2011 Bôcher Prize

Assaf Naor and Gunther Uhlmann received the 2011 AMS Maxime Bôcher Memorial Prize at the 117th Annual Meeting of the AMS in New Orleans in January 2011.

Citation: Assaf Naor

The Bôcher Prize is awarded to Assaf Naor for introducing new invariants of metric spaces and for applying his new understanding of the distortion between various metric structures to theoretical computer science, especially in the papers "On metric Ramsey type phenomena" (with Yair Bartal, Nathan Linial, and Manor Mendel, *Annals of Math.* (2) **162** (2005) no. 2, 643–709); "Metric cotype" (with Manor Mendel, *Annals of Math.* (2) **168** (2008), no. 1, 247–298); and "Euclidean distortion and the sparsest cut" (with Sanjeev Arora and James R. Lee, *J. Amer. Math. Soc.* **21** (2008), no. 1, 1–21). The prize also recognizes Naor's remarkable work with J. Cheeger and B. Kleiner on a lower bound in the sparsest cut problem. The bound follows from a quantitative version of Cheeger and Kleiner's beautiful differentiation theorem for Lipschitz functions with values in L^1, which was itself motivated by this application, as envisioned by Naor and J. R. Lee.

Assaf Naor

Biographical Sketch: Assaf Naor

Assaf Naor was born in Rehovot, Israel, on May 7, 1975. He received his Ph.D. from the Hebrew University in Jerusalem under the direction of Joram Lindenstrauss. He was a postdoctoral researcher at the Theory Group of Microsoft Research in Redmond, Washington, from 2002 to 2004, and a researcher at the Theory Group from 2004 to 2006. Since 2006 he has been a faculty member at the Courant Institute of Mathematical Sciences of New York University, where he is a professor of mathematics and an associated faculty member in computer science. He received the Bergmann Memorial Award (2007), the European Mathematical Society Prize (2008), the Packard Fellowship (2008), and the Salem Prize (2008), and he was an invited speaker at the International Congress of Mathematicians (2010). Naor's research is focused on analysis and metric geometry and their interactions with approximation algorithms, combinatorics, and probability.

Response: Assaf Naor

I am immensely grateful for the Bôcher Memorial Prize. Above all, I wish to thank my collaborators, especially Manor Mendel, with whom a decade-long collaboration and friendship has resulted in exciting, sometimes unexpected, discoveries. I thank my advisor Joram Lindenstrauss for his inspiration and encouragement. I am also grateful to Gideon Schechtman for being my unofficial advisor, collaborator, and friend, and for the mentoring and friendship of Keith Ball.

In 1964 Lindenstrauss published a seminal paper entitled "On nonlinear projections in Banach spaces". This paper contains results showing that Banach spaces exhibit unexpected rigidity phenomena: the existence of certain nonlinear mappings that are quantitatively continuous (e.g., uniformly continuous or Lipschitz) implies that certain linear properties are preserved. Subsequent work of Enflo on Hilbert's fifth problem in infinite dimensions and then a beautiful 1976 theorem of Ribe ushered in a new era in metric geometry.

Ribe's theorem in particular showed that local linear properties of Banach spaces are preserved under uniform homeomorphisms and can thus be conceivably formulated in a way that involves only distance computations, ignoring entirely the linear structure. Of course, if this could be done, one could then investigate which metric spaces have these properties, even if the geometric spaces in question have nothing to do with linear spaces (e.g., Riemannian manifolds, graphs, or discrete groups). Since at roughly the same time a deep theory of quantitative local linear invariants of Banach spaces was flourishing, this raised the possibility that this powerful linear theory could be applied in much greater generality to geometric problems that are ubiquitous in mathematics. Undoubtedly many mathematicians noticed the great potential here, but to the best of my knowledge this program was first formulated in writing by Bourgain in 1986.

The 1980s witnessed a remarkable surge of activity in this area, yielding several theorems on general metric spaces that are inspired by previously known linear results: these include metric space versions of the theory of Rademacher type, a nonlinear Dvoretzky theorem, and extension theorems for Lipschitz functions, as well as important results such as Bourgain's embedding theorem and Bourgain's metrical characterization of superreflexivity. Key players in this 1980s "golden age" include Ball, Bourgain, Gromov, Johnson, Lindenstrauss, Milman, Pisier, Schechtman, and Talagrand.

One reason that might explain why the 1980s golden age ended is that there were several stubborn problems that led to an incomplete theory, and one needed new definitions of concepts, such as a notion of Rademacher cotype for metric spaces, in order to proceed. One of the papers that is mentioned in the citation formulated a definition of metric cotype and proved several accompanying theorems and applications showing that it is a satisfactory notion of cotype for metric spaces. This work was a result of years of intensive effort and required insights that involve geometry, combinatorics, and harmonic analysis.

The quantitative structural theory of metric spaces also received renewed impetus due to the important 1995 paper of Linial, London, and Rabinovich that exhibited the usefulness of these problems (specifically Bourgain's embedding theorem) to the design of efficient approximation algorithms for NP-hard problems. The general theme here is that a natural way to understand the structure of a metric space is by representing it as faithfully as possible as points in a well-understood normed space, or as a superposition of simpler structures such as trees. The relevance of such problems to computer science ranges from "obvious" to roundabout and surprising: many algorithmic tasks, such as nearest neighbor search and clustering, are by definition questions on metric spaces, while metric structures sometimes appear in problems that do not have a priori geometry in them, such as the structures that arise from continuous relaxations of combinatorial optimization problems. This general philosophy of embedding theory is responsible for many important approximation algorithms and data structures. The work contained in the two other papers that are mentioned in the citation has been partially inspired by these applications: sharp nonlinear Dvoretzky theorems are applied to online algorithms and data structures; and new embedding methods manage to asymptotically determine (up to lower order terms) the most nonEuclidean finite subset of L^1 (completing the 1969 work of Enflo), and at the same time they yield the best known approximation algorithm for the sparsest cut problem with general demands. The methods behind these results are in essence multiscale analysis, combined with a variety of probabilistic techniques.

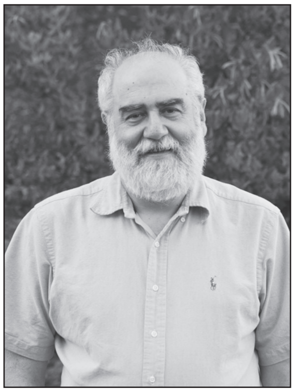
The quantitative structure theory of metric spaces is flourishing, with important new results appearing frequently. New algorithmic applications are constantly being discovered, and deep connections are being made with harmonic analysis, group theory, and geometric measure theory. There is still a lot that we do not know in the original program that is motivated by Ribe's theorem, including missing metric invariants that have yet to be defined. Experience shows that future progress will be difficult but fruitful, and the fact that many talented mathematicians and computer scientists are working in this field suggests that exciting developments are yet to come. I view the Bôcher Memorial Prize as a recognition of the achievements of this field and as an encouragement to the many researchers who developed the theory thus far to continue their efforts to uncover the hidden structure of metric spaces.

Citation: Gunther Uhlmann

The Bôcher Prize is awarded to Gunther Uhlmann for his fundamental work on inverse problems and in particular for the solution to the Calderón problem in the papers "The Calderón problem with partial data" (with Carlos E. Kenig and Johannes Sjöstrand, *Annals of Math.* (2) **165** (2007) no. 2, 567–591) and "The Calderón problem with partial data in two dimensions" (with Oleg Yu. Imanuvilov and Masahiro Yamamoto, *J. Amer Math. Soc.* **23** (2010), no. 3, 655–691). The prize also recognizes Uhlmann's incisive work on boundary rigidity with L. Pestov and with P. Stepanov and on nonuniqueness (also known as cloaking) with A. Greenleaf, Y. Kurylev, and M. Lassas.

Biographical Sketch: Gunther Uhlmann

Gunther Uhlmann was born in Quillota, Chile, in 1952. He studied mathematics as an undergraduate at the Universidad de Chile in Santiago, gaining his *Licenciatura* degree in 1973. He continued his studies at MIT, where he received a Ph.D. in 1976 under the direction of Victor Guillemin. He held postdoctoral positions at MIT, Harvard, and the Courant Institute. In 1980 he became assistant professor at MIT and then moved in 1985 to the University of Washington, where he was appointed Walker Family Endowed Professor in 2006. Since 2010 he has also held the Endowed Excellence in Teaching Chair at the University of California, Irvine. Uhlmann received a Sloan Research Fellowship in 1984 and a Guggenheim Fellowship in 2001. Also in 2001 he was elected a corresponding member of the Chilean Academy of Sciences. He has been a fellow of the Institute of Physics since 2004. He was elected to the American Academy of Arts and Sciences in 2009 and elected a SIAM fellow in 2010. He was an invited speaker at ICM in Berlin in 1998 and a plenary speaker at ICIAM in Zurich in 2007. He was named a Highly Cited Researcher by ISI in 2004.



Gunther Uhlmann

Response: Gunther Uhlmann

I am greatly honored by being named a corecipient of the 2011 Bôcher Memorial Prize. I would like to start by thanking the collaborators who are named in the citation; it was a great pleasure to work with them. I would also like to acknowledge my many other collaborators and my graduate students and postdocs who have enriched my life both professionally and personally. Many people were very influential in my career. Warren Ambrose made it possible for me to go to graduate school at MIT, and he was a continuous source of support and encouragement, especially in my early years in the United States. Herbert Clemens also helped me to come to the United States, and he has been an example to emulate in my life. My advisor Victor Guillemin taught me so much—he has a contagious enthusiasm for mathematics. Richard Melrose shared with me many times his great insight, and he has been a true friend. I met Alberto Calderón during my graduate studies at MIT; he is my mathematical hero, such an original mathematician. The year I was at Courant, I had the great fortune of meeting Louis Nirenberg. He taught me many things in mathematics and is the kindest person I have ever met—a wonderful role model for anybody to follow. I also treasured the friendship I started with Cathleen Morawetz during my stay at Courant. Most of all I have had the unwavering support of my family, Carolina, Anita, and Eric. Without them this would not have been possible. Carolina would have been so proud. This prize is for her.

About the Prize

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–1910. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the AMS, or the paper should have been published in a recognized North American journal. The prize is given every three years and carries a cash award of US$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2011 prize, the members of the selection committee were Alberto Bressan, Reese Harvey, and David Jerison.

Previous recipients of the Bôcher Prize are G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), Thomas Wolff (1999), Daniel Tataru (2002), Terence Tao (2002), Fanghua Lin (2002), Frank Merle (2005), and Charles Fefferman, Carlos Kenig, and Alberto Bressan (2008).