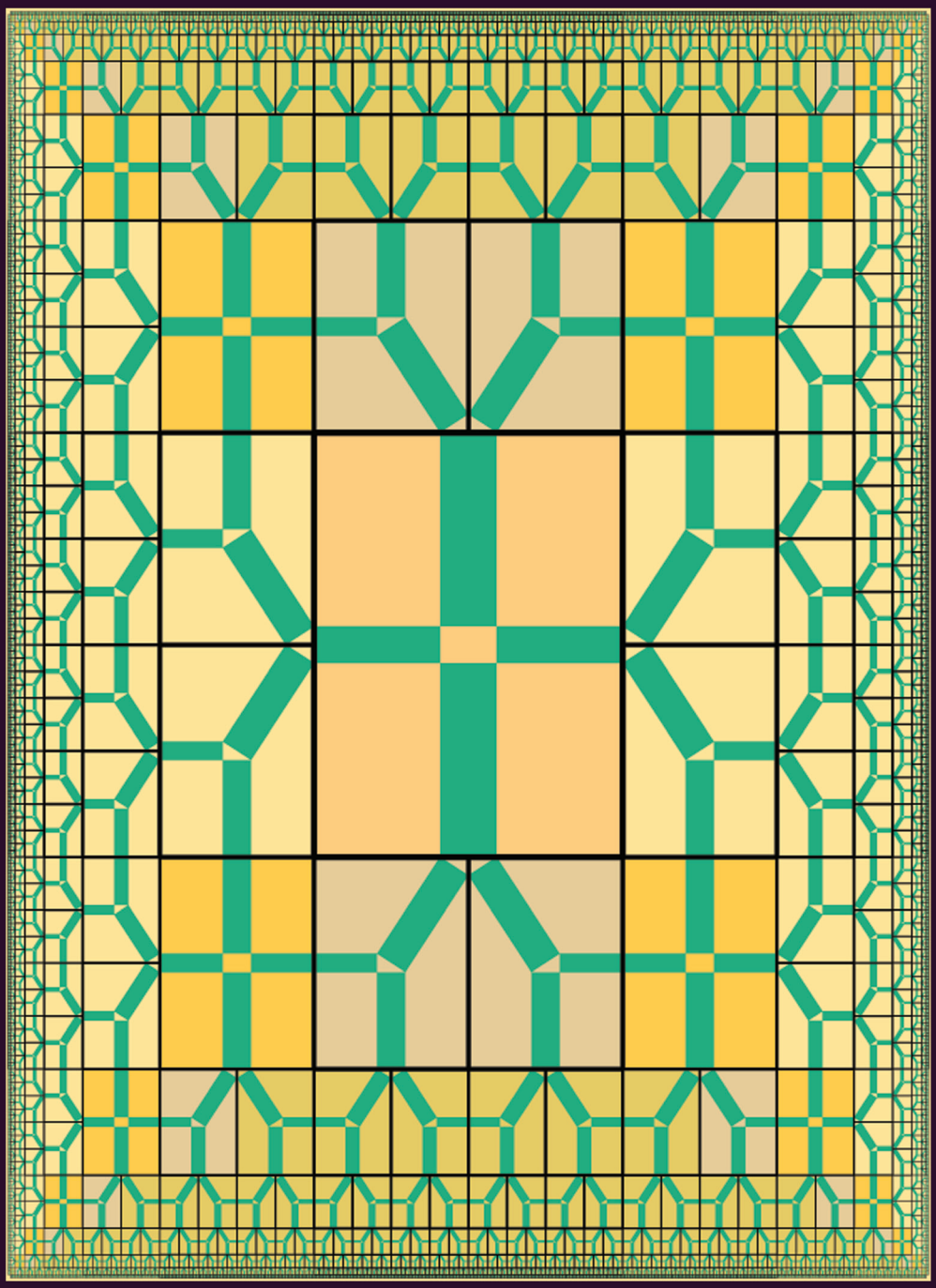

Fractal origami (see page 630)

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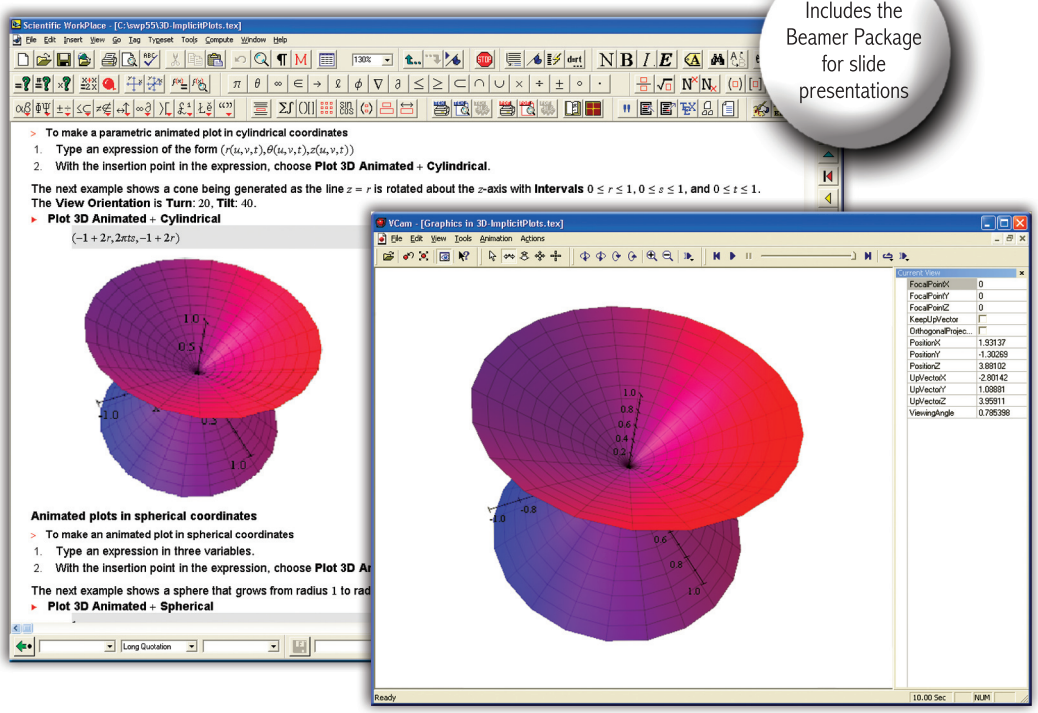

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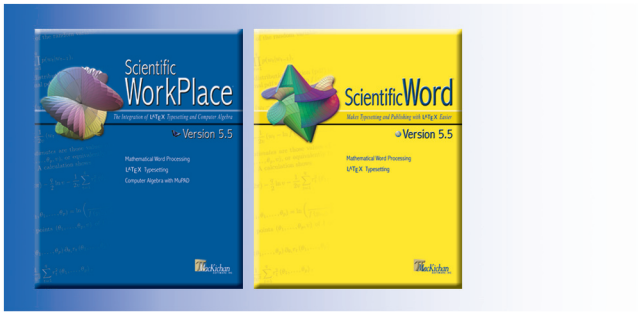

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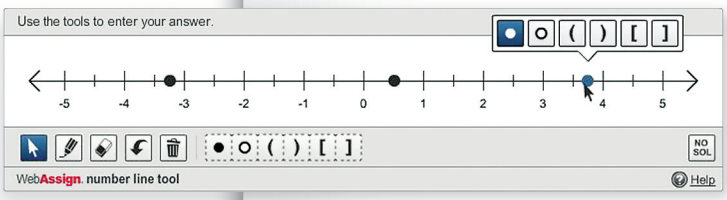

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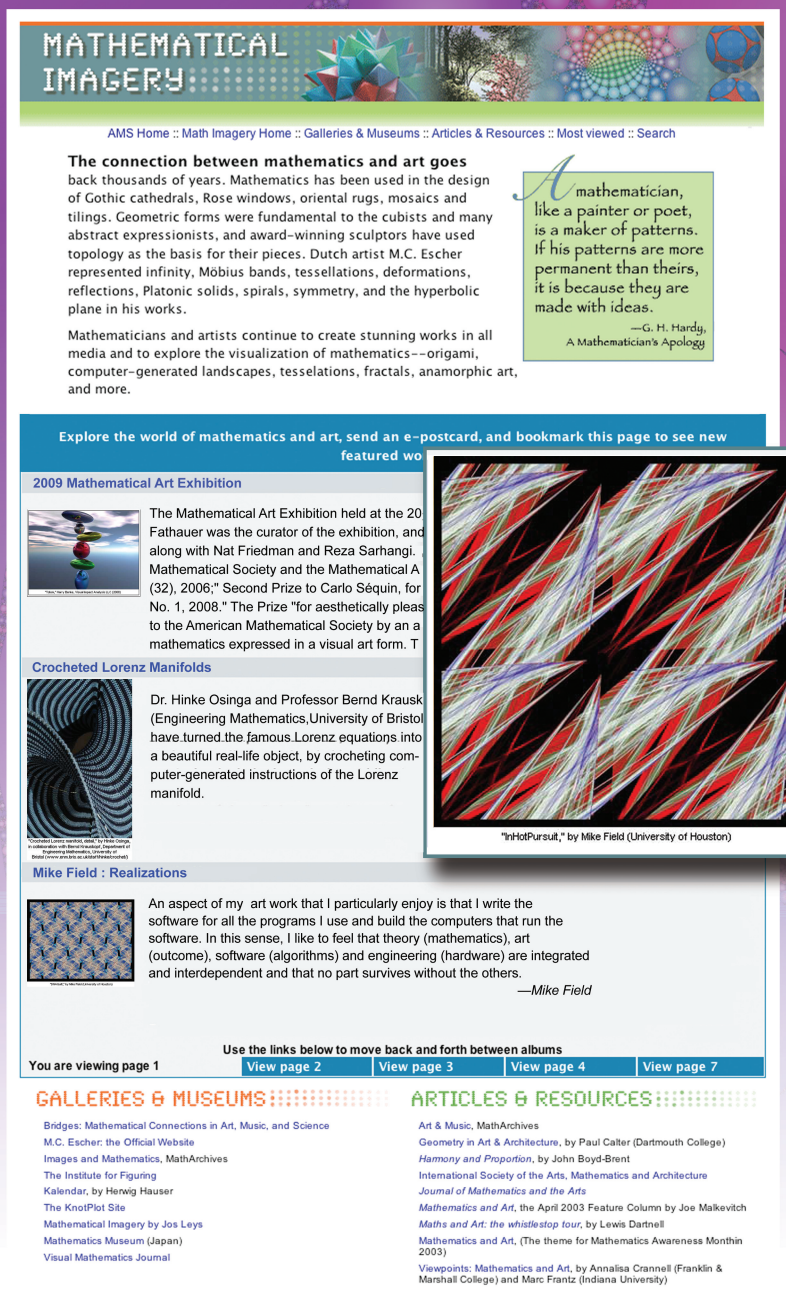

MATHEMATICAL IMAGERY

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The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Dutch artist M.C. Escher represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works.

Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics--origami, computer-generated landscapes, tesselations, fractals, anamorphic art, and more.

A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.

—G. H. Hardy, A Mathematician's Apology

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2009 Mathematical Art Exhibition

The Mathematical Art Exhibition held at the 20 Fathauer was the curator of the exhibition, and along with Nat Friedman and Reza Sarhangi. Mathematical Society and the Mathematical A (32), 2006;" Second Prize to Carlo Séquin, for No. 1, 2008." The Prize "for aesthetically pleas to the American Mathematical Society by an a mathematics expressed in a visual art form. T

Crocheted Lorenz Manifolds

Dr. Hinke Osinga and Professor Bernd Krausk (Engineering Mathematics,University of Bristol have turned the famous Lorenz equations into a beautiful real-life object, by crocheting computer-generated instructions of the Lorenz manifold.

"InHotPursuit," by Mike Field (University of Houston)

Dear Bill,

Here's one of the e-postcards from the site.

Annette

Mike Field : Realizations

An aspect of my art work that I particularly enjoy is that I write the software for all the programs I use and build the computers that run the software. In this sense, I like to feel that theory (mathematics), art (outcome), software (algorithms) and engineering (hardware) are integrated and interdependent and that no part survives without the others.

—*Mike Field*

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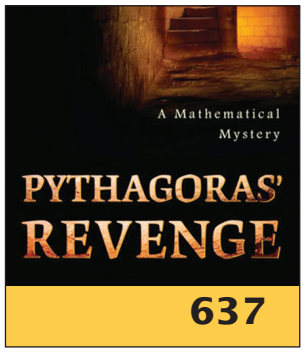

This month offers an eclectic selection of articles for the *Notices*. Origami, education, eigenvalues of the Laplacian, and special effects in the movies are the topics of our feature articles. Each illustrates the role of mathematics in our everyday lives. Each is fascinating in its own way.

—Steven G. Krantz
Editor

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of the American Mathematical Society

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I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz
Editor

A New Education Column for the *Notices*

Although many of us define ourselves primarily as research mathematicians, we are all of us first and foremost educators. We all teach, we all interact with students, we all direct Ph.D. theses. Teaching pays the bills, and is the most visible endeavor on our plates. The dean may have a vague idea of our scholarly presence, and perhaps a fleeting notion of what our research programs are about, but he is intensely aware of our teaching—both its quantity and its quality. He knows when it is good and effective, and he knows when it is bad. When I was chair of the math department, the dean of Arts & Sciences regularly said to me that "Teaching calculus is your main product. Attend to it." In the same breath he sometimes reminded me that the dean at the University of Rochester fifteen years ago shut down the Mathematics Graduate Program. My dean did not say this in an attempt to be friendly

Today we are all interested in teaching issues. Sometimes this is because we have our own kids in school. Sometimes it is because we are caught up in various teaching controversies. Sometimes it is because we have a particular agenda in the teaching game. Whatever the reason, teaching is a part of life, and an important and essential one. We all have a vested interest. We all teach virtually every day of the week. We all interact with students on a regular basis. It is worthwhile to examine the fundamentals of this activity, and to endeavor to develop and improve our teaching presence.

Since the *Notices of the American Mathematical Society* is the organ of record of the Society, this periodical should be a forum for important discussion of new teaching ideas. And old ones too. Like mathematical theorems, good ideas in teaching have a life of their own. And they are worth preserving and developing. Innovative techniques in teaching deserve to be explored and examined. The uses of new hardware and software should be discussed. New ideas need to be examined and weighed.

Discussions of education issues should be incisive and on point, but they should also be lively. With this thought in mind, we wish to launch here a regular *Notices* column about education issues. This column will include K–12 education, college and university education, graduate and postgraduate education, and international educational issues. The fundamental axiom is that there are no holds barred, save those dictated by scholarly decorum. We want to hear all opinions about everything educational, expressed concisely and forcefully. A typical column will be about 1,200 words. Some columns will appear by invitation and others will be offered by the author.

We have assembled a distinguished board of editors for the education column: David Bressoud of Macalester College, Roger Howe of Yale University, Karen King of New York University, and William McCallum of the University of Arizona. These renowned scholars and educators will help me to solicit and select columns for the *Notices*, and to edit them.

Our goal with this education column is to foster dialogue and debate. We are not looking for "feel good" pieces. We want columns that will elicit a strong reaction, and that will give rise to letters to the editor and to future columns. The development of new teaching techniques and new curricula is an ongoing process, and one that requires constant attention. Anticipating that this column will be a productive contribution to that dialogue, we seek contributions from working mathematicians, from educators, from administrators, from teachers, and from anyone with an active interest in the development of means to educate our youth.

Everyone knows that there is a teaching reform movement, and a traditionalist movement, and perhaps several variations and nuances on these two main themes. Teaching is a very personal process, and there is no "right" point of view, and no party line. My own feeling is that teaching is an art and a process, one that has been developing for at least three thousand years. Each teacher develops the craft of teaching for himself or herself, to suit the values and the techniques with which he/she is comfortable. Today we use some of the techniques (like lectures and Socratic dialogue) that have been used since the beginning, and we have many new ones as well. Group learning, self-discovery, Online learning, computer simulations, and interactive learning are just some of the ideas that are being explored today. Electronic media make possible a number of new types of texts and teaching tools. There are many aspects of teaching to consider, and several of these are new and unfamiliar. I hope that our education column will be an active forum for acquainting the mathematics community with new developments and new ideas and new perspectives. It is an exciting prospect, and I invite you to participate.

The first education column, by Herb Wilf, will appear in the June/July 2010 issue of the *Notices*.

—*Steven G. Krantz*
Editor, Notices of the AMS

Origami and Partial Differential Equations

Bernard Dacorogna, Paolo Marcellini, and Emanuele Paolini

Origami is the ancient Japanese art of folding paper. Even if origami is mainly an artistic product, it has received a great deal of attention from mathematicians, because of its interesting algebraic and geometrical properties.

We present a new mathematical model of origami with a double purpose. On one hand we give an analytical approach which provides a new perspective to the existing algebraic and geometrical models. On the other hand we use origami as a tool to exhibit explicit solutions to some systems of partial differential equations.

Mechanical properties of paper, as a material, are simple. A sheet of paper is rigid in tangential directions. Indeed, it cannot be stretched, compressed, or sheared. If a sheet of paper is constrained on a plane, it would only be possible to achieve rigid motions, i.e., rotations and translations of the whole sheet. On the other hand, in the normal direction it can be easily folded. This is due to the infinitesimal thickness of the paper and to the elastic properties of the fibers composing the material.

The physical properties we have described so far can be defined from a mathematical point of view. To this aim we let $\Omega \subset \mathbb{R}^2$ be a two-dimensional domain (usually a rectangle), which is the reference configuration for the sheet of paper. An origami is a suitable immersion of the sheet of paper in the three-dimensional space. Hence it can be identified with a map u (see Figures 1 and 2)

$$u\colon \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3.$$

The tangential rigidity can be expressed by requiring that u is a *rigid map* of Ω into $\mathbb{R}^3$. In other words the gradient $Du(x)$ of the map u is an *orthogonal* 3×2 matrix, meaning that,

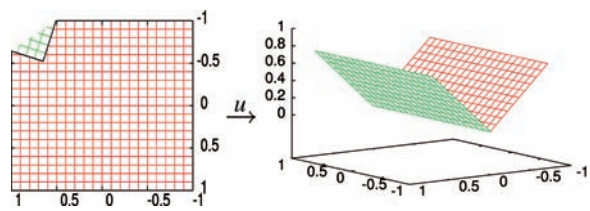

Figure 1. The map $u(x,y) = (x, \frac{y}{\sqrt{2}}, \frac{|y|}{\sqrt{2}})$ describes a 90-degree folding in $\mathbb{R}^3$ of the square paper Ω in the left-hand side. The gradient of u is a 3×2 *orthogonal* matrix.

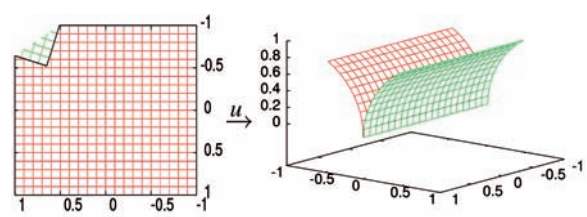

Figure 2. The map $u(x,y) = (x, \mathrm{sign}(y)(1-\cos y), \sin|y|)$, is not piecewise linear. However, the gradient is a 3×2 orthogonal matrix.

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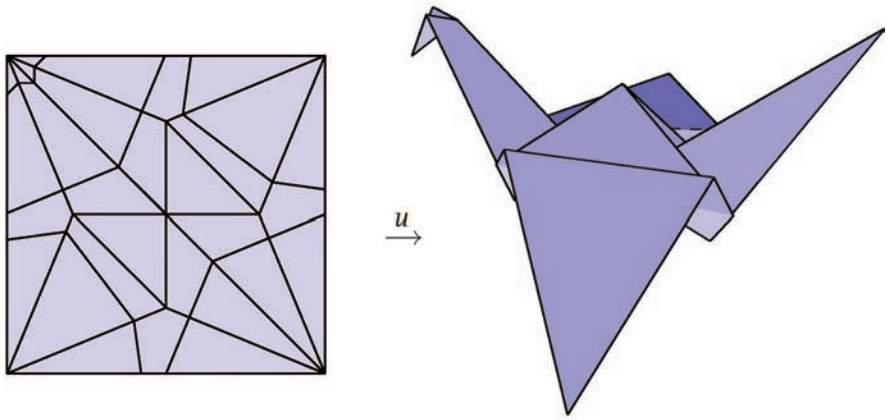


Figure 3. On the right: the crane is the most famous origami; on the left: the corresponding singular set.

for every $x \in \Omega$,

$$Du(x) \in O(3,2) := \{M \in \mathbb{R}^{3\times 2} : M^t M = I\}.$$

Since $Du^t Du = I$, the rank of Du is maximal; hence the immersion is locally one-to-one. Thus, if u is sufficiently regular, it is a *local isometry* between $\mathbb{R}^2$ and the manifold $u(\Omega) \subset \mathbb{R}^3$ with the metric induced by the ambient space $\mathbb{R}^3$. Therefore the Gauss curvature of $u(\Omega)$ coincides with the Gauss curvature of Ω, which is zero, and $u(\Omega)$ is a developable surface (see [15]).

Since origami is a folded paper, the map u cannot be everywhere smooth; it is only *piecewise* smooth. Folding creates discontinuities in the gradient. Since we do not allow cutting the sheet of paper, u is, however, a continuous map. The singular set $\Sigma_u \subset \Omega$, which is the set of discontinuities of the gradient Du, is called *crease pattern* in the origami context. Usually this set is composed by straight segments (as in Figure 3), but it is also possible to make origami with curved folds (see Figure 4).

This formulation of an origami as a map $u: \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3$ also allows us to consider *modular origami*, which corresponds to a domain Ω which is not connected (several sheets of paper), as for instance in Figure 5.

A special attention will be given to the so-called *flat origami*. A flat origami is defined as a map whose image is contained in a plane; it can be represented, up to a change of coordinates, as a map

$$u: \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2.$$

In this case it can be proved that the folding lines must be straight segments which satisfy some *angle condition* at every vertex of the singular set Σ_u. More precisely, at any vertex of the singular set Σ_u, exactly $2k$ consecutive angles $\alpha_1, \cdots, \alpha_{2k}$ meet with the property that

$$\alpha_1 + \alpha_3 + \cdots + \alpha_{2k-1} = \alpha_2 + \alpha_4 + \cdots + \alpha_{2k} = \pi.$$

We will see (cf. Theorem 4.1) that such condition is enough to guarantee, on a simply connected open set Ω, that a given union of segments Σ is actually the singular set Σ_u of some rigid map u.

Our general definition allows us, in a natural way, to consider in the higher-dimensional case a map $u: \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$, where the gradient $Du(x)$ is an orthogonal $m \times n$ matrix. That is, $Du(x) \in O(m,n)$, where

$$O(m,n) := \{M \in \mathbb{R}^{m\times n} : M^t M = I\}.$$

When $m = n$ we simply write $O(n)$. In this case our mathematical global approach also allows us to consider *flat* origami in higher dimensions ($u: \Omega \subset \mathbb{R}^n \to \mathbb{R}^n, Du(x) \in O(n)$), which will also be called *hyper-origami*. As an example, for $n = 3$, we will describe, in the section "Fractal Construction", how to "fold" a cube.

Closely related is the more difficult problem of finding a map with orthogonal gradient satisfying some boundary condition. For example, when $n = m = 2$, we consider the Dirichlet problem

$$(1) \qquad \begin{cases} Du(x) \in O(2) & \text{for a.e. } x \in \Omega \\ u(x) = 0 & \text{for } x \in \partial\Omega. \end{cases}$$

At first sight one could think that, by means of repeated folding, a sheet of paper could in principle be crunched to a single point, say the origin. To convince oneself that this is not possible it is enough to look at the equation. An origami folded to a single point y_0 $(= 0)$ is in fact represented by the constant map $u \equiv y_0$, which, however, has gradient $Du \equiv 0 \notin O(2)$.

In order to solve (1), we have to take advantage of the fact that the boundary does not have full dimension. Anyway, the construction cannot be that simple; in fact, it is impossible to find a map solving the system and which is *smooth* on some neighborhood of a point of the boundary. Indeed,

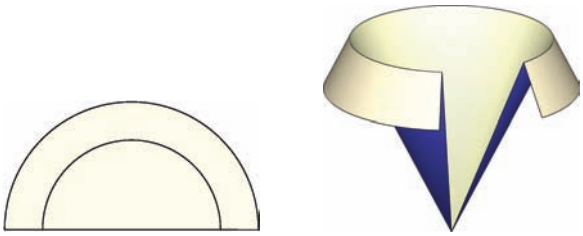


Figure 4. A circular crease can be folded to obtain a cone.

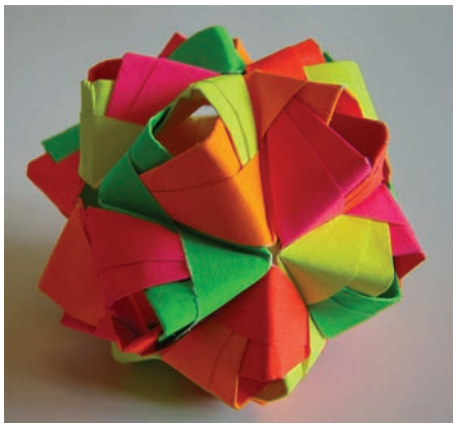


Figure 5. A picture of a *modular* origami representing an *icosahedron*, built with 30 identical copies of a folded square, linked together in a symmetrical way.

let u be a smooth map which satisfies $Du \in O(2)$ or equivalently

$$|Du|^2 = 2\,|\det Du| = 2.$$

Since $\det Du = \pm 1$, by continuity the Jacobian has a sign, say $\det Du = 1$. Therefore the equation $|Du|^2 = 2 \det Du$, with the notation $u(x,y) = \binom{v(x,y)}{w(x,y)}$, can be easily transformed into the Cauchy-Riemann system

$$\begin{cases} v_x - w_y = 0 \\ v_y + w_x = 0 \end{cases}$$

with furthermore $|Dv| = |Dw| = 1$. Denoting by τ, ν, respectively, the tangent and normal unit vectors on $\partial\Omega$, we thus have, up to sign,

$$\langle Dv; \tau \rangle = \langle Dw; \nu \rangle, \qquad \langle Dw; \tau \rangle = \langle Dv; \nu \rangle.$$

Since $v = w = 0$ on $\partial\Omega$, we also obtain $Dv = Dw = 0$, which contradicts the fact that $|Dv| = |Dw| = 1$.

Thus any solution to the differential problem (1) is Lipschitz continuous but not of class C^1 near the boundary; therefore it assumes in a *fractal way* the homogenous boundary datum $\varphi = 0$ (see Figure 6). The map u will be explicitly defined at every $(x,y) \in \Omega$, and it will be piecewise affine, with infinitely many pieces, in accord with its fractal nature near the boundary of Ω.

Using this property of the singular set we are able to construct some examples of fractal origami which satisfy some "nonisometric" boundary conditions. In general, solutions to (1) may be very irregular, up to the point that the singular set can be everywhere dense.

In conclusion, origami is seen here as a tool for finding simple solutions in a context of systems of partial differential equations where infinitely many solutions exist, none of them being of class C^1, most of them having very irregular gradient. General existence theories have been developed by means of convex integration and by Baire category method to obtain Lipschitz continuous solutions. Simpler scalar problems can be treated with the viscosity theory, obtaining explicit representation formulas. This theory cannot, however, be applied in the context considered here, due to the lack of a maximum principle. Nevertheless, origami allows us to present particularly simple solutions with a recursive structure. It is a challenging issue to adapt the method proposed here to more general systems of first-order differential systems such as those encountered in geometry, optimal design, and nonlinear elasticity (see the section: "Implicit Partial Differential Equations: Conclusion").

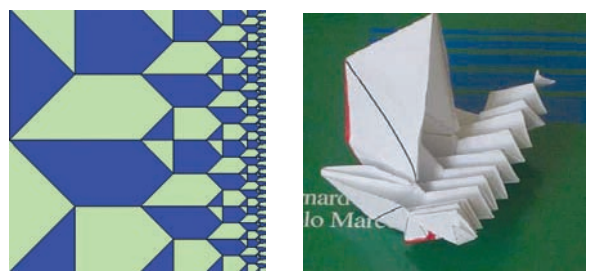


Figure 6. On the right-hand side a solution to the Dirichlet problem (1) actually folded with paper; it gives rise to a fractal shape. In the left-hand side the singular set of the same map is represented in a region near the boundary.

Some Other Topics About Origami

Many mathematicians interested in geometry or algebra (for example, in group theory, Galois theory, graph theory…) have studied origami constructions. An important issue is the geometrical construction of numbers. In this aspect origami turns out to be more powerful than the classical *rule and compass* constructions (see [1], [5], [6] or [17]). In order to determine what can be constructed through origami, it is important to formalize the rules. These are known as *Huzita axioms* and have been proposed by Hatori, Huzita, Justin, and Lang. Here are the seven axioms.

Axiom 1: Given two points P_1 and P_2, there is a unique fold passing through both of them.

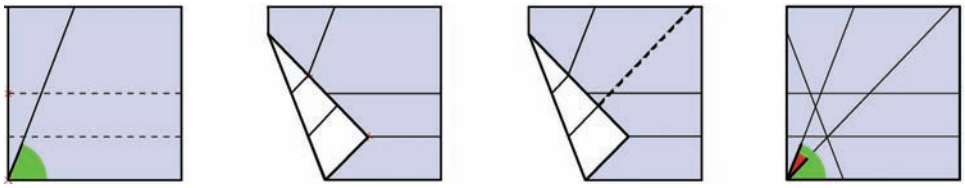


Figure 7. Trisecting an angle. The resulting red angle is a third of the given green angle.

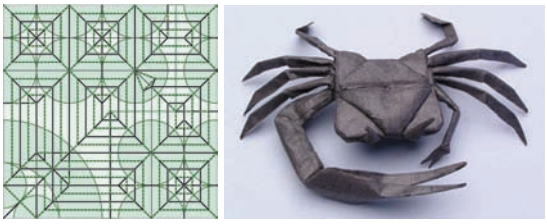
Axiom 2: Given two points P_1 and P_2, there is a unique fold placing P_1 onto P_2.
Axiom 3: Given two lines L_1 and L_2, there is a fold placing L_1 onto L_2.
Axiom 4: Given a point P and a line L, there is a unique fold perpendicular to L passing through P.
Axiom 5: Given two points P_1 and P_2 and a line L, there is a fold placing P_1 onto L and passing through P_2.
Axiom 6: Given two points P_1 and P_2 and two lines L_1 and L_2, there is a fold placing P_1 onto L_1 and P_2 onto L_2.
Axiom 7: Given a point P and two lines L_1 and L_2, there is a fold placing P onto L_1 and perpendicular to L_2.

For a slightly different approach to these axioms and for the connection to Galois theory, we refer to the book by Cox [6].

Origami constructions, contrary to rule-and-compass ones, allow one to trisect any angle (see Figure 7), to double a cube, or to construct a regular heptagon.

Another topic, more closely related to what we discuss in this paper, is the problem of finding an algorithm to decide whether a given crease pattern can actually be folded to generate a *physical origami*, i.e., a rigid map without interpenetration. The problem turns out to be NP-hard (nondeterministic polynomial-time hard), see [4].

Other interesting research is devoted to constructing crease patterns to fold to some given three-dimensional models with elaborated shape. With suitable software, Lang [21] has been able to fold realistic models of different kinds of animals.



(Image(s) courtesy of Robert J. Lang, www.langorigami.com.)

Figure 8. On the left-hand side the "ball packing technique" to create the crease pattern of the model (*crab*) shown on the right (by R. T. Lang).

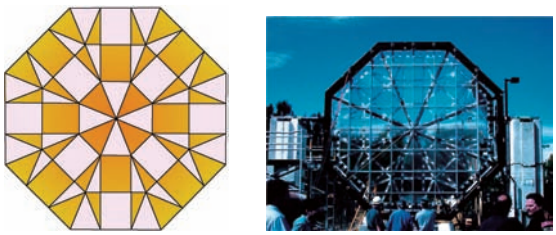
One of the main points turns out to be that of finding the best possible *packing* of given disks into the paper rectangle (see the left-hand side in Figure 8); in fact, we notice that every origami is a *short* map, namely

$$|u(x) - u(y)| \leq |x - y|, \quad \forall\, x, y \in \Omega;$$

this means that the distance of two points in the folded model cannot be larger than the distance of the two points in the unfolded model. This implies, for example, that the preimage of the claws of the crab in Figure 8 must contain the whole circle centered in the preimage of the tip of the claw with radius given by the length of the claw.

Recently in [19] some numerical algorithms have been found to reproduce three-dimensional curved models. Some of them have the peculiar property that the crease pattern is mostly composed of curved lines, whereas classical origami only uses straight lines. Curved creases have also been considered by Huffman [15].

Another interesting application comes from aerospace engineering. A team of scientists at the Lawrence Livermore National Laboratory in the United States are building a large space telescope. To put it into action in space, the telescope will be folded and unfolded following the rules of origami. The picture in Figure 9, taken from [14], represents a prototype of the telescope.



(Courtesy/LLNL.)

Figure 9. The unfolded space telescope and, on the left, the corresponding singular set.

Mathematical Origami as a Rigid Map

In this presentation an *origami* will be modeled by the notion of *rigid map*. A map $u\colon \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ is said to be a *rigid map* if it is Lipschitz continuous and $Du(x) \in O(m, n)$ for a.e. $x \in \Omega$.

From the definition it follows that, given any two vectors $\sigma, \tau \in \mathbb{R}^n$, we have

$$\begin{aligned}\langle Du(x)\sigma, Du(x)\tau\rangle_{\mathbb{R}^m} &= \langle (Du(x))^t Du(x)\sigma, \tau\rangle_{\mathbb{R}^n} \\ &= \langle \sigma, \tau\rangle_{\mathbb{R}^n},\end{aligned}$$

where $\langle\cdot,\cdot\rangle_{\mathbb{R}^n}$ is the scalar product in $\mathbb{R}^n$. In particular u leaves invariant the metric on the tangent space.

Since u is Lipschitz continuous it is, by the classical Rademacher theorem, differentiable almost everywhere in Ω. We call the *singular set* of u the set Σ_u where the map u is not differentiable. We now quote two classical results. The first one says that isometries are rigid maps with empty singular sets.

Theorem 3.1 (Cartan-Dieudonné). *Let $\Omega \subset \mathbb{R}^n$ be an open connected set and $u\colon \Omega \to \mathbb{R}^m$ be an isometry:*

$$|u(x) - u(y)| = |x - y|, \quad \forall x, y \in \Omega.$$

Then $m \geq n$ and u is affine.

The second result applies to the case $m = n$. In this case a rigid map corresponds to a *flat origami*. The theorem states that a flat and smooth origami can only be trivial.

Theorem 3.2 (Liouville). *Let Ω be a connected open subset of $\mathbb{R}^n$. If $u\colon \Omega \to \mathbb{R}^n$ is of class C^1 and satisfies $Du \in O(n)$, then u is affine.*

From now on we will only consider *piecewise C^1 rigid maps*, which are maps $u\colon \Omega \to \mathbb{R}^n$ satisfying the following properties: (i) Σ_u is relatively closed in Ω; (ii) u is of class C^1 in every connected component of $\Omega \setminus \Sigma_u$; (iii) every compact set $K \subset \Omega$ intersects only a finite number of connected components of $\Omega \setminus \Sigma_u$.

Reconstruction from the Crease Pattern

As above, when $n = m$ we are in the *flat case*, and we consider *flat origami*, i.e., piecewise C^1 rigid maps $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n$. In this context we can prove that, if Ω is a convex polyhedral set, then every connected component of $\Omega \setminus \Sigma_u$ is also a convex polyhedral set. The singular set Σ_u is the union of the faces bounding the convex regions, hence it is itself a $(n-1)$-dimensional polyhedral complex.

Every $(n-1)$-dimensional facet of Σ_u is the interface between two different regions, say A_1 and A_2. On each region the map u is affine, $u(x) = M_i x + q_i$ for $x \in A_i$, and it is not difficult to show that the two affine maps are related to each other by

$$M_2 = M_1 S, \qquad M_1 = M_2 S, \tag{2}$$

where $S \in \mathbb{R}^{n\times n}$ is the matrix representing a symmetry with respect to the $(n-1)$-dimensional plane containing the facet we are considering (thus $\det S = -1$). In particular we have

$$\det M_2 = -\det M_1,$$

and the sign of the determinant gives a *coloration* to $\Omega \setminus \Sigma_u$.

In particular we see that if M_1 is given, M_2 is uniquely determined. From these considerations it follows that, if Ω is also connected, once we know the affine map u in a single region, we can recover it in every other region.

However, not every polyhedral complex Σ is the singular set of some rigid map u. Notice, in particular, that if e is a $(n-2)$-dimensional edge of Σ, then the number of $(n-1)$-dimensional facets containing e must be even. This is due to the 2-coloration property of the complement of Σ_u. So we denote with $f_1, \cdots, f_{2k}$ the faces containing e. If we denote with α_i the angles between faces f_i and f_{i+1}, by means of condition (2) we can show that the following *angle condition* holds (see Figure 10):

$$\sum_{i=1}^{k} \alpha_{2i-1} = \sum_{i=1}^{k} \alpha_{2i} = \pi. \tag{3}$$

This condition is well known in the study of flat origami. When $n = 2$ this condition was first observed by Kawasaki [18].

Figure 10. The angle condition. The sum of angles with the same color is equal to π.

Hence, up to now, we know that the singular set Σ_u of a piecewise C^1 rigid map u is a polyhedral set such that $\Omega \setminus \Sigma_u$ is 2-colorable; moreover, every $(n-2)$-dimensional edge is contained in an even number of $(n-1)$-dimensional facets spaced so that the sum of angles on the regions of one color is equal to the sum of angles for the regions of the other color.

It is interesting to know that the reverse implication, stated in the following theorem, holds (see [10]).

Theorem 4.1 (Recovery theorem). *Let Ω be a simply connected open subset of $\mathbb{R}^n$. Let $\Sigma \subset \Omega$ be a locally finite polyhedral set satisfying the angle condition (3) on every $(n-2)$-dimensional facet. Then*

(i) there exists a rigid map u such that $\Sigma = \Sigma_u$ is the singular set of Du;

(ii) u is uniquely determined once we fix the value $y_0 = u(x_0)$ and the Jacobian $J_0 = Du(x_0)$ at a point $x_0 \in \Omega \setminus \Sigma$.

The theorem means that if we construct a set Σ satisfying the conditions just mentioned, there always exists a map u such that $\Sigma = \Sigma_u$. Moreover, the map u is uniquely determined up to compositions (on the left) with rigid motions of the whole space.

In the literature about flat folding (see, for instance, [2], [4]), the problem of reconstructing an origami from its singular set (which is also called *crease pattern*) has already been considered. However, a more strict definition of origami should be used to treat *physical origami*. In a physical origami the *interpenetration problem* is addressed, i.e., the problem that physically a noninjective map cannot be folded without cutting the paper. To be more precise, since the thickness of the paper can anyway be neglected, we admit noninjective maps as long as they are the limit of injective ones.

This restriction makes the angle condition not sufficient to guarantee the global reconstruction of a physical origami. An example is given by the singular set in Figure 11 which satisfies the angle condition, hence it is the singular set of some map u which, however, cannot be folded without cutting the paper. In this more strict setting it is shown (see [4]) that the problem of deciding whether a crease pattern can be flat-folded is NP-hard.

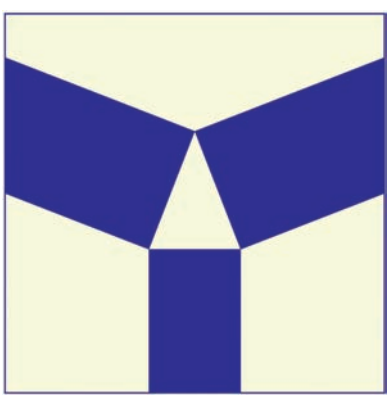


Figure 11. A singular set of a rigid map which cannot actually be folded without cutting the paper.

Fractal Construction

We now turn our attention to the more difficult problem of finding a solution to the following *implicit partial differential system*

$$(4) \qquad \begin{cases} Du(x) \in O(n) & \text{for a.e. } x \in \Omega \\ u(x) = 0 & \text{for } x \in \partial\Omega \end{cases}$$

in the case when Ω is a rectangle in $\mathbb{R}^2$ (or parallelepiped in $\mathbb{R}^n$). Now we know that a solution is represented by a flat origami. However, the Dirichlet boundary condition is, in principle, incompatible with the first-order differential system, as already mentioned in the introduction.

We first discuss the case $n = 2$. As we already said, only a fractal construction can ensure the boundary condition $u = 0$. Figure 6 gives an idea of the construction that we are going to develop. If Ω is a square, we can divide it in infinitely many squares, which are smaller and smaller, while we approach the boundary of Ω. Then it is enough to consider a *base map* u_0 defined on one of these squares. This map will be translated, rotated, and rescaled to fit any other square. To assure that the gluing of the squares gives a continuous map, we need the base map to have prescribed *recursive* boundary conditions. More precisely we require that, on the right-hand side of the base square, the map is defined so that it reproduces twice the values of the left-hand side, rescaled by half; i.e.,

$$\begin{aligned} u_0(1, y) &= u_0(0, 2y) && \text{for } y \in [0, 1/2], \\ u_0(1, y) &= u_0(0, 2y - 1) && \text{for } y \in [1/2, 1]; \end{aligned}$$

while on the upper and lower sides we only need periodic boundary conditions:

$$u_0(x, 0) = u_0(x, 1) \qquad \text{for } x \in [0, 1].$$

If the map assumes at least once the value 0 on every square in the net, then by its Lipschitz continuity (every rigid map is 1-Lipschitz continuous) it can be extended to the boundary $\partial\Omega$ with the 0 value.

The base module can thus be found by using the angle condition (3) and the recovery theorem 4.1. In this way we are able to find solutions to the above problem. More generally we might consider linear boundary conditions, such as $u(x, y) = (\lambda x, \mu y)$ on $\partial\Omega$. The corresponding singular set is shown in Figure 12.

Notice that we can easily pose the problem in higher dimension by considering *n-dimensional flat origami*, namely *hyper-origami*. Again we subdivide the domain in smaller and smaller cubes (or parallelepipeds). On the base domain we need to find an n-dimensional origami which fulfills the recursive boundary condition. For $n = 3$ we can imagine this operation as the folding of a rigid three-dimensional box. In Figure 13, we present the "instructions" for folding the base parallelepiped to obtain the zero boundary condition. By gluing together the base modules we obtain the 3-dimensional singular set.

Finally we note that one can solve (4) with *symmetric* rigid maps, i.e., maps u which are gradients of functions $v\colon \Omega \subset \mathbb{R}^2 \to \mathbb{R}$. The problem (4) when $n = 2$ then becomes

$$\begin{cases} D^2 v(x) \in O(2) & \text{for a.e. } x \in \Omega \\ Dv(x) = 0 & \text{for } x \in \partial\Omega. \end{cases}$$

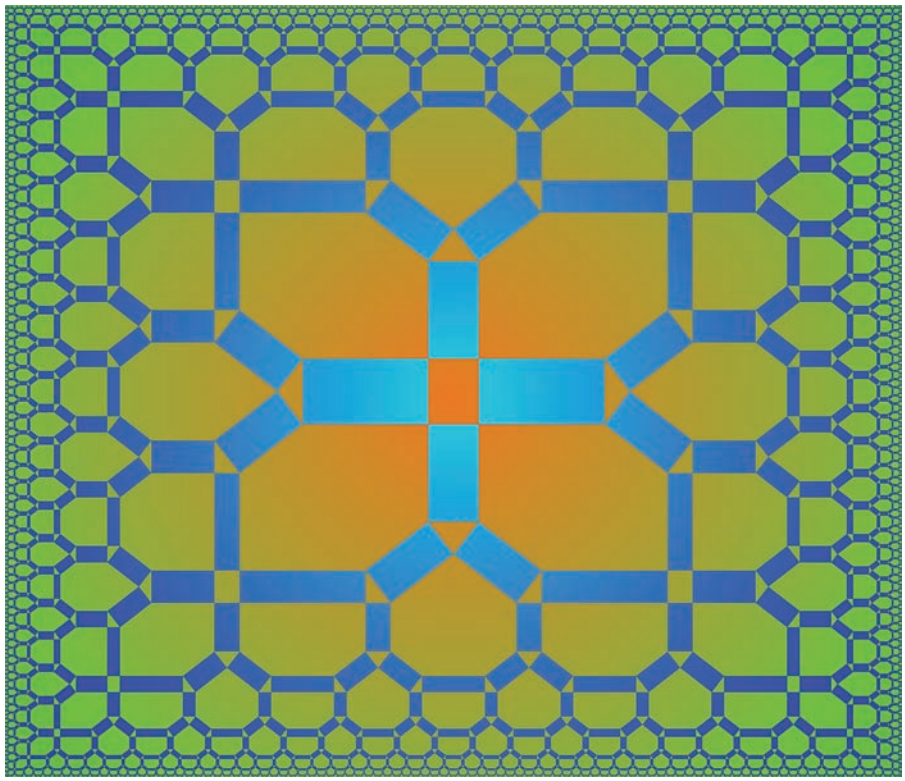


Figure 12. A fractal construction for the linear boundary data.

It turns out (see [11]) that the corresponding angle condition (3) is, in this case, much more restrictive in the sense that at every vertex of $\Sigma_u = \Sigma_{Dv}$ exactly four consecutive angles $\alpha^+, \alpha^-, \beta^+, \beta^-$ meet with

$$\alpha^+ + \beta^+ = \pi \qquad \text{and} \qquad \alpha^- = \beta^- = \frac{\pi}{2}.$$

We call this property "*second-order angle condition*". The singular set represented in Figure 12 satisfies the second-order angle condition, in contrast with the singular set of Figure 6.

Implicit Partial Differential Equations: Conclusion

What we have done so far is to find explicit solutions to Dirichlet problems, as in the section "Fractal Construction". This kind of problem belongs to a wider class, called *implicit partial differential equations*. Namely we look for Lipschitz continuous solutions to

$$(5) \qquad \begin{cases} F(x, u(x), Du(x)) = 0 & \text{for a.e. } x \in \Omega \\ u(x) = \varphi(x) & \text{for } x \in \partial\Omega, \end{cases}$$

where $u, \varphi : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ and $F : \Omega \times \mathbb{R}^m \times \mathbb{R}^{m\times n} \to \mathbb{R}$ (also called Hamiltonian).

Note that, as in (4), the problem is apparently ill posed, since we want to solve a *first-order* PDE and at the same time prescribe full Dirichlet condition on the boundary. Indeed, except for very special cases, no C^1 solution is to be expected, and only Lipschitz solution may exist. Curiously enough, if there is a solution of (4) or more generally of (5), then, in general, there are infinitely many solutions. Let us review some of the techniques for solving such equations and see how the constructions we have proposed for origami may shed light on some issues related to implicit PDEs.

Let us start with the scalar case $m = 1$ which has received considerable attention since the work of E. Hopf, notably by Kruzkov, Lax, and Oleinik. In this case the equation

$$F(x, u, Du) = 0$$

is sometimes called *Hamilton-Jacobi equation*. Crandall-Lions further developed in this context the important tool of *viscosity solution*. The beauty and the importance of the viscosity method is that, at the same time, it gives existence of solutions and a way of selecting one among the infinitely many solutions of (5). It is, however, of crucial importance in this context that the Hamiltonian F be *convex* in the variable Du. For example, the eikonal equation

$$\begin{cases} |Du(x)| = 1 & \text{for a.e. } x \in \Omega \\ u(x) = 0 & \text{for } x \in \partial\Omega \end{cases}$$

has, for convex domains Ω, the viscosity solution

$$u(x) = \operatorname{dist}(x, \partial\Omega),$$

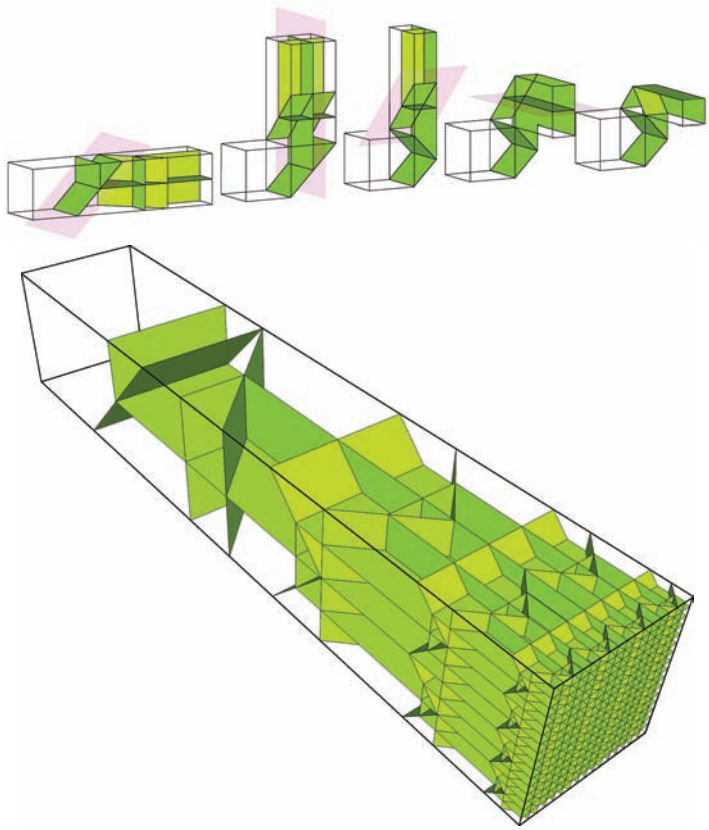


Figure 13. A *"hyper-origami"*, precisely a *three-dimensional flat origami* which solves the Dirichlet problem (4) **for** $n = 3$ **with zero boundary condition. At the top an elementary brick is represented. It has a self-similar property which allows recursively gluing it together with half-sized homothetic copies of itself to obtain the pattern represented at the bottom.**

although, as said earlier, there are infinitely many other solutions, one of them being

$$u(x) = -\operatorname{dist}(x, \partial\Omega).$$

However, as soon as the Hamiltonian F is *nonconvex* (with respect to the last variable) and still in the scalar case $m = 1$, or in the *vectorial* context $n, m \geq 2$ (as for origami), the viscosity theory does not apply. Two other main theories have been developed to deal with these cases, and, although different, they give essentially the same results (for detailed references see [9]). However, both theories are purely existential and do not give a simple criterion for selecting one solution among the infinitely many as did the viscosity theory. Let us briefly discuss both of them.

(i) The theory of *convex integration* proposed by Gromov is one of them. It was originally introduced to give a new proof of the celebrated Nash-Kuiper theorem on isometric embedding, which is closely related to the problem discussed in the present paper. The theory of convex integration has been developed in the context of implicit partial differential equations by Müller-Sverak and others.

(ii) The *Baire category method* is the other tool and has been introduced by Cellina, Bressan-Flores, and De Blasi-Pianigiani for the scalar case and generalized to the vectorial context by Dacorogna-Marcellini.

We also point out that, in the scalar case $m = 1$, and for Hamiltonians F, not necessarily convex, that do not depend on lower-order terms (i.e., $F(Du) = 0$), explicit solutions can easily be obtained and are known as "*pyramids*", following the work of Cellina and Friesecke. This was a source of inspiration to construct explicit solutions of (4) to Cellina-Perrotta and to the work presented here. Contrary to the finite element method used in numerical analysis to build approximated solutions, here, by means of piecewise affine maps with appropriate grid, we are able to exhibit exact solutions.

Since the two approaches described above are only existential, it is then an important issue to single out one solution among the infinitely many. It is what we achieved by means of origami for the special problem (4). There are many other problems related, for example, to geometry, optimal design,

or nonlinear elasticity where an actual explicit construction would be important.

Among them a problem which is significant in applications to microstructures in crystallographic models is the one of *potential wells* (see, for example, [3]). When we consider the case of two wells, it consists in finding maps u whose gradient satisfies

$$\begin{cases} Du(x) \in SO(n)A \cup SO(n)B, & \text{a.e. in } \Omega \\ u(x) = \varphi(x), & \text{on } \partial\Omega, \end{cases}$$

where A and B are two given $n \times n$ matrices and $SO(n)$ is the set of *special orthogonal* matrices; i.e., those with determinant equal to $+1$. This is a natural generalization of the problem considered here (see (4)), since

$$O(n) = SO(n)I \cup SO(n)I_-,$$

where I is the identity matrix and $I_- = \mathrm{diag}(-1, 1, \ldots, 1)$.

Finally, implicit partial differential equations, because of the existence of infinitely many solutions, are particularly challenging for numerical analysts. In the problem studied in the section "Fractal Construction", one could imagine, as has been suggested by one of the referees, finding algorithms that mimic the origami construction.

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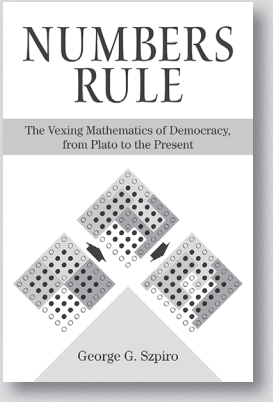

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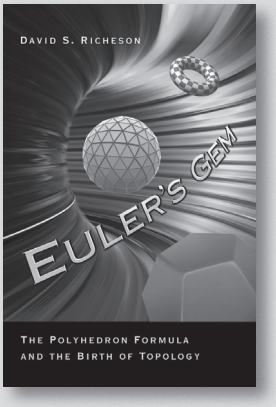

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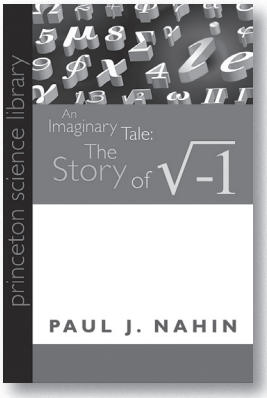

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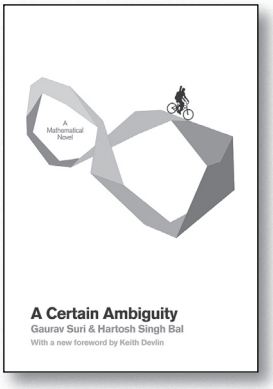

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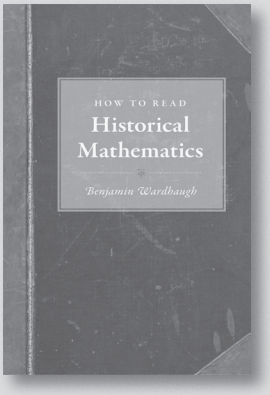

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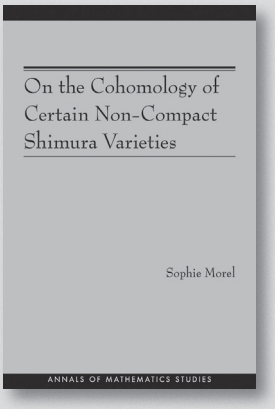

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What Is Mathematics For?

Underwood Dudley

A more accurate title is "What is mathematics education for?" but the shorter one is more attention-getting and allows me more generality. My answer will become apparent soon, as will my answer to the subquestion of why the public supports mathematics education as much as it does.

So that there is no confusion, let me say that by "mathematics" I mean algebra, trigonometry, calculus, linear algebra, and so on: all those subjects beyond arithmetic. There is no question about what arithmetic is for or why it is supported. Society cannot proceed without it. Addition, subtraction, multiplication, division, percentages: though not all citizens can deal fluently with all of them, we make the assumption that they can when necessary. Those who cannot are sometimes at a disadvantage.

Algebra, though, is another matter. Almost all citizens can and do get through life very well without it, after their schooling is over. Nevertheless it becomes more and more pervasive, seeping down into more and more eighth-grade classrooms and being required by more and more states for graduation from high school. There is unspoken agreement that everyone should be exposed to algebra. We live in an era of universal mathematical education.

This is something new in the world. Mathematics has not always loomed so large in the education of the rising generation. There is no telling how many children in ancient Egypt and Babylon received training in numbers, but there were not many. Of course, in ancient civilizations education was not for everyone, much less mathematical education. Literacy was not universal, and I suspect that many who could read and write could not subtract or multiply numbers. The ancient Greeks, to their glory, originated real mathematics, but they did not do it to fill classrooms with students learning how to prove theorems. Compared to them, the ancient Romans were a mathematical blank. The Arab scholars who started to develop algebra after the fall of Rome were doing it for their own pleasure and not as something intended for the masses. When Brahmagupta was solving Pell's equation a millennium before Pell was born, he did not have students in mind.

Of course, you may think, those were the ancients; in modern times we have learned better, and arithmetic at least has always been part of everyone's schooling. Not so. It may come as a surprise to you, as it did to me, that arithmetic was not part of elementary education in the United States in the colonial period. In *A History of Mathematics Education in the United States and Canada* (National Council of Teachers of Mathematics, 1970) we read

> Until within a few years no studies have been permitted in the day school but spelling, reading, and writing. Arithmetic was taught by a few instructors one or two evenings a week. But in spite of the most determined opposition, arithmetic is now being permitted in the day school.

Opposition to arithmetic! *Determined* opposition! How could such a thing be? How could society function without a population competent in arithmetic? Well, it did, and it even thrived. Arithmetic was indeed needed in many occupations, but those who needed it learned it on the job. It was a system that worked with arithmetic then and that can work with algebra today.

Arithmetic did make it into the curriculum, but, then as now, employers were not happy with what the schools were turning out. Patricia Cline Cohen, in her estimable *A Calculating People: The Spread of Numeracy in Early America* (U. of Chicago Press, 1983; Routledge paperback, 1999) tells us that

Underwood Dudley is a mathematician, retired from DePauw University. His email address is ddunx46135@yahoo.com.

> Prior to this act [1789] arithmetic had not been required in the Boston schools at all. Within a few years a group of Boston businessmen protested to the School committee that the pupils taught by the method of arithmetic instruction then in use were totally unprepared for business. Unfortunately, the educators in this case insisted that they were doing an adequate job and refused to make changes in the program.

Both sides were right. It is impossible to prepare everyone for every possible occupation and it is foolish to try. Hence many school leavers will be unprepared for many businesses. But mathematics teachers, then as now, were doing an adequate job.

A few years ago I was at a meeting that had on its program a talk on the mathematics used by the Florida Department of Transportation. There is quite a bit. For example, the Florida DoT uses Riemann sums to determine the area of irregular plots of land, though it does not call the sums that. After the talk I asked the speaker what mathematical preparation the DoT expects in its new hires. The answer was, none at all. The DoT has determined that it is best for all concerned to assume that the background of its employees includes nothing beyond elementary arithmetic. What employees need, they can learn on the job.

There seems to be abroad in the land the delusion that skill in algebra is necessary in the world of work and in everyday life. In *Moving Beyond Myths* (National Academy of Sciences, 1991) we see

> *Myth:* Most jobs require little mathematics.
> *Reality:* The truth is just the opposite.

I looked very hard in the publication for evidence for that assertion, but found none. Perhaps the NAS was equating mathematics with arithmetic. Many people do this, as I have found in asking them about how, or if, they use mathematics. Almost always, the "mathematics" they tell me about is material that appears in the first eight grades of school.

Algebra, though, is mentioned explicitly in *Everybody Counts* (National Research Council, 1989):

> Over 75 percent of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination.

I find that statement extraordinary. I will take my telephone Yellow Pages, open it at random, and list in order the first eight categories that I see:

> Janitor service, Janitors' equipment and supplies, Jewelers, Karate and other martial arts, Kennels, Labeling, Labor organizations, Lamps and lamp shades.

In which six is algebra required, even for training or license? I again looked very hard for evidence in the NRC's publication but couldn't find any.

It may be that no evidence is presented because none is needed: everybody knows that algebra is needed for all sorts of jobs. For example, there was an algebra book whose publisher advertised that it contained

> "Career Applications"—Includes explanations, examples, exercises, and answers for work in electronics; civil/chemical engineering; law enforcement; nursing; teaching; and more. Shows students the relationship of chapter concepts and job skills—with applications developed through interviews and market research in the workplace that ensure relevance.

Of course I requested an examination copy, and the publisher graciously sent me one. To return the favor, I will refrain from naming the publisher or the author. The career applications were along the lines of

> In preparation for the 2002 Winter Olympic Games in Salt Lake City, several people decide to pool their money and share equally the $12000 expense of renting a four-bedroom house in Salt Lake City for two weeks. The original number of people who agreed to share the house changed after two people dropped out of the deal because they thought the house was too small. Those left in the deal must now pay an additional $300 each for the rental. How many people were left?

Exactly what career this applied to was not specified. Nor was it mentioned that the best way to solve this problem is to find a member of the group and ask. The answer should be forthcoming. If the person's reply is the conundrum in the text, the member of the group should be beaten about the head until he or she promises to behave in a more civilized manner.

This is not to say that the problem is not a good one. It is a good one, a very good one, and one that students should try to solve. Students should be made to solve many word problems, the more the better. The reason for solving them, though, is not that they will arise in their careers.

Another text, whose author and publisher I will not name—alas, still in print in its third edition—asserts

> This text aims to show that mathematics is useful to virtually everyone. I hope that users will complete the course with greater confidence in their ability to solve practical problems.

Here is one of the practical problems:

> An investment club decided to buy $9000 worth of stock with each member paying an equal share. But two members left the club, and the remaining members had to pay $50 more apiece. How many members are in the club?

Do you detect the similarity to the career application in the first text? The two problems are the same, with different numbers. The second is not practical, any more than the first comes up as part of a job.

The reason that this problem—well worth doing by students—appears in more than one text is that it is a superb problem, so superb that has been appearing in texts for hundreds of years, copied from one author to another. If you want a problem that makes students solve a quadratic equation, here it is.

I keep looking for uses of algebra in jobs, but I keep being disappointed. To be more accurate, I used to keep looking until I became convinced that there were essentially none. For this article I searched again and found a website that promised applications of "college algebra" to the workplace. The first was

> You are a facilities manager for a small town. The town contains approximately 400 miles of road that must be plowed following a significant snowfall. How many plows must be used in order to complete the job in one day if the plows can travel at approximately 7 miles per hour when engaged?

This is another textbook "application" made up, I think, by its writer with no reference to external reality. (It's a big small town that has 400 miles of streets.) The facilities manager knows how many plows there are and can estimate how many more, if any, are needed. The next problem, I think, did arise outside of the head of a textbook writer:

> How much ice cream mix and vanilla flavor will it take to make 1000 gallons of vanilla ice cream at 90% overrun with the vanilla flavor usage rate at 1 oz. per 10 gallon mix? (90% overrun means that enough air is put into the frozen mix to increase its volume by 90%.)

Though dressed up with x's and y's, the solution amounts to calculating that you need $1000/(1 + .9) = 526.3$ gallons of mix to puff up into 1000 gallons of ice cream, so you will need $526.3/10 = 52.6$ ounces of flavor.

The employee adding the flavor will not need algebra, nor will he or she need to think through this calculation. There will be a formula, or rule, that gives the result, and that is what happens on the job. Problems that arise on the job will be for the most part problems that have been solved before, so new solutions by workers will not be needed.

I am glad that we do not have to depend on workers' ability to solve algebra problems to get through the day because, as every teacher of mathematics knows, students don't always get problems right. The chair of the department of a Big Ten university once observed, probably after a bad day, that it was possible for a student to graduate with a mathematics major without ever having solved a single problem correctly. Partial credit can go a long way. This was in the 1950s, looked on by many as a golden age of mathematics education.

In one of those international tests of mathematical achievement appeared the problem of finding which of two magazine subscriptions was cheaper: 24 issues with (a) the first four issues free and $3 each for the remainder or (b) the first six issues free and $3.50 each for the remainder. This is not a tough problem, so I leave its solution to you. As easy as it is, only 26% of United State eighth-graders could do it correctly. That percentage was above the international average of 24%. Even the Japanese eighth-graders could manage only 39%. No doubt when the eighth-graders become adults they will be better at solving such problems, but even so I do not want them having to solve problems that when solved incorrectly can do me harm.

Though people know that they do not use algebra every day, or even every month, many seem to think that there are hosts of others who do. Perhaps they have absorbed the textbook writers' insistence on the "real world" uses of algebra, even though the texts actually demonstrate that there are none. Were uses of algebra widespread in the world of work, all textbook writers would have to do is to ask a few people about their last applications of algebra, turn them into problems, and put them in their texts. If 75% of all jobs require algebra, they could get a problem from three of every four people they ask. However, such problems do not appear in the texts. We get instead the endlessly repeated problems about investment clubs losing two members and all of the other chestnuts, about cars going from A to B and farmers fencing fields

and so on, that I lack the space to display. The reason that problems drawn from everyday life do not appear in the texts is not that textbook authors lack energy and initiative; it is that they do not exist.

Though they may not use algebra themselves, people are solidly behind having everyone learn algebra. Tom and Ray Magliozzi, the brothers who are hosts of National Public Radio's popular "Car Talk" program, like to pose as vulgarians when they are actually nothing of the kind. On one program, brother Tom made some remarks against teaching geometry and trigonometry in high school. I doubt very much that he was serious. Whether he was serious or not does not affect the content of his remarks or the reaction of listeners. The reaction was unanimous endorsement of mathematics. When mathematics is attacked, people leap to its defense.

In his piece Tom alleged that he had an octagonal fountain in his back yard that he wanted to surround with a border and that he needed to calculate the length of the side of the concentric octagon. After succeeding, using, he said, the Pythagorean theorem, he reflected

> That this was maybe the second time in my life—maybe the first—that I had occasion to use the geometry and trigonometry that I had learned in high school. Furthermore, I had never had occasion to use the higher mathematics that the high school math had prepared me for.
>
> *Never*!
>
> Why did I—and millions of other students—spend valuable educational hours learning something that we would never use?
>
> Is this education? Learning skills that we will never need?

After some real or pretended populism ("The people who run the education business are money-grubbing self-serving morons"), he concluded that

> The purpose of learning math, which most of us will never use, is only to prepare us for further math courses—which we will use even less frequently than never.

There were answers, quite a few of them, posted at the "Car Talk" website. All disagreed with Tom's conclusion, which actually has elements of truth. (A reply that started with "I agree" might be thought to be a counterexample, but the irony that followed was at least as heavy as lead.) One response included

> Perhaps you've had only one opportunity to use geometry in your life, but there are a number of occupations in which it's a must. Myself, I'm pleased that my house was designed and built by people who were capable of calculating the correct rise of a roof for proper drainage or the number of cubic feet of concrete needed for a strong foundation.

Here is the common error of supposing that problems once solved must be solved anew every time they are encountered. House builders have handbooks and tables, and use them. Indeed, houses, as well as pyramids and cathedrals, were being built long before algebra was taught in the schools and, in fact, before algebra.

Another common misconception occurs in another response:

> You sure laid a big oblate spheroid shaped one when you went on your tirade against having to learn geometry, trigonometry and other things mathematical.
>
> Who uses this stuff? Geologists, aircraft designers, road builders, building contractors, surgeons and, yes, even radio broadcast technicians (amplitude modulation and frequency modulation are both based on manipulating wave forms described by trig functions—don't get me started on alternating current).
>
> So, Tommy, get a life. The only people who don't use these principles every day are those who can't do and can't teach, and thus are suited only for lives as politicians or talk show hosts.

People seem to think that because something involves mathematics it is necessary to know mathematics to use it. Radio does indeed involve sines and cosines, but the person adjusting the dials needs no trigonometry. Geologists searching for oil do not have to solve differential equations, though differential equations may have been involved in the creation of the tools that geologists use.

I am not saying that mathematics is never required in the workplace. Of course it is, and it has helped to make our technology what it is. However, it is needed very, very seldom, and we do not need to train millions of students in it to keep businesses going. Once, when I was an employee of the Metropolitan Life Insurance Company, I was given an annuity rate to calculate. Back then, insurance companies had rate books, but now and then there was need for a rate not in the book.

Using my knowledge of the mathematics of life contingencies, I calculated the rate. When I gave it to my supervisor he said, "No, no, that's not right. You have to do it *this* way." "But," I said, "that's three times as much work." Yes, I was told, but that's the way that we calculate rates. My knowledge of life contingencies got in the way of the proper calculation, done the way it had been done before, which any minimally competent employee could have carried out.

It may be that there could arise, say, a partial differential equation that some company needed to solve, the likes of which it had never seen before. If so, there are plenty of mathematicians available to do the job. They'd work cheap, too.

Jobs do not require algebra. I have expressed this truth many times in talks to any group who would listen, and it was not uncommon for a member of the audience to tell me, after the talk or during it, that I was wrong and that he used algebra or calculus in his job all the time. It always turned out that he used the mathematics because he wanted to, not because he had to.

Even those who are not burdened with the error that algebra is necessary to hold many jobs support the teaching of algebra. Everyone supports the teaching of algebra. The public wants more mathematics taught, to more students. The requirements keep going up, never down.

The reason for this, I am convinced, is that the public knows, or senses, that mathematics develops the power to reason. It shows, better than any other subject, how reason can lead to truth. Of course, other sciences exhibit the power of reason, but there's all that overhead—ferrous and ferric, dynes and ergs—that has to be dealt with. In mathematics, there is nothing standing between the problem and the reasoning.

Economists reason as well, but sometimes two economists reason to two different conclusions. Philosophers reason, but never come to *any* conclusion. In mathematics problems can be solved, using reason, and the solutions can be checked and shown to be correct. Reasoning needs to be learned, and mathematics is the best way to learn it.

People grasp this, perhaps not consciously, and hence want their children to undergo mathematics. Many times people have told me that they liked mathematics (though they call it "math") because it was so definite and it was satisfying to get the right answer. Have you not heard the same thing? They liked being able to reason correctly. They knew that the practice was good for them. No one has ever said to me, "I liked math because it got me a good job."

We no longer have the confidence in our subject that allows us to say that. We justify mathematics on its utility in the world of getting and spending. Our forebears were not so diffident. In 1906 J. D. Fitch wrote

> Our future lawyers, clergy, and statesmen are expected at the University to learn a good deal about curves, and angles, and number and proportions; not because these subjects have the smallest relation to the needs of their lives, but because in the very act of learning them they are likely to acquire that habit of steadfast and accurate thinking, which is indispensible in all the pursuits of life.

I do not know who J. D. Fitch was, but he was correct. Thomas Jefferson said

> Mathematics and natural philosophy are so peculiarly engaging and delightful as would induce everyone to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of a body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law.

In 1834, the Congressional Committee on Military Affairs reported

> Mathematics is the study which forms the foundation of the course [at West Point]. This is necessary, both to impart to the mind that combined strength and versatility, the peculiar vigor and rapidity of comparison necessary for military action, and to pave the way for progress in the higher military sciences.

Here is testimony from a contemporary student:

> The summer after my freshman year I decided to teach myself algebra. At school next year my grades improved from a 2.6 gpa to a 3.5 gpa. Tests were easier and I was much more efficient when taking them and this held true in all other facets of my life. To sum this up: algebra is not only mathematical principles, it is a philosophy or way of thinking, it trains your mind and makes otherwise complex and overwhelming tests seem much easier both in school and in life.

Anecdotal evidence to be sure, but then all history is a succession of anecdotes.

That is what mathematics education is for and what it has always been for: to teach reasoning, usually through the medium of silly problems. In the Rhind Papyrus, that Egyptian textbook of mathematics c. 1650 BC, we find

> Give 100 loaves to five men so that the shares are in arithmetic progression and the sum of the two smallest is 1/7 of the three greatest.

The ancient Egyptians were a practical people, but even so this eminently unpractical problem was thought to be worth solving. (The shares are 1 2/3, 10 5/6, 20, 29 1/6, and 38 1/3.) George Chrystal's *Algebra* (1886) has on page 154 more than fifty problems, all with the instruction "Simplify", including

$$\frac{\dfrac{\dfrac{1}{x^2}+\dfrac{1}{y^2}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}-\dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x^2}+\dfrac{1}{y^2}}}{\left(\dfrac{8}{\left(\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}\right)\left(\dfrac{x^2}{y^2}+\dfrac{y^2}{x^2}-2\right)}\right)}$$

There is no reason given, anywhere in his text, why anyone would want to simplify such things. It was obvious. That is how algebra is learned. As for the reason for learning algebra, that was obvious as well, and it was not for jobs. (The answer to the problem—what fun Chrystal must have had in making it up—is -1.)

I am not so unrealistic as to advocate that textbook writers start to produce texts with titles like *Algebra, a Prelude to Reason*. That would not fly. We do not want to make unwilling students even more unwilling. We cannot go back to texts like Chrystal's. But could we perhaps *tone it down* a little? Can we be a little less insistent that mathematics is essential for earning a living?

What mathematics education is for is not for jobs. It is to teach the race to reason. It does not, heaven knows, always succeed, but it is the best method that we have. It is not the only road to the goal, but there is none better. Furthermore, it is worth teaching. Were I given to hyperbole I would say that mathematics is the most glorious creation of the human intellect, but I am not given to hyperbole so I will not say that. However, when I am before a bar of judgment, heavenly or otherwise, and asked to justify my life, I will draw myself up proudly and say, "I was one of the stewards of mathematics, and it came to no harm in my care." I will not say, "I helped people get jobs."

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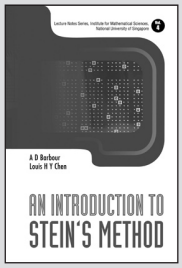

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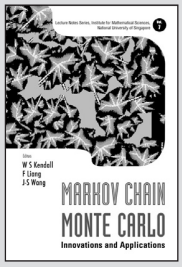

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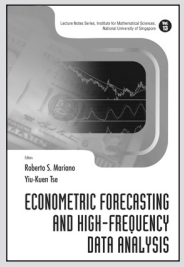

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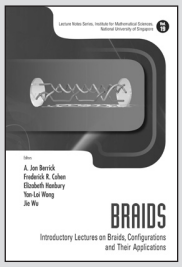

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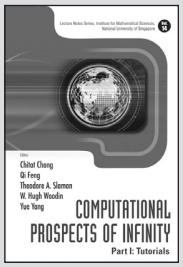

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Applied Mathematics and Scientific Computing in the Visual Effects Industry

Aleka McAdams, Stanley Osher, and Joseph Teran

Whether it's an exploding fireball in *Star Wars: Episode 3* or a swirling maelstrom in *Pirates of the Caribbean: At World's End*, special effects leveraging numerical simulations can be seen in a wide range of Hollywood blockbusters. Although previously considered too involved and prohibitively expensive for applications like movie special effects, as computers get faster and architectures evolve, simulation of such phenomena is now much more practical. Moreover, as the bar has been raised for increasingly realistic effects and even for computer-generated imagery to blend seamlessly with live performances, physically based simulation has become not only tractable but an invaluable tool for creating realistic virtual worlds in movies and video games.

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Some of the physical phenomena commonly simulated in movie and video game special effects include water, fire, smoke, explosions, rigid body dynamics, and the deformation of elastic bodies. The governing equations for these processes are most often in the form of a system of partial differential equations. The development of algorithms for solving such equations with the computer is one of the cornerstones of applied mathematics and scientific computing. In fact, some techniques (most notably, level set methods) have completely revolutionized the industry and have even been honored with technical Academy Awards.

In this article we will describe some of the most compelling applications of applied math and scientific computing in the visual effects industry. Specifically we will talk about the techniques used to simulate every imaginable component of the digital environment, as well as the interactions between these components within a scene. Furthermore, we will discuss some of the ways in which physical simulation techniques for

special effects differ from those developed for more classical applications in physics and engineering. Particularly, there are many cases in which the artistic vision of a scene requires a high level of controllability in the outcome of a simulation. To this end, special effects simulation tools, while physically based, must be able to be dynamically controlled in an intuitive manner in order to ensure both believability and the quality of the effect. We will highlight techniques from computational fluid dynamics, computational solid dynamics, rigid body simulation, and collision detection and resolution.

Computational Fluid Dynamics

Computational fluid dynamics (CFD) techniques are used to simulate a number of phenomena. Obviously, crashing waves and oceans can leverage CFD, but explosions, fireballs, and smoke effects all make use of CFD nowadays as well. Before the use of CFD, computer generated (CG) special effects such as explosions were driven by force fields applied to passive unconnected particles, producing less than realistic results. However, with the combination of improved hardware and faster algorithms, realistic CFD-based special effects for smoke, fire, water, and other fluids have become much more prevalent.

The governing equations for fluid dynamics can be derived from the principle of the conservation of mass and momentum. Although compressible fluid models have been used for some special effect applications (e.g., explosions and shock waves), these models are more difficult to solve numerically. As such, practitioners tend to use incompressible fluid models whenever possible, so we will overview their derivation here.

From an Eulerian frame of reference, i.e., a reference frame which is independent of the fluid's motion, an incompressible fluid's velocity is governed by the Navier-Stokes equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f} \qquad (1)$$
$$\nabla\cdot\mathbf{v} = 0,$$

where ρ is density, p is pressure, μ is viscosity, and $\mathbf{f}$ represents outside forces.

The first equation can be derived directly from the conservation of momentum (with some simplifications from the conservation of mass and incompressibility), but perhaps a more intuitive derivation begins with Newton's Second Law, $F = ma$. The left side of the equation can be thought of as the "mass times acceleration" term with ρ corresponding to mass and $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}$, or the material derivative of $\mathbf{v}$, corresponding to acceleration. The forces acting on the fluid can be divided into two types: those from internal stresses and those acting on the fluid from outside (e.g., gravity). For an incompressible fluid, the internal forces can be described by $-\nabla p + \mu\nabla^2\mathbf{v}$.

The second equation can be derived from the conservation of mass and the assumption that the fluid is incompressible. By the conservation of mass, the material derivative of density, i.e., the change in density of a parcel of advected fluid through time, is zero:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0.$$

However, for an incompressible fluid, ρ is a positive constant, so this simplifies to $\nabla\cdot\mathbf{v} = 0$.

A number of numerical schemes are used in CFD to solve these equations. We can classify many methods commonly used in computer graphics into two categories: splitting or projection methods typically solved on a grid, and gridless methods typically solved using a smoothed particle hydrodynamics (SPH) algorithm.

Smoothed particle hydrodynamics (SPH) were initially developed for simulation of astrophysical problems. Müller et al. introduced the notion of using SPH to simulate liquids to computer graphics [25]. Particle methods such as SPH are good at simulating very active fluids (e.g., splashing water, smoke) in real time, making them excellent candidates for video game simulations, and more complex and high resolution particle simulations can be used to create realistic effects for motion pictures as well. Furthermore, this particle representation of the fluid makes it easy to determine the boundary between the simulated fluid and air.

As the name suggests, SPH represents the fluid as particles whose attributes (e.g., velocity, pressure, density) are spatially smoothed using a radially symmetric smoothing kernel $W(\mathbf{r})$. Any scalar quantity A is thus interpolated to a location $\mathbf{x}$ to produce the smoothed field

$$A_s(\mathbf{x}) = \sum_j m_j \frac{A_j}{\rho_j} W(|\mathbf{x} - \mathbf{x}_j|),$$

where j iterates over all of the particles, m_j is the mass of the jth particle, $\mathbf{x}_j$ is its location, ρ_j the density, and A_j the field value at $\mathbf{x}_j$. Derivatives on A can then be computed by differentiating W, e.g.,

$$\nabla A_s(\mathbf{x}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(|\mathbf{x} - \mathbf{x}_j|).$$

Applying SPH to the Navier-Stokes equations then is a matter of representing the quantities in (1) with these weights. Furthermore, since $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}$ in fact represents the time derivative of a fluid particle's velocity in an Eulerian frame, we can simplify this expression when using a particle representation with the material derivative $\frac{D\mathbf{v}}{Dt}$. Some complications arise when using SPH, particularly, the force terms (i.e., ∇p and $\nabla^2\mathbf{v}$)

are not symmetric, but these can be alleviated by using different approximations. (See [25] for one such approximation.)

Many grid-based methods use finite differencing on a staggered grid where the pressure is defined on cell centers and velocity components are defined on cell faces. The timestepping scheme then depends on the Helmholtz-Hodge decomposition, which states that any vector field $\mathbf{v}^*$ can be decomposed into $\mathbf{v}^* = \mathbf{v} + \nabla p$ where $\mathbf{v}$ is divergence-free and p is a scalar field. With this, the problem can be broken into two basic steps. In the first step, the velocity is advected (i.e., it is moved along the velocity field) and forces and diffusion are applied by solving

$$\rho\left(\frac{\partial \mathbf{v}^*}{\partial t} + (\mathbf{v}^* \cdot \nabla)\mathbf{v}^*\right) = \mu\nabla^2\mathbf{v}^* + \mathbf{f}$$

with initial data

$$\mathbf{v}^*|_{t=t^n} = \mathbf{v}^n.$$

This candidate velocity $\mathbf{v}^*$ is then made divergence-free by solving

$$\nabla^2 p = \nabla \cdot \mathbf{v}^* \tag{2}$$
$$\mathbf{v} = \mathbf{v}^* - \nabla p$$

for $\mathbf{v}$, the projection of $\mathbf{v}^*$ onto the divergence-free subspace. (See [34] for a frequently used discretization to these equations.)

While the Navier-Stokes equations and these discretizations are enough to determine much of the behavior of a fluid, additional quantities must be solved for and specialized treatment is needed to better capture the behavior of different types of fluids. We will look into grid-based methods for smoke, fire, and water in more detail.

Smoke

Made up of fine particles suspended in heated air, smoke can be modeled as an inviscid (i.e., $\mu = 0$) incompressible fluid. In addition to (1), the smoke density ρ_s (which is different from the constant air density) and the gas temperature T are evolved through the velocity field via

$$\frac{\partial \rho_s}{\partial t} + (\mathbf{v} \cdot \nabla)\rho_s = 0 \tag{3}$$

and

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T = 0, \tag{4}$$

respectively. The force $\mathbf{f}$ is typically defined as a function of T and ρ_s and is used to capture the effects of buoyancy.

The finite difference scheme described in [34] can produce numerical dissipation and diffusion (i.e., modeled quantities such as velocity can lose energy and be smoothed out due to interpolation), especially on low-resolution grids. In the case of smoke, this can damp out the pluming and other interesting effects one would wish to capture in smoke. While a higher-resolution grid may be able to capture some of these effects, this would be computationally expensive. Instead, Fedkiw et al. introduced vorticity confinement techniques (created by Steinhoff for extremely turbulent flow fields about helicopters [35]) for smoke which are able to counteract this dissipation even on low-resolution grids [11]. Additional techniques to add even more turbulence or speed up computations have also been explored [31, 23]. Examples of smoke CFD special effects can be seen in *Terminator 3* [14] and *Star Wars: Episode 3* [15].

Fire

In many cases, fire can be modeled as an incompressible fluid in a way very similar to smoke. Again, vorticity confinement can be used to produce turbulent fireballs and swirling flames, and additional combustion particles can be added to the simulation to spark new flames [15].

While these combustion particles can produce gas expansion effects for fireballs and exploding spaceships, additional tools are necessary to simulate true fuel-based combustion such as a burning building or a gas-powered blowtorch. Nguyen et al. introduced the idea of modeling the gaseous fuel and the flaming gaseous products as separate (but coupled) incompressible fluids. The reaction front where fuel turns into fire is defined by a level set function which evolves according to the fuel velocity [26] (more about level set functions and their evolution can be found in the section "The Level Set Method"). These techniques were used to create the dragon's flaming breath in *Harry Potter and the Goblet of Fire*.

Water

Of course, one of the more obvious applications of CFD to special effects is the simulation of water-based phenomena. The state-of-the-art in water simulation techniques used in the movies are by and large based on the particle level set method introduced by Foster and Fedkiw [12], further refined by Enright et al. [10] and for which Fedkiw shared an Academy Award for Technical Achievement. Just as the reaction front for fire is defined by a level set function, the surface of the water can be defined in the same way. However, numerical dissipation and diffusion can cause the level set defined surface to lose volume in high curvature regions. The particle level set method provides a means to retain this volume by defining the water surface as a combination of a level set function and particles.

Several improvements and artist controls to this method have been introduced, and a number of films feature particle level set-based special effects, including the turbulent seas and flooding

Figure 1. (top) CG ocean simulated using the particle level set method in *Poseidon* [13]. (bottom) The surface of the tarmonster in *Scooby Doo 2* is modeled as a level set function and is evolved dynamically using CFD [40].

waters of *Poseidon* [13], the storm surge in *The Day After Tomorrow* [19], and the maelstrom in *Pirates of the Caribbean: At World's End*.

In Hollywood, CFD techniques aren't limited to modeling run-of-the-mill gases and fluids. The melting Terminatrix in *Terminator 3* [36] and the mud, beer, and other fluids in *Shrek* were simulated with particle level set methods. Additionally, CFD-inspired techniques have been used to simulate nonfluids such as hair (see the subsection "Cloth and Hair").

The Level Set Method

First introduced by Osher and Sethian in 1988 [29], the level set method provides a dynamic implicit representation of surfaces (see also [8] and [9] for interesting precursors). In this method, a closed curve (or set of curves) in $\mathbb{R}^2$ or closed surface(s) in $\mathbb{R}^3$ can be implicitly defined by the zero isocontour of a "level set function" which is defined throughout space. The implicitly defined surface can then be evolved by an underlying flow, handling topological changes such as merging and break-up automatically.

The most commonly used level set function is the "signed distance function". As the name suggests, the function is defined by the signed distance to the implicit surface where the sign is negative inside the surface and positive outside. For example, the function $\phi(x, y) = \sqrt{x^2 + y^2} - 1$ is the signed distance function for the unit circle. Signed distance functions have a number of properties which make them very useful for defining object surfaces in physical simulations. The fast-marching method of Tsitsiklis [28] makes initializing a signed distance function from an explicit representation (such as a triangulated surface) computationally inexpensive. By checking the sign of the function evaluated at a point, one can quickly determine if the point is inside or outside of the object. Furthermore, the surface normal (useful in collision handling) is defined by $\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|}$, and the curvature of the surface is simply $\kappa = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|}$.

The power of the level set method for two phase incompressible flows was originally shown in the computational mechanics literature by Sussman et al. [37]. Here, the idea of using an Eikonal equation-based reinitialization at every time step was first introduced, along with accurate advection schemes for the level set evolution. The method allowed for natural topology changes associated with pinching and merging in contrast with explicit representations of the boundary (e.g., by parameterized surfaces), where topology changes are more tricky.

One widely used application of the level set method in special effects is the particle level set method for fluid simulation (see the section "Water"). This method uses a combination of particles and a level set to define the water surface. The level set function, ϕ, can be evolved with the fluid velocity field, $\mathbf{v}$, by integrating $\phi_t + \mathbf{v} \cdot \nabla\phi = 0$. Changes in the fluid surface's topology (as occurs in the presence of splashing or air pocket formation/destruction) are automatically defined by the implicit surface. Numerical dissipation and diffusion can cause volume loss in high-curvature regions of the fluid surface, producing visible artifacts such as disappearing droplets. Here, the particle representation can compensate for this loss, and fast marching allows for easy surface reconstruction. Conversely, since the level set function is defined throughout space, the implicit surface can fill in low-curvature regions where the particle number is too low to properly resolve the surface. The convenient definition of curvature for level sets makes determining high/low curvature regions automatic. Additionally, this significantly simplifies the inclusion of effects such as surface tension which depend directly on the surface curvature.

Solids

Between complicated meshing problems and physics-based solids simulations, applied math

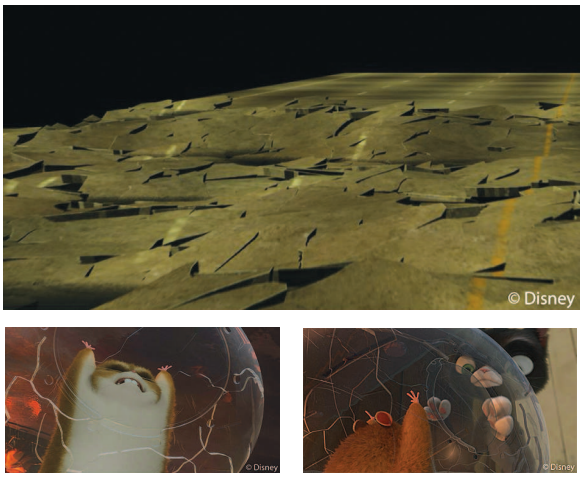

Figure 2. Geometric fracturing of Rhino's ball (bottom left, bottom right) and a roadway (top) in *BOLT* [18].

plays an integral role in CG sequences involving more than just fluids. Whether it's your favorite Pixar animated character or the apocalyptic cityscapes of *The Day After Tomorrow*, virtually every CG solid has an explicit representation as a meshed surface or volume. Mesh generation, deformation, and topological changes are all mathematically complex challenges faced by CG engineers. Artists turn to physics-based simulations in scenes where physical effects such as inertia can provide more realism or the complexity of the scene would be difficult to control by hand. Flesh simulations can endow CG characters with realistically bulging muscles and rippling fat. Hair simulators can tackle the complexity of thousands of interacting hairs, and rigid body simulations can control the dynamics of an exploding spacecraft.

Geometry, Meshing, and Fracture

Mesh generation is the first problem encountered when creating a new CG object. Artists typically work with surface meshes, which are created either by combining and deforming primitives or by converting laser-scanned data from a model into a triangulated or quadrilateral surface. One commonly used algorithm for building a triangulated surface and level set function from scanned data was introduced by Curless and Levoy [7]. First, the data from each scan is converted to a level set function defined over a discrete cubic lattice whose individual cells we will refer to as voxels. Voxels near the surface then take on signed distance function values, and voxels away from the surface are either marked as "empty" (between the surface and the sensor) or "unseen". The level set functions from each scan are then combined to form a single level set function. A surface extraction method such as marching cubes [22] is used to construct the triangulated surface at the zero-isocontour of the level set function. The surface is also extracted on boundaries between empty and unseen regions, which effectively fills any holes in the scan (Figure 3).

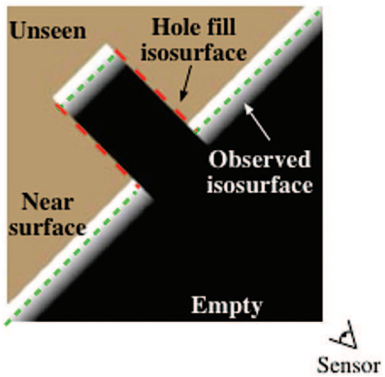

Figure 3. Extracting a surface from laser-scanned data [7].

Whereas artist versions of most CG objects are in the form of triangulated surfaces, tetrahedralized volume forms of the object are needed for most simulations. The first step of many algorithms is to define a body-centered cubic (BCC) or face-centered cubic (FCC) lattice on a uniform or octree grid. This initial lattice tetrahedralizes space with well-conditioned elements. The tetrahedralized volume is then modified to conform to the triangulated surface. The isosurface stuffing algorithm by Labelle and Shewchuk produces a well-conditioned mesh by snapping some vertices to the triangulated surface and refining others using a precomputed stencil [21].

Additionally, in scenes where an elaborately shaped triangulated surface cracks or shatters, it is necessary to somehow create new fragment meshes defined on the broken pieces. For the movie *BOLT*, Hellrung et al. [18] developed an algorithm which resolves every fragment exactly into a separate triangulated surface mesh and allows straightforward transfer of texture and look properties of the unfractured model. This method takes advantage of a cutting algorithm by Sifakis et al. [32] which cuts tetrahedralized volumes by duplicating cut elements and embedding the cut surface in these elements. In the embedding process, the cut elements are divided into "material" and "void" sections, defining regions inside and outside of the object, respectively [32]. In the first step of the triangulated surface algorithm, they generate a tetrahedral mesh that fully covers the object to be fractured. The triangulated surface of the unfractured object itself is used as the first cut in an application of the cutting algorithm,

Figure 4. Davy Jones rigid simulation rig from *Pirates of the Caribbean: Dead Man's Chest* [6].

effectively sectioning the background tetrahedral volume into "material" and "void" regions. The fracture surface is then applied as the second cut, resulting in the separation of the material volume into separate fragments. The cutting algorithm computes the triangulated boundary of every volumetric fragment in such a way that every triangle of a fragment is contained inside a triangle either of the uncut object or of the fracture surface.

Rigid Body Simulations

The simplest objects to simulate are rigid bodies. As the name suggests, these objects can only follow rigid motions (i.e., rotations and translations). Rigid bodies can be divided into two types: unconstrained rigid bodies, such as the pieces from a breaking up fighter plane in *Pearl Harbor*, and rigid bodies connected by joints which constrain their degrees of freedom (known as articulated rigid bodies), such as the battle droids in *Star Wars: Episode I* or Davy Jones' beard in *Pirates of the Caribbean: Dead Man's Chest* [6].

Each rigid body can be described by a state vector $X(t)$ consisting of its position $x(t)$, orientation $R(t)$ (or more typically a unit quaternion representing orientation), momentum $p(t)$, and angular momentum $L(t)$. By differentiating $X(t)$, we get to the known quantities that drive the simulation, force $F(t)$ and torque $\tau(t)$:

$$\frac{d}{dt}X(t) = \begin{pmatrix} v(t) \\ \omega(t)R(t) \\ F(t) \\ \tau(t) \end{pmatrix}.$$

Here, the velocity v is related to momentum by $mv(t) = p(t)$, where m is the mass of the body, and angular velocity ω is related to angular momentum by $I(t)\omega(t) = L(t)$, where $I(t)$ is the inertia tensor. Using an ordinary differential equations solver to numerically integrate $X'(t)$, the state vector (and thus the rigid body's position and orientation) can be updated at each timestep.

Figure 5. Hair's high geometric complexity presents a challenge to researchers. (left) A close-up of real hair. (right) Simulated hair [24].

The motion of articulated rigid bodies is constrained by the joints that connect the pieces. Various solution techniques exist for enforcing these constraints. One class of methods uses generalized coordinates to reduce the degrees of freedom, disallowing motions not permitted by the joint. By handling the constraints locally, these methods are fast but can break down in the presence of contact and collisions with other objects. These constraints can be enforced in a more global manner by using Lagrange multipliers [4] or impulses [39].

Deformable Object Simulations

Artists turn to deformable object simulations to handle complex systems such as hair, fur, and cloth or to give realistic responses to a wide range of objects, from CG food in *Ratatouille* to flesh simulations for characters in *Van Helsing*. Since the deformable object can stretch and bend, partial differential equations are needed to model the material dynamics. The object's behavior is usually described in terms of the relationship between its undeformed (or material) configuration B_0 and its deformed configuration at time t, B_t. This relationship is defined by $x(t) = \phi(X, t)$, where ϕ is a function which maps points $X \in B_0$ to points $x(t) \in B_t$.

The governing equations for these simulations come from continuum mechanics. The first, which is derived from Newton's second law, is the equation of motion, $\rho\ddot{\phi} = \nabla \cdot \mathbf{P} + \mathbf{f}$. Here, ρ is the density, $\mathbf{P}$ is the first Piola-Kirchoff stress tensor, and $\mathbf{f}$ represents external forces. Depending on the model, another stress tensor may be used in the place of $\mathbf{P}$. The second governing equation depends on the constitutive model which defines the stress-strain relationship for the material; in other words, it defines how the material deforms when stressed.

The linear elasticity model is the simplest constitutive model and is based on Hooke's law of elasticity. In this model, there is a linear relationship between stress and strain, typically denoted by $\sigma = \mathbf{C} : \varepsilon$, where σ is the Cauchy stress tensor,

ε is the infinitesimal strain tensor, and $\mathbf{C}$ is the fourth-order tensor of material stiffness. When the material is isotropic and homogeneous, this relationship can be simplified to $\sigma = 2\mu\varepsilon + \lambda tr(\varepsilon)I$, where μ and λ are the Lamé parameters and I is the identity matrix. Valid for small deformations, while it does not accurately model large deformations, it (and its rotation-invariant analogue) is still useful for a lot of graphics applications due to its easy computation. The neo-Hookean constitutive model is a generalization of linear elasticity for large deformations, and other even more general models exist for modeling soft rubber-like objects.

Just as important as the constitutive model to a simulation is the choice of discretization. Typically the simulated object will already have an explicitly meshed form (see the section "Geometry, Meshing, and Fracture"). The finite element method (FEM) [27] or finite volume method (FVM) [38] can then be applied to this mesh (or a triangulated/tetrahedralized version of this mesh) to numerically integrate the governing equations. Since high-order accuracy is usually not needed, linear interpolating functions are used for FEM, and σ and $\mathbf{f}$ are assumed to be constant on each element, so all integrands in the FEM or FVM formulations are constant and easy to compute.

Figure 6. (top) Penny's hair from *BOLT* is simulated using a clumping technique. Only a few hundred hairs were simulated, and the rest are added at render time [disneyanimation.com]. (bottom) All hairs are simulated in this hybrid volumetric/geometric collision algorithm [24].

Collision Detection and Handling

Rarely is an object simulated in isolation, so additional work is necessary to detect and handle contact and collisions with other objects. This problem can be tricky even in the simple scenario of two convex rigid bodies interacting. In highly complex collision scenarios (e.g., hair simulations), this problem can become intractable if standard geometric collision algorithms are used, so additional machinery is needed. In this section, we will describe the framework for a standard geometric collision detection and handling algorithm and highlight some recent work tackling the complexity of this problem with cloth and hair.

Geometric Collision Detection. Collisions can be detected either continuously or at the end of each simulation timestep. Since results only need to be physically plausible, not necessarily physically accurate, continuous collision detection and resolution are not typically used. For volumetric objects, collisions are detected by checking for any intersections at the end of the timestep. This type of detection can miss collisions where objects completely pass through each other in one step, but it is usually assumed that the simulation timesteps are small enough to avoid these situations.

Rigid body collisions are most easily checked by using level set representations of the objects. Intersections between two implicitly defined surfaces can then be found by testing sample points (e.g., vertices of its triangulated surface representation) on one object with the level set function on the other. Of course, this check is not sufficient for edge-face collisions since all vertices are outside of the implicit surface. On high-resolution meshes this case can be ignored, but additional checks are necessary for low-resolution meshes.

This implicit surface-detection method cannot detect self-collisions for deformable objects or be used for objects without a level set representation (e.g., open surfaces and curves in 3D). In these cases, point-face and edge-edge collisions can be detected by considering the linear interpolations between time n and time $n+1$ configurations. Define a tetrahedron by the four vertices in a point-face or edge-edge pair. A collision is detected if at any point between time n and time $n+1$, the linearly interpolated positions of the four points become coplanar, i.e., the volume of the tetrahedron is 0. The time at which intersection occurs can then be determined by solving a cubic equation in time [5].

In both cases, acceleration structures are used to reduce the number of intersection tests that need to be performed. Structures such as bounding box hierarchies are used to eliminate element pairs

from the detection list which are too far away from each other.

Geometric Collision Handling. Although continuous collision resolution, in which collisions are resolved in the order in which they occur, can produce more accurate and stable results, it can also be prohibitively expensive when a large number of collisions need to be processed. Instead, less accurate (though physically plausible) solutions are found by simultaneously resolving all collisions detected in the timestep between time n and $n+1$.

The first step in collision resolution is to determine the normal direction between two colliding elements, $\mathbf{n}$, and the relative velocities of the two objects at the collision point, u_{rel}. When $u_{rel} \cdot \mathbf{n} < 0$, the objects are colliding, and something must be done to prevent interpenetration. In the absence of friction, the collision is then resolved by applying impulses to the colliding objects in the normal direction so that the new normal relative velocity is $u'_{rel} \cdot \mathbf{n} = -\epsilon u_{rel} \cdot \mathbf{n}$. Here, $\epsilon \in [0, 1]$ is the coefficient of restitution and determines how "bouncy" the collision is. When $\epsilon = 1$, the collision is said to be perfectly elastic, and kinetic energy is conserved. When $\epsilon = 0$, the collision is perfectly inelastic, and all kinetic energy is lost. The choice of ϵ depends on the materials being modeled. If collisions are resolved in a pairwise fashion (which is common to many collision algorithms), new collisions between different pairs may be created by these impulses, so this algorithm must be iterated over more than once.

When $u_{rel} \cdot \mathbf{n} = 0$, the objects are considered to be in contact. In this case, the forces between the objects must be resolved to prevent the objects from accelerating toward each other. This case is much more difficult, as any force or impulse applied that is too strong can cause the objects to bounce apart, and the problem can be complicated when many objects are stacked on top of each other. One method, proposed by Baraff, analytically calculates the contact forces between objects [2, 3]. Another method by Guendelman et al, uses impulses similar to those for collisions; however, by partially ordering the contact pairs to work from the ground up, unwanted bouncing is avoided [16]. It is also possible to determine the necessary impulses to resolve all collisions and contact at once. See [20] for one such example. Friction can be handled in similar ways by applying impulses or modifying forces in the tangential direction, and the development of physically accurate models with nice numerical representations is an active area of research today.

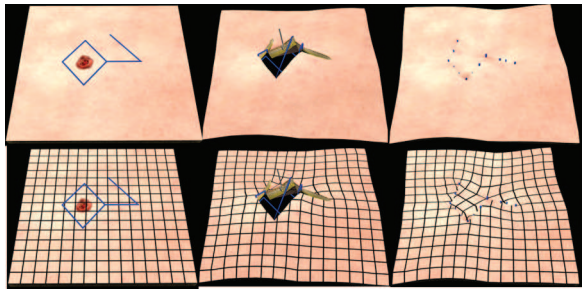

Figure 7. The images show a simulated malignant melanoma removal procedure using the framework of [33]. The bottom array of images uses an imposed surface texture to better visualize the post procedure equilibrium configuration of the tissue.

Cloth and Hair. Due to their complexity, CG cloth and hair/fur are almost always simulated rather than controlled by hand. However, this complexity also means that the standard geometric collision handling algorithms are not able to effectively resolve all collisions, and the cloth/hair can easily become tangled, producing visible artifacts.

For cloth, a modern approach is that of [5], which uses a three-stage process to ensure no collisions are missed. First, contact is preconditioned using penalty-based repulsions that are small enough to prevent visual artifacts. Then, in the second phase, collision impulses are iteratively applied, as in the subsection "Geometric Collision Handling". Third, rigid groups (where colliding objects are rigidly evolved) are used as a final safety net, postconditioning the collisions. One example of this algorithm used in practice is Yoda's robes in *Star Wars: Episode II*.

Hair simulation is even more complicated than cloth due to the massive number of hairs interacting and colliding. Instead of simulating each individual strand, the most common methods manage the complexity of many hairs interacting by simulating a smaller set of guide hairs (typically no more than several hundred) and interpolating a larger number of hairs for rendering. The use of sparse clumps often allows the use of inexpensive repulsion penalties as opposed to true geometric collision handling because guides are expected to have thickness. Examples of this technique include Penny's hair in *BOLT*. Alternatively, there have been several methods that treat every simulated hair as part of a fluid-like continuum volume [1, 17, 30]. These approaches naturally model hair interaction without explicit collisions.

More recently, McAdams et al. [24] explored a hybrid technique factoring hair computation into two parts: a coarse, highly coupled volumetric behavior, which is efficiently modeled by a continuum, and a finer, more locally coupled

Lagrangian particle simulation of hair. However, unlike previous continuum-based approaches that only simulate guide hairs that do not interact directly, this method simulates many hairs (several thousand) that are allowed to collide directly via the same geometric collision handling algorithm as used for cloth. The volumetric step works by projecting out any divergence in the hair's velocity field using the projection method as described in (2) for computational fluid dynamics. Geometric collision handling is only tractable because the volumetric step of the algorithm handles bulk collision behavior, leaving fewer collisions to be resolved.

Scientific Computing in Real Time

Simulations using numerical methods for partial differential equations are being used at an increasing rate due largely to improvements in processor speed and the availability of multicore processors. One very new example of this is real-time scientific computing—that is, simulations that run fast enough that a user can interact with them while the simulation is running. Perhaps the most obvious way to make real-time simulation possible is to reduce the complexity of the model by coarsening the representations of the simulated objects (e.g., using a coarser grid for fluid simulation or a lower resolution mesh for deformable object simulation). However, in many cases, the level of coarsening necessary for real-time application can produce unacceptably poor results. As such, one active area of research is in model reduction, where real-time low-resolution simulations make use of precomputed higher-resolution simulation information.

Such techniques will soon become commonplace in video games when creating scenes with natural phenomena. These techniques are also being used to simulate surgeries [33]. Figure 7 shows a finite element simulation of elastic soft tissues during a malignant melanoma procedure. The user makes incisions and deforms and sutures the tissue in real time while the simulator solves the governing equations.

Conclusion

With increased applicability of numerical methods in movies and video games, a new appreciation for applied math and scientific computing is emerging. This burgeoning aspect of the effects industry is serving as an exciting new frontier for applied mathematicians that uniquely combines the need for mathematical and computer science insights with the art of moviemaking. The nascent state of applied mathematics in the effects industry is driving research and development efforts to provide the algorithms and innovations needed to produce larger-scale and more impressive effects. That is, because scientific computing techniques are only recently becoming practical at a large scale in movies, their full efficacy and final place in the moviemaking process is far from determined. There will be efforts for years to come to fully realize the potential of applied math in the industry, and this can only be done by researchers with a strong background in mathematics, computer science, and physics.

Figure Credits

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Localized Eigenfunctions: Here You See Them, There You Don't

Steven M. Heilman and Robert S. Strichartz

The Laplacian Δ ($\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in the plane) is one of the most basic operators in all of mathematical analysis. It can be used to construct the important space-time equations of mathematical physics, such as the heat equation, the wave equation, and the Schrödinger equation of quantum mechanics. It has been studied from many points of view and in many different contexts (Riemannian manifolds, graphs, fractals, etc.). The eigenvalue equation $-\Delta u = \lambda u$ in a domain Ω with suitable boundary conditions (Dirichlet conditions $u|_{\partial\Omega} = 0$ and Neumann conditions $\frac{\partial u}{\partial n}|_{\partial\Omega} = 0$ are the most common) defines both the eigenvalue λ (in many cases a nonnegative real number) and the eigenfunction u. The set of all eigenvalues (usually a discrete set) is called the *spectrum* of the Laplacian on Ω, and there is a vast literature on the relationship between the spectrum and the geometry of Ω. See for example [K], [GWW] and [Z2], [HZ]. The spectrum contains a lot of information about the domain, and teasing out this information, including relationships between quantum and classical mechanics on the domain, has been a fascinating, ongoing, and highly nontrivial enterprise. For example, the famous Weyl law gives the asymptotic size of the kth eigenvalue as $k \to \infty$. There are also interesting and intricate estimates on the size of the first and second nontrivial eigenvalues. Needless to say, there are no connections between estimates of small and large eigenvalues.

Eigenfunctions are also fascinating objects. Clearly these are more complicated objects, so it can be expected that it is more difficult to say things about them. Many of the classical special functions and orthogonal polynomials can be related to eigenfunctions of the Laplacian for specific geometries. There have been many recent studies concerning eigenfunctions associated to large eigenvalues. In this note we invite you to look at some startling pictures of some specific "localized" eigenfunctions associated to small eigenvalues.

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Examples of Localization

How should we define a "localized" eigenfunction? We would be tempted to say it is an eigenfunction with support Ω_1 that is considerably smaller than all of Ω. But it is well known that eigenfunctions are real analytic functions and hence cannot vanish on any open set. So we must be content with saying

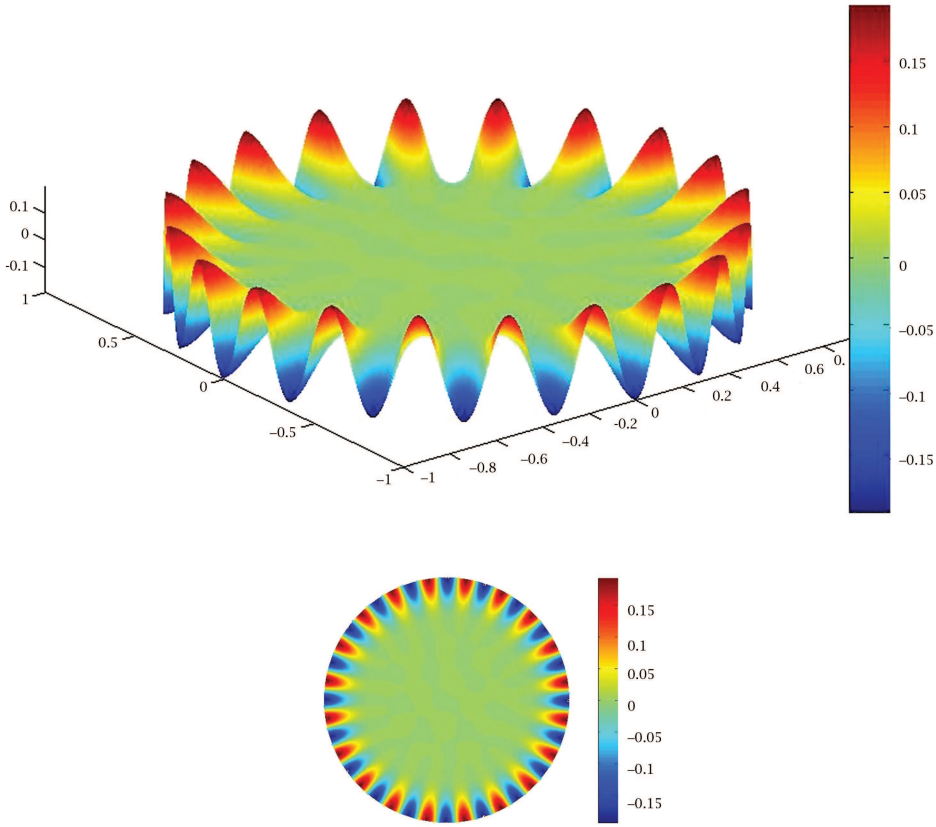

Figure 1. Localized circle eigenfunction.

that the function is "very small" on the complement of Ω_1. This is of course not a mathematical definition, although it might be acceptable to a physicist or a justice of the Supreme Court. One could make it into a precise definition with a parameter ϵ to quantify the statement "very small", but this just raises the question: How small does ϵ have to be to make the statement interesting? In this note we will show you some pictures to try to convince you that there are surprising examples where ϵ is smaller than you might expect.

Localized eigenfunctions have been observed before. As usual, physicists know more than mathematicians in the subject but with less certainty (this is a kind of uncertainty principle). Regardless, the cross-pollination in this subject between these two groups over the past century merits recognition and esteem. For high-frequency eigenfunctions, relations between eigenfunction localization and billiard dynamics have been studied. A nonexhaustive list includes treatments of (non)localization on: closed stable geodesics [BL] and closed unstable geodesics [H, C2, BZ, HH]. Other results address dichotomies [Be], numerical aspects [Ba, Bä], rarity [Šn, C1, Z1], and near nonexistence [L] of such (phase space) localized eigenfunctions, as $\lambda \to \infty$.

In the low-frequency realm, no deep explanation for eigenfunction localization seems to exist. Low-frequency, or "ground state", eigenfunctions have been widely studied (see for example [P, BMP]). However, the authors can only find scattered examples of low-frequency localization, such as: near a fractal boundary [RSH], in narrow channels between domains [CH], and in square pairs with irrational ratios of frequency oscillations (Example 3 in [JMS]). As is well known, an eigenfunction is itself an eigenfunction in each of its nodal domains (with appropriate boundary conditions). Therefore, results for low-frequency eigenfunctions can inform higher-frequency studies.

A simple example of an eigenfunction on the disc localized to a neighborhood of the boundary circle is shown in Figure 1. These examples tend to involve eigenfunctions with eigenvalues rather high up in the spectrum and domains with special types of billiard flows. In contrast, our examples occur low down in the spectrum and are consequences of symmetry considerations. We work with Neumann boundary conditions because they are natural (the weak formulation of the eigenvalue equation is $-\int_\Omega (\nabla u \cdot \nabla v)\, dx = \lambda \int_\Omega uv\, dx$ for all reasonable test functions v, without imposing any boundary conditions) and they were essential

Figure 2. Two rooms with a passage.

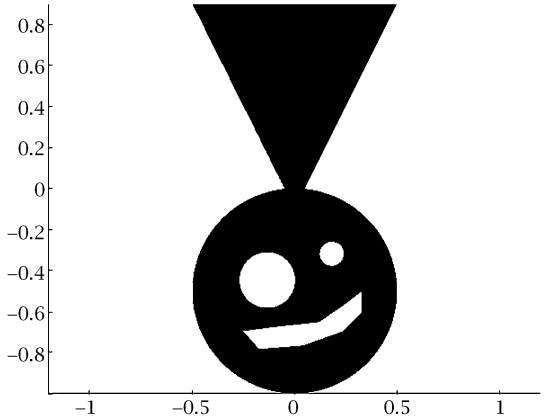

Figure 3. Smiley domain, height $h = 0.1$.

in our work on approximating fractal Laplacians with ordinary planar Laplacians [BHS], [HS]. Similar examples with Dirichlet boundary conditions also exist. It was our coauthor Tyrus Berry who first serendipitously discovered examples of localized eigenfunctions on sawtooth-shaped domains as reported in [BHS], but these examples did not play any role in the theory developed there. By coincidence, many localized eigenfunctions of fractal Laplacians have been known since the work of Fukushima and Shima [FS], and these can also be explained by symmetry considerations [BK]. See [St1, St2] for expository accounts of this phenomenon.

Our examples can be thought of as modified versions of "rooms and passages" domains [CH]. If $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$ consists of two rooms, Ω_1, Ω_2, and a very short narrow connection passage Ω_3 (see Figure 2), then it is not surprising that there are Dirichlet eigenfunctions on Ω that are very close to Dirichlet eigenfunctions on Ω_1 extended to be zero on $\Omega_2 \cup \Omega_3$. In our examples we will join two rooms $\Omega = \Omega_1 \cup \Omega_2$ that intersect in a small, but not very small, piece.

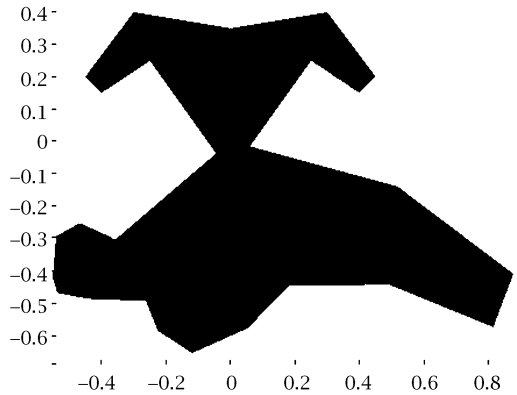

Figure 4. Cow domain, height $h = 0.1$.

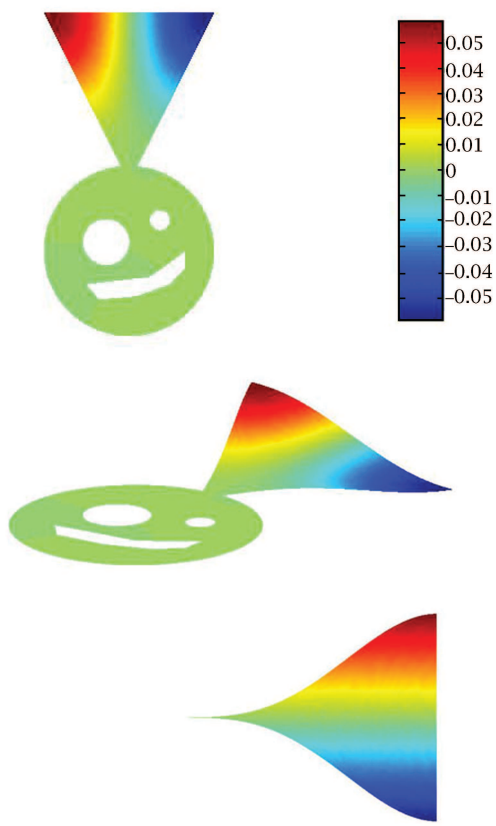

Figure 5. Smiley, fifth eigenfunction.

Figure 3 shows one example, the "smiley face", and Figure 4 shows another, the "cow". The main idea is that Ω_1 must possess an axis of symmetry, and the rooms join together at one end of this axis. In other words, there is a line L such that the reflection R in L preserves Ω_1, $R\Omega_1 = \Omega_1$, and L passes through $\Omega_1 \cap \Omega_2$. Suppose that u_1 is a Neumann eigenfunction of the Laplacian on Ω_1 that is skew-symmetric with respect to R, $u_1(Rx) = -u_1(x)$. Such eigenfunctions occur throughout the

spectrum, since indeed every eigenspace splits into functions that have symmetry and skew-symmetry with respect to R (most eigenspaces are 1-dimensional and are of one or the other symmetry type). Then u_1 vanishes along L. If the point p where L intersects $\partial\Omega_1$ is a corner point, then u_1 and ∇u_1 vanish at p, so u_1 is relatively small near p. (For example, $u_1(x, y) = \cos \pi x - \cos \pi y$ at the origin if Ω_1 is the unit square.) So it is not surprising that there is an eigenfunction u on Ω that is close to u_1 on Ω_1 and close to zero off Ω_1.

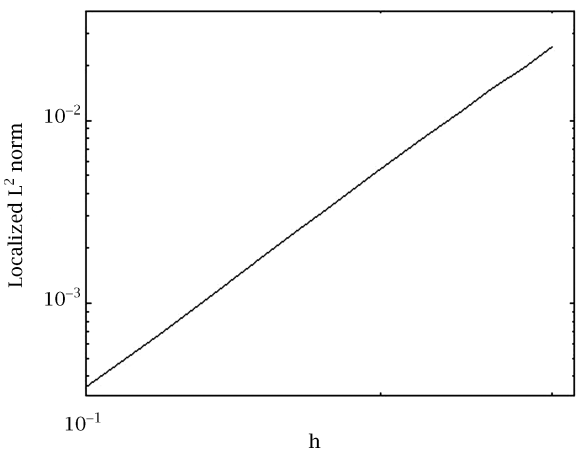

Figure 6. Localization in L^2 norm, for Figure 5 example.

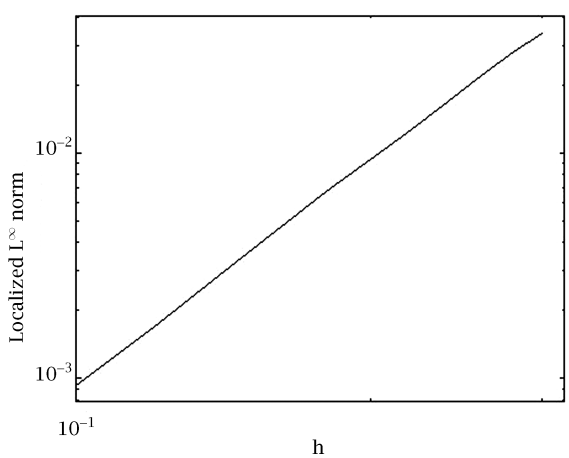

Figure 7. Localization in L^∞ norm, for Figure 5 example.

Such reasoning does not yield a sharp estimate for how localized u is, so we look at some experimental evidence. We use Matlab to numerically approximate some eigenfunctions on our domains using the finite element method. Figures 5–12 show the results. Note that we normalize the eigenfunctions to have L^2 norm on Ω equal to 1, and we can measure the localization either by the L^2 norm on $\Omega \setminus \Omega_1$, a kind of average localization, or the L^∞ norm on $\Omega \setminus \Omega_1$, a uniform localization. In Figures 6, 7, 9, and 10 we show both of these as a function of the aperture size h for the connection for each domain. (Here h is the distance from the line $\{y = 0\}$ to the lowest vertex of the triangular "top" of the given domain; see Figs. 3 and 4). These are log-log plots, suggesting a power law relationship over the given range of h values. In the tables below we give the best fit power law for two eigenfunctions in each domain. Note that the powers vary considerably in these four examples.

Summary Table: Smiley domain

	Eigfcn 5	Eigfcn 12
L^2 Localization	$y = 11.254x^{3.9087}$	$y = 249.06x^{2.4636}$
L^∞ Localization	$y = 4.1735x^{3.2959}$	$y = 90.552x^{2.3553}$

Summary Table: Cow domain

	Eigfcn 4	Eigfcn 11
L^2 Localization	$y = 119.65x^{3.0889}$	$y = 2096.5x^{2.8028}$
L^∞ Localization	$y = 31.615x^{2.6075}$	$y = 676.08x^{2.5700}$

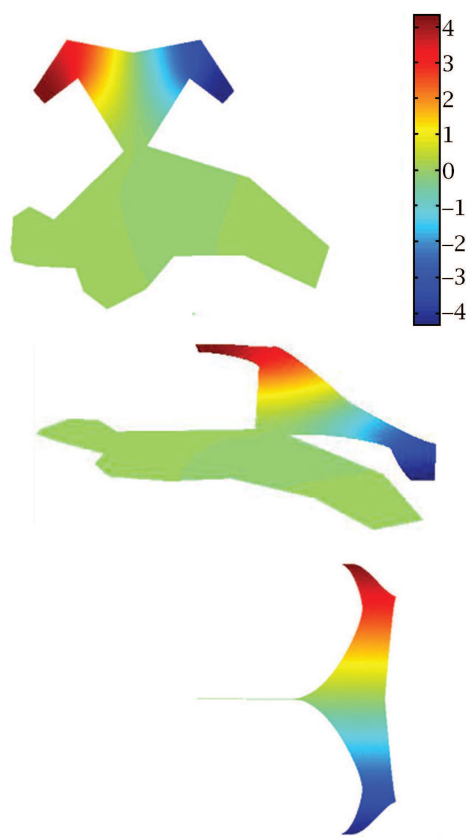

Figure 8. Cow, fourth eigenfunction.

Conclusion

Our examples show how easy it is to find surprisingly localized eigenfunctions. The domains do not have to have any special properties beyond the symmetry of the Ω_1 piece (breaking the symmetry even slightly makes the examples disappear). We do not have to go very high up in the spectrum. As mathematicians it is natural for us to want a theorem that explains the examples, or at least a conjectured theorem. Perhaps there is such a theorem, and a perceptive reader might be able to find one, but at present we don't see any. Or we might suggest that there is more to mathematics than just theorems. This might sound like a radical suggestion, or perhaps it is just common sense.

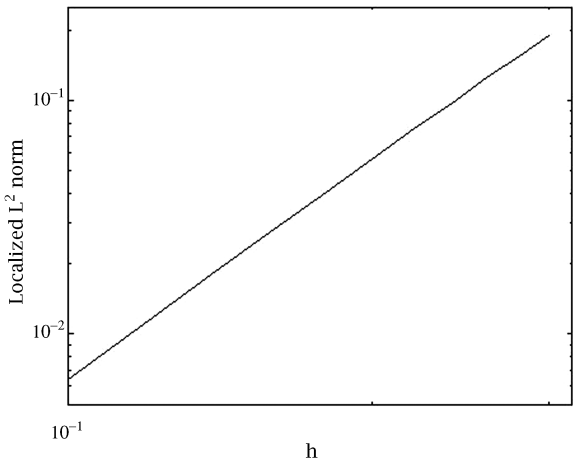

Figure 9. Localization in L^2 norm, for Figure 8 example.

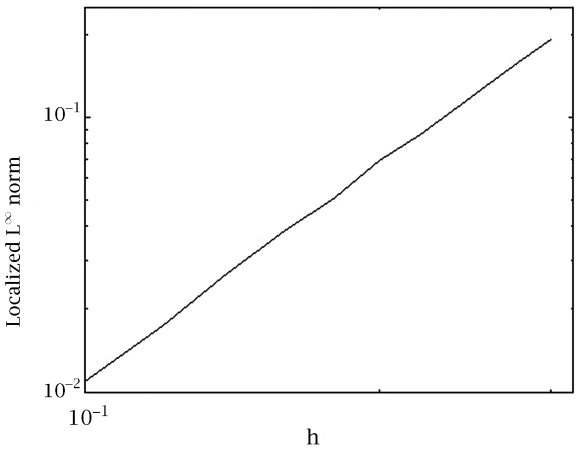

Figure 10. Localization in L^∞ norm, for Figure 8 example.

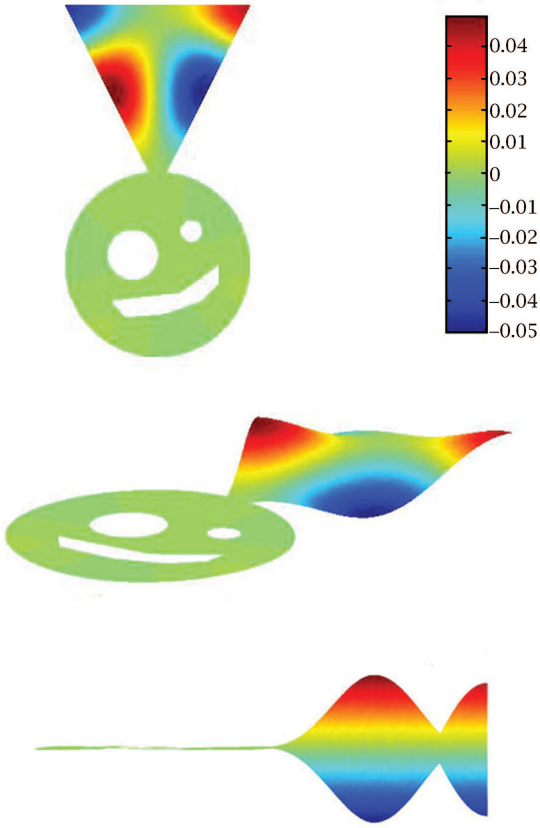

Figure 11. Smiley, twelfth eigenfunction.

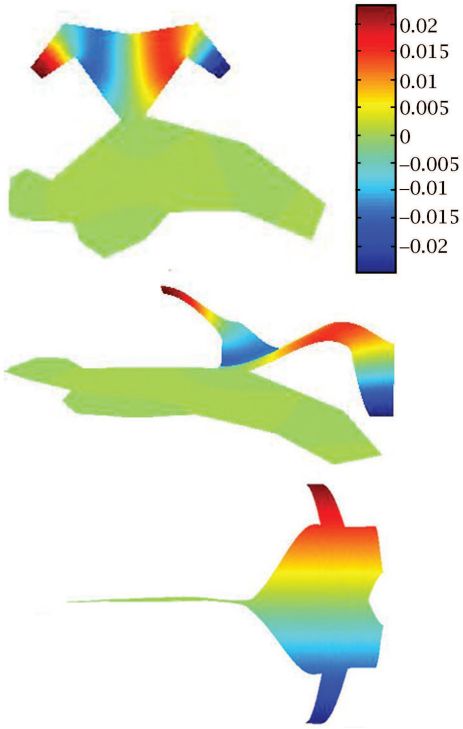

Figure 12. Cow, eleventh eigenfunction.

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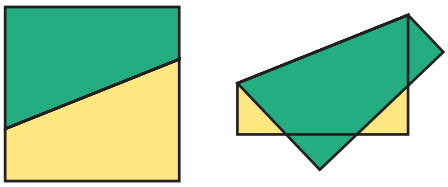
Fractal Origami

The cover image is a variant of Figure 12 in this issue's article by Bernard Dacorogna, Paolo Marcellini, and Emanuele Paolini on some mathematical aspects of origami.

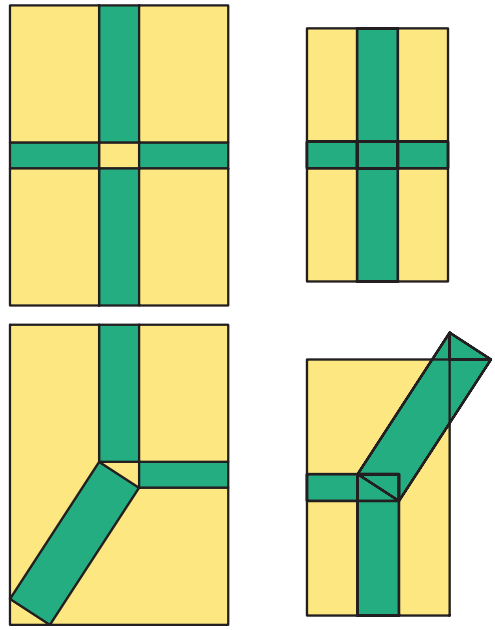
The connection between the image and origami might not be apparent. A partition of a planar region Ω into any number of convex polygons is said to satisfy the *Kawasaki angle condition* if at every vertex there is an even number of polygons meeting at each vertex, and the alternating sum of angles there vanishes. A partition satisfying this condition determines a map from Ω into the plane with the property that maps on neighboring polyhedra are obtained by reflection. The Kawasaki condition implies that this specification is consistent. I call such maps **fold maps**. The simplest nontrivial fold map is the basic origami move.



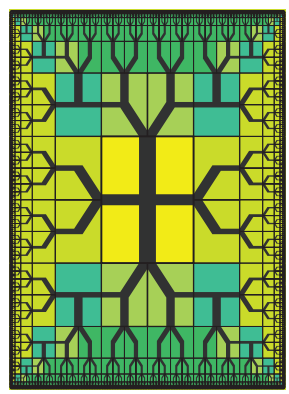
In this and other figures, I follow Dacorogna et al. in distinguishing those domains that change orientation from those that don't.

A fold map F is contracting in the sense that $|F(P) - F(Q)| \leq |P - Q|$. A curious question arises: *given a domain Ω and a boundary path map Φ from its boundary $\partial\Omega$ satisfying some contraction condition, does there exist a partition whose associated fold map agrees with Φ on the boundary?* According to Dacorogna et al., this is so if the boundary map satisfies the strict contraction condition $|F(P) - F(Q)| \leq \lambda|P - Q|$ for some $\lambda < 1$. Any map satisfying other than a very simple boundary constraint must come from an infinite partition, and one might expect that the behaviour of the partition near the boundary will be interesting. In Figure 12 of their article, and on the cover of this issue, are illustrated partitions that give rise to a linear boundary map in the case where Ω is a rectangle.

This seems to be the only large class with an explicit construction. In both these cases, the partition is by rectangles of different sizes but of only two basic designs. It is easy to draw the unadorned picture without much trouble. It is made up of rectangles of various sizes, but up to similarity exactly the two basic designs shown above, which Dacrorogna et al. call the *base regions*: It is relatively easy to draw the figure in a straightforward way, since the pattern of rectangles is easy to lay out, and the region at the top occurs precisely on diagonals. But it is somewhat more interesting to do something that



reflects more directly the recursive branching structure. The regions can be assembled in a directed acyclic graph with the central region as root and edges going out from the center to smaller and smaller regions. This can be pruned to become a tree, one associated to a finite state diagram of five states, marked by distinct colors in this figure:



Using this finite state machine to traverse the tree makes possible what might be called "intelligent" drawing. *What other boundary conditions on fold maps can be realized by finite state diagrams? What can one say in general about the relationship between a boundary condition and the structure of a partition whose fold map satisfies it?*

—Bill Casselman
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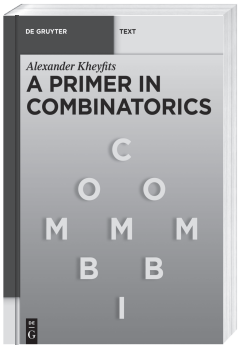

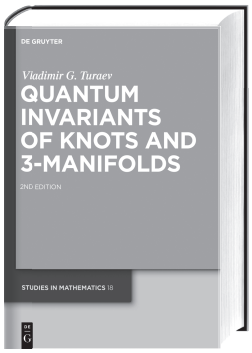

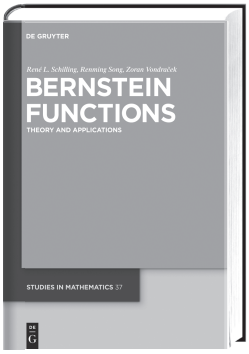

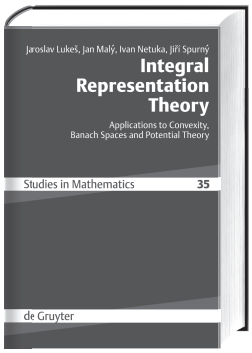

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WHAT IS...

a Perverse Sheaf?

Mark Andrea de Cataldo and Luca Migliorini

Manifolds are obtained by gluing open subsets of Euclidean space. Differential forms, vector fields, etc., are defined locally and then glued to yield a global object. The notion of sheaf embodies the idea of gluing. Sheaves come in many flavors: sheaves of differential forms, of vector fields, of differential operators, constant and locally constant sheaves, etc. A locally constant sheaf (*local system*) on a space X is determined by its monodromy, i.e., by a representation of the fundamental group $\pi_1(X,x)$ into the group of automorphisms of the fiber at $x \in X$: the sheaf of orientations on the Möbius strip assigns $-\mathrm{Id}$ to the generators of the fundamental group $\mathbb{Z}$. A sheaf, or even a map of sheaves, can be glued back together from its local data: exterior derivation can be viewed as a map between sheaves of differential forms; the gluing is possible because exterior derivation is independent of the choice of local coordinates.

The theory of sheaves is made more complete by considering *complexes of sheaves*. A complex of sheaves K is a collection of sheaves $\{K^i\}_{i\in\mathbb{Z}}$ and maps $d^i : K^i \to K^{i+1}$ subject to $d^2 = 0$. The i-th *cohomology sheaf* $\mathcal{H}^i(K)$ is $\ker d^i / \operatorname{im} d^{i-1}$. The (sheafified) de Rham complex $\mathcal{E}$ is the complex with entries the sheaves $\mathcal{E}^i$ of differential i-forms and with differentials $d : \mathcal{E}^i \to \mathcal{E}^{i+1}$ given by the exterior derivation of differential forms. By the Poincaré lemma, the cohomology sheaves are all zero, except for $\mathcal{H}^0 \simeq \mathbb{C}$, the constant sheaf.

The de Rham theorem, stating that the cohomology of the constant sheaf equals closed forms modulo exact ones, points to the fact that $\mathbb{C}$ and $\mathcal{E}$ are cohomologically indistinguishable from each other, even at the local level. The need to identify two complexes containing the same cohomological information via an isomorphism leads to the notion of *derived category* ([2]): the objects are complexes and the arrows are designed to achieve the desired identifications. The inclusion of complexes $\mathbb{C} \subseteq \mathcal{E}$ is promoted by decree to the rank of isomorphism in the derived category because it induces an isomorphism at the level of cohomology sheaves.

While the derived category brings in a thick layer of abstraction, it extends the reach and flexibility of the theory. One defines the cohomology groups of a complex and extends to complexes of sheaves the ordinary operations of algebraic topology: pull-backs, push-forwards, cup and cap products, etc. There is also a general form of duality for complexes ([2]) generalizing classical Poincaré duality.

Perverse sheaves live on spaces with singularities: analytic spaces, algebraic varieties, PL spaces, pseudo-manifolds, etc. For ease of exposition, we limit ourselves to sheaves of vector spaces on complex algebraic varieties and to perverse sheaves with respect to what is called *middle perversity*. In order to avoid dealing with pathologies such as sheaves supported on the Cantor set, one imposes a technical condition called *constructibility*. Let us just say that the category D_X of *bounded constructible complexes* of sheaves on X sits in the derived category and is stable under the various topological operations mentioned above. If K is in D_X, only finitely many of its cohomology sheaves are nonzero and, for every i, the set $\operatorname{supp} \mathcal{H}^i(K)$, the closure of the set of points at which the stalk is nonzero, is an algebraic subvariety.

A *perverse sheaf* on X is a bounded constructible complex $P \in D_X$ such that the following holds for $K = P$ and for its dual $P^\vee$:

$$\dim_{\mathbb{C}} \operatorname{supp} \mathcal{H}^{-i}(K) \le i, \quad \forall i \in \mathbb{Z}. \tag{1}$$

A map of perverse sheaves is an arrow in D_X.

The term "sheaf" stems from the fact that, just as in the case of ordinary sheaves, (maps of) perverse sheaves can be glued; as to "perverse", see below. The theory of perverse sheaves has its roots

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in the two notions of intersection cohomology and of $\mathcal{D}$-module. As we see below, perverse sheaves and $\mathcal{D}$-modules are related by the Riemann-Hilbert correspondence.

It is time for examples. If X is nonsingular, then $\mathbb{C}_X[\dim X]$, i.e., the constant sheaf in degree $-\dim_{\mathbb{C}} X$, is self-dual and perverse. If $Y \subseteq X$ is a nonsingular closed subvariety, then $\mathbb{C}_Y[\dim Y]$, viewed as a complex on X, is a perverse sheaf on X. If X is singular, then $\mathbb{C}_X[\dim X]$ is usually not a perverse sheaf. On the other hand, the intersection cohomology complex (see below) is a perverse sheaf, regardless of the singularities of X. The extension of two perverse sheaves is a perverse sheaf. The following example can serve as a test case for the first definitions in the theory of $\mathcal{D}$-modules. Let $X = \mathbb{C}$ be the complex line with origin $o \in X$, let z be the standard holomorphic coordinate, let $\mathcal{O}_X$ be the sheaf of holomorphic functions on X, let a be a complex number, and let D be the differential operator $D : f \mapsto zf' - af$. The complex P_a

$$0 \longrightarrow P_a^{-1} := \mathcal{O}_X \xrightarrow{D} P_a^0 := \mathcal{O}_X \longrightarrow 0 \tag{2}$$

is perverse. If $a \in \mathbb{Z}^{\geq 0}$, then $\mathcal{H}^{-1}(P_a) = \mathbb{C}_X$ and $\mathcal{H}^0(P_0) = \mathbb{C}_o$. If $a \in \mathbb{Z}^{<0}$, then $\mathcal{H}^{-1}(P_a)$ is the extension by zero at o of the sheaf $\mathbb{C}_{X\setminus o}$ and $\mathcal{H}^0(P_a) = 0$. If $a \notin \mathbb{Z}$, then $\mathcal{H}^{-1}(P_a)$ is the extension by zero at o of the local system on $X \setminus o$ associated with the branches of the multivalued function z^a and $\mathcal{H}^0(P_a) = 0$. In each case, the associated monodromy sends the positive generator of $\pi_1(X \setminus o, 1)$ to $e^{2\pi i a}$. The dual of P_a is P_{-a} (this fits well with the notions of adjoint differential equation and of duality for $\mathcal{D}$-modules). Every P_a is the extension of the perverse sheaf $\mathcal{H}^0(P_a)[0]$ by the perverse sheaf $\mathcal{H}^{-1}(P_a)[1]$. The extension is trivial (direct sum) if and only if $a \notin \mathbb{Z}^{>0}$.

A local system on a nonsingular variety can be turned into a perverse sheaf by viewing it as a complex with a single entry in the appropriate degree. On the other hand, a perverse sheaf restricts to a local system on some dense open subvariety. We want to make sense of the following slogan: *perverse sheaves are the singular version of local systems.* In order to do so, we discuss the two widely different ideas that led to the birth of perverse sheaves about thirty years ago: *the generalized Riemann-Hilbert correspondence* (RH) and *intersection cohomology* (IH) ([3]).

(RH) Hilbert's 21st problem is concerned with Fuchs-type differential equations on a punctured Riemann surface Σ. As one circuits the punctures, the solutions are transformed: the sheaf of solutions is a local system on Σ (see (2)).

The 21st problem asked whether any local system arises in this way (it essentially does). The sheafification of linear partial differential equations on a manifold gives rise to the notion of *$\mathcal{D}$-module*. A *regular holonomic* $\mathcal{D}$-module on a complex manifold M is the generalization of the Fuchs-type equations on Σ. The sheaf of solutions is now replaced by a *complex of solutions*, which, remarkably, belongs to D_M. In (2), the complex of solutions is P_a, the sheaf of solutions to $D(f) = 0$ is $\mathcal{H}^{-1}(P_a)$, and $\mathcal{H}^0(P_a)$ is related to the (non)solvability of $D(f) = g$. Let $\mathcal{D}^b_{r,h}(M)$ be the bounded derived category of $\mathcal{D}$-modules on M with regular holonomic cohomology. RH states that the assignment of the (dual to the) complex of solutions yields an equivalence of categories $\mathcal{D}^b_{r,h}(M) \simeq D_M$. Perverse sheaves enter the center of the stage: they correspond via RH to regular holonomic $\mathcal{D}$-modules (viewed as complexes concentrated in degree zero).

In agreement with the slogan mentioned above, the category of perverse sheaves shares the following formal properties with the category of local systems: it is Abelian (kernels, cokernels, images, and coimages exist, and the coimage is isomorphic to the image), stable under duality, Noetherian (the ascending chain condition holds), and Artinian (the descending chain condition holds), i.e., every perverse sheaf is a finite iterated extension of *simple* (no subobjects) perverse sheaves. In our example, the perverse sheaves (2) are simple if and only if $a \in \mathbb{C} \setminus \mathbb{Z}$.

What are the simple perverse sheaves? Intersection cohomology provides the answer.

(IH) The *intersection cohomology groups* of a singular variety X with coefficients in a local system are a topological invariant of the variety. They coincide with ordinary cohomology when X is nonsingular and the coefficients are constant. These groups were originally defined and studied using the theory of geometric chains in order to study the failure, due to the presence of singularities, of Poincaré duality for ordinary homology, and to put a remedy to it by considering the homology theory arising by considering only chains that intersect the singular set in a controlled way. In this context, certain sequences of integers, called *perversities*, were introduced to give a measure of how a chain intersects the singular set, whence the origin of the term "perverse". The intersection cohomology groups thus defined satisfy the conclusions of Poincaré duality and of the Lefschetz hyperplane theorem.

On the other hand, the intersection cohomology groups can also be exhibited as the cohomology groups of certain complexes in D_X: the *intersection complexes* of X with coefficients in the local system. It is a remarkable twist in the plot of this story that the simple perverse sheaves are precisely the intersection complexes of the irreducible subvarieties of X with coefficients given by simple local systems!

We are now in a position to clarify the earlier slogan. A local system L on a nonsingular subvariety $Z \subseteq M$ gives rise to a regular holonomic

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$\mathcal{D}$-module supported over the closure $\overline{Z}$. The same L gives rise to the intersection complex of $\overline{Z}$ with coefficients in L. Both objects extend L from Z to $\overline{Z}$ across the singularities $\overline{Z} \setminus Z$. By RH, the intersection complex is precisely the complex of solutions of the $\mathcal{D}$-module.

A pivotal role in the applications of the theory of perverse sheaves is played by the *decomposition theorem*: let $f : X \to Y$ be a proper map of varieties; then the intersection cohomology groups of X with coefficients in a simple local system are isomorphic to the direct sum of a collection of intersection cohomology groups of irreducible subvarieties of Y, with coefficients in simple local systems. For example, if $f : X \to Y$ is a resolution of the singularities of Y, then the intersection cohomology groups of Y are a direct summand of the ordinary cohomology groups of X. This "as-simple-as-possible" splitting behavior is the deepest known fact concerning the homology of complex algebraic varieties and maps. It fails in complex analytic and in real algebraic geometry. The decomposition of the intersection cohomology groups of X is a reflection in cohomology of a finer decomposition of complexes in D_Y. The original proof of the decomposition theorem uses algebraic geometry over finite fields (perverse sheaves make perfect sense in this context). For a discussion of some of the proofs see [1].

One striking application of this circle of ideas is the fact that the intersection cohomology groups of projective varieties enjoy the same classical properties of the cohomology groups of projective manifolds: the Hodge (p, q)-decomposition theorem, the hard Lefschetz theorem, and the Hodge-Riemann bilinear relations. This, of course, in addition to Poincaré duality and to the Lefschetz hyperplane theorem mentioned above.

The applications of the theory of perverse sheaves range from geometry to combinatorics to algebraic analysis. The most dramatic ones are in the realm of representation theory, where their introduction has led to a truly spectacular revolution: proofs of the Kazhdan-Lusztig conjecture, of the geometrization of the Satake isomorphism, and, recently, of the fundamental lemma in the Langlands program (see the survey [1]).

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The Housekeeper and the Professor

Reviewed by Koji Fujiwara

The Housekeeper and the Professor
Yoko Ogawa
Picador, 2009
US$14.00, Paperback, 192 pages
ISBN-13: 978-0312427808

This book is a best-selling Japanese novel that has now been translated into English. It is about a sixty-four-year-old mathematician. He had been a professor seventeen years earlier, but then he had a car accident and his brain was damaged, in particular the part associated with memory. Now his memory lasts only eighty minutes, although he remembers things from before the accident. His field of research is number theory, but he is no longer active in research because of his brain damage.

Because of his memory problem, it is almost impossible for him to have an ordinary life. Because he does not remember anything, everything seems to him to happen suddenly. It was natural for him to withdraw from society and from relationships with other people. He became very lonely.

The professor lives with his sister-in-law, who has been a widow for many years. She lives in the main house and he in a tiny separate house. They are from a rich family and live on the income from the real estate they own. They don't have much interaction, at least after the accident. The sister-in-law hires a housekeeper to take care of the professor. The housekeeper comes every day in the morning and makes breakfast, does laundry, cleans the house, and makes dinner. She is twenty-nine years old, has never married, and has a boy of age ten.

In Japan, it's unusual to hire a housekeeper. It's done only in very rich families. It's also pretty unusual for a woman to have a child without being married.

The housekeeper comes to the professor's house for the first time in March 1992. The book is about the relationship shared by the professor, the housekeeper, and her son.

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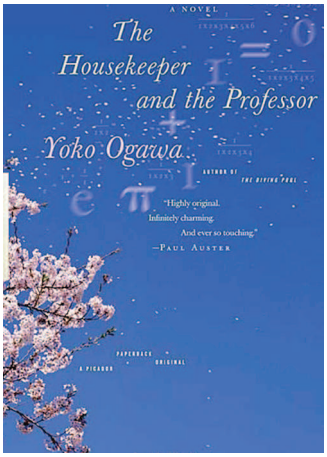


The professor loves mathematics, especially numbers. That's nearly the only thing he cares about, but he also likes baseball, even though he does not follow the results. His favorite team is the Tigers, and his favorite player is Enatsu, a pitcher. Both the Tigers and Enatsu exist in real Japanese professional baseball leagues. Enatsu is a legendary pitcher who retired in 1984—which means that the professor does not know that Enatsu has retired, so he keeps asking how Enatsu is doing. Perhaps one of the typical images of Enatsu is that he was really an excellent pitcher, at least when he was young, and he would remember all the details of his games and talk passionately about baseball for hours. On the other hand, he is a bit extreme and does not see anything much other than baseball. When he retired from the Japanese baseball league in 1984 at age thirty-six, although he was in bad shape, he went to the United States to have a tryout in the U.S. major leagues, and he failed. He was similar to Don Quixote. Later, in 1993, he got arrested for stimulant drug use and spent two years in a jail. Please don't misunderstand; I like Enatsu. I'm only saying this because I think what baseball is to Enatsu is a bit like what mathematics is to the professor. Quite a few real stories about Japanese baseball are quoted in the book. Because of that, I felt as if the professor really existed when I was reading the book. However, enjoying the book does not require knowing about Japanese baseball.

When the housekeeper comes to the professor's house each morning, he does not remember her. But he has many small pieces of paper clipped to his jacket to remind him of essential information. One of them is about her. Every morning when she comes, he identifies her with the face on one of the pieces of paper, and then he asks one of his routine questions: how old are you?, what is your shoe size?, etc. The question is always about numbers. If she says her shoe size is 24 (about 6 in U.S. sizes), then he says that's 4 factorial. Then she asks what factorial is, and

he answers. That's a typical conversation between them.

The professor soon learns that the housekeeper has a ten-year-old son, and he thinks it is outrageous (his reactions are often out of balance) that the son has to be at home for dinner without his mother. The professor tells the housekeeper to bring her son to the house. This is how the relationship among the three people starts.

The book does not contain any big drama. But every little thing in the professor's life is unusual and difficult for the people around him because of his memory problem. In one of the biggest events in the book, they go out to watch a Tigers game. The mother and the son try hard to prevent the professor from noticing that Enatsu retired long ago, so that the professor will not be disappointed. In addition to his memory problem, his inclination for numbers, especially prime numbers, is so strong that his life also tends to be slightly comical or awkward. For example, if the housekeeper comes back from a shopping trip that takes longer than eighty minutes, he will ask her shoe size again.

I don't know what typical images people have of mathematicians, but, being a mathematician and knowing many of them, I can say that the professor is not typical. On the other hand, I can certainly think of a few mathematicians who are like him and who are unusually into numbers. For example, I remember one professor from the time I was visiting the United States as a postdoc. Although I do geometry, I was taking a course on automorphic forms for fun. I could tell that the professor teaching the course had an unusual interest in and talent for numbers. He remembered amazingly well certain very big and important numbers. He was a nice person and a good teacher, and I admired his passion, too. At the same time, I was discouraged that I don't have such a special feeling for numbers. I thought I had better stay away from them: Leave them to number theorists. (Eventually he received a Fields Medal. So I did not have to be discouraged.)

Some people have said that the character of the professor in this novel is based on Erdős. It is true that a book on Erdős is mentioned as a reference at the end of the book. However, the professor reminds me of Ramanujan. I remember the famous story about Hardy telling Ramanujan that the number of the taxi he had just taken was 1729. Ramanujan immediately said, "It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways":

$$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$

The professor tells many stories about math and numbers to the housekeeper and her son. One of their favorite things is the famous formula of Euler,

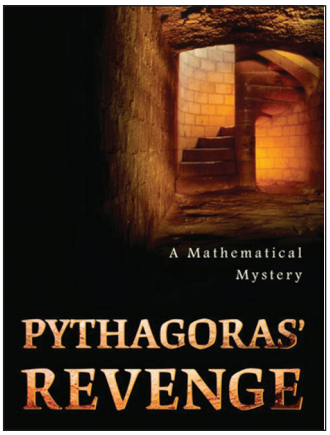
$$e^{\pi i} = -1.$$

They knew π but did not know i and e, so I don't think they really got the meaning of the formula. Recently, a student in my freshman calculus course came and asked me if/how this formula is true/proved. I told him "You have just learned Taylor series, so just plug πi into the Taylor series of e^x, then use the Taylor series for sine and cosine. That's all." He looked disappointed. I'm not sure that's the kind of answer he expected. I wanted to show him a geometric explanation using the complex number plane, but he has not learned that yet. But maybe that's not the answer he wanted to hear either. It seems this formula has a strong charm to attract people from various backgrounds. Maybe this is one example of the fact that the beauty and mystery of math can reach many people in different ways. In a sense, I envy them, because, probably like many other mathematicians, I do and try to enjoy math mainly in the style of definition-theorem-proof. By the way, a direct English translation of the title of the book is *Math Formulas the Professor Loved*.

It's an intriguing question how someone like the professor, with such a short memory, could build relationships with other people. When you see someone you know, like family members, friends, colleagues, or neighbors, you already have some feelings about them. Those feelings are mostly based on the experiences you have had with them, but you don't have to remember all of those experiences each time. It seems that feelings, thoughts, and ideas from past events are filed in your brain and produce something abstract, and this abstraction is more important for you than the individual feelings, thoughts, and ideas. But you need to file them in the correct folder for the abstraction process to occur, and there is a folder associated to each person you know. I don't know much about current brain studies concerning memory, so I will stop here. Anyway, when I was reading the book, I felt the relationships and the feelings shared by the three characters were real and convincing. The author is very successful on this point.

The author of the book, Yoko Ogawa, is popular in Japan. She received the Akutagawa Prize in 1990, the most prestigious prize for young authors of Japanese literature. She wrote this book in 2004, and it was one of the best-selling books of the year. Since then, it has appeared as a film, a TV show, a radio show, and a comic book. I saw the film first and liked it. Then I read the original Japanese book and now the English version for the review. The translation into English is good.

I'm sure many people will enjoy reading this book. It's easy to read, and it does not matter how much the reader knows about math. When I reread the book for review, I wondered if the way the characters build their relationship is typically Japanese—that is, indirect and slow. But I think their way is maybe universal. They are vulnerable and therefore cautious and thoughtful. What is nice and encouraging is that, although none of them has a strong position in society, they support each other and manage to have a good time. Math helps them do it.



Book Review

The Full Potential of Storytelling:

Review of *Pythagoras' Revenge: A Mathematical Mystery*

Reviewed by Javier Fresán

Pythagoras' Revenge: A Mathematical Mystery
Arturo Sangalli
Princeton University Press, 2009
US$24.95, 224 pages
ISBN: 978-1-4008-2990-3

What is the difference between *The Name of the Rose* and *The Da Vinci Code?* Leaving the literary abyss aside, naive readers might find many contact points between two of the most successful historical whodunits ever written. But as soon as they try to go further, they find that Umberto Eco's splendid reconstruction of the labyrinths of scholastic thought turns out to be as important as the murders themselves. By contrast, what underlies the treasure hunt of Professor Robert Langdon is "dust, gloom and nothingness", to give it a more substantial description than it deserves. In the novel under review, neo-Pythagorean sects searching for a reincarnation of their Master coexist with scholars absorbed in the translation of ancient manuscripts. Nevertheless, the author's wish to teach and delight makes the story closer to *The Name of the Rose* than to *The Da Vinci Code*.

Arturo Sangalli received a Ph.D. in mathematics from the Université de Montréal and, for twenty years, conducted research in universal algebra. Then he developed an interest in popularization of science that took the form of many contributions to *New Scientist* before the publication of his first book. With a title partially borrowed from Oscar Wilde, *The Importance of Being Fuzzy: And Other Insights from the Border between Math and Computers* attempted to illuminate some of the mathematical ideas behind the suggestive terminology of fuzzy logic, neural networks, and genetic algorithms. Initially *Pythagoras' Revenge* was intended to discuss the tyranny of numbers in modern societies in the same style as Sangalli's previous book. But, as if by magic, it became instead a work of fiction.

Few things are certain about Pythagoras' life and teachings, since all preserved sources were written down centuries after his death. According to the earliest chronicles, he was born on the Aegean island of Samos around 570 B.C. and traveled widely during his youth. After many years of apprenticeship—in which, some sources claim, Pythagoras may have met the priests of Egypt, the naked philosophers of India, and the Babylonian astronomers—he settled in the Greek colony of Croton, where he founded a fraternity to share his extensive knowledge with a select minority. In a way, the term "unconventional epicureans", coined for the assassins of Julius Caesar by historian Arnaldo Momigliano, is also valid to describe this ascetic sect whose followers exerted a powerful political influence.

The conviction that "number is the root of all things" played a central role in the doctrine of the Pythagoreans, probably due to their discovery that simple proportions between the length of strings explain musical harmony. Far from being a mere tool in human affairs, integers extended throughout the "music of the spheres" that governed the laws of universe. That is the reason why the emergence of irrational numbers was not celebrated as a step toward a more accurate worldview but rather considered a major threat to the equilibrium of the cosmos. If one trusts the legend, the discoverer of creatures outside the realm of reason, such as the square root of two or the diagonal of the pentagon, would have been drowned at sea after having divulged the secret to people who did not belong to the Order.

Javier Fresán will soon start a Ph.D. thesis in arithmetic geometry at Université Paris 13. His website is `http://jfresan.wordpress.com`.

The author thanks Joakim Andén, Iñaki Arbeloa, Michael Eickenberg, and Guillermo Rey for useful discussions, and especially Allyn Jackson for asking him to review the book and for improving the English of the text.

Today all is number but for different reasons. As a by-product of the triumph of technology, most of our routines depend heavily on computer systems, decisions based on statistics, or numerical parameters designed to encapsulate chaotic realities. Without the never-ending chains of 0s and 1s—for which Hollywood has over and over again shown a weakness—emails, DVDs, and GPS would just disappear. Even when technology works properly, a measurement mistake may have tragic consequences. A traveler coming from pre-Socratic Greece could think that ours is a "brave new world" in which all mysteries of numbers have been revealed. But after the necessary time to recover from the drastic change, he would realize that a complete comprehension of irrational numbers is still far away.

After Cantor's breakthrough, we know that almost all numbers are irrational. However, it is extremely difficult to decide whether a given one is. It is surprising that mankind had to wait until the middle of the eighteenth century to have a proof of the irrationality of π, a ubiquitous constant in Greek mathematics. Or that the values of the zeta function at odd integers remained virgin territory for more than two hundred years. It is still more worrying that almost nothing can be said about numbers that do not admit a simple representation, for instance as the zeros of a well-behaved entire function. In a world where numbers have gained power, we have no definite answer to the most obvious question: what are they? That is one of the possible interpretations of Pythagoras' revenge to which the title of this book alludes.

Two scholars fantasizing about action outside of academia each have information that the other needs. One is a young American mathematician, Jule Davidson, who secretly envies his sister, who works freelance as a consultant in computer security. The closest thing he can afford to her exciting trips around the world is to solve riddles on the Internet. Across the ocean, the Oxford fellow Elmer Galway has not inherited his father's passion for adventure. Instead of following his father's steps as a renowned archaeologist, Galway devotes the little time not taken up by the duties of his classical history chair to help an antiquarian book dealer. The innocent-looking hobbies of Davidson and Galway will lead them to a sect that tries to find a scroll explaining how to recognize the "extraordinarily gifted man" into whom Pythagoras would have reincarnated.

The only point on which all specialists agree is that the first man to call himself a philosopher left no writings, partly for the sake of secrecy and partly because the zeitgeist promoted oral means of transmitting knowledge, as Plato would show later in *Phaedrus*. But what if Pythagoras had compiled his fundamental teachings, foreseeing the downfall of the brotherhood? What if some of his disciples had felt the need of getting the odd circumstances of the Master's death straight? In 1947 a collection of religious and historical scrolls of huge significance was discovered in the northwest shore of the Dead Sea. Fifty years later, a construction worker found an unknown edition of one of the masterpieces of Spanish literature while knocking down a wall. So it would not be a stretch to imagine that the bowels of a medieval Italian basilica could hide a manuscript referring to the lost original source.

This is the starting point of a thriller that takes the reader from suburban Chicago at the end of the twentieth century to the island of Croton in 445 B.C. None of the ingredients of the plot are very original: there are indeed thousands of novels speculating about sects, lost books, or "beautiful minds". What is new is how the writer masters a manifold of registers to produce a sound story in which culture is not accessory but central. This is evident from the first chapters, where the suspense of a computer countdown helps Sangalli to explain the difficulty of proving that some mathematical problems have no solution. In the meantime, one of the characters discovers, during a seductive conversation with Professor Galway's assistant, that the humanities also have their Achilles' heel: "books full of lies", as Cicero called the work of unscrupulous copyists.

In most cases dialogues allow the author to talk about science in a very natural manner. For instance, the reader hardly notices that he is being taught the basics of philosophy of proof during one of Jule Davidson's encounters with a member of the sect. On another occasion, the excuse to introduce the idea of complexity is a lecture of an internationally famous problem-solver, who claims to have proven that randomness is at the heart of mathematics. At that point, the explanation of the main feature of Pythagoras' revenge is transparent even for those without any scientific background. Sangalli is aware that it is difficult to provide a satisfactory definition for randomness. So it is remarkable how he profits from readers' probable astonishment at this difficulty to guide them through Kolmogorov's notion of complexity and physicists' attempts to generate random numbers from atoms.

Mathematicians probably would have appreciated a more detailed discussion of some of the topics treated in *Pythagoras' Revenge*. For example, the story of the unsolvable Fifteen Puzzle, which in spite of its success was never patented because it was compulsory to submit a "working model", is a great missed metaphor of the meaning of logical consistency. Also conjectures about normal numbers, such as the possibility of compressing Jorge Luis Borges' *Library of Babel* in the digits of π, would have fit in perfectly with the passage in which Alan Turing and Gregory Chaitin go on stage. But it does not matter, for what remains after the end of this page-turner is Sangalli's impressive capacity to communicate mathematics. Let us take this book as a reminder to capitalize on the full potential of scientific storytelling.

Ask Professor Nescio

Editor's Note: Graduate students, early career faculty, and other mathematicians may have professional questions that they are reluctant to pose to colleagues, junior or senior. The *Notices* advice column, "Ask Professor Nescio", is a place to address such queries. Nomen Nescio is the pseudonym of a distinguished mathematician with wide experience in mathematics teaching, research, and service. Letters to Professor Nescio are edited to protect the identity of the questioner and to appeal to general interest. They are also edited, in some cases, to recast questions to be of more general interest and so that all questions are first person. Some letters may be edited composites of several submitted questions.

—Steven G. Krantz

Dear Professor Nescio,

I left academic mathematics in December 1993 after working part-time for two years. I had positions at Wake Forest College and the University of North Texas, ending in 1991. I published five papers, three of which are in the *Journal of Approximation Theory*. After working for an investment management firm in Atlanta, I accepted a job with a wireless engineering firm in Maryland, and wound up working on Wall Street starting in 2000. At present I'm working in one of those evil hedge funds as a quantitative analyst. And, before we go any further, I just haven't earned the kinds of bonuses you read about in the newspapers—mine have always been much lower than average.

I have excellent references for teaching and research. I also have a few ideas for math papers, and have pursued these as time permits, but don't have anything ready to submit.

I am forty-nine years old, and I would like to know if you think it's feasible to transition back to the academy. This is where I wanted to be in the first place. There are a few universities recruiting in Hong Kong, and my family and I are interested in moving there (my wife is from Hong Kong), so I would like to know just how feasible it is to be hired for an overseas position at my age and with a limited publication record.

Sincerely,
—Seeking Work

Dear Seeking,

Yours is a perplexing case and one where Professor Nescio fears to tread without an appropriate caveat. While I can and will speak about your prospects in the U.S, you should take whatever I have to say about the market in Hong Kong with a certain degree of skepticism; I know nothing about the academic marketplace there. Nevertheless, I'll try to help, but my comments must be of a general and abstract nature. I will rely on my knowledge of human nature, but, as we all know, some of what we perceive as the nature of human beings is actually a consequence of the culture in which we ourselves are raised.

My suspicion is that your chances of success, even if you seek a position in the U.S., will be less if you go after a job where research in approximation theory is the prime requirement for the position. Why do I think this? First, from what you said, you received your Ph.D. in 1989. In 1990 there began one of those cyclical drops in the number of available academic positions. You may have suffered from this while on the market the year before. In any case, your research does not seem to have been sufficiently strong to overcome this drawback. (I hope this is not too brutal. Professor Nescio prefers a tactful course, but it strikes me that your situation demands total frankness. There are many good mathematicians, and publishing in a journal with the stature of the one you cited is a good recommendation. In and of itself, however, it does not make you rise above the pack. On the other hand, perhaps there were other reasons you did not secure a full-time position. Nevertheless I am forced to respond only on the basis of what I read and my experience.) In seeking a research position, you will compete against people who have less known about them—but the reference letters will portray them as full of promise. With most mathematicians I know this will be believed more readily and optimistically than whatever is said about the research of a 1989 degree recipient.

So you would be better advised to seek a position that values research but emphasizes teaching. In such an endeavor I think you should underline something that makes you unique, or at least rare in the pool of applicants. On that score your experience on Wall Street rather than your knowledge of approximation theory should be a major asset.

Many universities in the U.S. seek to provide their students with grounding in financial mathematics; perhaps this holds in Hong Kong as well. If you apply to such an institution, your competition may be able to discuss their background in stochastic processes, but, I suspect, few will have had your financial experience. Indeed it is Professor Nescio's observation that the more successful academic financial programs employ several people from the financial sector as adjuncts.

In the U.S. a person in your situation would have difficulty securing an academic position, especially given the state of the economy and its documented effects on universities. My suspicion is that this will hold as well in Hong Kong. I would suggest you cast a wide net and do not discount the Middle East.

Please inform Professor Nescio what happens in your mid-course correction.

—Good luck,
Professor Nescio

Dear Professor Nescio,

I have a difficult issue with one of my papers, which is based on my Ph.D. thesis. This paper was submitted to a well-established journal in September of 2005, and received (after several demands) a positive but somewhat unofficial answer in early 2007. I therefore recorded it as accepted in my resume and got a permanent position in a good university in September 2007.

Shortly thereafter I met the editor at a conference. He explained to me the problems he had with the referees and with the journal's page limit, and confirmed to me I was correct to put the paper in my CV.

In the middle of 2008 the editor informed me he had sent the definitive acceptance letter to the editor-in-chief. But, after that, I still got no news of the paper, and at the beginning of 2009 I started to contact the editor-in-chief directly. It appeared that he had no information on my paper.

I submitted this paper more than four years ago, and I still have no official acceptance letter or clear hope to see it published. I start to be tempted to withdraw it and submit it to another journal, but that would make my previous resume false and I am reluctant to give up an acceptance, even unofficial. What would you advise me to do in this situation?

Sincerely,
—Wants to Get Published

Dear Wants To Get Published,

Your story is not one I have ever encountered. I certainly know of papers where the refereeing process was abysmally long; indeed I have been the victim of such irresponsibility. I have also taken on the role of someone called and asked to rescue the editor after a paper has languished for well over a year. But four years with the nod from a referee and an editor and to still have the paper languishing? Unprecedented! Professor Nescio feels outrage rising inside him. It is impossible for me to fathom what is happening in the minds of the editors; there must be an untold story here.

What to advise? First, a phone conversation with the editor-in-chief is called for. Explain that promotion and tenure considerations have entered the picture. (You did not mention this in the letter but the timing leads Professor Nescio to assume it does. I hope you have other papers that have appeared.) How a mathematician can ignore this situation I do not know. Bluntly ask whether the editor wants you to withdraw the paper. If possible I would locate the publisher of the journal and assert that what has happened is a mark against the company or university—it is. Also solicit the help of your adviser. Get him/her to call the editor-in-chief. You might even ask your department chair or some senior mathematician at your current university who acts as your mentor and/or confidant to contact the editor-in-chief on your behalf. (I leave it to you to decide if this might complicate your tenure consideration more than leaving the paper in limbo.)

In short, I would advise pulling out all the stops. Nevertheless you should be aware of one of the less attractive facets of human nature that will be in play here: When you are in trouble, you are usually there all by yourself. True, most will give up their seat in a crowded bus to an older person or make a contribution to a worthy and noble cause; frankly, they have little at stake in doing so. But asking someone to dramatically inconvenience themselves, put their reputation on the line, or jeopardize their relationship with another in the profession may not go over well no matter how righteous the cause. Indeed, how someone responds to such a request is the true test of friendship. You should never confuse cordiality or collegiality with friendship, a mistake Professor Nescio has committed on more than one occasion. Most advisers maintain warm feelings about their thesis students and, moreover, your thesis reflects on your adviser. This is why I counsel starting with him/her.

This is similar to having been left at the altar, and you are entitled to as much righteous indignation as you can muster. Like the jilted bride or groom, I think you will discover this experience is a singularity in your life—devastating but unlikely to ever happen again. Undoubtedly it will color your approach to the profession. Nevertheless accept the assurance of Professor Nescio that most of the behavior of our professional colleagues is more straightforward and savory.

—Good luck,
Professor Nescio

Dear Professor Nescio,

I am in a tenure-track position in a community college, but I am planning to move on to a four-year college later. I am concerned that, once I get tenure here, it would be hard for me to get a position elsewhere. I heard that you can make a deal to reduce years to get tenure. Could you please give me some advice on how to change schools? Thank you very much.

—Searching for a Better Life

Dear Searcher for a Better Life,

There are several issues here that are worth commenting on. To be sure, Professor Nescio is empathetic with your situation. Indeed, though the circumstances differ, there are many others who seek to improve their academic situation.

First there is the issue of searching for a job. There is much to write about here, so much that Professor Nescio will not start. Many sources exist, and a good place to start is at the Young Mathematicians Network (`http://concerns.youngmath.net/`). Another that I stumbled across while counseling a student and that seems very thorough is `http://www.gwu.edu/~math/graduate/jobweb.html`. I am sure there are many others.

There is also the issue of trying to find a job at a four-year college while at a community college. I am afraid that the flow of mathematicians in academia is analogous to the flow of water; it easily flows downward, frequently spreads sideways, and only with some extra help can it flow upwards. It happens that mathematics faculty at community colleges get positions at four-year institutions, but usually something that will push them up the incline is required. Outstanding teaching evaluations and/or involvement in student-oriented activities will be most helpful. Here is a point about getting a job to always remember, whether that job is at a four-year college or a research university: there has to be something that makes you stand out as a candidate.

When a research university searches for an analyst, for example, after an initial screening of the applicants that eliminates those whose specialty is not what the search committee seeks and those whose credentials do not match the institution's self-image and expectations, there may be as few as a dozen applicants on the short list. Already we see at work the principle I just stated, but it is highly unlikely that a dozen people will be invited for an interview on the campus. How does the committee get down to the three to five candidates they invite to campus? There is no unique answer to this, but whatever the explanation is, one can be certain that there was something going for the interviewees that singled them out in the eyes of the committee. It might be the existence of a grant, the personal acquaintance of a committee member, involvement of the candidate in highly specialized and desirable activities, or something else from an extremely long list that parallels the list of all the things departments and mathematicians do.

Now the process for filling a position at a four-year institution is similar even if the specific considerations differ radically. For example, a four-year place might emphasize that nebulous quality "collegiality": how easy is it to imagine having this person at a faculty meeting; will they contribute to a committee's work? Research departments that I know seldom consider these things, even though it might sow future problems. In some ways this makes the competition at a four-year institution stiffer. The number of applicants who can theoretically make the short list is a lot higher.

So what is it that is going to make your application stand out? What is it about your time at the community college that makes you a better candidate than someone with a newly minted degree? Don't discount personal contacts. They can be extremely helpful unless you are a total bore.

Regarding your reference to making a deal to reduce the time to tenure, I would not make this a central theme in your approach. You are already asking for a variance in the normal recruiting procedure, which is to look at fresh doctoral recipients; this itself presents a nontrivial task, especially in these times. To add to the hurdles, for the department to overcome any insistence on a shorter time to tenure will complicate things further. Professor Nescio does not want to take on the role of moralizer, but spending more years before you get tenure may just be the price you pay for changing your career path. In addition, an institution that pays careful attention to teaching and collegiality will frequently want to see you in action on *their home turf* for the full probationary period. So politely ask the question, but don't push the issue.

The conclusion is that your task is not impossible but it is daunting, especially in today's market. Nevertheless, the longer you wait, the more difficult it will be to be successful.

—Good luck,
Professor Nescio

Bombieri and Tao Receive King Faisal Prize

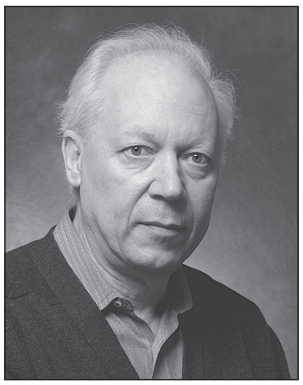


Photo by Cliff Moore.

Enrico Bombieri

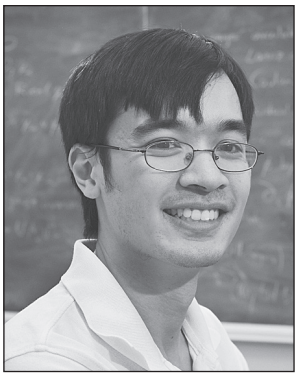


Photo by Reed Hutchinson/UCLA.

Terence Tao

The King Faisal Foundation annually presents the King Faisal International Prize to reward dedicated men and women whose contributions make a positive difference, including to scientists and scholars whose research results in significant advances in specific areas that benefit humanity. The prize carries a cash award of approximately US$200,000.

Two mathematicians shared the 2010 prize in science: Enrico Bombieri and Terence Tao. The science prize is given alternately in mathematics, chemistry, physics, and biology.

Enrico Bombieri is IBM Von Neumann Professor at the Institute for Advanced Study, Princeton University. He has made pioneering contributions to various branches of mathematics. His work is characterized by originality, power, and clarity of exposition, addressing fundamental and difficult problems in number theory, algebraic geometry, complex analysis, and minimal surfaces. His work ranges from analytic number theory to algebra and algebraic geometry and the partial differential equations of minimal surfaces.

A major contribution has been his work on the large sieve and its application to the distribution of prime numbers. Given an arithmetic progression, the large sieve provides information about the distribution of an arbitrary finite set of integers. Bombieri applied this improved large-sieve method to prove what is now called "Bombieri's mean value theorem", which concerns the distribution of primes in arithmetic progressions.

In 1974 he was awarded the Fields Medal, the highest award in mathematical sciences, for his major contributions in the primes, in univalent functions and the local Bieberbach conjecture, in theory of functions of several complex variables, and in theory of partial differential equations and minimal surfaces—in particular, to the solution of Bernstein's problem in higher dimensions.

In the past decade, his main contributions have been in the active area of Diophantine approximation and Diophantine geometry, exploring questions of how to solve equations and inequalities in integers and rational numbers. Many topics related to prime number theory have potential practical applications to cryptography and security of data transmission and identification.

Bombieri received the 2008 AMS Doob Prize for the book *Heights in Diophantine Geometry*, written with Walter Gubler. The citation read, in part: "The book is a research monograph on all aspects of Diophantine geometry, both from the perspective of arithmetic geometry and of transcendental number theory.… One gets the sense that every lemma, every theorem, every remark has been carefully considered, and every proof has been thought through in every detail. There are well-chosen illuminating examples throughout every chapter. The book is a masterpiece in terms of its original approach, its unrivalled comprehensiveness, and the sheer elegance of the exposition. There can be no doubt that this book will become the basis for the future development of this central subject of modern mathematics."

Bombieri received his Ph.D. from the University of Milan in 1963. He has held positions at the University of Milan, the University of Cagliari, the University of Pisa and Scuola Normale Superiore, Pisa, as well as at the Institute for Advanced Study. Along with the Fields Medal and the Doob Prize, other awards include the Feltrinelli Prize (1976), the Balzan Prize (1980), and the Cavaliere di Gran Croce al Merito della Repubblica, Italy (2002). He is a member of the Accademia Nazionale, Rome; the Accademia Nazionale dei Lincei, Italy; and Academia Europaea and a foreign member of the Royal Swedish Academy and of the French Académie des Sciences. He is a member of the U.S. National

Academy of Sciences and a fellow of the American Academy of Arts and Sciences.

Terence Tao is professor of mathematics at the University of California Los Angeles. He is a world-renowned mathematician working in a number of branches of mathematics, including harmonic analysis, partial differential equations, combinatorics, number theory, and signal processing. He is known for his highly original solutions of very difficult and important problems and for his technical brilliance in the use of the necessary mathematical machinery. Working with Ben Green, he proved there are arbitrarily long arithmetic progressions of prime numbers—a result now known as the Green–Tao theorem.

Tao has shown extraordinary mathematical abilities since early childhood. Born in Adelaide, Australia, in 1975, he was taking university-level mathematics courses at the age of nine, after having scored a 760 on the mathematics section of the SAT at the age of eight. He competed in his first International Mathematical Olympiad in 1986 when he was ten years old. Over three years he won a bronze, silver, and gold medal. At thirteen, he was the youngest gold medalist in the history of the competition and remains so to this day. He received his bachelor's and master's degrees at the age of seventeen from Flinders University. He completed his Ph.D. at age twenty-one at Princeton University under Elias Stein. He was appointed full professor at UCLA at age twenty-four, the youngest to hold that rank at the university.

Tao's areas of research include harmonic analysis, partial differential equations, combinatorics, and number theory. He has held Sloan Foundation (1999–2001) and Packard (1999–2006) fellowships. He was awarded the Salem Prize in 2000 for his work in L^p harmonic analysis and on related questions in geometric measure theory and partial differential equations. He received the Bôcher Prize in 2002 for his breakthrough on the problem of critical regularity in Sobolev spaces of the wave maps equations. In 2006 he received the Fields Medal for his contributions to partial differential equations, combinatorics, harmonic analysis, and additive number theory; he was awarded the SASTRA Ramanujan Prize in the same year. In 2007 he was awarded a MacArthur Fellowship and the Ostrowski Prize, and he received the Waterman Award in 2008. He is a Fellow of the Royal Society, the Australian Academy of Sciences (corresponding member), the National Academy of Sciences (foreign member), and the American Academy of Arts and Sciences. He is an associate editor of the *American Journal of Mathematics* (2002–) and of *Dynamics of Partial Differential Equations* (2003–) and an editor of the *Journal of the American Mathematical Society* (2005–2010) and of *Analysis & PDE* (2007–).

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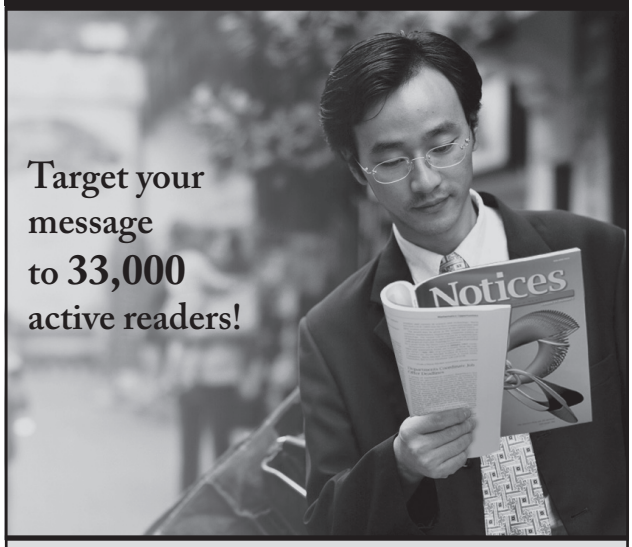

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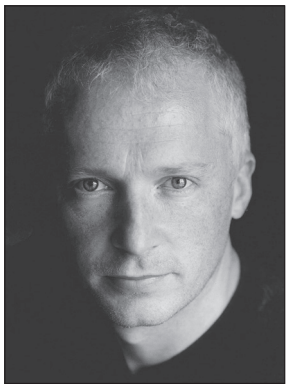
JPBM Communications Award

The 2010 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the Joint Mathematics Meetings in San Francisco, California, in January 2010.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences. JPBM represents the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The award carries a cash prize of US$1,000.



Marcus du Sautoy

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitemore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), Claire and Helaman Ferguson (2002), Robert Osserman (2003), Barry Cipra (2005), Roger Penrose (2006), Steven H. Strogatz (2007), Carl Bialik (2008), and George Csicsery (2009).

The 2010 JPBM Communications Award was presented to Marcus du Sautoy. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipient's response on receiving the award.

Citation

The 2010 JPBM Communications Award is made to Marcus du Sautoy, the Simonyi Professor for the Public Understanding of Science and a professor of mathematics at the University of Oxford.

For the past fifteen years du Sautoy has complemented his love of mathematical discovery with a passion for communicating mathematics to a broad public. He has reached hundreds of thousands through his books, television shows, and hundreds of articles and appearances in newspapers, magazines, television, and radio. His 2003 book on the Riemann hypothesis, entitled *The Music of the Primes*, is a best seller which has been translated into ten languages. In his 2008 book, *Symmetry: A Journey into the Patterns of Nature*, du Sautoy guides the reader through groups and symmetry, from Babylonia to Moonshine theory, while at the same time giving an engaging glimpse into mathematicians' minds. His four-part television series, *The Story of Maths*, presents a fascinating look at the development of mathematics from the design of the pyramids in Egypt to Perelman's proof of Poincaré's conjecture. Whether it is talking about Beckham's choice of number on a sports radio program, explaining the work of the Abel prize winner on Norwegian television, writing a weekly math column for the *London Times*, hosting a television game show based on math puzzles, or delivering the Royal Institution Christmas Lectures, Marcus du Sautoy invariably seizes opportunities to make mathematics more accessible and more appealing.

Biographical Sketch

Marcus du Sautoy is the Charles Simonyi Professor for the Public Understanding of Science and professor of mathematics at the University of Oxford and a Fellow of New College. His research seeks to understand the world of symmetry by using the concept of a zeta function, a classical tool from number theory, and involves a wide spectrum of techniques, from p-adic Lie groups to model theory, from algebraic geometry to analytic methods. In 2001 he won the Berwick Prize of the London Mathematical Society, which is awarded every two years to reward the best mathematical research made by a mathematician under the age of forty. In 2009 he was awarded the Royal Society's Faraday Prize, the United Kingdom's premier award for excellence in communicating science. He is the author of numerous academic articles and books on mathematics and has been a visiting professor at the École Normale Supérieure in Paris, the Max Planck Institute in Bonn, the Hebrew University in Jerusalem, and the Australian National University in Canberra. His presentations on mathematics, which include "Why Beckham chose the 23 shirt", have played to a wide range of audiences: from theater directors to bankers, from diplomats to

prison inmates. Marcus du Sautoy plays the trumpet and football. Like Beckham, he also plays in a prime number shirt, no. 17, for Recreativo FC, based in the Hackney Marshes. Born in 1965, he lives in London with his wife, three children, and cat Freddie Ljungberg.

Response

Mathematics is about discovery, but it is also about communication. Those new discoveries don't begin to breathe until you bring them alive in the minds of others. The more minds that appreciate the beauty of our discoveries, the more vibrant and healthy our subject will be. I was inspired to become a mathematician because of people in the past like Martin Gardner, Christopher Zeeman, and George Gamow, who were keen to communicate beyond the confines of our research community. My efforts to spread the excitement and beauty of mathematics to as wide an audience as possible is my way of paying back those who made the effort to excite me.

One of the books that inspired me as a student was G. H. Hardy's beautiful book *A Mathematician's Apology*. It is a book I love but also hate because his opening sentence has cast a shadow over those who might want to communicate the joy of doing mathematics. He writes: "It is a melancholy experience for a professional mathematician to find himself writing about mathematics. The function of mathematicians is to do something, to prove new theorems, to add to mathematics and not to talk about what he or other mathematicians have done." It is important that mathematicians prove Hardy wrong, and indeed Hardy himself is the counterexample to his own statement, being a brilliant mathematician and communicator.

I would like to thank all those in the mathematical community who have been extraordinarily supportive over the years in my efforts to excite people about mathematics. It is really important to know that your community is supportive of what you do. Prizes like the JPBM Communications Award are important in sending out the message that our community values communication, and I am very honored to have received the award for 2010.

MAA Prizes Presented in San Francisco

At the Joint Mathematics Meetings in San Francisco, California, in January 2010, the Mathematical Association of America (MAA) presented several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education, in one particular aspect or many, whether in a short period or over a career.

Kenneth A. Ross, professor emeritus at the University of Oregon, received the 2010 Gung and Hu Award. Ross has made many contributions to mathematics over a long career. From 1971 to 1980 he served the American Mathematical Society (AMS) as associate secretary for the Western Section, and in 1984 he was elected secretary of the Mathematical Association of America (MAA). When the responsibilities of the secretary of the MAA were split in 1989, he became the MAA's first associate secretary and continued to be responsible for the MAA presence in the national joint AMS-MAA meetings.

He was elected president of the MAA in 1994 and later became chair of the Response Group on the NCTM Standards, furnishing insightful reports to NCTM that reflected a consensus of opinion among mathematicians. He is skilled at bringing people together to talk and iron out their differences. He cochaired the advisory board on renovating part of MAA headquarters for use as a conference center and set the agenda for the facility. He has been editor of the Carus Monograph Series for the MAA and is currently a member of the editorial board of the MAA's Spectrum Series. He served on the Board of Governors for many years, as well as on many committees concerned with finance, publications, meetings, membership, awards, and prizes. Recently he served as associate editor of *Mathematics Magazine* and on committees on the Silver and Gold Banquets, on Bylaws Revision, and on the MAA Centennial in 2015. He has also been active in the MAA's Pacific Northwest Section. He gave invited

talks to at least twenty-one MAA sections as a national officer of MAA.

Ross's publications include *Abstract Harmonic Analysis* (with Edwin Hewitt); *Elementary Analysis: The Theory of Calculus*; *Discrete Mathematics* (with Charles Wright); *Sidon Sets* (with Jorge M. Lopez); and *A Mathematician at the Ball Park*, which demonstrates his lifelong passion for baseball. He received his Ph.D. from the University of Washington in 1960. His mathematical interests include abstract commutative harmonic analysis, elementary probability, and expository writing.

Haimo Awards for Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2010 Haimo Awards were presented to Curtis Bennett, Michael Dorff, and Allan J. Rossman.

Curtis Bennett has been teaching at the collegiate level for almost twenty years, the last six at Loyola Marymount University and earlier at Bowling Green State University and Michigan State University, where he developed some of his ideas about teaching. Students at all levels and with a variety of career goals credit him not only with helping them increase their mathematical skills but also with giving them increased self-confidence. His approaches and methods encourage students to "think like mathematicians." To accomplish this, he designs or selects problems, projects, and exercises with a particular educational goal in mind. He embraces and experiments with technology. At Loyola Marymount, Bowling Green State, and Michigan State, he developed courses to help future high school teachers deepen their understanding of secondary mathematics and draw clear connections between the high school mathematics topics and upper-level collegiate courses in algebra, analysis, and geometry. As an academic advisor, he helps students discover new passion for mathematics and to recognize and have more confidence in their abilities in mathematics. Bennett is also passionate about collaborating with colleagues and sharing his work related to teaching with others across the country. This is evidenced by his work as a two-time Carnegie Scholar; by his work in MAA's workshops on Preparing Mathematicians to Educate Teachers (PMET) in 2004 and 2005; by his work in helping young scholars and mathematicians at other institutions develop their ideas about teaching; and by his minicourses, presentations, and organization of special sessions at national meetings of the MAA. He was a cofounder of the Young Mathematicians Network and served on its editorial board for three years. He works in a variety of fields, including the study of geometries associated to groups of Lie type, combinatorics, and the scholarship of teaching and learning.

Michael Dorff has made many contributions to the teaching and learning of mathematics in his classrooms at Brigham Young University (BYU), regionally in public education, and nationally through his work with students and faculty related to research by undergraduates in mathematics. His undergraduate teaching at BYU is characterized by his care for the students while having high expectations for them, even in large lecture classes of beginning calculus, and by efforts to help the students see the bigger picture. For example, in his large lecture calculus classes, he often has "commercial breaks" in which he makes a short presentation about something designed to help students get excited about mathematics, such as a presentation about "soap bubbles and the universe" or a contest in which the students make proposals for the "most beautiful mathematical equation". Recently he has been organizing seminars for math majors, minors, and prospective majors to learn about career opportunities in the mathematical sciences, and his department has reported a 22 percent increase in the number of majors since he began the seminars. Regionally, he has worked with BYU's Center for the Improvement of Teaching Education and Schooling, which brings public school teachers and university faculty together with the goal of improving numeracy among young children. He has developed a funded program with other faculty for professional development of K–6 teachers in local school districts. For several years, he has been leading an REU program at BYU to bring undergraduates from across the country to do research in mathematics. Nearly all of the students have gone on to graduate school. Even more important, however, is his founding of the Center for Undergraduate Research in Mathematics (CURM), in which fifteen faculty members go to BYU to help them learn to mentor undergraduates in doing research in mathematics on their own campuses. He is chair of the MAA Committee on Early Career Mathematicians and Subcommittee on Research by Undergraduates.

Allan Rossman is professor of statistics at California Polytechnic State University, San Luis Obispo. His excellence as a teacher is exemplified by his student-centered approach, his innovation, his endless enthusiasm for the subject, and his desire for his students to obtain the deepest understanding and appreciation possible of the subjects they are studying. His teaching is described as providing just the right amount of guidance in classroom activities, thereby empowering students to discover important ideas and connections for themselves. He has created innovative curricular changes in statistics courses and in the major overall. He

helped develop an orientation course for incoming statistics majors, providing them with an overview of the statistics discipline, examples of statistical practice, and broad understanding of statistical ideas, while developing their communication skills. He helped develop a more advanced two-course, data-oriented sequence for mathematics and statistics majors. The changes he helped bring about have contributed to Cal Poly's reputation as having a unique undergraduate statistics major. He provides extensive support material and opportunities for success while demanding deep understanding. He has strong rapport with the students, who give him outstanding evaluations and who appreciate his passion and contagious love for the subject. Nationally, through many workshops for faculty, including the MAA's Statistical Thinking with Active Teaching Strategies (STATS) workshops, he has reached hundreds of statistics educators, primarily mathematicians, and given them tools to rethink their own statistics courses. In addition to helping faculty develop effective pedagogical strategies, he has participated in developing recommendations for curriculum and assessment in statistics education, including the recent GAISE reports from the American Statistical Association (ASA). He is a Fellow of the American Statistical Association and served as program chair for the Joint Statistical Meetings in 2007. He has been coeditor of *Stats: The Magazine for Students of Statistics* and chaired the ASA-MAA Joint Committee on Undergraduate Statistics. He is past president of the International Association for Statistical Education and recently began to serve as chief reader for the Advanced Placement Program in Statistics.

Chauvenet Prize

The Chauvenet Prize recognizes a member of the MAA who has written an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy.

Brian J. McCartin of Kettering University received the 2010 Chauvenet Prize for his article "*e*: The master of all", *Mathematical Intelligencer*, **28** (2006), no. 2, 10–21. This article is an engaging, expansive exposition of the ubiquity of the number *e* in mathematics. Whatever a reader's background, whether a beginning calculus student or a master versed in all manner of limits, series, transforms, and operators, everyone will find something new about *e* in this article. For example, did you know that the polar golden spiral, $r=e^{\cot(\alpha\theta)}$, $\alpha>0$, has the peculiar property that each line through the origin meets the spiral in a common angle? Johann Bernoulli was so enamored with this property that he arranged to have the curve etched on his tombstone. With wit and clarity and long experience as an applied mathematician, McCartin has given the mathematical community a well-crafted dossier for the number *e*.

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. Mathematical monographs at the undergraduate level, histories, biographies, works of mathematical fiction, and anthologies are among those types of books eligible for the prize. The prize was given for the first time in 2007, the 300th anniversary of the birth of Leonhard Euler.

David S. Richeson of Dickinson College was awarded the 2010 Euler Prize for his book *Euler's Gem: The Polyhedron Formula and the Birth of Topology* (Princeton University Press, 2008). The renowned formula, $V-E+F=2$, where V, E, and F are the numbers of vertices, edges, and faces of a polyhedron, is elegant, concise, and surprising. Previous attempts to explain the beauty of this simple formula and to explore its depth pale by comparison to Richeson's extraordinary narrative. It is entertaining, scholarly, and informative, not only to the general reader but also to professional mathematicians. It conveys an understanding of what the formula says and also addresses the question of what "polyhedron" means and what it suggests to those who explore "shapes" in general, and it leads eventually to questions addressed in the development of the vast subject of modern topology. The material is put into historical perspective, and an amazing amount of actual mathematics is included. With short chapters, lucid explanations, and descriptions that are related to ordinary everyday experience, the implications of the formula become clear to the reader. The exposition is amazingly friendly, and the prose is a joy to read. Tantalizing questions are raised and answered. And these questions go all the way up to questions answered by Thurston and Perelman. There are excellent notes and references, and the book clearly reflects careful research on the part of the author.

Certificates of Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2010 are: Benjamin Freed (Clarion University), Allegheny Mountain Section; Michael Dorff (Brigham Young University), Intermountain Section; Elizabeth Mayfield (Hood College), Maryland–District of Columbia–Virginia Section; Amy Cohen (Rutgers University), New Jersey Section; John Watson (Arkansas Tech University), Oklahoma–Arkansas Section; and Janet Beery (University of Redlands), Southern California–Nevada Section.

—From MAA announcements

AWM Awards Given in San Francisco

The Association for Women in Mathematics (AWM) presented two awards at the Joint Mathematics Meetings in San Francisco, California, in January 2010.

Schafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents. Schafer passed away in September of 2009.

The 2010 Schafer Prize was awarded to Hannah Alpert of the University of Chicago and Charmaine Sia of the Massachusetts Institute of Technology. Alpert, a junior at the University of Chicago and a Goldwater Scholar, coauthored a paper on topological graph theory before she began college. After her freshman year, she attended the Willamette Valley Research Experience for Undergraduates (REU), and she spent the summer of 2009 at the REU in Duluth. Remarkably, she has written and submitted for publication three single-authored papers in three different areas based on her work there. In one, she determined the k-ranking numbers of 3-by-n grid graphs, using "innovative" methods that also "give tremendous insight into the general case." She has been invited to present the results of another, on finite phase transitions in countable abelian groups, in a graduate seminar. Alpert's mentors paint a consistent picture of a remarkably mature young mathematician, one who is a creative problem solver with a "formidable talent". Over and over, she has solved challenging open problems in elegant and fully original ways. One letter writer compares her to a Nobel Prize winner he taught; others describe her as "incredible", "fantastic", and "destined to become a first-rate mathematician".

Charmaine Sia is a senior at the Massachusetts Institute of Technology, where she has excelled in both undergraduate and graduate classes. She has a perfect undergraduate transcript. To quote one of her recommenders, "Charmaine absorbs mathematics like a sponge." Another one writes, "I have never seen a student with as voracious an appetite for knowledge." She has won three bronze medals at the International Mathematical Olympiad (IMO) and a top-75 ranking in the Putnam Mathematical Competition. She has written four papers, two of which are single-authored. She spent the past three summers in undergraduate research programs, starting with SPUR at MIT in 2007, where she won the prize for best research in the program for her work on zero-sum problems in finite group theory. The next summer she participated in the Duluth REU program, where she wrote two papers, one on classifying the orbits of special groups under the Hurwitz action and the other on game chromatic numbers of products of graphs. Both papers have been published in professional journals. In the summer of 2009, Sia participated in the SMALL research program at Williams College, where she coauthored two papers on knot theory. In the words of her teachers and mentors, Sia is an "astonishing" student who "has distinguished herself in every possible way" and is "already a mature mathematician" with "immense potential". She is expected to become an outstanding research mathematician.

Additionally, the accomplishments of four outstanding senior mathematics majors were recognized: Anna Lieb, University of Colorado, Boulder, was runner-up for the prize. Honorable mentions were awarded to Megan Bernstein, University of California, Berkeley; Ruthi Hortsch, University of Michigan; and Laura Starkston, Harvard University.

Louise Hay Award

Established in 1991, the Louise Hay Award for Contributions to Mathematics Education recognizes outstanding achievements in any area of mathematics education. Louise Hay was widely recognized for her contributions to mathematical logic and her devotion to students.

The 2010 award was presented to Phyllis Z. Chinn of Humboldt State University.

Chinn's career has been marked by an eagerness to enliven students, teachers, undergraduates, and her colleagues with the excitement of mathematics and by a principled conviction that the best way to accomplish this is through discovery learning. At Humboldt State University she established the "Expanding Your Horizons" conference to introduce middle grades girls to mathematics, science, and engineering, and she coached high school students for the Mandelbrot Math Competition. She has developed courses for prospective and practicing elementary, middle, and high school teachers in

problem solving, school mathematics from an advanced standpoint, calculus, and graph theory. She directed two professional development programs for K–12 teachers, the Redwood Area Math Project and the North Coast Mathematics and Science Initiative. Perhaps her most influential work was through Project PROMPT, which engaged college and university faculty in rethinking the content and pedagogy for prospective elementary and middle school teachers. This project spawned similar projects in Louisiana, Texas, and Oklahoma.

Chinn's creative mathematical spark has led her to discover fascinating research questions in graph theory arising from Cuisenaire rods. She has an Erdős number of one. She is also a juggler, and she says that juggling teaches the merits of practice and persistence and illustrates the usefulness of algorithms. She has been an advocate for women in mathematics and science throughout her career. She developed and taught courses on women in science and mathematics. In the words of Diane Johnson, a professor of mathematics at Humboldt State University, Phyllis was a "proud and successful mother…the first woman tenured in our department, and…a mentor and inspiration to those of us who have followed her."

—From AWM announcements

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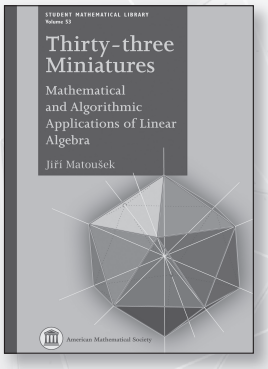

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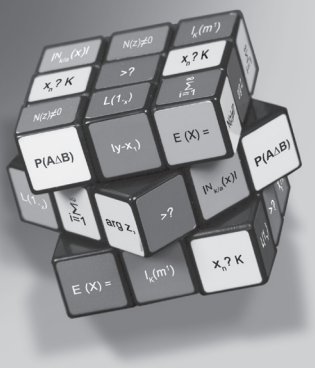

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2010 Award for an Exemplary Program or Achievement in a Mathematics Department

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006. The purpose is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible.Through the generous support of an anonymous donor, the award carries a cash prize of US$5,000.

The award is presented by the AMS Council acting on the recommendation of a selection committee. For the 2010 award, the members of the selection committee were: Steven A. Bleiler (chair), Amy Cohen, William B. Jacob, Michael E. Moody, and Roger A. Wiegand.

The previous recipients of the award are Harvey Mudd College (2006), the University of California, Los Angeles (2007), the University of Iowa (2008), and the University of Nebraska, Lincoln (2009).

The recipient of the 2010 Award for an Exemplary Program or Achievement in a Mathematics Department is the Mathematics Department at North Carolina State University. What follows is the selection committee's citation.

Citation

The American Mathematical Society is pleased to recognize the Department of Mathematics at North Carolina State University with the 2010 Award for an Exemplary Program or Achievement by a Mathematics Department. Emerging from a strong field of nominees, the Department of Mathematics at North Carolina State has clearly distinguished itself through its many activities and programs.

The department offers the largest REU (Research Experiences for Undergraduates) program in the U.S., including a special program REU + for undergraduates from traditionally underrepresented groups. The department also exhibits a strong commitment to outreach to other departments and disciplines, as well as to business, industry, and government, including outreach requirements in the major and graduate programs. It is further distinguished by a well-thought-out program of activities to increase student participation in their professional development, and by the effective and encouraging environment, complete with integrated professional experiences, that the department has developed for its advanced degree programs, which nicely complements the department's long-standing and continuing commitment to diversity in the mathematics work force.

For these reasons and more, the Society is proud to recognize the Mathematics Department at North Carolina State University with this year's award.

2010 Mathematics Programs that Make a Difference

Each year the AMS Committee on the Profession (CoProf) selects outstanding programs to be designated as *Mathematics Programs that Make a Difference*. For 2010 the honored programs are the Department of Computational and Applied Mathematics (CAAM), Rice University, and the Summer Program in Quantitative Sciences, Harvard School of Public Health.

CoProf created the Mathematics Programs that Make a Difference designation in 2005 as a way to bring recognition to outstanding programs that successfully address the issue of underrepresented groups in mathematics. Each year CoProf identifies two exemplary programs that:

1) aim to bring more individuals from underrepresented minority backgrounds into some portion

of the pipeline beginning at the undergraduate level and leading to an advanced degree in mathematics, or retain them in the pipeline;

2) have achieved documentable success in doing so; and

3) are replicable models.

Previously designated Mathematics Programs that Make a Difference are: the graduate program at the University of Iowa and the Summer Institute in Mathematics for Undergraduates/Research Experience for Undergraduates at Universidad de Puerto Rico, Humacao (2006); Enhancing Diversity in Graduate Education (EDGE) and the Mathematical Theoretical Biology Institute (2007); the Mathematics Summer Program in Research and Learning (Math SPIRAL) at the University of Maryland and the Summer Undergraduate Mathematical Science Research Institute at Miami University (Ohio) (2008); and the Department of Statistics at North Carolina State University and the Department of Mathematics at the University of Mississippi (2009).

The selection committee for the 2010 Mathematics Programs that Make a Difference consisted of: Gerard Buskes, Donald Cole, Jackie Hughes-Oliver, Susan Loepp (chair), Francis Edward Su, Michelle Wachs, and Kim Weems.

Below are CoProf's citations, followed by brief descriptions of the programs prepared by *Notices* staff.

Department of Computational and Applied Mathematics (CAAM), Rice University

Citation

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Department of Computational and Applied Mathematics at Rice University for its significant efforts to encourage students from underrepresented groups to continue in the study of mathematics.

The department's unwavering commitment to students through individual guidance and support has created an exceptionally welcoming community in which students thrive. Over the last twenty-five years, the department has graduated thirty-four underrepresented minority Ph.D.s. Graduates of the program have distinguished careers in government labs, industry, and academia. The department of Computational and Applied Mathematics at Rice University has made, and continues to make, a remarkable contribution to the national effort to produce more minority Ph.D.s in the mathematical sciences.

The AMS commends the members of the Department of Computational and Applied Mathematics at Rice University for their high level of commitment and their successful efforts to improve the diversity of the profession of mathematics in the United States.

Program Description

The Department of Computational and Applied Mathematics (CAAM) at Rice University has been one of the most successful departments in the nation in mentoring and producing mathematical sciences doctorates drawn from underrepresented minority groups. In the past twenty-five years, the department has produced thirty-four Ph.D.s from these groups. Additionally, CAAM has produced forty-three female Ph.D.s.

The twelve members of the CAAM faculty are involved in cutting-edge research in inverse problems, discrete and continuous optimization, computational neuroscience, partial differential equations (PDE), PDE-constrained optimization, and large scale numerical linear algebra. The interdisciplinary nature of the department has played a role in making CAAM attractive to groups traditionally underrepresented in mathematics.

Much of the department's success in the training of Ph.D.s from underrepresented minority groups has come through the leadership of Richard Tapia, the recipient of the 2004 AMS Award for Distinguished Public Service to Mathematics. During his more than thirty years at Rice University, Tapia has personally mentored dozens of students from underrepresented groups, many of whom have gone on to outstanding positions in academia and industry. His enthusiasm and high standards have helped to make CAAM a place where students of all backgrounds can thrive.

CAAM has two women professors, Liliana Borcea and Bèatrice Rivière, who serve as role models for women students, and Tapia too has worked to increase the number of women students in the department. Although the department does not have a program specifically aimed at women, it has been very successful in producing women Ph.D.s: An article in the August 2004 *Notices* that presented statistics about which departments were producing the most women Ph.D.s in mathematics pointed to CAAM as one of the standouts, with 45.5 percent of its Ph.D.s going to women in the years between 1995 and 2003.

CAAM has played a central role in promoting education diversity across the Rice campus and in the state of Texas. One example is the Rice University Center for Excellence and Equity in Education, which was founded by Tapia and which seeks to promote greater participation of underrepresented groups in the sciences and to encourage academic excellence for all. Tapia also established Alliances for Graduate Education and the Professoriate, which unites several colleges and universities in Texas in the common mission of significantly increasing the number of underrepresented minority students earning the Ph.D. and positioning them to become leaders in science, technology, engineering, and mathematics.

Tapia's work on promoting diversity in mathematics, engineering, and the sciences has brought a national spotlight to the CAAM department. Among his many honors are the Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring (1996) and the Lifetime Mentor Award from the American Association for the Advancement of Science (1997). He is a founder of the national organization SACNAS (Society for the Advancement of Chicanos and Native Americans in Science).

Website: `http://www.caam.rice.edu/`

Summer Program in Quantitative Sciences, Harvard School of Public Health

Citation

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Summer Program in Quantitative Sciences, Harvard School of Public Health, for its significant efforts to encourage students from underrepresented groups to continue in the study of mathematics and statistics.

The program's strong, personal connection with students has resulted in remarkable success. Talented undergraduate students from underrepresented minority groups are recruited and mentored for four weeks during the summer. The program includes research projects, training in biostatistics, and opportunities to network with faculty members and graduate students. Graduates of the program hold leadership positions in the biostatistics community and have received prestigious fellowships and grants. Established in 1992, the program is one of the first of its kind and has served as a model for similar programs throughout the country. The program has made a remarkable contribution to the national effort to produce more minority students pursuing careers in biostatistics and public health.

The AMS commends the faculty and staff members of the Harvard School of Public Health for their high level of commitment and their successful efforts to improve the diversity of the profession of the mathematical sciences in the United States.

Program Description

The Summer Program in Quantitative Sciences at the Harvard School of Public Health (HSPH) is designed to attract mathematically talented students from underrepresented minority groups to consider graduate school and careers in biostatistics and public health. Since 1994, when the program first began admitting students, 153 students have participated, and twenty have enrolled in HSPH for graduate school. The program receives funding from the National Institutes of Health.

Each year, between six and twelve minority students have participated in the program, and it is also open to students from other groups underrepresented in graduate education in public health, such as first-generation college students, low-income college students, and handicapped students. An essential element of the program's success is its small size, which permits the development of personal connections with all of the students. Lasting four weeks, the program includes an introductory course in biostatistics and statistical computing that meets each day for lectures and computer labs; typically the course is taught by a minority statistician who is a student in the HSPH Biostatistics Department. There is an afternoon lecture series on epidemiology, health and social behavior, environmental health, and current research in biostatistics. In addition, students engage in small-group research projects that are based on studies being conducted by scientists at HSPH or at the Harvard Medical School.

Career development sessions bring students together with faculty and career advisors to discuss the importance of the GRE examination, tips for applying to graduate school, and issues specific to minority students. Participants also take a practice GRE test. Field trips to such places as the genotyping facility at the Channing Laboratory of Brigham and Women's Hospital expose the students to the vast array of research activities and educational options relating to biostatistics.

Of the 131 program participants known to have received their undergraduate degrees, at least eighty-seven (66 percent) have completed graduate degrees or gone on to pursue graduate studies, at least sixty-seven (51 percent) have pursued graduate study related to health or medical school, and forty (31 percent) have gone on to pursue graduate training in statistics or biostatistics. Currently, two students are in graduate programs at the Harvard School of Public Health, and twenty have received either master's or doctorate degrees at the HSPH.

Program Director: Rebecca Betensky, Professor of Biostatistics, Harvard School of Public Health

Program Coordinator: Catherine Haskell

Website: `http://www.hsph.harvard.edu/biostats/diversity/summer/`

NCSU Math Department Wins AMS Award

Allyn Jackson

In May 2009 the mathematics department at North Carolina State University moved into its new building, SAS Hall. Named in honor of a donation from the statistical software company SAS, the new building provides an exceptional environment for faculty, students, and staff. "It is a great place to work, with quiet corridors for students and faculty," said Michael Shearer, one of the faculty members who worked to convince the university that the department needed a new home. "With modern and spacious visitor, teaching, conference, and seminar spaces, we have the physical environment that is already attracting leading scholars." The building is not just functional but beautiful as well. The entrance is a four-story atrium with a spectacular fabric mobile incorporating the golden ratio, designed by a team from the world-renowned NC State College of Design. SAS corporation lent the university around fifty works of art that decorate the hallways and public spaces. A logarithmic spiral begins on the floor inside the building and extends to the outside plaza.

The new building is a symbol of the department's creative spirit and a bellwether for its future. Even while in its old building, the department trained its focus on the essentials: stimulating excellent research and bringing out the best in students at all levels. As a result, by the time SAS Hall was ready for occupancy, the department had become a dynamic, thriving enterprise that could really capitalize on the new facilities. In 2010 the AMS chose NCSU to receive the annual Award for Exemplary Program or Achievement in a Mathematics Department.

Allyn Jackson is senior writer and deputy editor of the Notices. *Her email address is* axj@ams.org.

The Seeds of Success

The seeds of the department's current success were sown about fifteen years ago, when it decided to emphasize bringing domestic students into its graduate program. One motivation was that some grants held by NCSU faculty had come from defense agencies or other sources that required students supported on the grants to be U.S. citizens. "But in part it was really a cultural change," explained the current department head, Aloysius Helminck. The faculty "really felt this was important and that we needed to make a difference." Because the domestic students were talented but came with varied backgrounds for graduate school, the department realized that it had to provide careful mentoring and a supportive environment. It also realized that the students would benefit from education and training that prepared them for a variety of career paths both within and outside of academia. The department began to structure vertically integrated activities for students, even before VIGRE formally began (VIGRE stands for the influential National Science Foundation program Vertical Integration of Research and Education).

The NCSU mathematics department really took off about five years ago, when Helminck became department head. He secured several substantial grants that gave the department the means to fulfill its aspirations in education and research. Helminck's leadership has helped to build a cohesive vision for the department that the entire faculty can rally around. "You have to create a culture in which people believe this is the right direction for the department so that there is broad faculty participation," Helminck said. "You start with a few people and you show that a program is successful, then one by one you have faculty start to participate…They can totally change their minds. Those who were initially opposed are the biggest

Photos courtesy of the NCSU Mathematics Department.

Participants in the 2009 NC State REU in Mathematics (shown in the main lobby of the new mathematics/statistics building, SAS Hall).

supporters now, because they see the programs really make a difference.''

One of the distinctive characteristics of the NCSU mathematics department is the way it blends interdisciplinary research with foundational research, said faculty member Michael Singer. "The research aspect is very interesting but for training students it really makes a big difference that we are able to offer students both a basic foundation in mathematics, as well as the possibility to be trained to pursue a host of careers in the mathematical sciences,'' he said. "We have enormous breadth and depth. I think this attracts a lot of different students and yields well-trained mathematical scientists. The diversity that you see in the research is translated into diversity of course offerings and research experiences for our students.'' There is also an unusual level of cooperation between theoretical and applied mathematics, Shearer noted. "The applied side is unusually broad, including specialists in scientific computing and modeling who do not prove theorems, but relate models and simulations to practical applications,'' he said.

Research Experiences for Students

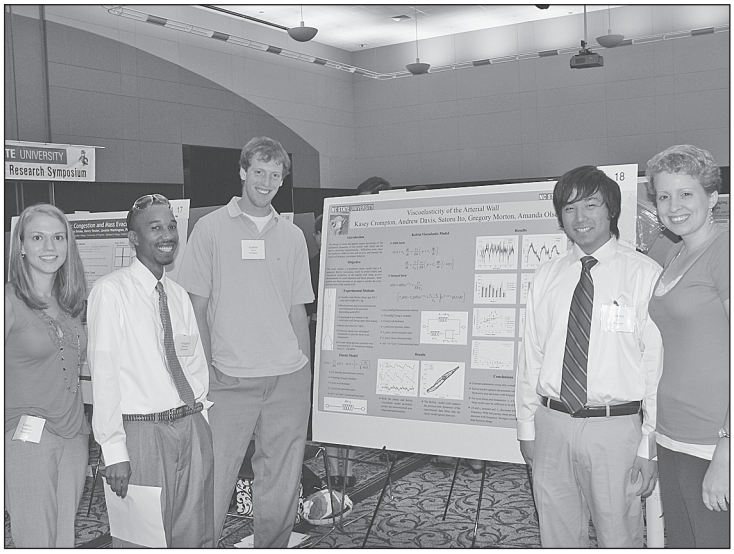
The blend of interdisciplinary and foundational research has allowed the department to launch extremely successful research-experience programs for its students. At the undergraduate level, the department has two programs: Research Experiences for Undergraduates (REU), which is one of the largest REUs in the nation, and REU+, aimed at students traditionally underrepresented in the mathematical sciences. Both take place over the summer and are aimed at students from outside NCSU (the department's math majors are encouraged to apply to REUs on other campuses). Students work on a wide range of problems, such as ultrasound imaging in cardiovascular tissue, gravity quantization, and data mining. Many projects result in research papers, and one, in electron gun design, even led to a patent application. The active participation not only of faculty but also of researchers from industry makes for a rich, vibrant atmosphere. "The students who come out of the [REUs] are all highly invigorated, and they want to keep on working'' in mathematics, Helminck remarked. "A lot of them want to apply to NC State for graduate school... They really feel that it is a life-changing experience.''

One measure of the success of the NCSU REU programs is the percentage of students who have subsequently entered graduate programs. From the 2005, 2006, and 2007 REU and REU+ groups, 100 percent, 77 percent, and 79 percent, respectively, have gone on to graduate school. The REU programs have serious participation by NCSU faculty and also give graduate students early experience as mentors of undergraduates—experience that has proven useful on the job market later on.

The department also has a counterpart program to the REUs called REG, Research Experience for Early Graduate Students. Students typically participate in this 10-week summer program at the end of their first year in graduate school, while still engaged in coursework but before taking qualifying exams. In the REG, a student might work solely with a faculty member or be part of an interdisciplinary team. Projects are typically

designed to be completed by the end of the summer, and in some cases they result in a publication for the student. Students learn background for the research as well as research strategies and critical oral and written communication skills. In many graduate programs, students are so consumed with coursework that "they lose sight of why they went to graduate school, which was to do research, to become a research mathematician," Helminck said. The REG "seems to invigorate a lot of students… They study harder and become very motivated." Students begin to form collegial groups with others interested in the same research areas, and they form ties to faculty early on, easing the path to choosing an advisor. The REG has also improved the retention rate: 96 percent of participating students have gone on to take and pass all three qualifying exams, whereas before the program began, this rate was roughly 70 percent.

The department's qualifying exam structure is quite unusual: There are fifteen exam topics in all, and more are to be added. The department wanted the exam structure to reflect the increasingly interdisciplinary nature of mathematical research. So, for example, a student interested in biomathematics might take qualifying exams in that subject as well as in partial and in ordinary differential equations. Background in such core subjects as algebra, analysis, and geometry is provided through coursework. "All the students still get to see a broad spectrum of math, but [the exam structure] helps them focus on studying the subject in which they are going to do research," Helminck explained. "We have found that this really helps in cutting down the time to degree."



In the REU program, teams of students work with faculty and graduate students to solve real-world problems. In this 2008 photo are (from left to right) Kasey Crompton (University of South Carolina-Columbia), Gregory Morton (Morehouse College), Andrew Davis (Clarkson University), Satoru Ito (NC State University), and Amanda Olsen (LaGrange College). This project was led by Dr. M. Olufsen and graduate student D. Valdez-Jasso and was named one of the poster session prize winners at the 2009 JMM.

Emphasizing Professional Development

Graduate students are offered a variety of professional development activities. For example, the university has a scholarship program through which a student can observe a professor teaching a course one semester and then teach the course solo the following semester, under the guidance of the professor. Some students in the program have even taught upper-level undergraduate courses. Graduate students participate in seminars in which they give talks and receive supportive critiques of their presentations. There are also "Research Training Modules" that provide a forum to discuss a range of professional issues, such as how to apply for a job, how to find an advisor, and what to do at a conference. These modules were developed as part of an NSF Research Training Group grant on mathematics of materials, but they are advertised and available to all students and postdocs. The department makes efforts to send its students to conferences, and a former graduate student even endowed a travel fund for graduate students. The professional development activities are useful not only for those who embark on academic careers but also for the increasing number who take positions in industry or in laboratories.

There are various opportunities for graduate students to get a taste of industrial and applied research. The newly established Center for Quantitative Sciences in Biomedicine facilitates project-oriented research in the biomedical sciences and provides a basis for novel graduate and postgraduate training at the interface of the quantitative and biomedical sciences. In addition, each year several graduate students participate in off-campus internships at institutions including Merck Pharmaceutical, the National Institute of Environmental Health Sciences, Harvard Medical School, MIT Lincoln Lab, Lawrence Livermore Lab, Boeing, the Hammer Institute, and Aerospace Corporation. These internship experiences are important in preparing students for the job market and often lead to postdoctoral and employment offers.

Students are also encouraged to participate in summer activities such as the Industrial Mathematical and Statistical Modeling Workshop, which is held annually at NCSU. This workshop is cosponsored by the Statistical and Applied Mathematical Sciences Institute, one of the seven national mathematics institutes supported by the NSF, and NCSU's Center for Research in Scientific Computation (CRSC), which brings together mathematics faculty members, faculty from other

departments on campus, and disciplinary scientists from industry and government laboratories. Founded in 1986 and currently directed by H. Tom Banks, the CRSC is a great asset to the department and has helped support various departmental programs.

Inspiring Commitment

All of this attention to students demands great commitment on the part of faculty. How does the department inspire this commitment? One way is through tangible rewards. "It factors into promotion, into tenure, into raises," Singer said. "It is part of what [faculty members] are evaluated on. You get what you measure." At the same time, changes that made the department more student-centered have also stepped up faculty commitment. For example, about a decade ago the department revamped its honors program, so that talented undergraduates were matched with outstanding mentors, and this change raised the faculty's enthusiasm about its students. Mathematics is a popular major at NC State, attracting diverse and high-quality students; about sixty-five students graduate with a B.A. or B.S. each year, and about one-third attend graduate school. "The faculty cannot help being inspired by our best students, who go on to graduate work at top-rated universities, in mathematics and related fields," Shearer remarked. At the graduate level, the professional development programs provide a forum in which faculty give advice and answer questions about careers, applications, organizing the final year of graduate study, and so on. "The faculty are inspired by seeing the students benefit from these sessions," he said.

There is no question that faculty morale is also raised by such tangible benefits as the new building and the grants the department holds. In addition to those held by individual faculty members, the department has grants from such NSF programs as Research Experiences for Undergraduates, Mentoring Through Critical Transition Points, Research Training Groups, and S-STEM, as well as from the National Security Agency. Monetary support for graduate students, including through the summer REG program, helps the students feel valued and included. Many graduate students are supported on research assistantship positions from faculty grants. Those who are supported on teaching assistantships are not simply let loose in front of a classroom but are given careful guidance in developing their teaching skills.

On top of all the achievements mentioned so far, the department also excels in the recruitment and retention of students from underrepresented groups. One indication of this success is the percentage of female graduate students, which has steadily increased from its historic average of around 33 percent to its current level of 49 percent. In addition, in the last ten years more than ten African-American students earned Ph.D.s in the department. If that number sounds low, one must consider that between 2002 and 2007 the number of African-Americans receiving mathematical sciences Ph.D.s across the nation has ranged from ten to seventeen each year. The department has also awarded Ph.D.s to several students of Hispanic origin, as well as to two students of Native American origin. The picture is similarly impressive at the undergraduate level, where approximately one-quarter of the graduates come from underrepresented minorities. Among schools that are not historically black institutions, the NCSU department consistently ranks high in producing graduates who are from underrepresented minorities.

The department is stepping into the national arena to increase diversity in the mathematical sciences. Helminck spearheaded a new program, funded in late 2009 by the NSF, to provide postdoctoral positions specifically for minority Ph.D.s. The program allies the NSF-funded mathematics institutes and seven universities with good track records in producing Ph.D.s drawn from underrepresented minorities. The program will provide four three-year postdoctoral positions consisting of two years at one of the universities and one year at one of the institutes. "We believe this program will be very important in getting more minority faculty in Group I and Group II institutions," Helminck said.

This is a department that manages to do it all—research in a broad range of areas, outstanding teaching and mentoring, strong ties to industry, and a welcoming environment for underrepresented groups. A recent evaluation called the department "a nationally and internationally visible jewel". What is the secret to its success? There is none, said Singer. "It's not something magical. It's commitment and hard work."

Mathematics People

Koenker Awarded Parzen Prize

Roger Koenker of the University of Illinois has been awarded the 2010 Emanuel and Carol Parzen Prize for Statistical Innovation "in honor of his significant innovations in statistical theory that have transformed practice by the creation of methods and software for conditional quantile regression and L_1 estimation" and "for outstanding and influential research contributions in statistical science and economics; for pioneering and expositing quantile regression: theory, computation, real data applications; for global leadership as a mentor and research on diverse statistical methods: regularization, time series modeling, density estimation."

The Parzen Prize is awarded in even-numbered years by the Department of Statistics at Texas A&M University to North American statisticians who have made outstanding and influential contributions to the development of applicable and innovative statistical methods.

—Department of Statistics, Texas A&M University

Goldbring and Sargsyan Awarded ASL Sacks Prize

Isaac Goldbring and Grigor Sargsyan, both of the University of California Los Angeles, have been awarded the 2009 Sacks Prize of the Association for Symbolic Logic (ASL) for the most outstanding doctoral dissertations in mathematical logic. Goldbring received his Ph.D. in 2009 from the University of Illinois at Urbana-Champaign under the supervision of Lou van den Dries. According to the prize citation, in his thesis, *Nonstandard Methods in Lie Theory,* he "applies model theory to a fundamental problem from topological group theory…the main result replaces an incorrect proof in a widely cited paper from 1957 using totally new ideas." Sargsyan received his Ph.D. in 2009 from the University of California Berkeley under the supervision of John Steel. According to the citation, his thesis, *A Tale of Hybrid Mice*, contains "uncountably many new ideas" in inner model theory.

The Sacks Prize was created to honor Gerald Sacks of the Massachusetts Institute of Technology and Harvard University for his unique contribution to mathematical logic. It consists of a cash award plus five years' free membership in the ASL.

—From an ASL announcement

Aizenman Awarded Heineman Prize

The 2010 Dannie Heineman Prize for Mathematical Physics has been awarded to Michael Aizenman of Princeton University "for his development of the random current approach to correlations which has had an impact on a wide variety of problems, especially his rigorous non-perturbative proof of the triviality of ϕ^4 field theory."

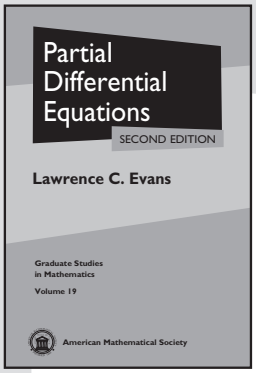
The prize carries a cash award of US$10,000 and is presented in recognition of outstanding publications in the field of mathematical physics. The prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Institute of Physics (AIP) and the American Physical Society (APS). The prize is presented annually.

—From an APS announcement

National Academy of Engineering Elections

The National Academy of Engineering (NAE) has elected sixty-eight new members and nine foreign associates. Among them are five whose work involves the mathematical sciences. They are: William D. Gropp, Paul and

AMERICAN MATHEMATICAL SOCIETY



TEXTBOOK

Partial Differential Equations

Second Edition

Lawrence C. Evans, *University of California, Berkeley, CA*

This is the second edition of the now definitive text on partial differential equations (PDE). It offers a comprehensive survey of modern techniques in the theoretical study of PDE with particular emphasis on nonlinear equations. Its wide scope and clear exposition make it a great text for a graduate course in PDE. For this edition, the author has made numerous changes, including

- **a new chapter on nonlinear wave equations,**
- **more than 80 new exercises,**
- **several new sections,**
- **a significantly expanded bibliography.**

About the First Edition:

I have used this book for both regular PDE and topics courses. It has a wonderful combination of insight and technical detail. … Evans' book is evidence of his mastering of the field and the clarity of presentation.

—Luis Caffarelli, *University of Texas*

It is fun to teach from Evans' book. It explains many of the essential ideas and techniques of partial differential equations … Every graduate student in analysis should read it.

—David Jerison, *MIT*

Graduate Studies in Mathematics, Volume 19; 2010; 749 pages; Hardcover; ISBN: 978-0-8218-4974-3; List US$93; AMS members US$74; Order code GSM/19.R

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Cynthia Saylor Professor of Computer Science, University of Illinois, Urbana-Champaign, elected "for contributions to numerical software in the area of linear algebra and high-performance parallel and distributed computation"; ALI MOSLEH, professor of mechanical engineering, University of Maryland, College Park, "for contributions to the development of Bayesian methods and computational tools in probabilistic risk assessment and reliability engineering"; GEORGE F. PINDER, director, Research Center for Groundwater Remediation Design and professor of civil and environmental engineering and of mathematics, University of Vermont, Burlington, "for leadership in groundwater modeling applied to diverse problems in water resources"; and WILLIAM R. PULLEYBLANK, director of mathematical sciences, T. J. Watson Research Center, "for contributions to the theory and methods of optimization and leadership in their application to business problems." Elected as a foreign associate was DANIE G. KRIGE, independent consultant, South Africa, "for development of statistical methods and their application to resource valuation."

—From an NAE announcement

Austin Selected as Clay Research Fellow

TIMOTHY AUSTIN of the University of California Los Angeles has been awarded a 2010 Clay Research Fellowship by the Clay Mathematics Institute (CMI). He received his B.A. from Trinity College, Cambridge University, in 2005 and will receive his Ph.D. in mathematics from UCLA this year under the guidance of Terence Tao. His research interests include ergodic theory, metric geometry, and geometric group theory. In his recent work, he has developed new techniques for the analysis of certain nonconventional ergodic averages associated with the phenomenon of multiple recurrence and has shown how to construct examples of infinite discrete groups with various novel geometric properties. He has been appointed to a five-year fellowship term.

Clay Research Fellows are appointed for terms ranging from two to five years; graduating doctoral students and mathematicians within three years of receiving the doctoral degree are eligible for the fellowships. The primary selection criteria for the fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader.

—From a CMI announcement

Mathematics Opportunities

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period in intervals not shorter than three consecutive months. The Research Instructorship option provides a combination of full-time and half-time support over a period of three academic years, usually one academic year full-time and two academic years half-time. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The deadline for proposals is **October 20, 2010**. See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

—From an NSF announcement

International Mathematics Competition for University Students

The Seventeenth International Mathematics Competition (IMC) for University Students will be held July 24-30, 2010, at American University in Blagoevgrad, Bulgaria. Participating universities are invited to send several students and one teacher; individual students are welcome. Students completing their first, second, third, or fourth years of university education are eligible. The competition will consist of two sessions of five hours each. Problems will come from the fields of algebra, analysis (real and complex), and combinatorics. The working language will be English. Participants are encouraged to register online by **June 15, 2010**. See the website `http://www.imc-math.org.uk/` or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44-20-7679-7322; email: `j.jayne@ucl.ac.uk`.

—John Jayne, University College London

News from the Bernoulli Center

The Bernoulli Center (CIB) was founded in 2002 and is funded jointly by the Swiss National Science Foundation and the Swiss Federal Institute of Technology in Lausanne. Its mission is to support research in mathematics and its applications, to organize and host thematic programs, to provide a supportive and stimulating environment for researchers, and to launch and foster collaborations between mathematicians working in different areas, as well as between mathematicians and other scientists.

The Center is calling for proposals for four one-semester programs to be held during the period July 1, 2012, to June 30, 2014. Each thematic program consists of a six-month period (January 1–June 30 or July 1–December 31) of concentrated activity in a specific area of current research interest in the mathematical sciences. In exceptional cases, one-year and three-month programs will also be considered. Letters of intent must be submitted by **December 1, 2010**. For more details see the website `http://bernoulli.epfl.ch/new/index.php`.

—From a Bernoulli Center announcement

Inside the AMS

Epsilon Awards for 2010

The AMS Epsilon Fund for Young Scholars was established in 1999 to provide financial assistance to summer programs in the United States and Canada for mathematically talented high school students. These programs have provided mathematically talented youngsters with their first serious mathematical experiences. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

The AMS has chosen ten summer mathematics programs to receive Epsilon grants for activities in the summer of 2010. The grants will support program expenses and student scholarships and, in some cases, scholarships only. The programs were chosen on the basis of mathematical excellence and enthusiasm. Award amounts were governed by the varying financial needs of each program and totaled US$100,000.

The 2010 grants are awarded to: All Girls/All Math, University of Nebraska, Lincoln; Lamar Achievement in Mathematics Program (LAMP), Lamar University; MathPath, Macalester College; PROMYS (Program in Mathematics for Young Scientists), Boston University; PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayagüez Campus; Research Science Institute, Massachusetts Institute of Technology; Stanford University Mathematics Camp (SUMaC), Stanford University; Stony Brook Mathematics Camp, State University of New York at Stony Brook; Texas State Honors Summer Math Camp, Texas State University; Young Scholars Program, University of Chicago.

The grants for summer 2010 are paid for by the AMS Epsilon Fund for Young Scholars. The AMS Epsilon Fund for Young Scholars has been funded by contributions of AMS members and friends; the goal of the endowment is to provide at least US$100,000 in support each summer.

For further information about the Epsilon Fund for Young Scholars, visit the website `http://www.ams.org/giving-to-ams/`or contact `development@ams.org`. Information about how to apply for Epsilon grants is available at `http://www.ams.org/employment/epsilon.html`. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at `http://www.ams.org/employment/mathcamps.html`.

—AMS Development Office

Deaths of AMS Members

Diomedes J. Barcenas, professor, University de Los Andes, Merida, Venezuela, died on November 13, 2009. Born on January 28, 1949, he was a member of the Society for 15 years.

Christopher I. Byrnes, professor, Washington University, St. Louis, died on February 11, 2010. Born on June 28, 1949, he was a member of the Society for 35 years.

Thomas E. Caywood, from Peoria, Arizona, died on December 19, 2008. Born on May 9, 1919, he was a member of the Society for 68 years.

Lester E. Dubins, professor, University of California, Berkeley, died on February 11, 2010. Born on April 27, 1920, he was a member of the Society for 55 years.

Alan J. Goldman, professor, Johns Hopkins University, died on February 13, 2010. Born on March 2, 1932, he was a member of the Society for 57 years.

Juergen P. Herzberger, professor, University of Oldenburg, Germany, died on November 22, 2009. Born on December 17, 1940, he was a member of the Society for 21 years.

Mendel B. Kapilevich, from Brighton, Massachusetts, died on August 28, 2007. Born on March 1, 1923, he was a member of the society for 19 years.

Norman J. Levitt, professor, Rutgers University, died on October 24, 2009. Born on August 27, 1943, he was a member of the society for 41 years.

Ilppo Simo Louhivaara, professor, Free University of Berlin, died on December 19, 2008. Born on October 29, 1927, he was a member of the Society for 47 years.

Charles K. Megibben, professor, Vanderbilt University, died March 5, 2010. Born in October 1936, He was a member of the Society for 48 years.

Yoko Nakagawa, professor, Yamaguchi Prefectural University, died in the summer of 2009. Born on July 27, 1946, she was a member of the Society for 39 years.

George Piranian, professor emeritus, University of Michigan, died on August 31, 2009. Born on May 2, 1914, he was a member of the Society for 67 years.

Gerhard Ringel, professor emeritus, University of California, Santa Cruz, died on June 24, 2008. Born on October 28, 1919, he was a member of the Society for 37 years.

Daniel Jay Rudolph, professor, Colorado State University, died on February 4, 2010. Born on October 3, 1949, he was a member of the Society for 31 years.

Bernard Sherman, professor, New Mexico Institute of Mining and Technology, died on February 28, 2008. Born on January 4, 1919, he was a member of the Society for 67 years.

Hans Troger, professor, Vienna University of Technology, died February 22, 2010. Born on March 11, 1943, he was a member of the Society for 38 years.

Jacques Troue, from Montreal, Canada, died on December 24, 2001. Born on October 6, 1919, he was a member of the Society for 41 years.

B. S. Yadav, professor, University of Delhi, died February 24, 2010. Born on July 30, 1931, he was a member of the Society for 10 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 23, 2010: Proposals for 2010 NSF-CBMS Regional Conferences. See the website `http://www.cbmsweb.org/NSF/2011_call.htm` or contact: Conference Board of the Mathematical Sciences, 1529 Eighteenth Street, NW, Washington, DC 20036; telephone: 202-293-1170; fax: 202-293-3412; email: `lkolbe@maa.org` or `rosier@georgetown.edu`.

May 1, 2010: Applications for May review for National Academies Postdoctoral and Senior Research Associateship Program. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—*November 2009, p. 1320*

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AMS Officers and Committee Members—*October 2009, p. 1133*

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NRC Board on Mathematical Sciences and Their Applications—*March 2010, p. 423*

NRC Mathematical Sciences Education Board—*April 2010, p. 541*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2010, p. 272*

Program Officers for Federal Funding Agencies—*October 2009, p. 1126 (DoD, DoE); December 2007, p. 1359 (NSF); December 2009, p. 1464 (NSF Mathematics Education)*

Program Officers for NSF Division of Mathematical Sciences—*November 2009, p. 1313*

May 1, 2010: Applications for the fall 2010 program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship program of the National Academies. See http://sites.nationalacademies.org/PGA/policyfellows/index.htm or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

May 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html, telephone: 703-934-0163, or email: awm@awm-math.org. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

May 15–June 15, 2010: Proposals for DMS Workforce Program in the Mathematical Sciences. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233.

June 1, 2010: Applications for NSF's Enhancing the Mathematical Sciences Workforce in the Twenty-First Century (EMSW21) program. See http://www.nsf.gov/pubs/2005/nsf05595/nsf05595.htm.

June 15, 2010: Registration for International Mathematics Competition for University Students. See "Mathematics Opportunities" in this issue.

July 31, 2010: Nominations and applications for the 2010 Monroe H. Martin Prize. Contact R. Roy, Director, Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742-2431.

August 1, 2010: Applications for August review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

September 30, 2010: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT) program; by invitation only. See http://www.nsf.gov/pubs/2010/nsf10523/nsf10523.htm?WT.mc_id=USNSF_25.

October 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 20, 2010: Proposals for NSF Postdoctoral Research Fellowships. See "Mathematics Opportunities" in this issue.

November 1, 2010: Applications for November review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

December 1, 2010: Letters of intent for proposals for thematic programs at the Bernoulli Center. See "Mathematics Opportunities" in this issue.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Archimedes Codex: How a Medieval Prayer Book Is Revealing the True Genius of Antiquity's Greatest Scientist, by Reviel Netz and William Noel. Da Capo Press, October 2007. ISBN 978-03068-1580-5. (Reviewed September 2008.)

The Calculus of Friendship: What a Teacher and Student Learned About Life While Corresponding About Math, by Steven Strogatz. Princeton University Press, August 2009. ISBN-13: 978-06911-349-32.

Chez les Weils (French), by Sylvie Weil. Buchet-Chastel, January 2009. ISBN-13: 978-22830-236-93.

Crocheting Adventures with Hyperbolic Planes, by Daina Taimina. A K Peters, March 2009. ISBN-13: 978-15688-145-20.

Decoding the Heavens: A 2,000-Year-Old Computer—and the Century-Long Search to Discover Its Secrets, by Jo Marchant. Da Capo Press, February 2009. ISBN-13: 978-03068-174-27.

Ernst Zermelo: An Approach to His Life and Work, by Heinz-Dieter Ebbinghaus. Springer, April 2007. ISBN-13: 978-3-540-49551-2. (Reviewed August 2009.)

Gaming the Vote (Why Elections Aren't Fair and What We Can Do About It), by William Poundstone. Hill and Wang, February 2009. ISBN-13: 978-08090-489-22.

The Housekeeper and the Professor, by Yoko Ogawa. Picador, February 2009. ISBN-13: 978-03124-278-01. (Reviewed in this issue.)

Logicomix: An Epic Search for Truth, by Apostolos Doxiadis and Christos Papadimitriou. Bloomsbury USA, September 2009. ISBN-13:978-15969-145-20.

Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart Menzler-Trott, Craig Smorynski (translator), Edward R. Griffor (translator). AMS-LMS, November 2007. ISBN-13: 978-0-8218-3550-0.

The Mathematical Mechanic: Using Physical Reason to Solve Problems, by Mark Levi. Princeton University Press. ISBN: 978-0691140209.

Mathematicians: An Outer View of the Inner World, by Mariana Cook, Princeton University Press, June 2009. ISBN13: 978-0-691-13951-7.

Mathematicans Fleeing from Nazi Germany: Individual Fates and Global Impact, by Reinhard Siegmund-Schultze. Princeton University Press, July 2009. ISBN 978-0-691-14041-4.

Mathematicians of the World, Unite!: The International Congress of

Mathematicians: A Human Endeavor, by Guillermo P. Curbera. A K Peters, March 2009. ISBN-13: 978-15688-133-01.

Mathematics and Common Sense: A Case of Creative Tension, by Philip J. Davis. A K Peters, October 2006. ISBN 1-568-81270-1. (Reviewed June/July 2009.)

Mathematics in Ancient Iraq: A Social History, by Eleanor Robson. Princeton University Press, August 2008. ISBN-13: 978-06910-918-22. (Reviewed March 2010.)

Mathematics in India, by Kim Plofker. Princeton University Press, January 2009. ISBN-13: 978-06911-206-76. (Reviewed March 2010.)

Mathematics in 10 Lessons: The Grand Tour, by Jerry P. King. Prometheus Books, May 2009. ISBN: 978-1-59102-686-0.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, by Victor J. Katz et al. Princeton University Press, July 2007. ISBN-13: 978-0-6911-2745-3.

The Millennium Prize Problems, edited by James Carlson, Arthur Jaffe, and Andrew Wiles. AMS, June 2006. ISBN-13: 978-08218-3679-8. (Reviewed December 2009.)

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396743.

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN-13: 978-06187-846-08. (Reviewed October 2009.)

Out of the Labyrinth: Setting Mathematics Free, by Robert Kaplan and Ellen Kaplan. Oxford University Press, January 2007. ISBN-13: 978-0-19514-744-5. (Reviewed June/July 2009.)

A Passion for Discovery, by Peter Freund. World Scientific, August 2007. ISBN-13: 978-9-8127-7214-5.

Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century, by Masha Gessen. Houghton Mifflin Harcourt, November 2009. ISBN-13: 978-01510-140-64.

Picturing the Uncertain World: How to Understand, Communicate, and Control Uncertainty Through Graphical Display, by Howard Wainer, Princeton University Press, April 2009. ISBN-13: 978-06911-375-99.

Pioneering Women in American Mathematics: The Pre-1940 Ph.D.'s, by Judy Green and Jeanne LaDuke. AMS, December 2008. ISBN-13:978-08218-437-65.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03. (Reviewed February 2010.)

The Princeton Companion to Mathematics, edited by Timothy Gowers (June Barrow-Green and Imre Leader, associate editors). Princeton University Press, November 2008. ISBN-13: 978-06911-188-02. (Reviewed November 2009.)

Proofs from THE BOOK, by Martin Aigner and Günter Ziegler. Expanded fourth edition, Springer, October 2009. ISBN-13: 978-3-642-00855-9

Pythagoras' Revenge: A Mathematical Mystery, by Arturo Sangalli. Princeton University Press, May 2009. ISBN-13: 978-06910-495-57. (Reviewed in this issue.)

Recountings: Conversations with MIT Mathematicians, edited by Joel Segel. A K Peters, January 2009. ISBN-13: 978-15688-144-90.

Roger Boscovich, by Radoslav Dimitric (Serbian). Helios Publishing Company, September 2006. ISBN-13: 978-09788-256-21.

Sacred Mathematics: Japanese Temple Geometry, by Fukagawa Hidetoshi and Tony Rothman. Princeton University Press, July 2008. ISBN-13: 978-0-6911-2745-3.

Solving Mathematical Problems: A Personal Perspective, by Terence Tao. Oxford University Press, September 2006. ISBN-13: 978-0-199-20560-8. (Reviewed February 2010.)

Sphere Packing, Lewis Carroll, and Reversi, by Martin Gardner. Cambridge University Press, July 2009. ISBN: 978-0521756075.

Strange Attractors: Poems of Love and Mathematics, edited by Sarah Glaz and JoAnne Growney. A K Peters, November 2008. ISBN-13: 978-15688-134-17. (Reviewed September 2009.)

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77.

Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature, by Michael Field and Martin Golubitsky. Society for Industrial and Applied Mathematics, second revised edition, May 2009. ISBN-13: 978-08987-167-26.

Teaching Statistics Using Baseball, by James Albert. Mathematical Association of America, July 2003. ISBN-13: 978-08838-572-74. (Reviewed April 2010.)

Tools of American Math Teaching, 1800–2000, by Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts. Johns Hopkins University Press, July 2008. ISBN-13: 978-0801888144. (Reviewed January 2010.)

The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter That Made the World Modern, by Keith Devlin. Basic Books, September 2008. ISBN-13: 978-0-4650-0910-7.

Why Does E=mc²? (And Why Should We Care?), by Brian Cox and Jeff Forshaw. Da Capo Press, July 2009. ISBN-13: 978-03068-175-88.

Zeno's Paradox: Unraveling the Ancient Mystery Behind the Science of Space and Time, by Joseph Mazur. Plume, March 2008 (reprint edition). ISBN-13: 978-0-4522-8917-8.

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Dear Friends and Colleagues,

During 2009 your generous contributions helped the Society and our profession in many ways. Thank you for your support.

In 2009, the Epsilon Fund, the endowment whose income supports the Young Scholars program, stands at a level where it can annually provide grants to support ten separate programs that touch approximately 600 talented and highly motivated mathematics students every year. We are very grateful for the numerous individual contributions indicated in the following pages. We continue to place a very high priority on supporting the programs that bring mathematically talented high school students together and introduce them to mathematical research.

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The Executive Committee and Board of Trustees have established the Thomas S. Fiske Society to honor those who have made provisions for the AMS in their estate plans. For further information contact the Development Office at 800-321-4AMS, or `development@ams.org`.

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ε Thaleia Zariphopoulou
ε Thomas Zaslavsky
ε Tien Yu Zhao
ε*David E. Zitarelli
ε Steven M. Zucker
ε John A. Zweibel
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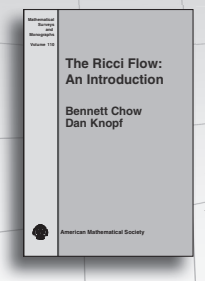
RICCI FLOW TITLES FROM THE AMS



The Ricci Flow: An Introduction

Bennett Chow, *University of California, San Diego, CA*, and **Dan Knopf**, *University of Texas, Austin, TX*

An introduction to the Ricci flow program connected to Thurston's geometrization conjecture, with a fast-track guide to the material

Mathematical Surveys and Monographs, Volume 110; 2004; 325 pages; Hardcover; ISBN: 978-0-8218-3515-9; List US$88; AMS members US$70; Order code SURV/110

Ricci Flow and Geometrization of 3-Manifolds

John W. Morgan, *Stony Brook University, NY*, and **Frederick Tsz-Ho Fong**, *Stanford University, CA*

An overview of how to use Ricci flow to establish the Poincaré Conjecture and the more general Geometrization Conjecture for all 3-manifolds

University Lecture Series, Volume 53; 2010; 150 pages; Softcover; ISBN: 978-0-8218-4963-7; List US$41; AMS members US$33; Order code ULECT/53

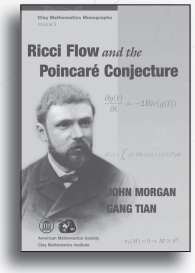
Ricci Flow and the Sphere Theorem

TEXTBOOKS FROM THE AMS

Simon Brendle, *Stanford University, CA*

An alternative approach to Ricci flow techniques, from a rising young star in geometric analysis

Graduate Studies in Mathematics, Volume 111; 2010; 176 pages; Hardcover; ISBN: 978-0-8218-4938-5; List US$47; AMS members US$38; Order code GSM/111



Ricci Flow and the Poincaré Conjecture

John Morgan, *Columbia University, New York, NY*, and **Gang Tian**, *Princeton University, NJ, and Peking University, Beijing, China*

Full details of a complete proof of the important Poincaré Conjecture, using geometric arguments presented by Grigory Perelman

Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

Clay Mathematics Monographs, Volume 3; 2007; 521 pages; Hardcover; ISBN: 978-0-8218-4328-4; List US$69; AMS members US$55; Order code CMIM/3

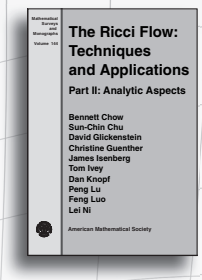
The Ricci Flow: Techniques and Applications

Part I: Geometric Aspects

Bennett Chow, **Sun-Chin Chu**, **David Glickenstein**, **Christine Guenther**, **James Isenberg**, **Tom Ivey**, **Dan Knopf**, **Peng Lu**, **Feng Luo**, and **Lei Ni**

A clear, detailed explanation of advanced material to assist in using Ricci flow to study the geometry and topology of three-dimensional manifolds

Mathematical Surveys and Monographs, Volume 135; 2007; 536 pages; Hardcover; ISBN: 978-0-8218-3946-1; List US$109; AMS members US$87; Order code SURV/135



The Ricci Flow: Techniques and Applications

Part II: Analytic Aspects

Bennett Chow, **Sun-Chin Chu**, **David Glickenstein**, **Christine Guenther**, **James Isenberg**, **Tom Ivey**, **Dan Knopf**, **Peng Lu**, **Feng Luo**, and **Lei Ni**

A detailed but accessible account of the analytic techniques and results of Ricci flow

Mathematical Surveys and Monographs, Volume 144; 2008; 458 pages; Hardcover; ISBN: 978-0-8218-4429-8; List US$109; AMS members US$87; Order code SURV/144

The Ricci Flow: Techniques and Applications

Part III: Geometric-Analytic Aspects

Bennett Chow, **Sun-Chin Chu**, **David Glickenstein**, **Christine Guenther**, **James Isenberg**, **Tom Ivey**, **Dan Knopf**, **Peng Lu**, **Feng Luo**, and **Lei Ni**

A discussion of geometric-analytic topics in Ricci flow, covering much of Perelman's ideas and explaining technical aspects useful for singularity analysis

Mathematical Surveys and Monographs, Volume 163; 2010; 517 pages; Hardcover; ISBN: 978-0-8218-4661-2; List US$113; AMS members US$90; Order code SURV/163

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Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at `http://www.ams.org/cgi-bin/mathcal-submit.pl`. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at `http://www.ams.org/mathcal/`.

May 2010

* 3–5 **Simons Lectures in Mathematics**, Massachusetts Institute of Technology, Cambridge, Massachusetts.
Lecturer: Andrei Okounkov (Princeton University) will give 3 lectures on "Applied Noncommutative Geometry" (Algebra and geometry of random surfaces; Noncommutative geometry and Painlevé equations; Towards probabilistic mirror symmetry).
Information: `http://math.mit.edu/news/simons`.

* 6–7 **4th International Conference on Information Security and Cryptology**, Middle East Technical University, Ankara, Turkey.
Focus: The conference focuses on a different topic each year and this year's topic is "Data Privacy". A. Accepted contributions will appear in the conference proceedings. B. ISC'10 will feature the following invited talks: 1. Berry Schoenmakers (`http://www.win.tue.nl/~berry/`), Secure Multiparty Computation: A Showcase for Modern Cryptography; 2. Jianying Zhou (`http://icsd.i2r.webbreaka-star.edu.sg/staff/jianying/`), Non-repudiation Protocols and Applications. We kindly invite you to participate in the conference (with or without a paper) to share your knowledge with others.
Deadline: The last date for submission of full papers is: April 1, 2010.
Information: `http://www.iscturkey.org/indexen.php`.

* 15–16 **Young Researchers Meet (Mathematics and Computer Science)**, Brown University, Providence, Rhode Island.
Description: A large number of new Indian Institutes of Science Education & Research (IISERs), IITs and Central Universities have been started recently in India. All these institutions and existing academic institutions have a fairly large requirement of outstanding faculty members in mathematics and computer science. In view of this, it is proposed to organize a Young Researchers Meet at Brown University, Providence, Rhode Island, from May 15-16, 2010. The primary goal of this meeting is to inform junior faculty, post-docs and senior graduate students in the U.S. about academic opportunities available in India and motivate them to return to India. A delegation from India representing a wide cross-section of institutes/universities will make presentations on various aspects of an academic career in India.
Deadline: For application: April 12, 2010.
Information: Please see website for further details: email: `yrmbrown@dam.brown.edu`; `http://www.dam.brown.edu/people/menon/yrmbrown.html`

* 28–30 **International Symposium on Geometry and Topology in Memory of C.-C. Hsiung**, Lehigh University, Bethlehem, Pennsylvania.
Description: This symposium celebrates the life and mathematical legacy of Chuan-Chih Hsiung (1916-2009), geometer, professor at Lehigh University, and founder of the *Journal of Differential Geometry*.
Speakers: Werner Ballmann, University of Bonn; Simon Brendle, Stanford University; Kenji Fukaya, Kyoto University; Richard Hamilton (to be confirmed), Columbia University; Michael Hopkins, Harvard University; Alina Marian, University of Illinois at Chicago; D.H. Phong,

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

Columbia University; I. M. Singer, MIT; Y.-T. Siu, Harvard University; Xu-Jia Wang, Australia National University.
Information: http://www.lehigh.edu/~dlj0/geotop.html.

* 31–June 4 **International Conference on Advances in Partial Differential Equations and Their Applications**, Fudan University, Shanghai, China.
Description: The aim of the conference is to bring together experts who work on diverse frontiers of nonlinear PDEs and their applications to geophysical fluid dynamics to survey recent progress and current challenges, and to discuss how new ideas and methods could advance the field in coming years. In addition, the conference will bring many internationally acclaimed researchers working in such an active area to China. It is anticipated that the conference will attract a large number of young researchers in China and from other countries and contribute significantly to the advancement of international applied and computational mathematics community. We invite mathematicians and scientists working in the related areas to participate in this conference. In particular, we encourage women and minorities to attend the conference. Partial funding from U.S. NSF may become available for women and monorities from the U.S.
Information: http://www.indiana.edu/~fluid/shanghai/shanghai.html.

June 2010

* 4–9 **XIIth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena resort, Varna, Bulgaria.
Description: This conference can be considered a continuation of previous meetings on Geometry and Mathematical Physics which took place in Bulgaria–Zlatograd (1995) and Varna (1998-2009).
Aim: To bring together experts in Classical and Modern Differential Geometry, Complex Analysis, Mathematical Physics and related fields to assess recent developments in these areas and to stimulate research in related topics.
Principal Speakers: Vladimir Dragovic, Geometry and Integrability in Rigid Body Dynamics; Andrei Ludu, Differential Geometry of Moving Surfaces, Solitons and Filaments.
Information: http://www.bio21.bas.bg/conference/.

* 7–10 **Istanbul Contact Geometry and Topology Workshop**, Istanbul Center for Mathematical Sciences at Bogazici University, Istanbul, Turkey.
Description: There will be some talks and mini courses on Contact Geometry and Topology.
Confirmed speakers: Kenneth Lee Baker (Univ. Miami), Vincent Colin (Univ. Nantes), David Gabai (Princeton Univ.), Hansjörg Geiges (Univ. Köln), Paolo Ghiggini (Univ. Nantes), Jeremy Van Horn-Morris (American Inst. Math.)
Information: http://www.imbm.org.tr/20100607icgt.html.

* 10–14 **Symmetry Plus Integrability 2010—The First International Conference on Integrable Systems and Nonlinear Waves on the Gulf of Mexico, In Honor of Yuji Kodama's 60th Birthday**, South Padre Travelodge, South Padre Island, Texas.
Description: This conference will be held on the occasion of the 60th birthday of Professor Yuji Kodama. The conference will focus on integrable systems and nonlinear waves.
Topics: Soliton, continuous and discrete integrable systems, water waves, nonlinear optics and photonics, random matrices and integrable systems, integrable systems and number theory, dispersionless integrable systems, higher dimensional integrable systems.
Plenary Speakers: Mark J. Ablowitz (University of Colorado), Anthony Bloch (University of Michigan), Masanobu Kaneko (Kyushu University, Japan), Boris Konopelchenko (University of Salento, Lecce, Italy), Alexander Mikhailov (University of Leeds, U.K.), Yoshimasa Nakamura (Kyoto University, Japan)
Information: http://www.math.utpa.edu/kmaruno/SPI2010.html.

* 14–16 **IMA Special Program: Interdisciplinary Research Experience for Undergraduates**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota.
Description: The IMA summer REU gives students an experience in working on an interdisciplinary project involving mathematics. Students will work in teams of three with a faculty advisor and a postdoctoral mentor. This is a hands-on experience. There will be a few formal lectures. However, students will spend most of their time doing actual research. Students are expected to produce a poster and a publication-quality written report, and to give an oral presentation by the end of the 5-week period.
Information: http://www.ima.umn.edu/2009-2010/SW6.14-7.16.10/.

* 14–18 **Summer school "p-adic cohomologies and arithmetic applications"**, Mathematical Research Institute, University of Sevilla (IMUS), Sevilla, Spain.
Description: The School is addressed to Ph.D. students and young post-doc researchers working on Number Theory, Algebraic Geometry and related areas. There will be four courses: (1) "Introduction to rigid cohomology" by Bernard Le Stum (University of Rennes). (2) "Dwork cohomology and applications" by Steven Sperber (University of Minnesota). (3) "Overconvergent F-isocrystals and rigid cohomology on curves" by Nobuo Tsuzuki (Tohoku University). (4) "An introduction to Dwork's p-adic analytic representation of an additive character" by C. Douglas Haessig (University of Rochester).
Information: For more information, including information on registration, financial support, accommodations, and transportation, see: http://www.imus.us.es/padic2010/.

* 16–20 **Fourth International Conference on Recent Advances in Applied Dynamical Systems**, Zhejiang Normal University, Jinhua, Zhejiang, China.
Description: The main subjects of the conference will be bifurcation theory and applications of dynamical systems and related topics. The topics include, but are not limited to local and global bifurcation, computational and theoretical aspects of bifurcation and chaos in biology, economics, engineering, fluid dynamics, neural networks, physics and other nonlinear dynamical systems.
Information: For details about the conference, please visit the website: http://nonlsci.zjnu.net.cn/ of the Center for Dynamical Systems and Nonlinear Sciences, Zhejiang Normal University, or contact: 2010jinhua@gmail.com.

* 19–24 **Automorphism Groups of Topological Structures**, The Eilat Campus of Ben Gurion University of the Negev, Israel.
Description: The purpose of the conference is to bring together mathematicians working in all fields of mathematics dealing with Automorphism Groups of Topological Structures, in order to exchange ideas and present results of current research. Specific topics to be covered in this meeting are: 1) Recovery of spaces from their groups of homeomorphisms. 2) Topological characterization of the homeomorphism groups of various concrete spaces. 3) Realization of topological groups as groups of automorphisms of concrete spaces. 4) Extreme amenability of automorphism groups and homeomorphism groups. 5) The Urysohn universal space. 6) Structure theory of general and free topological groups.
Information: http://www.math.bgu.ac.il/~arkady/Eilat2010_conference/main_page.html.

* **Second Mathematical Chemistry Workshop of the Americas**, Universidad de los Andes, Bogota, Colombia.
Description: The event will bring together leading researchers in the field of Mathematical Chemistry, which includes the application of different branches of mathematics to chemistry, e.g., graph theory, topology, order theory, mathematical modeling, statistics, and some other mathematical areas. The meeting will also cover topics like chemoinformatics, environmental toxicology, quantitative structure-activity relationship (QSAR) and quantitative molecular similarity

analysis (QMSA), bioinformatics (including genomics and proteomics), and other relevant fields.
Information: Visit: http://sites.google.com/site/2mathchemamericas/.

* 19–26 **XIVth International Summer Conference on Probability and Statistics (ISCPS-2010), Seminar of Statistical Data Analysis (SDA-2010), Workshop on Branching Processes and Applications (WBPA-2010)**, Sozopol, Bulgaria.
Description: The Conference and the Seminar are organized by: Institute of Mathematics and Informatics of Bulgarian Academy of Sciences; Faculty of Mathematics and Informatics of Sofia University; Bulgarian Statistical Society. We plan to organize several sessions in the field of: Stochastic Processes; Limit Theorems; Financial Mathematics; Combinatorial Analysis; Theoretical and Applied Statistics; Applications in Biology, Industry, Economics and Education of Statistics.
Information: http://stochastics.fmi.uni-sofia.bg.

* 21–23 **Conference on Special Functions & their Applications–CSFA 2010**, School of Mathematics and Allied Sciences (SOMAAS) Jiwaji University, Gwalior (MP), India.
Description: CSFA 2010 is being organized by the School of Mathematics & Allied Sciences, Jiwaji University, Gwalior (MP), India. During the three day conference a symposium on computational & biological mathematics will also be organized. The organizers invite papers for presentation.
Information: http://www.ssfaindia.webs.com/conf.htm.

* 21–July 2 **Summer School on Mathematical String Theory 2010**, Virginia Tech, Blacksburg, Virginia.
Description: This is a summer school for grad student and postdocs in mathematics and physics, covering topics of interest to both, ranging from E_n algebras, topological chiral homology, and aspects of derived categories, to topics in compactifications of string theories.
Speakers: Dima Arinkin (UNC-CH), Arend Bayer (Connecticut), John Francis (Northwestern), Josh Guffin (U-Penn), Simeon Hellerman (IPMU), Ilarion Melnikov (Potsdam), and Peter Zograf (Steklov, St. Petersburg).
Information: http://www.phys.vt.edu/mp10.

* 22–24 **The 6th East Asia SIAM Conference in conjunction with the Applied Mathematics International Conference AMIC2010**, Kuala Lumpur, Malaysia.
Description: The EASIAM Conference is an annual meeting in the series of conferences. EASIAM provides a platform for exploration and application of mathematics to science, industry, engineering and technology in the East and South East Asia. Previous EASIAM meetings were held in Hong Kong, Sapporo, Xiamen, Daejeon, and Brunei. The goal of the conference is to promote research that will lead to effective mathematical and computational methods and techniques for science, engineering, industry and society.
Information: http://math.um.edu.my/easiam/.

* 28–30 **International Symposium on Voronoi Diagrams in Science and Engineering (ISVD2010)**, Laval University, Quebec City, Canada.
Description: The ISVD 2010 which is devoted to one of the most fascinating data structures in Computational Geometry—the Voronoi diagrams and its applications, is a multidisciplinary event hosting scientists working on Voronoi Diagrams and on their applications in engineering including, geospatial information sciences (GIS), visualization, urban planning, computer science, geology, robotics, bioinformatics, high-performance computing, physics, chemistry and other applications.
Key issues: Applications of the Voronoi/Delaunay methodology in sciences and engineering, Theoretical aspects of Voronoi diagrams/Delaunay methodology, Computational and implementation aspects of Voronoi diagrams, Voronoi Art.
Information: http://isvd2010.scg.ulaval.ca.

* 28–July 2 **Fifth Pacific Rim Conference on Mathematics**, Stanford University, Stanford, California.
Description: This meeting has been held every three years since 1998 at universities on the Pacific Rim. This fifth meeting is the first to be held in North America. It will cover a wide range of topics and is designed to foster cooperation among mathematicians at Pacific Rim institutions.
Sessions: The ten sessions are in the areas of algebraic geometry, combinatorics, differential geometry, electromagnetic waves, functional analysis, kinetic equations and gas dynamics, mathematical imaging, nonlinear partial differential equations, number theory, and random systems and PDEs.
Support: Some support is available for advanced graduate students and postdoctoral level mathematicians.
Information: http://math.stanford.edu/PRCMindex.html.

July 2010

* 11–16 **Path Integrals 2010**, Howard University's Blackburn Center, Washington, DC (USA).
Description: Professors Arthur Jaffe and J. Zinn-Justin have agreed to join us during the opening session on Sunday afternoon July 11. Professor Jaffe will discuss the important role of path integrals in the successes of constructive quantum field theory and Professor J. Zinn-Justin will discuss the importance of path integrals in condensed matter and particle physics. During the conference banquet, Professor Cecile DeWitt-Morette will discuss the history of path integrals and the impact on quantum field theory.
Other invited speakers: Professors Wolfhard Janke, John Klauder, H. H. Kleinert, N. Kumano-go and Jean-Claude Zambrini. Those interested in giving a contributed talk can submit abstracts on the website.
Information: http://sites.google.com/site/pathintegrals2010/home.

* 12–22 **Regulators III**, Barcelona, Spain.
Description: A regulator is a realization from motivic cohomology to "more computable" cohomology theories (e.g., Hodge, Betti, etale), Over the past three decades, the subject of regulators has evolved considerably on the mathematical scene, through its interaction with algebraic cycles and algebraic K-theory, Hodge theory, arithmetic algebraic geometry, number theory, topology (as in Lawson homology), and mathematical physics. This conference brings together leading experts from various backgrounds, to present the latest developments in the field. Graduate students, post-doctoral fellows, as well as interested researchers, are encouraged to apply by registering on-line.
Information: http://www.imub.ub.es/regulatorsIII/.

* 12–August 13 **A minisemester on evolution of interfaces**, Hokkaido University, Sapporo, Japan.
Description: We are organizing a series of research seminars and tutorial lectures with particular focus on evolution of interfaces. Two intensive activities each lasting one week are planned.
Organizers: Y. Giga, H. Ishii, T. Funaki, Y. Tonegawa, R. V. Kohn, P. Rybka.
Intensive Activities: (1) Tutorial Lectures and Interdisciplinary Conference: "Mathematical Aspects of Crystal Growth" (July 26–July 30, 2010).
Tutorial lecturers include: R. E. Caflisch, D. Margetis, R. Monneau.
Invited lecturers include: J. Evans, H. Mitake, T. Ohtsuka, G. Sazaki, R. Sekerka, K. Sudoh, P. G. Vekilov, A. Yagi.
(2) Tutorial Lectures and International Workshop: "Singular Diffusion and Evolving Interfaces" (August 2–August 6, 2010).
Tutorial lecturers include: J. Manfredi, J. M. Mazón.
Invited lecturers include: W. Allard, S. Esedoglu, H. Garcke, J. S. Guo, P. Mucha, T. Nishikawa, K. Shirakawa, N. Yamazaki.
Registration: For registration, program and financial support see our web page.
Information: http://www.math.sci.hokudai.ac.jp/sympo/minisemester/index_en.html.

* 16–23 **International Conference on Spectral Geometry, Preceded by Pre-conference Minicourses**, Dartmouth College, Hanover, New Hampshire.
Description: The primary themes of the conference will be "Inverse spectral geometry on compact manifolds and orbifolds" and "Quantum unique ergodicity". There will be two plenary lectures on each of these themes presented by Victor Guillemin and Peter Sarnak, respectively. In addition to spectral problems on Riemannian manifolds, there will be a secondary focus on the related topic of quantum graphs, in which the one-dimensional Laplacian acts along edges, and waves scatter at vertices.
Travel/Support: Pending NSF approval, we anticipate that we will be able to provide travel support for some participants, with preference going to graduate students and recent Ph.D.s. More senior mathematicians without other sources of support are also eligible to apply. The target date to apply for support is April 15; later applications will be considered as long as funds remain available.
Information: `http://math.dartmouth.edu/~specgeom`.

* 19–23 **16th International Conference on Difference Equations and Applications ICDEA2010**, Faculty of Physics and Mathematics of University of Latvia, Riga, Latvia.
Description: The purpose of the conference is to bring together both experts and novices in the theory and applications of difference equations and discrete dynamical systems. The special emphasis of the meeting will be on the applications of difference equations on mathematical biology.
Plenary speakers: Josef Hofbauer, Jim Cushing, Luis Alseda, Robert Devaney, Jean Mahwin, Aleksandr Sharkovski
Scientific Committee: Andre Vanderbauwhede, Hal Smith, Gerry Ladas, Michael Misiurewicz, Andrejs Reinfelds.
Information: `http://icdea2010.lu.lv`.

* 25–30 **25th Summer Conference on Topology and its Applications**, Jan Kochanowski University in Kielce, Poland.
Description: The 25th Summer Topology Conference is an annual event bringing together an international audience of researchers in general topology and related fields. There will be special sessions in the following subjects: general/set-theoretic topology, geometric topology, continuum theory, dynamical systems, topological algebra, topology in computer science, topology in functional analysis.
Information: `http://www.ujk.edu.pl/~topoconf/`.

* 26–29 **Mixed Integer Programming 2010**, Georgia Institute of Technology, Atlanta, Georgia.
Description: The Mixed Integer Programming (MIP) workshop series brings together the integer optimization research community in an annual meeting, to discuss very recent developments in the field. It consists of a single track of invited talks, which are scheduled to leave ample time for discussion and interaction among the participants.
Registration/Support: To encourage participation of students and junior researchers, no registration fee is charged, and financial support is available. The workshop features a poster session.
Deadline: The deadline for poster abstract submissions and applications for travel support is March 31, 2010.
Information: `http://www2.isye.gatech.edu/mip2010/`.

August 2010

* 2–5 **IMA Hot Topics Workshop: Integral Equation Methods, Fast Algorithms and Applications**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, MN 55455
Description: The main objective of this workshop is to have timely reviews and discussions of recent results and current issues in (1) the mathematical foundations of integral-equation methods and associated fast algorithms, (2) implementations and performance assessments of such methods on a variety of computer architectures, and (3) existing and new application areas. A plethora of computational techniques already exists, and includes tree-codes, pre-corrected fast Fourier transforms, Ewald procedures, and the many classes of fast multipole methods. This workshop will help clarify the relative advantages of existing schemes, and elucidate the trajectories and applications of many methods now under development.
Information: `http://www.ima.umn.edu/2009-2010/SW8.2-5.10/`.

* 2–7 **Geometry, Topology, and Dynamics in Negative Curvature**, Raman Research Institute, Bangalore, India.
Description: This is a satellite conference for the 2010 ICM. The conference will focus on mathematics centered around negatively curved (or more generally, non-positively curved) spaces.
Information: `http://www.icts.res.in/program/gtdnc`.

* 2–11 **Mathematical Modeling in Industry XIV - A Workshop for Graduate Students**, Center for Mathematical Research (CIMAT) in Guanajuato, Gto., México.
Description: The IMA is holding a 10-day workshop on Mathematical Modeling in Industry. This event will take place at CIMAT in Guanajuato, Mexico. The workshop is designed to provide graduate students and qualified advanced undergraduates with first hand experience in industrial research. Students will work in teams of up to 6 students under the guidance of a mentor from industry. The mentor will help guide the students in the modeling process, analysis and computational work associated with a real-world industrial problem. A progress report from each team will be scheduled during the period. In addition, each team will be expected to make an oral final presentation and submit a written report at the end of the 10-day period.
Information: `http://www.ima.umn.edu/2009-2010/MM8.2-11.10/`.

* 3–5 **International Conference on Mathematical Applications in Engineering: (ICMAE, 2010)**, Kuala Lumpur, Malaysia.
Conference Objective: Mathematics provides the methodology and framework of rigorous thoughts for engineering developments in the changing world. The aim of the conference is to stimulate collaboration between active researcher, educators, mathematicians and engineers representing educational institutions, government agencies and engineering related organizations. The prime objective is to act as the main forum to exchange, examine, promote and discuss issues related to the latest trend, importance, needs and future developments of mathematics within its own domain as well as its effective applications in engineering.
Conference Topics: Papers are invited in topics that cover, but are not limited to: Actuarial & Financial Mathematics, Algebra(s) and Applications, Complex Analysis, Differential Equations and Applications, Discrete Mathematics, Dynamic System, Experimental Design, Graph Theory, Industrial Mathematics, Linear Algebra and Applications.
Information: `http://www.iiu.edu.my/icmae`.

* 16–21 **Workshop on Operator Algebras and Conformal Field Theory**, University of Oregon, Eugene, Oregon.
Description: This will be an educational workshop aimed at exploring the foundations of conformal field theory from the perspective of operator algebras. It will focus on Wasserman's construction of CFTs associated to the loop group SU(n), which he uses to study representations of central extensions of the loop group.
Information: `http://www.uoregon.edu/~njp/cft.html`.

* 29–September 12 **Summer School "Modern Problems of Wavelet Analysis and its Applications"**, Kursk State University, Kursk, Russia.
Description: The 2-week course in the Continuous and Discrete Wavelet Transform covers the following topics: Time-frequency localization of wavelet functions, multiresolution analysis and variational methods for wavelet constructions; PDE-based methods for the continuous wavelet transform; multiresolution signal processing; wavelets and topology of chaotic phase curves.
Deadlines: Application with the possibility of a DAAD grant support: April 30, 2010. Application without a financial support: June 15, 2010

Detailed description of the school: http://www.kursksu.ru/pages/sums_mpwaa.
Description of Go East Program for participants' support: http://www.daad.de/ausland/foerderungsmoeglichkeiten/ausschreibungen/12996.de.html.
Information: http://www.kursksu.ru/pages/sums_mpwaa.

September 2010

* 6–10 **Dynamics Days Europe 2010**, University of Bristol, Bristol, United Kingdom.
Description: Dynamics Days Europe is a major annual international conference aimed at bringing together theoretical and experimental scientists from a variety of disciplines, including nonlinear dynamics, theoretical physics, fluid dynamics and mathematics. University of Bristol is proud to host the Dynamics Days Europe 2010, which will take place on 6-10 September 2010 in Bristol, United Kingdom, starting on Monday morning and finishing on Friday noon. Conference will consist of invited presentations, mini-symposia, a limited number of contributed talks, and all participants are encouraged to submit poster presentations.
Information: http://www.enm.bris.ac.uk/anm/ddays10/index.html.

* 13–15 **IMA Hot Topics Workshop: Medical Device-Biological Interactions at the Material-Tissue Interface**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota.
Description: This workshop is to bring together a group of scientists, mathematicians, and physicians to discuss the device-biological interactions at the material-tissue interfaces from the perspective of medical efficacy and scientific mechanisms. Medical, engineering, and mathematic tools will be used. The goal of the workshop is to identify challenges and opportunities in the fields to stimulate collaborations among physicians, scientists, and mathematicians. Topics of the workshop include 1. Biological reactions to foreign material (implant), including blood clotting and inflammation reactions; 2. Bacterial colonization at implant surface and biofilm formation; 3. Responses of tissues to the electric stimulations and sensing; 4. Mechanical interactions between implants and the tissues: in vivo loading conditions and their effects on tissues (e.g., atrophy and hypertrophy).
Information: http://www.ima.umn.edu/2010-2011/SW9.13-15.10/.

* 19–22 **Applied Statistics 2010 (AS2010)**, Ribno (Bled), Slovenia.
Description: The conference, organized in Ribno in the vicinity of the beautiful Lake Bled, will provide an opportunity for researchers in statistics, data analysts, and other professionals from various statistical and related fields to come together, present their research, and learn from each other. Cross-discipline and applied paper submissions are especially welcome. A three days program consists of invited paper presentations and contributed paper sections from diverse topics. Selected papers will be published in *Advances in Methodology and Statistics*, a peer-reviewed journal of the Statistical Society of Slovenia.
Information: http://conferences.nib.si/AS2010.

October 2010

* 8–10 **Yamabe Symposium on "Geometry and Low-Dimensional Topology"**, University of Minnesota, Minneapolis, Minnesota.
Confirmed speakers: Toby Colding, Massachusetts Institute of Technology; Kenji Fukaya, Kyoto University; David Gabai, Princeton University; Ko Honda, University of Southern California; Claude LeBrun, State University of New York at Stony Brook; Melissa Liu, Columbia University; Yi Ni, California Institute of Technology; and Ron Stern, University of California at Irvine.
Support: Support from the National Science Foundation will be used to defray workshop expenses for a number of participants, with highest preference given to younger scientists (grad students, postdocs, young faculty or researchers at most five years after Ph.D.), although all active people are eligible. Women and minorities are especially encouraged to apply.
Deadline: The application deadline for full consideration for funding is: August 12, 2010.
Information: http://www.math.umn.edu/yamabe/.

* 18–22 **AIM Workshop: Algebraic systems with only real solutions**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of different versions of the B. and M. Shapiro Conjecture and related questions in real algebraic geometry.
Information: http://aimath.org/ARCC/workshops/realsolutions.html.

* 25–29 **AIM Workshop: The geometry of the outer automorphism group of a free group**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the outer automorphism group of the free group, Out(F); in particular, its geometry and its inherent asymmetry.
Information: http://aimath.org/ARCC/workshops/outerauto.html.

* 27–30 **Fourth International Workshop on Differential Algebra and Related Topics (DART-IV)**, Key Laboratory of Mathematics Mechanization, Chinese Academy of Sciences, Beijing, China.
Description: This workshop on Differential Algebra and Related Topics is the fourth in the series. Based on the foundational works of Ritt and Kolchin since the 1930s, differential algebra has evolved into an extremely rich subject during the last two decades. From traditional studies of differential equations using classical algebraic methods, differential algebra now embodies symbolic computation as well as abstract category and scheme theories. It has interactions with algebraic geometry, combinatorics, mathematical logic, computability, complexity analysis, differential algebraic equations on manifolds, Diophantine geometry, arithmetic differential equations, and more. Related algebraic structures, difference algebra, difference and differential schemes, Rota-Baxter algebra, and quantum differential operator algebra have made much progress, too. Many of these and other topics, as well as applications to cellular biology and control theory, will be presented at this workshop.
Information: http://mmrc.iss.ac.cn/~dart4/index.html.

December 2010

* 2–4 **4th Global Conference on Power Control and Optimization (PCO'2010)**, Damai Puri Resort, Kucing, Sarawak, Malaysia.
Description: Scope of the conference is contemporary and original research and educational development in the area of electrical power engineering, control systems and methods of optimization. Proposals for holding special sessions, tutorial, exhibition and workshop are invited from prospective authors, industrial bodies and academicians and should be addressed to the conference secretariat.
Organizing Committee: Nader Barsoum (Chairman); Pandian Vasant; Rabi W. Yousif; Jeffrey F. Webb.
Keynote Speakers: Milun Babic (Serbia); Tsu Tian Lee (Taiwan); Mingcong Deng (Japan); T. S. Zhao (Hong Kong; Yew Soon Wong (Singapore); Alan Oxley (Malaysia); M. Ahmad Al-Nimr (Jordan); Robert Grubbstrom (Sweden); Vladimir Fedorov (Russia); Wolfgang Borutzky (Germany); Vitezslav Benda (Czech Republic); Wilhelm Höflinger (Austria).
Deadline: Submission of camera-ready paper: 10/15/2010.
Information: Please contact Dr. Kisho and Dr. Vasant at: pcoglobal@gmail.com; http://www.pcoglobal.com; Venue: http://www.damaipuriresort-kuching.com.

* 6–10 **AIM Workshop: Waves and multiscale processes in the tropics**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will bring university-based mathematicians and physicists together with scientists from operational climate modeling centers to discuss the

new insights and the mathematical basis for tropical multiscale processes coming from observations and from theory, and the consequences of such insights for the simulation of these processes in fully complex global climate models.
Information: http://aimath.org/ARCC/workshops/multiscale.html.

* 13–17 **AIM Workshop: Random Matrices**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on recent developments on limiting distributions concerning both local and global properties of the spectrum of a random matrix.
Information: http://aimath.org/ARCC/workshops/randommatrices.html.

* 15–16 **The International Conference on Mathematics Education Research**, Malacca, Malaysia.
Description: International Conference on Mathematics Education Research 2010 (ICMER2010) is organized by Laboratory of Innovation in Mathematics Education from the Institute for Mathematical Research, Universiti Putra Malaysia. The aim of the conference is to open up a discussion platform for the analysis, development, exchange and critique of ideas on recent practices and research in mathematics education. It carries the theme of Towards Excellence in Mathematics Education.
Information: http://einspem.upm.edu.my/icmer2010.

January 2011

* 3–5 **ICMS 2011: International Conference on Mathematical Sciences in Honour of Professor A. M. Mathai**, St. Thomas College Pala, Kottayam 686574, Kerala, India.
Description: The conference is organized at the occasion of the 75th birthday of A.M. Mathai, a world renown mathematician and statistician with seminal achievements in mathematical statistics, applied statistics, probability, astrophysics, and information theory. The conference will schedule invited and contributed papers in all these research fields. The conference will also review the past achievements of the Centre for Mathematical Sciences, Pala, India, that is currently headed by Professor A.M. Mathai.
Information: http://www.cmsintl.org/.

* 17–21 **AIM Workshop: Deformation theory, patching, quadratic forms, and the Brauer group**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on the interaction between algebraic geometry and the structure theory of fields, particularly the use of deformation theory and patching.
Information: http://aimath.org/ARCC/workshops/deformbrauer.html.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

July 2011

* 11–15 **The 10th International Conference on Finite Fields and their Applications**, Ghent, Belgium.
Topics: Of interest include, but are not limited to: Field-theoretic aspects of finite fields: structure of finite fields, normal bases, polynomials, primitive elements, number-theoretic aspects of finite fields, exponential sums, arithmetic theory of function fields. Computational aspects: algorithms and complexity, polynomial factorization, decomposition and irreducibility testing, sequences and functions. Applications: algebraic coding theory, cryptography, affine and projective geometry over finite fields, algebraic geometry over finite fields, finite incidence geometry, designs and configurations, difference sets, combinatorics, generalisations of finite fields.
Information: http://cage.ugent.be/fq10/.

* 26–29 **Conference in Harmonic Analysis and Partial Differential Equations in honour of Eric Sawyer**, Fields Institute, Toronto, Canada.
Description: The meeting will concentrate on Analysis and Partial differential Equations, specifically in relation to the diverse set of topics Dr. Eric T. Sawyer has worked on and influenced during his rich and prolific career: Weighted norm inequalities, Several complex variables, Unique continuation, Singular integrals, fractional integrals, Elliptic equations and systems, Degenerate equations, Monge-Ampère equations, Number theory and Combinatorics.
Information: http://www.fields.utoronto.ca/programs/scientific/11-12/PDE/.

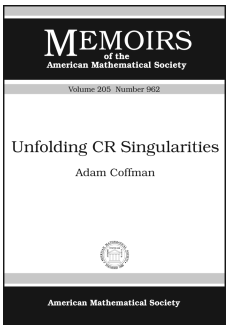
August 2011

* 1–5 **Conference in Honour of Søren Asmussen—New Frontiers in Applied Probability**, Sandbjerg Estate, Sønderborg, Denmark.
Description: The conference honours one of the leading researchers in applied probability, Søren Asmussen, on the occasion of his 65th birthday. Professor Asmussen's research cuts across many of the major themes of modern applied probability, including queueing theory, insurance and financial mathematics, stochastic simulation, and stochastic control. His nearly 150 publications include major contributions to the theory of heavy tails, rare-event asymptotics and related simulation methods, matrix-geometric modelling, Markov processes, and regeneration. He has also written four major books: *Branching Processes* (co-authored with H. Hering), *Applied Probability and Queues*; *Ruin Probabilities*; and *Stochastic Simulation: Algorithms and Analysis* (co-authored with P. Glynn). His outstanding research contributions have been recognized through the Marcel F. Neuts Applied Probability Prize (1999), and the INFORMS Outstanding Publication Prizes in Simulation in 2002 and 2008.
Information: http://www.thiele.au.dk/asmussen.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Analysis



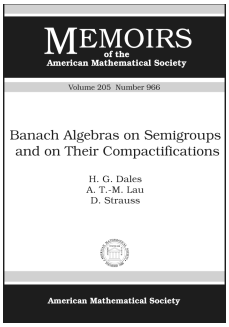
Unfolding CR Singularities

Adam Coffman, *Indiana University-Purdue University at Fort Wayne, IN*

Contents: *Unfolding CR singularities:* Introduction; Topological considerations; Local defining equations and transformations; A complexification construction; Real surfaces in $\mathbb{C}^2$; Real m-submanifolds in $\mathbb{C}^n$, $m < n$; Rapid convergence proof of the main theorem; Some other directions; Bibliography.

Memoirs of the American Mathematical Society, Volume 205, Number 962

April 2010, 90 pages, Softcover, ISBN: 978-0-8218-4657-5, LC 2010003516, 2000 *Mathematics Subject Classification:* 32V40, 32S30, 58K35, **Individual member US$40**, List US$67, Institutional member US$54, Order code MEMO/205/962



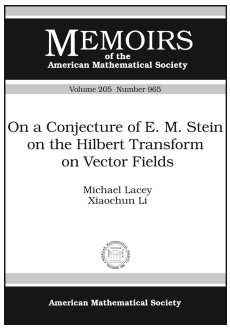
Banach Algebras on Semigroups and on Their Compactifications

H. G. Dales, *University of Leeds, England*, **A. T.-M. Lau**, *University of Alberta, Edmonton, AB, Canada*, and **D. Strauss**, *University of Leeds, England*

Contents: Introduction; Banach algebras and their second duals; Semigroups; Semigroup algebras; Stone–Čech compactifications; The semigroup $(\beta S, \square)$; Second duals of semigroup algebras; Related spaces and compactifications; Amenability for semigroups; Amenability of semigroup algebras; Amenability and weak amenability for certain Banach algebras; Topological centres; Open problems; Bibliography; Index of terms; Index of symbols.

Memoirs of the American Mathematical Society, Volume 205, Number 966

April 2010, 165 pages, Softcover, ISBN: 978-0-8218-4775-6, LC 2010003552, 2000 *Mathematics Subject Classification:* 43A10, 43A20; 46J10, **Individual member US$44**, List US$74, Institutional member US$59, Order code MEMO/205/966



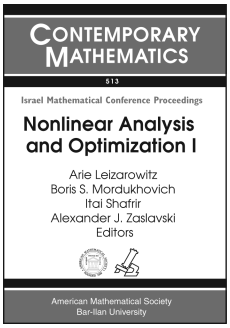
On a Conjecture of E. M. Stein on the Hilbert Transform on Vector Fields

Michael Lacey, *Georgia Institute of Technology, Atlanta, GA*, and **Xiaochun Li**, *University of Illinois, Urbana, IL*

Contents: Overview of principal results; Besicovitch set and Carleson's theorem; The Lipschitz Kakeya maximal function; The L^2 estimate; Almost orthogonality between annuli.

Memoirs of the American Mathematical Society, Volume 205, Number 965

April 2010, 72 pages, Softcover, ISBN: 978-0-8218-4540-0, LC 2010003519, 2000 *Mathematics Subject Classification:* 42A50, 42B25, **Individual member US$38**, List US$64, Institutional member US$51, Order code MEMO/205/965



Nonlinear Analysis and Optimization I

Nonlinear Analysis

Arie Leizarowitz, *Technion–Israel Institute of Technology, Haifa, Israel*, **Boris S. Mordukhovich**, *Wayne State University, Detroit, MI*, and **Itai Shafrir** and **Alexander J. Zaslavski**, *Technion–Israel Institute of Technology, Haifa, Israel*, Editors

This volume is the first of two volumes representing leading themes of current research in nonlinear analysis and optimization. The articles are written by prominent researchers in these two areas and bring the readers, advanced graduate students and researchers alike, to the frontline of the vigorous research in these important fields of mathematics.

This volume contains articles on nonlinear analysis. Topics covered include the convex feasibility problem, fixed point theory, mathematical biology, Mosco stability, nonexpansive mapping theory, nonlinear partial differential equations, optimal control, the proximal point algorithm and semigroup theory. The companion volume (Contemporary Mathematics, Volume 514) is devoted to optimization.

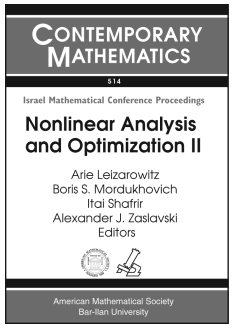
This item will also be of interest to those working in applications.

This book is co-published with Bar-Ilan University (Ramat-Gan, Israel).

Contents: A. S. Ackleh, **K. Deng**, and **Q. Huang**, Existence-uniqueness results and difference approximations for an amphibian juvenile-adult model; **S. Aizicovici**, **N. S. Papageorgiou**, and **V. Staicu**, Three nontrivial solutions for p-Laplacian Neumann problems with a concave nonlinearity near the origin; **V. Barbu**, Optimal stabilizable feedback controller for Navier-Stokes equations; **H. H. Bauschke** and **X. Wang**, Firmly nonexpansive and Kirszbraun-Valentine extensions: A constructive approach via monotone operator theory; **R. E. Bruck**, On the random product of orthogonal projections in Hilbert space II; **D. Butnariu**, **E. Resmerita**, and **S. Sabach**, A Mosco stability theorem for the generalized proximal mapping; **A. Cegielski**, Generalized relaxations of nonexpansive operators and convex feasibility problems; **Y. Censor** and **A. Segal**, Sparse string-averaging and split common fixed points; **T. Domínguez Benavides** and **S. Phothi**, Genericity of the fixed point property for reflexive spaces under renormings; **K. Goebel** and **B. Sims**, Mean Lipschitzian mappings; **T. Ibaraki** and **W. Takahashi**, Generalized nonexpansive mappings and a proximal-type algorithm in Banach spaces; **W. Kaczor**, **T. Kuczumow**, and **N. Michalska**, The common fixed point set of commuting nonexpansive mapping in Cartesian products of weakly compact convex sets; **L. Leuştean**, Nonexpansive iterations in uniformly convex W-hyperbolic spaces; **G. López**, **V. Martín-Marquez**, and **H.-K. Xu**, Halpern's iteration for nonexpansive mappings; **J. W. Neuberger**, Lie generators for local semigroups; **H.-K. Xu**, An alternative regularization method for nonexpansive mappings with applications.

Contemporary Mathematics, Volume 513

June 2010, approximately 276 pages, Softcover, ISBN: 978-0-8218-4834-0, LC 2009051904, 2000 *Mathematics Subject Classification:* 03-XX, 11-XX, 14-XX, 20-XX, 26-XX, 30-XX, 34-XX, 35-XX, 39-XX, 41-XX, 46-XX, 47-XX, 49-XX, 52-XX, 54-XX, 55-XX, 57-XX, 58-XX, 65-XX, 76-XX, 90-XX, **AMS members US$71**, List US$89, Order code CONM/513



Nonlinear Analysis and Optimization II

Optimization

Arie Leizarowitz, *Technion–Israel Institute of Technology, Haifa, Israel*, **Boris S. Mordukhovich**, *Wayne State University, Detroit, MI*, and **Itai Shafrir** and **Alexander J. Zaslavski**, *Technion–Israel Institute of Technology, Haifa, Israel*, Editors

This volume is the second of two volumes representing leading themes of current research in nonlinear analysis and optimization. The articles are written by prominent researchers in these two areas and bring the readers, advanced graduate students and researchers alike, to the frontline of the vigorous research in important fields of mathematics.

This volume contains articles on optimization. Topics covered include the calculus of variations, constrained optimization problems, mathematical economics, metric regularity, nonsmooth analysis, optimal control, subdifferential calculus, time scales and transportation traffic. The companion volume (Contemporary Mathematics, Volume 513) is devoted to nonlinear analysis.

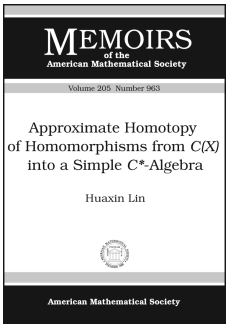
This item will also be of interest to those working in applications.

This book is co-published with Bar-Ilan University (Ramat-Gan, Israel).

Contents: J.-P. Aubin and **S. Martin**, Travel time tubes regulating transportation traffic; **R. Baier** and **E. Farkhi**, The directed subdifferential of DC functions; **Z. Balanov**, **W. Krawcewicz**, and **H. Ruan**, Periodic solutions to $O(2)$-symmetric variational problems: $O(2) \times S^1$- equivariant gradient degree approach; **J. F. Bonnans** and **N. P. Osmolovskii**, Quadratic growth conditions in optimal control problems; **J. M. Borwein** and **S. Sciffer**, An explicit non-expansive function whose subdifferential is the entire dual ball; **G. Buttazzo** and **G. Carlier**, Optimal spatial pricing strategies with transportation costs; **R. A. C. Ferreira** and **D. F. M. Torres**, Isoperimetric problems of the calculus of variations on time scales; **M. Foss** and **N. Randriampiry**, Some two-dimensional $\mathcal{A}$-quasiaffine functions; **F. Giannessi**, **A. Moldovan**, and **L. Pellegrini**, Metric regular maps and regularity for constrained extremum problems; **V. Y. Glizer**, Linear-quadratic optimal control problem for singularly perturbed systems with small delays; **T. Maruyama**, Existence of periodic solutions for Kaldorian business fluctuations; **D. Mozyrska** and **E. Pawłuszewicz**, Delta and nabla monomials and generalized polynomial series on time scales; **D. Pallaschke** and **R. Urbański**, Morse indexes for piecewise linear functions; **J.-P. Penot**, Error bounds, calmness and their applications in nonsmooth analysis; **F. Rampazzo**, Commutativity of control vector fields and "inf-commutativity"; **A. J. Zaslavski**, Stability of exact penalty for classes of constrained minimization problems in finite-dimensional spaces.

Contemporary Mathematics, Volume 514

June 2010, approximately 298 pages, Softcover, ISBN: 978-0-8218-4835-7, LC 2009051904, 2000 *Mathematics Subject Classification:* 03-XX, 11-XX, 14-XX, 20-XX, 26-XX, 30-XX, 34-XX, 35-XX, 39-XX, 41-XX, 46-XX, 47-XX, 49-XX, 52-XX, 54-XX, 55-XX, 57-XX, 58-XX, 65-XX, 76-XX, 90-XX, **AMS members US$71**, List US$89, Order code CONM/514



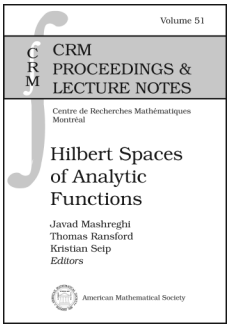
Approximate Homotopy of Homomorphisms from $C(X)$ into a Simple C^*-Algebra

Huaxin Lin, *University of Oregon, Eugene, OR*

Contents: Prelude; The basic homotopy lemma for higher dimensional spaces; Purely infinite simple C^*-algebras; Approximate homotopy; Super homotopy; Postlude; Bibliography.

Memoirs of the American Mathematical Society, Volume 205, Number 963

April 2010, 131 pages, Softcover, ISBN: 978-0-8218-5194-4, LC 2010003517, 2000 *Mathematics Subject Classification:* 46L05, 46L35, **Individual member US$41**, List US$69, Institutional member US$55, Order code MEMO/205/963



Hilbert Spaces of Analytic Functions

Javad Mashreghi and **Thomas Ransford**, *Université Laval, Montréal, QC, Canada*, and **Kristian Seip**, *Norwegian University of Science and Technology, Trondheim, Norway*, Editors

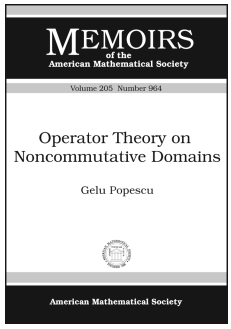
Hilbert spaces of analytic functions are currently a very active field of complex analysis. The Hardy space is the most senior member of this family. However, other classes of analytic functions such as the classical Bergman space, the Dirichlet space, the de Branges-Rovnyak spaces, and various spaces of entire functions, have been extensively studied. These spaces have been exploited in different fields of mathematics and also in physics and engineering. For example, de Branges used them to solve the Bieberbach conjecture. Modern control theory is another place that heavily exploits the techniques of analytic function theory. This book grew out of a workshop held in December 2008 at the CRM in Montréal and provides an account of the latest developments in the field of analytic function theory.

Titles in this series are co-published with the Centre de Recherches Mathématiques.

Contents: **J. A. Ball** and **V. Bolotnikov**, Canonical de Branges–Rovnyak model transfer-function realization for multivariable Schur-class functions; **N. Arcozzi**, **R. Rochberg**, and **E. Sawyer**, Two variations on the Drury–Averson space; **S. R. Garcia** and **W. T. Ross**, The norm of a truncated Toeplitz operator; **A. Boivin** and **C. Zhu**, Approximation in weighted Hardy spaces for the unit disc; **R. G. Douglas** and **J. Sarkar**, Some remarks on the Toeplitz corona problem; **E. Fricain** and **A. Hartmann**, Regularity on the boundary in spaces of holomorphic functions on the unit disk; **D. Khavinson** and **E. Lundberg**, The search for singularities of solutions to the Dirichlet problem: Recent developments; **O. El-Fallah**, **K. Kellay**, and **T. Ransford**, Invariant subspaces of the Dirichlet space; **J. Mashreghi**, **T. Ransford**, and **M. Shabankhah**, Arguments of zero sets in the Dirichlet space; **J. Zemánek**, Questions on Volterra operators; **D.-C. Chang**, **G. Dafni**, and **H. Yue**, Nonhomogeneous div-curl decompositions for local Hardy spaces on a domain; **R. Fournier** and **S. Ruscheweyh**, On the Bohr radius for simply connected plane domains; **A. Boivin** and **C. Zhu**, Completeness of the system $\{f(\lambda_n z)\}$ in $L^2_a[\Omega]$; **J. Mashreghi**, A formula for the logarithmic derivative and its applications; **H. Wulan** and **C. Xiong**, Composition operators on the minimal Möbius invariant space; **P. M. Gauthier**, Whether regularity is local for the generalized Dirichlet problem.

CRM Proceedings & Lecture Notes, Volume 51

May 2010, 214 pages, Softcover, ISBN: 978-0-8218-4879-1, 2000 *Mathematics Subject Classification:* 46E20, 46E22, 47B32, 31C25, **AMS members US$79**, List US$99, Order code CRMP/51



Operator Theory on Noncommutative Domains

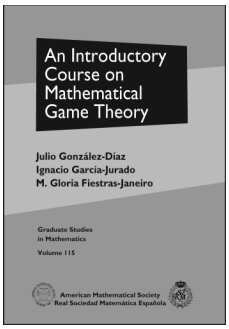
Gelu Popescu, *University of Texas at San Antonio, TX*

Contents: Introduction; Operator algebras associated with noncommutative domains; Free holomorphic functions on noncommutative domains; Model theory and unitary invariants on noncommutative domains; Commutant lifting and applications; Bibliography.

Memoirs of the American Mathematical Society, Volume 205, Number 964

April 2010, 124 pages, Softcover, ISBN: 978-0-8218-4710-7, LC 2010003518, 2000 *Mathematics Subject Classification:* 47A05, 47A56, 47A20, 46E40; 46L52, 46L07, 47A67, 47A63, 47A57, 47A60, **Individual member US$41**, List US$69, Institutional member US$55, Order code MEMO/205/964

Applications



An Introductory Course on Mathematical Game Theory

Julio González-Díaz, *Universidade de Santiago de Compostela, Spain*, **Ignacio García-Jurado**, *Universidade da Coruña, Spain*, and **M. Gloria Fiestras-Janeiro**, *Universidade de Vigo, Spain*

Game theory provides a mathematical setting for analyzing competition and cooperation in interactive situations. The theory has been famously applied in economics, but is relevant in many other sciences, such as political science, biology, and, more recently, computer science. This book presents an introductory and up-to-date course on game theory addressed to mathematicians and economists, and to other scientists having a basic mathematical background. The book is self-contained, providing a formal description of the classic game-theoretic concepts together with rigorous proofs of the main results in the field. The theory is illustrated through abundant examples, applications, and exercises.

The style is distinctively concise, while offering motivations and interpretations of the theory to make the book accessible to a wide readership. The basic concepts and results of game theory are given a formal treatment, and the mathematical tools necessary to develop them are carefully presented. Cooperative games are explained in detail, with bargaining and TU-games being treated as part of a general framework. The authors stress the relation between game theory and operations research.

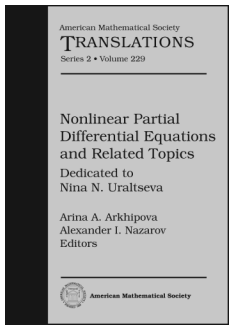
The book is suitable for a graduate or an advanced undergraduate course on game theory.

Contents: Introduction to decision theory; Strategic games; Extensive games; Games with incomplete information; Cooperative games; Bibliography; Notations; Index of authors; Index of solution concepts; Subject index.

Graduate Studies in Mathematics, Volume 115

June 2010, 324 pages, Hardcover, ISBN: 978-0-8218-5151-7, LC 2010000501, 2000 *Mathematics Subject Classification:* 91-01; 90C05, 05C57, **AMS members US$50**, List US$62, Order code GSM/115

Differential Equations



Nonlinear Partial Differential Equations and Related Topics

Dedicated to Nina N. Uraltseva

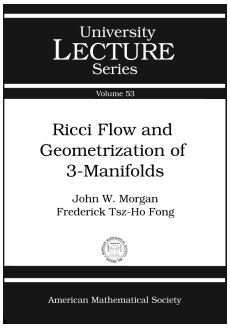
Arina A. Arkhipova and **Alexander I. Nazarov**, *St. Petersburg State University, Russia*, Editors

This book contains papers that engage a wide set of classical and modern topics in partial differential equations, including linear and nonlinear equations, variational problems, the Navier–Stokes system, and the Boltzmann equation. The results include existence and uniqueness theorems, qualitative properties of solutions, a priori estimates, and nonexistence theorems.

Contents: **J. Andersson**, **H. Shahgholian**, and **G. S. Weiss**, Regularity below the C^2 threshold for a torsion problem, based on regularity for Hamilton-Jacobi equations; **A. Arkhipova**, Signorini-type problem in $\mathbb{R}^N$ for a class of quadratic functionals; **M. Bildhauer** and **M. Fuchs**, A 2D-invariant of a theorem of Uraltseva and Urdaletova for higher order variational problems; **M. Bostan**, **I. M. Gamba**, and **T. Goudon**, The linear Boltzmann equation with space periodic electric field; **L. Caffarelli** and **L. Silvestre**, Smooth approximations of solutions to nonconvex fully nonlinear elliptic equations; **P. Constantin** and **G. Seregin**, Hölder continuity of solutions of 2D Navier-Stokes equations with singular forcing; **M. Giaquinta**, **P. Mariano**, **G. Modica**, and **D. Mucci**, Currents and curvature varifolds in continuum mechanics; **N. M. Ivochkina**, On classic solvability of the m-Hessian evolution equation; **N. V. Krylov**, About an example of N. N. Ural'tseva and weak uniqueness for elliptic operators; **V. Maz'ya** and **R. McOwen**, On the fundamental solution of an elliptic equation in nondivergence form; **G. Mingione**, Boundary regularity for vectorial problems; **A. Nazarov** and **A. Reznikov**, Attainability of infima in the critical Sobolev trace embedding theorem on manifolds; **M. V. Safonov**, Non-divergence elliptic equations of second order with unbounded drift; **V. V. Zhikov** and **S. E. Pastukhova**, Global solvability of Navier-Stokes equations for a nonhomogeneous non-Newtonian fluid.

American Mathematical Society Translations—Series 2 (*Advances in the Mathematical Sciences*), Volume 229

June 2010, approximately 257 pages, Hardcover, ISBN: 978-0-8218-4997-2, 2000 *Mathematics Subject Classification:* 35-06, 35F20, 35G20, 35J60, 35K55, 35Q30, 35J15, 35J20, 35J50, **AMS members US$95**, List US$119, Order code TRANS2/229



Ricci Flow and Geometrization of 3-Manifolds

John W. Morgan, *Stony Brook University, NY*, and **Frederick Tsz-Ho Fong**, *Stanford University, CA*

This book is based on lectures given at Stanford University in 2009. The purpose of the lectures and of the book is to give an introductory overview of how to use Ricci flow and Ricci flow with surgery to establish the Poincaré Conjecture and the more general Geometrization Conjecture for 3-dimensional manifolds. Most of the material is geometric and analytic in nature; a crucial ingredient is understanding singularity development for 3-dimensional Ricci flows and for 3-dimensional Ricci flows with surgery. This understanding is crucial for extending Ricci flows with surgery so that they are defined for all positive time. Once this result is in place, one must study the nature of the time-slices as the time goes to infinity in order to deduce the topological consequences.

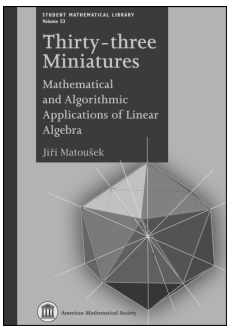
The goal of the authors is to present the major geometric and analytic results and themes of the subject without weighing down the presentation with too many details. This book can be read as an introduction to more complete treatments of the same material.

Contents: *Overview:* Lecture 1; Lecture 2; Lecture 3; Lecture 4; Lecture 5; Summary of Part 1; *Non-collapsing results for Ricci flows:* Lecture 6; Lecture 7; Lecture 8; Lecture 9; Lecture 10; Lecture 11; Lecture 12; *κ-solutions:* Lecture 13; Lecture 14; Lecture 15; Lecture 16; Lecture 17; Lecture 18; Lecture 19; *The canonical neighborhood theorem:* Lecture 20; Lecture 21; Lecture 22; *Ricci flow with surgery:* Lecture 23; Lecture 24; Lecture 25; Lecture 26; *Behavior as $t \to \infty$:* Lecture 27; Lecture 28; Lecture 29; Lecture 30; Lecture 31; Lecture 32; Bibliography.

University Lecture Series, Volume 53

May 2010, 150 pages, Softcover, ISBN: 978-0-8218-4963-7, LC 2010003310, 2000 *Mathematics Subject Classification:* 57M50, 57M40; 35K40, 53C21, **AMS members US$33**, List US$41, Order code ULECT/53

Discrete Mathematics and Combinatorics



Thirty-three Miniatures

Mathematical and Algorithmic Applications of Linear Algebra

Jiří Matoušek, *Charles University, Prague, Czech Republic*

This volume contains a collection of clever mathematical applications of linear algebra, mainly in combinatorics, geometry, and algorithms. Each chapter covers a single main result with motivation and full proof in at most ten pages and can be read independently of all other chapters (with minor exceptions), assuming only a modest background in linear algebra.

The topics include a number of well-known mathematical gems, such as Hamming codes, the matrix-tree theorem, the Lovász bound on the Shannon capacity, and a counterexample to Borsuk's conjecture, as well as other, perhaps less popular but similarly beautiful results, e.g., fast associativity testing, a lemma of Steinitz on ordering vectors, a monotonicity result for integer partitions, or a bound for set pairs via exterior products.

The simpler results in the first part of the book provide ample material to liven up an undergraduate course of linear algebra. The more advanced parts can be used for a graduate course of linear-algebraic methods or for seminar presentations.

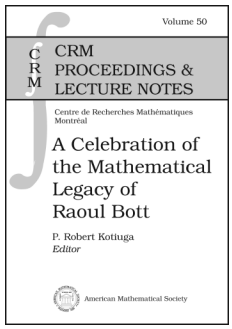
This item will also be of interest to those working in algebra and algebraic geometry.

Contents: Fibonacci numbers, quickly; Fibonacci numbers, the formula; The clubs of Oddtown; Same-size intersections; Error-correcting codes; Odd distances; Are these distances Euclidean?; Packing complete bipartite graphs; Equiangular lines; Where is the triangle?; Checking matrix multiplication; Tiling a rectangle by squares; Three Petersens are not enough; Petersen, Hoffman–Singleton, and maybe 57; Only two distances; Covering a cube minus one vertex; Medium-size intersection is hard to avoid; On the difficulty of reducing the diameter; The end of the small coins; Walking in the yard; Counting spanning trees; In how many ways can a man tile a board?; More bricks—more walls?; Perfect matchings and determinants; Turning a ladder over a finite field; Counting compositions; Is it associative?; The secret agent and umbrella; Shannon capacity of the union: a tale of two fields; Equilateral sets; Cutting cheaply using eigenvectors; Rotating the cube; Set pairs and exterior products; Index.

Student Mathematical Library, Volume 53

June 2010, approximately 171 pages, Softcover, ISBN: 978-0-8218-4977-4, 2000 *Mathematics Subject Classification:* 05C50, 68Wxx, 15-01, **AMS members US$29**, List US$36, Order code STML/53

Geometry and Topology



A Celebration of the Mathematical Legacy of Raoul Bott

P. Robert Kotiuga, *Boston University, MA*, Editor

A five-day conference celebrating the legacy of Raoul Bott was held at the CRM on June 9–13, 2008. The conference focused on the extraordinary impact Bott had on both topology and interactions between mathematics, physics and technology. The conference was co-organized by the Clay Mathematics Institute and had support from the National Science Foundation (Award 0805925). Montreal was a natural venue for such an event since Raoul Bott obtained two degrees in electrical engineering at McGill University in the 1940s and an honorary doctorate from McGill in 1987. The fact that Bott's presence is still

fresh in the minds of all those involved made for a tremendous amount of enthusiasm and every attempt has been made to channel this energy into this book.

The contributions to this book come from three generations of Bott's students, coauthors, and fellow kindred spirits in order to cover six decades of Bott's research, identify his enduring mathematical legacy and the consequences for emerging fields. The contributions can be read independently. In order to help a whole to emerge from the parts, the book is broken into four sections and to make the book accessible to a wide audience, each section starts with easier-to-read reminiscences and works its way into more involved papers.

Titles in this series are co-published with the Centre de Recherches Mathématiques.

Contents: P. R. Kotiuga, Introduction; *Montréal, the 1940s, and mathematical prehistory:* **C. Bott**, My parents' Montréal years and growing up with Raoul as my father; **J. Lambek**, Raoul Bott, McGill, the 1940s; **P. R. Kotiuga**, Iron rings, Doctor Honoris Causa Raoul Bott, Carl Herz, and a hidden hand; **J. H. Hubbard**, The Bott–Duffin synthesis of electrical circuits; *Early students and colleagues:* **P. D. Lax**, **F. Hirzebruch**, **B. Mazur**, **L. Conlon**, **E. B. Curtis**, **H. M. Edwards**, **J. Huebschmann**, and **H. Shulman**, Raoul Bott as we knew him; **M. Atiyah**, Working with Raoul Bott: From geometry to physics; **H. M. Edwards**, The algorithmic side of Riemann's mathematics; **M. W. Hirsch**, Actions of Lie groups and Lie algebras on manifolds; **J. J. Kohn**, PDE from the point of view of multiplier ideals; **P. Baum**, Dirac operator and K-theory for discrete groups; **J. L. Heitsch**, The Lefschetz principle, fixed point theory, and index theory; **J. Cantwell** and **L. Conlon**, A new look at the theory of levels; **A. Haefliger**, On the space of morphisms between Étale groupoids; *Localization, equivariance and outgrowths of Morse theory and periodicity:* **J. A. Bernhard**, **N. Hingston**, **J. Stasheff**, and **V. Guillemin**, Raoul Bott as we knew him; **N. Hingston**, Loop products on connected sums of projective spaces; **J. A. Bernhard**, Equivariant cohomology and reflections; **L. Jeffrey**, Connectedness of level sets of the moment map for torus actions on the based loop group; **L. W. Tu**, Computing characteristic numbers using fixed points; **H. Hohnhold**, **S. Stolz**, and **P. Teichner**, From minimal geodesics to supersymmetric field theories; *Dualities and interactions with quantum field theory:* **C. Vafa**, Raoul Bott as my math teacher; **S. Lu**, A physics colloquium at McGill that changed my life; **E. Witten**, Geometric Langlands from six dimensions; **J. Block**, Duality and equivalence of module categories in noncommutative geometry; **G. R. Cavalcanti** and **M. Gualtieri**, Generalized complex geometry and T-duality; **D. S. Freed**, **M. J. Hopkins**, **J. Lurie**, and **C. Teleman**, Topological quantum field theories from compact Lie groups.

CRM Proceedings & Lecture Notes, Volume 50

May 2010, 403 pages, Softcover, ISBN: 978-0-8218-4777-0, LC 2010001317, 2000 *Mathematics Subject Classification:* 01-XX, 18-XX, 19-XX, 35-XX, 55-XX, 57-XX, 58-XX, 81-XX, **AMS members US$100**, List US$125, Order code CRMP/50

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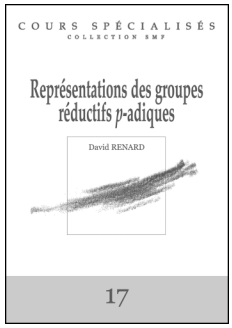
Algebra and Algebraic Geometry



Représentations des Groupes Réductifs p-adiques

David Renard, *Ecole Polytechnique, Palaiseau, France*

This book presents a part of the theory of (complex) representations of p-adic reductive groups. Starting from fundamentals accessible to graduate students, it culminates with the "Bernstein center" theory and the Langlands classification of smooth irreducible representations.

This book contains seven chapters. Chapters VI and VII are at the heart of the book. Chapter VI deals with the study of the category of smooth representations of a p-adic reductive group and the author establishes, among other things, Bernstein's decomposition theorem and the description of the center. Chapter VII deals with square integrable and temperate representations and contains the proof of Langland's classification theorem.

The first four chapters are more general and deal with: the study of algebras of idempotents, the one of locally compact totally discontinuous spaces and groups, smooth representations of the latter and specific representation classes (compact, unitary, square integrable). Chapter V is a reminder of structural results for reductive p-adic groups.

An appendix provides category theory notions necessary for reading this text.

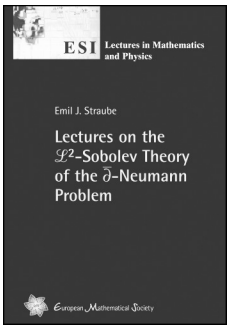
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Algèbres à idempotents; Espaces et groupes totalement discontinous; Représentations des groupes totalement discontinous; Représentations compactes, de carré intégrable, unitaires; Structure des groupes réductifs p-adiques; Représentations des groupes réductifs p-adiques; Classification de Langlands; Éléments de théorie des catégories; Théorème d'Amitsur et corollaires; Algèbre linéaire; Bibliographie.

Cours Spécialisés—Collection SMF, Number 17

January 2010, 332 pages, Hardcover, ISBN: 978-2-85629-278-5, 2000 *Mathematics Subject Classification:* 22E50, 20G05, **Individual member US$81**, List US$90, Order code COSP/17

Analysis



Lectures on the $\mathcal{L}^2$-Sobolev Theory of the $\bar{\partial}$-Neumann Problem

Emil J. Straube, *Texas A & M University, College Station, TX*

This book provides a thorough and self-contained introduction to the $\bar{\partial}$-Neumann problem, leading up to current research, in the context of the $\mathcal{L}^2$-Sobolev theory on bounded pseudoconvex domains in $\mathbb{C}^n$. It grew out of courses for advanced graduate students and young researchers given by the author at the Erwin Schrödinger International Institute for Mathematical Physics and at Texas A & M University.

The introductory chapter provides an overview of the contents and puts them in historical perspective. The second chapter presents the basic $\mathcal{L}^2$-theory. Following is a chapter on the subelliptic estimates on strictly pseudoconvex domains. The two final chapters on compactness and on regularity in Sobolev spaces bring the reader to the frontiers of research.

Prerequisites are a solid background in basic complex and functional analysis, including the elementary $\mathcal{L}^2$-Sobolev theory and distributions. Some knowledge in several complex variables is helpful. Concerning partial differential equations, not much is assumed. The elliptic regularity of the Dirichlet problem for the Laplacian is quoted a few times, but the ellipticity results needed for elliptic regularization in the third chapter are proved from scratch.

This item will also be of interest to those working in differential equations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; The $\mathcal{L}^2$-theory; Strictly pseudoconvex domains; Compactness; Regularity in Sobolev spaces; Bibliography; Index.

ESI Lectures in Mathematics and Physics, Volume 7

February 2010, 214 pages, Softcover, ISBN: 978-3-03719-076-0, 2000 *Mathematics Subject Classification:* 32-02, 32W05, 35N15, **AMS members US$45**, List US$56, Order code EMSESILEC/7

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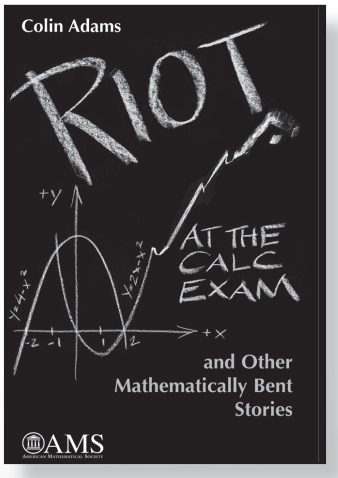

Riot at the Calc Exam and Other Mathematically Bent Stories

Colin Adams, *Williams College, Williamstown, MA*

This collection of humorous mathematics stories gives the reader a sense of what it would be like to participate in a mathematical world that is far from dreary. Several of the ideas for the stories were derived from lunchtime discussions at the humorous and friendly mathematics department at Williams College. The stories resonate with both mathematicians and laypeople with an interest in mathematics.

2009; 271 pages; Softcover;
ISBN: 978-0-8218-4817-3;
List US$32; AMS members US$26;
Order code MBK/62

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Departamento de Matemáticas,
Pontificia Universidad Católica de Chile,
Av. Vicuña Mackenna 4860,
Santiago, Chile;
fax: (56-2) 552-5916;
email: `mchuaqui@mat.puc.cl`

For full consideration, complete application materials must arrive by June 30, 2010.

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CYPRUS

CYPRUS
University of Cyprus
Department of Mathematics and Statistics

The Department of Mathematics and Statistics of the University of Cyprus, invites applications for one position in the field of Probability-Statistics at the rank of Lecturer or Assistant Professor. The official languages of the University are Greek and/or Turkish. For the above position knowledge of Greek is necessary. The deadline for applications is July 16th, 2010. For more information, see: `http://www.ucy.ac.cy/goto/mathstatistics/el-GR/Vacancies.aspx`.

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Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2010 rate is $3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: June/July 2010 issue–April 28, 2010; August 2010 issue–May 28, 2010; September 2010 issue–June 28, 2010; October 2010 issue–July 29, 2010; November 2010 issue–August 30, 2010; September 2010 issue–June 28, 2010.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices.* However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Albuquerque, New Mexico

University of New Mexico

April 17–18, 2010

Saturday – Sunday

Meeting #1059

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2010
Program first available on AMS website: March 4, 2010
Program issue of electronic *Notices*: April 2010
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Kenneth Bromberg, University of Utah, *The topology of deformation spaces of Kleinian groups.*

Ioana Dumitriu, University of Washington, *Random matrix theory, numerical linear algebra, and scientific computing: Border interactions.*

Steffen Rohde, University of Washington, *Random conformal maps.*

Special Sessions

Dyadic and Non-Dyadic Harmonic Analysis, **M. Cristina Pereyra**, University of New Mexico, and **Stephanie A. Salomone**, University of Portland.

Financial Mathematics: The Mathematics of Financial Markets and Structures, **Maria Cristina Mariani**, University of Texas at El Paso, **Ionut Florescu**, Stevens Institute of Technology, and **Maria P. Beccar-Varela**, University of Texas at El Paso.

Function Spaces, PDEs and Nonlinear Analysis, **Osvaldo Mendez**, **Behzad Rouhani**, and **Mohamed Amine Khamsi**, University of Texas at El Paso.

Geometric Combinatorics, **Art M. Duval**, University of Texas at El Paso, and **Jeremy Martin**, University of Kansas.

Geometric Function Theory, **Lukas Geyer**, Montana State University, and **Donald Marshall** and **Steffen Rohde**, University of Washington.

Geometric Structures and PDEs, **Charles Boyer** and **Dimiter Vassilev**, University of New Mexico.

Harmonic Analysis and Partial Differential Equations, **Matthew Blair**, University of New Mexico, and **Hart Smith**, University of Washington.

Kleinian Groups and Teichmüller Theory, **Kasra Rafi**, University of Oklahoma, **Hossein Namaze**, University of Texas, and **Kenneth Bromberg**, University of Utah.

Positivity in Noncommutative Settings, **Roger Roybal**, California State University Channel Islands, and **Terry Loring**, University of New Mexico.

Random Matrix Theory and Applications, **Ioana Dumitriu**, University of Washington, and **Raj Rao**, University of Michigan.

Selected Topics in Analysis and Numerics for PDEs, **Thomas Hagstrom**, Southern Methodist University, and **Stephen Lau** and **Jens Lorenz**, University of New Mexico.

Strongly-Nonlinear Phenomena: Theory and Applications to Nonlinear Optics, Hydrodynamics, Bose–Einstein Condensation and Biology, **Alejandro Aceves**, Southern Methodist University, and **Alexander Korotkevich** and **Pavel Lushnikov**, University of New Mexico.

Subjects in between Pure and Applied Mathematics, **Hanna Makaruk** and **Robert Owczarek**, Los Alamos National Laboratory.

Topics in Geometric Group Theory, **Matthew Day**, California Institute of Technology, **Daniel Peter Groves**, University of Illinois at Chicago, **Jason Manning**, SUNY at Buffalo, and **Henry Wilton**, California Institute of Technology.

Trends in Commutative Algebra, **Louiza Fouli**, New Mexico State University, and **Janet Vassilev**, University New Mexico.

Newark, New Jersey

New Jersey Institute of Technology

May 22–23, 2010

Saturday – Sunday

Meeting #1060

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: March 2010
Program first available on AMS website: April 8, 2010
Program issue of electronic *Notices*: May 2010
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Simon Brendle, Stanford University, *Hamilton's Ricci flow and the sphere theorem in geometry.*

Konstantin M. Mischaikow, Rutgers University, *Computational topology applied to the global dynamics of nonlinear systems.*

Ricardo H. Nochetto, University of Maryland, *Curvature driven flows in deformable domains.*

Richard E. Schwartz, Brown University, *Polygonal outer billiards.*

Special Sessions

Automorphic Forms, L-functions, and Applications, **Ameya Pitale**, American Institute of Mathematics, and **Anantharam Raghuram**, Oklahoma State University.

Biomembranes: Modeling, Analysis, and Computation, **Ricardo H. Nochetto** and **Dionisios Margetis**, University of Maryland.

Elliptic and Parabolic Problems in Geometry, **Simon Brendle**, Stanford University, and **Mu-Tao Wang**, Columbia University.

Expandable Computations, Algorithms, Methodologies and Experiments for Engineering Interpretation, **Mustapha S. Fofana**, Worcester Polytechnic Institute, **Marie D. Dahleh**, Harvard School of Engineering and Applied Sciences, Harvard University, and **Kenji Kawashima**, Precision and Intelligence Laboratory, Tokyo Institute of Technology.

Financial Mathematics, **Tim S.T. Leung**, Johns Hopkins University.

Graph Theory, **Nathan W. Kahl**, Seton Hall University, **Michael J. Ferrara**, University of Colorado at Denver, and **Arthur H. Busch**, University of Dayton.

Groups, Computations, and Applications, **Delaram Kahrobaei**, City University of New York.

Homology Theories for Knots and Skein Modules, **Mikhail Khovanov**, Columbia University, and **Jozef H. Przytycki** and **Radmila Sazdanovic**, George Washington University.

Invariants of Knots, Links, and 3-Manifolds, **Abhijit Champanerkar** and **Ilya S. Kofman**, College of Staten Island, CUNY, and **Philip J. P. Ording**, Medgar Evers College, CUNY.

Lie Algebras and Representation Theory, **Gautam Chinta**, City College, City University of New York, **Andrew Douglas**, New York City College of Technology, City University of New York, and **Bart Van Steirteghem**, Medgar Evers College, City University of New York.

Logic and Groups, **Peggy Dean**, **Claire Wladis**, and **Marcos Zyman**, Borough of Manhattan Community College, City University of New York.

Mathematical Neuroscience: Modeling, Analysis, and Simulations, **Horacio G. Rotstein**, New Jersey Institute of Technology.

Mathematics and Computations of Fluid Dynamics, **Yuan N. Young**, New Jersey Institute of Technology.

Mathematics of Optics and Matter Waves, **Roy Goodman**, New Jersey Institute of Technology.

Nonlinear Waves, **A. David Trubatch**, Montclair State University.

Recent Trends in Cayley Graphs to Model Interconnection Networks, **Daniela Ferrero**, Texas State University, and **Beth Novick**, Clemson University.

Teichmüller Theory, Hyperbolic Geometry, and Complex Dynamics, **Zheng Huang**, College of Staten Island, CUNY, and **Ren Guo**, University of Minnesota.

Topological and Computational Dynamics, **Jean-Philippe Lessard**, Institute for Advanced Study and Rutgers University, and **Konstantin M. Mischaikow**, Rutgers University.

Vortex Dynamics: Theory and Applications, **Denis Blackmore**, New Jersey Institute of Technology, **Morten Brøns**, Technical University of Denmark, and **Chjan Lim**, Rochester Polytechnic Institute.

Berkeley, California

University of California Berkeley

June 2–4, 2010

Wednesday – Friday

Meeting #1061

Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: April 2010
Program first available on AMS website: April 22, 2010
Program issue of electronic *Notices*: June 2010
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Alejandro Adem, University of British Columbia and PIMS, *Homotopy theory and spaces of representations.*

Peter W.-K. Li, University of California, Irvine, *What do we know about open manifolds?*

Ernesto Lupercio, CINVESTAV, *Title to be announced.*

Victor Perez Abreu, CIMAT, *On convolutions and infinite divisibility of probability measures.*

Alberto Verjovsky, IM-UNAM, *Title to be announced.*

Maciej Zworski, University of California Berkeley, *Random perturbations in discrete quantization.*

Special Sessions

Algebraic Topology and Related Topics, **Alejandro Adem**, University of British Columbia, **Gunnar E. Carlsson** and **Ralph L. Cohen**, Stanford University, and **Ernesto Lupercio**, CINVESTAV.

Analytic Aspects of Differential Geometry, **Nelia Charalambous**, ITAM, **Lizhen Ji**, University of Michigan, and **Jiaping Wang**, University of Minnesota.

Commutative Algebra and Representation Theory, **David Eisenbud** and **Daniel M. Erman**, University of California, Berkeley, **Jose Antonio de la Pena**, UNAM, and **Rafael Villareal**, Cinvestav-IPN.

Complex Analysis and Operator Theory, **Maribel Loaiza**, **Enrique Ramirez de Arellano**, and **Nikolai Vasilevski**, CINVESTAV, **Ilya M. Spitkovsky**, College of William & Mary, and **Kehe Zhu**, State University of New York at Albany.

Dynamical Systems, **Alberto Verjovsky**, IM-UNAM, and **Rodrigo Perez**, Indiana University-Purdue University, Indianapolis.

Graph Theory and Combinatorics with Emphasis on Geometric and Topological Aspects, **Gelasio Salazar**, Instituto de Fisica, Universidad Autonoma de San Luis Potosi, and **Dan S. Archdeacon**, University of Vermont.

Harmonic Analysis, Microlocal Analysis, and Partial Differential Equations, **Gunther Uhlmann**, University of Washington, and **Salvador Perez Esteva**, UNAM.

Low-Dimensional Topology, **Kenneth L. Baker**, University of Miami, and **Enrique Ramirez Losada**, CIMAT.

Singularity Theory and Algebraic Geometry, **David Eisenbud**, University of California, Berkeley, **Anatoly S. Libgober**, University of Illinois at Chicago, **Jose Seade**, UNAM, and **Xavier Gomez-Mont**, CIMAT.

Töplitz Operators and Discrete Quantum Models, **Alejandro Uribe**, University of Michigan, and **Maciej Zworski**, University of California, Berkeley.

Syracuse, New York

Syracuse University

October 2–3, 2010

Saturday – Sunday

Meeting #1062

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June/July 2010
Program first available on AMS website: August 19, 2010
Program issue of electronic *Notices*: October
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 15, 2010
For abstracts: August 10, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Alan Frieze, Carnegie-Mellon University, *Title to be announced.*

Yan Guo, Brown University, *Title to be announced.*

William Minicozzi, Johns Hopkins University, *Title to be announced.*

Andrei Zelevinsky, Northeastern University, *Title to be announced.*

Special Sessions

Advances in Theory and Applications of Evolution Equations (Code: SS 14A), **Tokia Diagana**, Howard University, and **Gaston Ní Guerekata**, **Alexander Pankov**, **Xuming Xie**, and **Guoping Zhang**, Morgan State University.

Analysis, Probability and Mathematical Physics on Fractals (Code: SS 9A), **Luke Rogers**, University of Connecticut,

Robert Strichartz, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Analytic Combinatorics (Code: SS 13A), **Miklos Bona**, University of Florida, and **Alex Iosevich**, University of Rochester.

Commutative Algebra and Algebraic Geometry (Code: SS 8A), **Anthony Geramita**, Queen's University, **Graham Leuschke** and **Claudia Miller**, Syracuse University, and **Michael Stillman**, Cornell University.

Difference Equations and Applications (Code: SS 2A), **Michael Radin**, Rochester Institute of Technology.

Geometric Analysis and Flows (Code: SS 12A), **William P. Minicozzi II**, Johns Hopkins University, **Xiaodong Cao**, Cornell University, and **Junfang Li**, University of Alabama at Birmingham.

Graphs Embedded in Surfaces, and Their Symmetries (Code: SS 4A), **Jack E. Graver** and **Mark E. Watkins**, Syracuse University.

Harmonic Analysis (Code: SS 10A), **Dmitriy Bilyk**, University of South Carolina, and **Svitlana Mayboroda**, Purdue University.

Lie Algebras and Representation Theory (Code: SS 15A), **David Hemmer**, State University of New York at Buffalo, and **Emilie Wiesner**, Ithaca College.

Mathematical Image Processing (Code: SS 5A), **Lixin Shen** and **Yuesheng Xu**, Syracuse University.

Nonlinear Analysis and Geometry (Code: SS 1A), **Tadeusz Iwaniec**, **Leonid V. Kovalev**, and **Jani Onninen**, Syracuse University.

Quasiconformal Mappings, Riemann Surfaces, and Teichmüller Spaces (in honor of Clifford J. Earle) (Code: SS 7A), **Yunping Jiang**, Queens College and The Graduate Center, City University of New York, and **Sudeb Mitra**, Queens College, City University of New York.

Representations of Algebras (Code: SS 6A), **Ed Green**, Virginia Polytechnic Institute, **Mark Kleiner** and **Dan Zacharia**, Syracuse University, and **Andrei Zelevinsky**, Northeastern University.

Several Complex Variables (Code: SS 3A), **Dan F. Coman** and **Evgeny A. Poletsky**, Syracuse University.

Topology and Combinatorics (Code: SS 11A), **Laura Anderson**, SUNY Binghamton, and **Patricia Hersh**, North Carolina State University.

Los Angeles, California

University of California Los Angeles

October 9–10, 2010

Saturday – Sunday

Meeting #1063

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2010
Program first available on AMS website: August 26, 2010
Program issue of electronic *Notices*: October 2010
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 22, 2010
For abstracts: August 17, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Greg Kuperberg, University of California Davis, *Title to be announced.*

Cris Moore, University of New Mexico, *Title to be announced.*

Stanley Osher, University of California Los Angeles, *Title to be announced.*

Terence Tao, University of California Los Angeles, *The cosmic distance ladder* (Einstein Public Lecture in Mathematics).

Melanie Wood, Princeton University, *Title to be announced.*

Special Sessions

Algebraic Structures in Knot Theory (Code: SS 8A), **Sam Nelson**, Claremont McKenna College, and **Carmen Caprau**, California State University Fresno.

Applications of Nonlinear PDE (Code: SS 5A), **Susan J. Friedlander** and **Igor Kukavica**, University of Southern California.

Automorphic Forms and Number Theory (Code: SS 12A), **William Duke**, University of California Los Angeles, **Ozlem Imamoglu**, ETH Zurich, and **Kimberly Hopkins**, University of California Los Angeles.

Combinatorics and Probability on Groups (Code: SS 3A), **Jason Fulman** and **Robert Guralnick**, University of Southern California, and **Igor Pak**, University of California Los Angeles.

Continuous and Discrete Dynamical Systems (Code: SS 11A), **Mario Martelli**, Claremont Graduate University, and **Robert Sacker**, University of Southern California.

Extremal and Probabilistic Combinatorics (Code: SS 4A), **Benny Sudakov**, University of California Los Angeles, and **Jacques Verstraete**, University of California San Diego.

Global Geometric Analysis (Code: SS 13A), **William Wylie**, University of Pennsylvania, **Joseph E. Borzellino**, California State University San Luis Obispo, and **Peter Petersen**, University of California Los Angeles.

Harmonic Analysis (Code: SS 9A), **Christoph Thiele**, University of California Los Angeles, and **Ignacio Uriarte-Tuero** and **Alexander Volberg**, Michigan State University.

Homotopy Theory and K-theory (Code: SS 10A), **Julie Bergner**, University of California Riverside, and **Christian Haesemeyer**, University of California Los Angeles.

Large Cardinals and the Continuum (Code: SS 2A), **Matthew Foreman**, University of California Irvine, **Alekos Kechris**, California Institute for Technology, **Itay Neeman**,

University of California Los Angeles, and **Martin Zeman**, University of California Irvine.

Mathematical Models of Random Phenomena (Code: SS 7A), **Mark Burgin**, University of California Los Angeles, and **Alan C. Krinik**, California State Polytechnic University Pomona.

Mathematics of Criminality (Code: SS 14A), **Andrea Bertozzi**, **Martin Short**, and **George Mohler**, University of California Los Angeles.

Recent Trends in Probability and Related Fields (Code: SS 6A), **Marek Biskup**, University of California Los Angeles, **Yuval Peres**, Microsoft Research, and **Sebastien Roch**, University of California Los Angeles.

Topology and Symplectic Geometry (Code: SS 1A), **Robert Brown** and **Ciprian Manolescu**, University of California Los Angeles, and **Stefano Vidussi**, University of California Riverside.

Notre Dame, Indiana

Notre Dame University

November 5–7, 2010

Friday – Sunday

Meeting #1064

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: September 2010
Program first available on AMS website: September 23, 2010
Program issue of electronic *Notices*: November 2010
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Laura DeMarco, University of Illinois at Chicago, *Title to be announced*.

Jordan Ellenberg, University of Wisconsin, *Title to be announced*.

David Fisher, Indiana University, *Title to be announced*.

Jared Wunsch, Northwestern University, *Title to be announced*.

Special Sessions

Algebraic and Topological Combinatorics (Code: SS 9A), **John Shareshian**, Washington University, and **Bridget Tenner**, DePaul University.

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), **Claudia Polini**, University of Notre Dame, **Alberto Corso**, University of Kentucky, and **Bernd Ulrich**, Purdue University.

Complex Analysis and Dynamical Systems (Code: SS 15A), **Laura DeMarco**, University of Illinois at Chicago, and **Jeffrey Diller**, University of Notre Dame.

Computability and Its Applications (Code: SS 11A), **Peter Cholak**, **Peter Gerdes**, and **Karen Lange**, University of Notre Dame.

Differential Geometry and its Applications (Code: SS 16A), **Jianguo Cao** and **Brian Smyth**, University of Notre Dame.

Geometry and Lie Theory (Code: SS 10A), **John Caine** and **Samuel Evens**, University of Notre Dame.

Groups, Representations, and Characters (Code: SS 4A), **James P. Cossey**, University of Akron, and **Mark Lewis**, Kent State University.

Hilbert Functions in Commutative Algebra and Algebraic Combinatorics (Code: SS 3A), **Fabrizio Zanello**, Michigan Technological University, **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Interdisciplinary Session on Deterministic and Stochastic Partial Differential Equations (Code: SS 5A), **Nathan Glatt-Holtz**, Indiana University, and **Vlad Vicol**, University of Southern California.

Nonlinear Evolution Equations (Code: SS 7A), **Alex Himonas** and **Gerard Misiolek**, University of Notre Dame.

Number Theory and Physics (Code: SS 8A), **Adrian Clingher**, University of Missouri St. Louis, **Charles Doran**, University of Alberta, **Shabnam N. Kadir**, Wilhelm Leibniz Universitat, and **Rolf Schimmrigk**, Indiana University.

Numerical Algebraic Geometry (Code: SS 13A), **Daniel J. Bates**, Colorado State University, **Jonathan D. Hauenstein**, Texas A&M University, **Andrew J. Sommese**, University of Notre Dame, and **Charles W. Wampler**, General Motors.

Quasigroups, Loops, and Nonassociative Division Algebras (Code: SS 6A), **Clifton E. Ealy**, Western Michigan University, **Stephen Gagola**, University of Arizona, **Julia Knight**, University of Notre Dame, **J. D. Phillips**, Northern Michigan University, and **Petr Vojtechovsky**, University of Denver.

Singularities in Algebraic Geometry (Code: SS 1A), **Nero Budur**, University of Notre Dame, and **Lawrence Ein**, University of Illinois at Chicago.

Topology, Geometry and Physics (Code: SS 14A), **Ralph Kaufmann**, Purdue University, and **Stephan Stolz**, University of Notre Dame.

Undergraduate Mathematics Education: A Vision for the 21st Century (Code: SS 12A), **Steven Broad**, St. Mary's College, **Nahid Erfan** and **Alex Himonas**, University of Notre Dame, and **Morteza Shafii-Mousavi**, Indiana University South Bend.

Richmond, Virginia

University of Richmond

November 6–7, 2010

Saturday – Sunday

Meeting #1065

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: September 2010
Program first available on AMS website: September 23, 2010
Program issue of electronic *Notices*: November 2010
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Matthew H. Baker, Georgia Institute of Technology, *Title to be announced.*

Michael J. Field, University of Houston, *Title to be announced.*

Sharon R. Lubkin, North Carolina State University, *Title to be announced.*

Stefan Richter, University of Tennessee, Knoxville, *Title to be announced.*

Special Sessions

Applications of Non-Archimedean Geometry (Code: SS 6A), **Matthew H. Baker**, Georgia Institute of Technology.

Codes and Designs (Code: SS 13A), **James A. Davis**, University of Richmond, and **Qing Xiang**, University of Delaware.

Computational and Applied Mathematics (Code: SS 11A), **Ludwig Kohaupt**, Beuth University, and **Mohammad Siddique**, Fayetteville State University.

Convexity and Combinatorics (Code: SS 9A), **Valeriu Soltan** and **James F. Lawrence**, George Mason University.

Differential Equations and Applications to Physics and Biology (Code: SS 8A), **Junping Shi**, College of William and Mary, and **Zhifu Xie**, Virginia State University.

Geometry of Banach Spaces and Connections with Other Areas (Code: SS 12A), **Frank Sanacory**, College at Old Westbury, and **Kevin Beanland**, Virginia Commonwealth University.

Kac-Moody Algebras, Vertex (Operator) Algebras, and Applications (Code: SS 7A), **William J. Cook**, Appalachian State University, and **Kailash C. Misra**, North Carolina State University.

Mathematical Models in Biology and Medicine (Code: SS 10A), **Lester Caudill**, University of Richmond.

Mathematics and the Arts (Code: SS 5A), **Michael J. Field**, University of Houston, **Gary R. Greenfield**, University of Richmond, and **Reza Sarhangi**, Towson University.

Minimum Rank Problems (Code: SS 3A), **Lon H. Mitchell**, Virginia Commonwealth University, and **Sivaram K. Narayan**, Central Michigan University.

Operator Theory (Code: SS 2A), **Stefan Richter**, University of Tennessee, and **William T. Ross**, University of Richmond.

Statistical Properties of Dynamical Systems (Code: SS 4A), **Michael J. Field** and **Matthew J. Nicol**, University of Houston.

Topics in Graph Theory (Code: SS 1A), **Daniel W. Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Pucon, Chile

December 15–18, 2010

Wednesday – Saturday

Meeting #1066

First Joint International Meeting between the AMS and the Sociedad de Matematica de Chile.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2010
Program first available on AMS website: N/A
Program issue of electronic *Notices*: N/A
Issue of *Abstracts*: N/A

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

AMS Invited Addresses

Ricardo Baeza, Universidad de Talca, Chile, *Title to be announced.*

Igor Dolgachev, University of Michigan, *Title to be announced.*

Andres Navas, Universidad de Santiago de Chile, *Title to be announced.*

Rodolfo Rodriguez, Universidad de Concepcion, *Title to be announced.*

Gunther Uhlmann, University of Washington, *Title to be announced.*

S. R. Srinivasa Varadhan, New York University, *Title to be announced.*

AMS Special Sessions

Arithmetic of Quadratic Forms and Integral Lattices, **Maria Ines Icaza**, Universidad de Talca, Chile, **Wai Kiu Chan**, Wesleyan University, and **Ricardo Baeza**, Universidad de Talca, Chile.

Automorphic Forms and Dirichlet Series, **Yves Martin**, Universidad de Chile, Chile, and **Solomon Friedberg**, Boston College.

Complex Algebraic Geometry, **Giancarlo Urzua** and **Eduardo Cattani**, University of Massachusetts.

Foliations and Dynamics, **Andrés Navas**, Universidad de Santiago de Chile, and **Steve Hurder**, University of Illinois at Chicago.

Group Actions: Probability and Dynamics, **Andres Navas**, Universidad de Santiago de Chile, and **Rostislav Grigorchuk**, University of Texas.

Inverse Problems and PDE Control, **Matias Courdurier**, Pontificia Universidad Católica de Chile, **Axel Osses**, Universidad de Chile, and **Gunther Uhlmann**, University of Washington.

Non-Associative Algebras, **Alicia Labra**, Universidad de Chile, and **Kevin McCrimmon**, University of Virginia.

Probability and Mathematical Physics, **Hui-Hsiung Kuo**, Louisiana State University, and **Rolando Rebolledo**, Pontificia Universidad Catolica de Chile.

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday

Meeting #1067

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Statesboro, Georgia

Georgia Southern University

March 12–13, 2011

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Harmonic Analysis and Partial Differential Equations (Code: SS 1A), **Paul A. Hagelstein**, Baylor University, **Alexander Stokolos**, Georgia Southern University, **Xiaoyi Zhang**, IAS Princeton and University of Iowa, and **Shijun Zheng**, Georgia Southern University.

Iowa City, Iowa

University of Iowa

March 18–20, 2011

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 16, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Worcester, Massachusetts

College of the Holy Cross

April 9–10, 2011

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 9, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Las Vegas, Nevada

University of Nevada

April 30 – May 1, 2011

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Advances in Modeling, Numerical Analysis and Computations of Fluid Flow Problems (Code: SS 2A), **Monika Neda**, University of Nevada Las Vegas.

Geometric PDEs (Code: SS 1A), **Matthew Gursky**, Notre Dame University, and **Emmanuel Hebey**, Université de Cergy-Pontoise.

Ithaca, New York

Cornell University

September 10–11, 2011

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 10, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lincoln, Nebraska

University of Nebraska-Lincoln

October 14–16, 2011

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2011
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Salt Lake City, Utah

University of Utah

October 22–23, 2011

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Port Elizabeth, Republic of South Africa

Nelson Mandela Metropolitan University

November 29 – December 3, 2011

Tuesday – Saturday
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday
Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 10, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention & Trade Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: **Georgia Benkart**, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: **Steven H. Weintraub**, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: **Matthew Miller,** Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

2010

April 17–18	Albuquerque, New Mexico	p. 685
May 22–23	Newark, New Jersey	p. 686
June 2–4	Berkeley, California	p. 687
October 2–3	Syracuse, New York	p. 687
October 9–10	Los Angeles, California	p. 688
November 5–7	Notre Dame, Indiana	p. 689
November 6–7	Richmond, Virginia	p. 690
December 15–18	Pucon, Chile	p. 690

2011

January 5–8	New Orleans, Louisiana Annual Meeting	p. 691
March 12–13	Statesboro, Georgia	p. 691
March 18–20	Iowa City, Iowa	p. 691
April 9–10	Worcester, Massachusetts	p. 692
April 30–May 1	Las Vegas, Nevada	p. 692
September 10–11	Ithaca, New York	p. 692
October 14–16	Lincoln, Nebraska	p. 692
October 22–23	Salt Lake City, Utah	p. 692
November 29–December 3	Port Elizabeth, Republic of South Africa	p. 693

2012

January 4–7	Boston, Massachusetts Annual Meeting	p. 693
March 10–11	Tampa, Florida	p. 693
March 30–April 1	Lawrence, Kansas	p. 693

2013

January 9–12	San Diego, California Annual Meeting	p. 693

2014

January 15–18	Baltimore, Maryland Annual Meeting	p. 694

2015

January 10–13	San Antonio, Texas Annual Meeting	p. 694

2016

January 6–9	Seattle, Washington Annual Meeting	p. 694

2017

January 4–7	Atlanta, Georgia Annual Meeting	p. 694

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 92 in the January 2010 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

Co-sponsored conferences:

May 24-29, 2010: From Carthage to the World, the Isoperimetric Problem of Queen Dido and its Mathematics Ramifications, Carthage, Tunisia (for more information please see http://math.arizona.edu/~dido/welcome.html).

June 12-July 2, 2010: Mathematics Research Communities, Snowbird, VT (for more information please see http://www.ams.org/amsmtgs/mrc.html).

June 17-19, 2010: Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal (for more information please see http://www.mat.uc.pt/~cmf/01MatrixTheory).

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–Max Lieblich, University of Washington

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–R News

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–MAA Reviews

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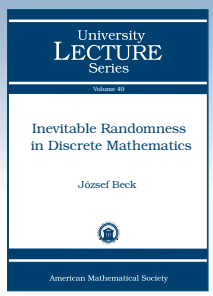

Inevitable Randomness in Discrete Mathematics

József Beck, *Rutgers, The State University of New Jersey, Piscataway, NJ*

Examples and rigorous proofs of the complexity law, offering new insights through the study of interesting concrete systems

Volume 49; 2009; 250 pages; Softcover; ISBN: 978-0-8218-4756-5; List US$59; AMS members US$47; Order code ULECT/49

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—EMS Newsletter

Volume 38; 2006; 162 pages; Softcover; ISBN: 978-0-8218-3644-6; List US$35; AMS members US$28; Order code ULECT/38

Lectures on Coarse Geometry

John Roe, *Pennsylvania State University, University Park, PA*

Volume 31; 2003; 175 pages; Softcover; ISBN: 978-0-8218-3332-2; List US$41; AMS members US$33; Order code ULECT/31

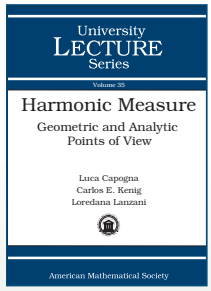
Lectures on Harmonic Analysis

Thomas H. Wolff (edited by Izabella Łaba, *University of British Columbia, Vancouver, Canada*, and **Carol Shubin,** *California State University Northridge*)

Lovely little book … deft and forceful writing … considerable selectivity … Proofs are tastefully either provided or sketched, so that the reader has a palpable sense of how the subject works … a delightful and satisfying reading experience, leaving the reader with a strong desire to push on and learn more … Both graduate students and experienced mathematicians will learn a great deal from this volume.

—Mathematical Reviews

Volume 29; 2003; 137 pages; Softcover; ISBN: 978-0-8218-3449-7; List US$33; AMS members US$26; Order code ULECT/29



Harmonic Measure

Geometric and Analytic Points of View

Luca Capogna, *University of Arkansas, Fayetteville, AR*, **Carlos E. Kenig**, *University of Chicago, IL*, and **Loredana Lanzani**, *University of Arkansas, Fayetteville, AR*

This book is a good introduction to an exciting new research area on the interface of harmonic analysis and geometric measure theory. The book is very well written, with clear explanations and useful pictures. It negotiates a fine compromise between brevity and detail as it presents a subject that is necessarily somewhat technical.

—Bulletin of the American Mathematical Society

Volume 35; 2005; 155 pages; Softcover; ISBN: 978-0-8218-2728-4; List US$37; AMS members US$30; Order code ULECT/35

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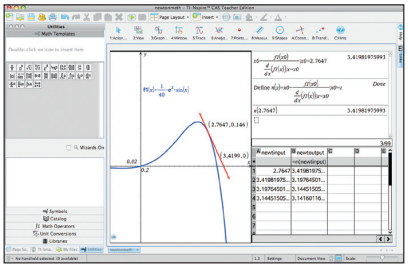

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