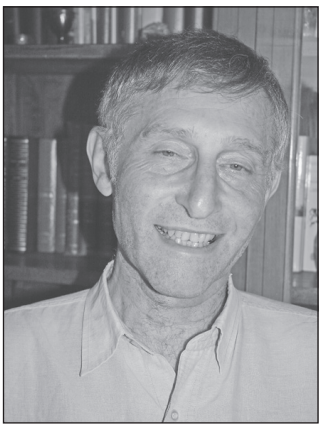
2011 Cole Prize in Number Theory

CHANDRASHEKHAR KHARE and JEAN-PIERRE WINTENBERGER received the 2011 AMS Frank Nelson Cole Prize in Number Theory at the 117th Annual Meeting of the AMS in New Orleans in January 2011.

Citation

The 2011 Frank Nelson Cole Prize in Number Theory is awarded to Chandrashekhar Khare and Jean-Pierre Wintenberger for their remarkable proof of Serre's modularity conjecture. In 1973 Jean-Pierre Serre made the audacious and influential conjecture that any irreducible two-dimensional representation of the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ that is odd (in the sense that the determinant of complex conjugation is -1 and not $+1$) arises from modular forms. This conjecture has many extremely important consequences: it implies that all odd rank 2 motives over $\mathbb{Q}$ are modular, it implies the Artin conjecture for odd two-dimensional representations of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, and, as Gerhard Frey and Serre realized in the mid-1980s, it implies Fermat's Last Theorem. Serre's conjecture has inspired much extremely important work. In the 1990s Wiles used ideas relating to Serre's conjecture to prove Fermat's Last Theorem and much of the Shimura-Taniyama conjecture. However, Serre's conjecture and the modularity of all odd rank 2 motives over $\mathbb{Q}$ still seemed completely out of reach. Serre's conjecture is essentially a statement about insoluble Galois groups, which had not been seriously touched in any previous work. In 2004 Khare and Wintenberger stunned the community when for the first time they found a plausible, and extremely beautiful, strategy to attack Serre's conjecture. See their paper "On Serre's conjecture for 2-dimensional mod p representations of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$" (*Annals of Math.* (2) **169** (2009), no. 1, 229–253). They continued to refine their strategy, while at the same time Mark Kisin made important and very original improvements to the modularity lifting theorems on which their strategy relies. Khare first proved the level-one case of Serre's conjecture in his paper "Serre's modularity conjecture: The level one case" (*Duke Math. J.* **134** (2006), no. 3, 557–589), and then Khare and Wintenberger completed the full proof of Serre's conjecture in their papers "Serre's modularity conjecture (I) and (II)" (*Invent. Math.* **178** (2009), no. 3, 485–504 and 505–586).

Chandrashekhar Khare



Jean-Pierre Wintenberger

Biographical Sketch

Chandrashekhar Khare was born in Mumbai, India, in 1967 and received his B.A. in mathematics from the University of Cambridge in 1989 and his Ph.D. from Caltech in 1995, where he worked with Haruzo Hida at UCLA and Dinakar Ramakrishnan at Caltech. From 1995 he worked at the Tata Institute for Fundamental Research in Mumbai. In 2002 he joined the faculty at the University of Utah before moving in 2007 to his current position as professor in the mathematics department at UCLA. He received the 2007 Fermat Prize from the Institut Mathématique de Toulouse, was a Guggenheim

Fellow in 2008, and received the Infosys Prize 2010 for Mathematical Sciences. He was an invited speaker in the Number Theory Section at the International Congress of Mathematicians held in Hyderabad, India, in August 2010.

Jean-Pierre Wintenberger was born in Neuilly-sur-Seine, near Paris, in 1954. He got his first thesis in 1978 and his Thése d'Etat (Habilitation) in 1984 in Grenoble, under the supervision of Jean-Marc Fontaine. He held the position of researcher in CNRS from 1978 to 1991, first in Grenoble, then in Orsay. He has been a professor at the Université de Strasbourg since 1991. He has been a member of the Institut Universitaire de France since 2007, received the Prix Thérèse Gautier from the French Academy of Science in 2008, and was an invited speaker in the Number Theory Section at the International Congress of Mathematicians held in Hyderabad, India, in August 2010.

Response

We are truly honored and very happy to be named as corecipients of the 2011 Cole Prize for Number Theory for our work on Serre's modularity conjecture. We thank the jury and AMS for this recognition of our work.

The conjecture is a beautifully simple and striking statement, as summarized in the citation. At the time it was made, in the 1970s, it must have seemed inaccessible. The precision with which it was formulated by J.-P. Serre, and the wealth of consequences he drew from it, attracted the efforts of many people.

Our work relies on the brilliant insights of many mathematicians. The celebrated work of A. Wiles in the 1990s provided a new tool, now called modularity lifting, with which to approach the conjecture. R. Taylor in the subsequent decade added several new insights, proving a potential version of Serre's conjecture which has had many strong consequences, some of which were used in our proof of the conjecture. As the citation mentions, the deeply original work of M. Kisin made the method of Wiles ever more versatile, and his work was crucially used in our proof. Another key development that is fundamental to our work is the version of modularity lifting theorems proved by C. Skinner and A. Wiles. Much of the work in this area is based on the pioneering work done in the 1970s and 1980s by J.-M. Fontaine, H. Hida, B. Mazur, K. Ribet, and J.-P. Serre on congruences between modular forms and local and global Galois representations. To all these mathematicians we are very grateful.

Serre's conjecture, once proved as a culmination of decades of work of many mathematicians, becomes a first step in linking linear, n-dimensional, finite characteristic representations of absolute Galois groups of number fields to automorphic forms.

Response from C. Khare

I am deeply grateful to my parents for the encouragement they gave me to indulge in a quixotic pursuit.

The institutions I have worked at—TIFR (Mumbai), University of Utah, and UCLA—have all provided very supportive environments at work. My wife, Rajanigandha, and my two children, Arushi and Vinayak, create a wonderful atmosphere at home. To all of them a heartfelt thanks!

Response from J.-P. Wintenberger

I have a thought for my parents, who were scientists and transmitted to me their curiosity, interest, and passion for science and research.

I wish to thank mathematicians who particularly influenced me by their works and personalities: J.-M. Fontaine, my advisor; A. Brumer; J. Coates; L. Illusie; M. Raynaud; and J.-P. Serre. I also wish to thank CNRS and Université de Strasbourg, who provided me the privilege of excellent conditions of work.

About the Prize

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years. The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years. These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole and has received contributions from Society members and from Cole's son, Charles A. Cole.

The Cole Prize carries a cash award of US$5,000. The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2011 prize, the members of the selection committee were Manjul Bhargava, Henryk Iwaniek, and Richard L. Taylor.

Previous recipients of the Cole Prize in Number Theory are H. S. Vandiver (1931), Claude Chevalley (1941), H. B. Mann (1946), Paul Erdős (1951), John T. Tate (1956), Kenkichi Iwasawa (1962), Bernard M. Dwork (1962), James B. Ax and Simon B. Kochen (1967), Wolfgang M. Schmidt (1972), Goro Shimura (1977), Robert P. Langlands (1982), Barry Mazur (1982), Dorian M. Goldfeld (1987), Benedict H. Gross and Don B. Zagier (1987), Karl Rubin (1992), Paul Vojta (1992), Andrew J. Wiles (1997), Henryk Iwaniec (2002), Richard Taylor (2002), Peter Sarnak (2005), and Manjul Bhargava (2008).