

2011 Doob Prize

PETER KRONHEIMER and TOMASZ MROWKA received the 2011 AMS Joseph Doob Prize at the 117th Annual Meeting of the AMS in New Orleans in January 2011. They were honored for their book *Monopoles and Three-Manifolds* (Cambridge University Press, 2007).

Citation

The study of three- and four-dimensional manifolds has been transformed by the development of gauge theories adapted from mathematical physics. The appearance of gauge-theoretic invariants of four-manifolds led to Donaldson's discovery of pairs of four-manifolds that were homeomorphic but not diffeomorphic. For three-manifolds, a generalization of Morse theory introduced by Floer gave a home to the solutions of the Yang-Mills equations and their topological interpretations. In the 1990s Seiberg and Witten developed a more direct approach to the riches of gauge invariants. The book by Kronheimer and Mrowka presents an ambitious and thorough account of these ideas and their consequences.

The construction of instanton homology by Floer begins with Morse theory and the anti-self-dual Yang-Mills equation. Substituting the Seiberg-Witten equations leads to three variants of Floer homology that the authors develop and explain. To do this they need substantial foundations in analysis, geometry, and topology. Some of this material—including basic Morse theory for manifolds with boundary, sharper compactness results, and functoriality for Floer theory—appears with details for the first time in this book.

Three-manifolds are a rich source of geometric phenomena, including foliations, contact structures, surgery, and knots. The potency of the monopole techniques is demonstrated in the final two chapters of the book, in which calculations and further work are discussed and all of these phenomena are related to Seiberg-Witten-Floer theory. Future researchers interested in manifold theory will surely develop these tools further. Their apprenticeship will be the book by Kronheimer and Mrowka. The authors deserve the Doob Prize for the breadth and depth of their exposition, as well as the care with which they make this rich and technical subject accessible.

Biographical Sketch

Born in London, Peter Kronheimer was educated at the City of London School and Merton College, Oxford. He obtained his B.A. in 1984 and his D.Phil.



Peter Kronheimer

in 1987 under the supervision of Michael Atiyah. After a year as a junior research fellow at Balliol and two years at the Institute for Advanced Study, he returned to Merton as fellow and tutor in mathematics. In 1995 he moved to Harvard, where he is now the William Caspar Graustein Professor of Mathematics. He is a recipient of the Förderpreis from the Mathematisches Forschungsinstitut, Oberwolfach, and the Whitehead Prize from the London Mathematical Society. He is a corecipient (with Tomasz Mrowka) of the Oswald Veblen Prize from the American Mathematical Society and was elected a fellow of the Royal Society in 1997. Peter lives in Newton, Massachusetts, with his wife, Jenny, and two sons, Matthew and Jonathan.

Tomasz Mrowka was born in State College, Pennsylvania, and lived in Kalamazoo, Michigan, and Amherst, New York, while following his father's academic career. He was an undergraduate at MIT from 1979 to 1983. In 1983 he entered graduate school at the University of California, Berkeley, and studied with Clifford Taubes and Robion Kirby. Taubes moved to Harvard in 1985, and Mrowka went along from 1985 to 1988 as



Tomasz Mrowka

a visiting graduate student, where he also studied with John Morgan. He received his Ph.D. from UC Berkeley in 1989. After graduate school he held postdoctoral positions at MSRI (1988–89), Stanford (1989–91), and Caltech (1991–92). He was promoted to full professor at Caltech in 1992 and remained on the faculty until 1996. He was a visiting professor at Harvard (spring of 1995) and at MIT (fall of 1995) before returning to MIT as a professor of mathematics in the fall of 1996. He was appointed to the Simons Professorship in

2007, and in 2010 the name of his chair changed to the Singer Professor of Mathematics.

Mrowka received the National Young Investigator Grant from the NSF in 1993 and was a Sloan Foundation Fellow from 1993 to 1995. In 2007 Kronheimer and Mrowka were jointly awarded the Veblen Prize in Geometry from the AMS. Mrowka is a fellow of the American Academy of Arts and Sciences, class of 2007, and was awarded a Guggenheim Fellowship for the 2009 academic year. Mrowka works mainly on the analytic aspects of gauge theories and applications of gauge theory to problems in low-dimensional topology.

Outside mathematics, Mrowka enjoys travel, good food, bicycling, swimming, hiking, and most of all spending time with his wife Gigliola Staffilani and twins Mario and Sofia.

Response

We are honored and delighted to hear that we have been awarded the Joseph L. Doob Prize. The book we wrote, *Monopoles and Three-Manifolds*, evolved out of our wish to better understand the construction of monopole Floer homology. At the time that we started writing it in the summer of 2000, we had been working for some time on a more ambitious project: we were trying to understand the Floer homology that would arise from the nonabelian monopole equations, with a view to using that understanding in a proof of the Property P conjecture for knots. While the substantial extra difficulties of the nonabelian case remained out of reach, we gradually realized that the usual (abelian) monopoles, though generally regarded as well understood in the folklore of the field, already had many aspects that had never been clearly treated in the literature. We embarked on writing some notes, with the aim of perhaps producing a short book on the subject. A year later, we had a manuscript of about 120 pages, and we reported to the National Science Foundation that the project was nearly complete. It remained “nearly complete” while continuing to grow over the next few years, until it had increased in size by a factor of six.

There were several reasons for this growth. One was that the scope of the project naturally increased as the field continued to develop. New applications of Floer homology were found; the work of Ozsváth and Szabó on Heegaard Floer homology revealed new structures that greatly influenced our approach to the exposition; and the calculation of several interesting examples came within reach. A second reason for the growth was that we were motivated to develop new approaches to several of the technical aspects of the theory (perturbations, gluing, and compactness results, among others) in order to write a text that would be applicable to other, similar problems in geometry. Many of the ideas and techniques that eventually emerged were not known to us when we began: examples

include the extensive use of a blown-up space as a setting for gluing theorems and calculations for a type of coupled Morse theory that seems to be of independent interest. Finally, the book grew because we were determined not to rely on the assumed understanding that such things could be done, but instead to show, wherever possible, exactly how to do these things.

While the Property P conjecture was eventually proved by other means, we are particularly pleased that the results and calculations that went into the manuscript have seen real application. Particularly notable was the proof by Cliff Taubes of the Weinstein conjecture in dimension three, a result which motivated us in the push to the finish. We also hope that we have written a book whose self-contained treatment of several topics in geometric analysis will be useful to students in nearby fields.

We would like to thank our families for their love, support, patience, and understanding during the long writing process. Many thanks to all the mathematicians whose ideas are reflected in the book and to the staff at Cambridge University Press for working with us as we completed the manuscript. Finally, we thank the American Mathematical Society and the selection committee for recognizing our work in this way.

About the Prize

The Doob Prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob (1910–2004). Paul Halmos (1916–2006) was Doob’s first Ph.D. student. Doob received his Ph.D. from Harvard in 1932 and three years later joined the faculty at the University of Illinois, where he remained until his retirement in 1978. He worked in probability theory and measure theory, served as AMS president in 1963–1964, and received the AMS Steele Prize in 1984. The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize of US\$5,000 is given every three years.

The Doob Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2011 prize, the members of the selection committee were Harold P. Boas, Andrew Granville, Robin Hartshorne, Neal I. Koblitz, and John H. McCleary.

The previous recipients of the Doob Prize are William P. Thurston (2005) and Enrico Bombieri and Walter Gubler (2008).