Peter Hilton: Codebreaker and Mathematician (1923–2010)

Jean Pedersen, Coordinating Editor

Photo courtesy of Jean Pedersen.

Peter Hilton at Santa Clara University (c. 1985).

Peter Hilton was born in London on April 7, 1923, to Elizabeth Freedman and a family physician, Mortimer Hilton, whose office was in Peckham. His parents hoped that he would become either a doctor (as his brother Sydney did) or a lawyer. Fate first intervened when he was run over by a Rolls Royce at the age of ten. During a long hospital stay he discovered that he could use the plaster cast on his left leg, which reached all the way to his navel, as a sort of white board; he solved mathematical problems on it, erasing them each morning. This is when, in his words, "It came to me that I really loved mathematics and thoroughly enjoyed doing it. I recall even having unkind thoughts about visitors who came to see if I was all right, as they would interrupt me when I was really enjoying what I was doing."

He attended St. Paul's School in Hammersmith, where, being enlightened educators, they left him to study what he wanted in the later years. With unexplained prescience about the unrest leading up to World War II, he decided to teach himself German. He then won a scholarship to Queen's College, Oxford, where he read mathematics. When he was eighteen, fate intervened for a second time. Peter began work at Bletchley Park in January 1942 (see Copeland's contribution below for more details); he thereby avoided service in the military, where, as Peter said at the time, "I would surely die—of boredom!" On Peter's eightieth birthday, Shaun Wylie, Peter's coauthor, wrote:

> "This young chap from Oxford joined us in Hut 8 at Bletchley, and soon made his mark. When he moved on to work on Fish, he did more than that; he dominated his section. . . he was brilliant at his job and enormous fun to be with."

Peter was awarded an M.A. at Oxford in 1948, a D.Phil. from Oxford in 1950 for his thesis, *Calculation of the Homotopy Groups of A_n^2-Polyhedra*, and a Ph.D. from Cambridge in 1952.

He married Margaret (Meg) Mostyn on September 14, 1949; they had two sons, Nicholas and Tim. Peter and Meg shared a love of theater, Meg professionally and Peter very much less formally.

Indirectly, Peter's love of theater led to our meeting. In 1977 I was asked to get George Pólya, with whom I had been studying informally for about ten years, and Peter together to do a presentation at a joint meeting of the Mathematical Association of America (MAA) and AMS in Seattle. They wanted Hilton to tell about "How Not to Teach Mathematics" and then have Pólya give "Some Rules of Thumb". When I wrote to Peter, he said "No!" He thought it would be more interesting (and fun) if he simulated a thoroughly bad demonstration of teaching mathematics, and he agreed to do that, instead, if I would be the moderator.

Jean Pedersen is professor of mathematics and computer science at Santa Clara University. Her email is `jpedersen@scu.edu`.

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The lectures took place on August 14, 1977. A repeat performance, which was televised [1], took place in the fall of 1978 at the San Diego meeting of the National Council of Teachers of Mathematics. Peter then agreed to come to Santa Clara University to give a colloquium talk. When he saw my office and heard about my interest in geometry, we discovered we both had a strong interest in polyhedra (from totally different points of view), and, as they say, the rest is history!

Our mutual interest lay in the mathematics involved with geometry. Peter got interested in an algorithm I had developed that concerned a systematic method of folding a straight strip of paper, say, adding-machine tape, that produced increasingly good approximations for any given rational multiple of π, at equally spaced points along an edge of the paper. We began working seriously on it in January of 1981 when we were both at the Eidgenössische Technische Hochschule (ETH) in Zürich. This naturally took us into number theory, polyhedral geometry, and combinatorics, and it continued for over thirty years until the time of Peter's death. The relevant mathematics is chronicled in our most recent book, *A Mathematical Tapestry: Demonstrating the Beautiful Unity of Mathematics* [2]. In Section 16.6 we wrote about "Pólya and ourselves—Mathematics, tea and cakes", which describes, as the title suggests, more about our interactions with each other and with Pólya.

An event that demonstrates the *joie de vivre* of collaborating with Peter involved the construction of a figure for one of our articles. We needed a star $\{\frac{11}{3}\}$-gon (i.e., the top edge of the tape would visit every third vertex of a bounding regular convex 11-gon). I had pulled the tape a little too tight and got a star $\{\frac{10}{3}\}$-gon. When I reported this mishap to Peter he said cheerfully, "Don't worry, we say in the beginning these are only approximations!" He never failed to see the humorous side of things.

Although Peter excelled at abstract thinking, he always applied mathematics to real life. He came to understand the principle of mathematical induction at the age of seven when drying dishes for his mother. He realized that the old trick of drying two plates at a time could, in theory, be extended by induction to n plates. As for applying mathematics to social situations, Sir Christopher Zeeman said in his letter on Peter's eightieth birthday:

> "From you I also learned how to talk politics, without endangering friendship. You taught me that if we disagreed about some conclusions then we should go back and examine our beliefs about the underlying facts. You acknowledged that if you believed in my version of the facts then you would share my conclusions, and you persuaded me that if I believed in your version of the facts then I would share your conclusions. It was a kind of mathematical approach; an agreement about the proofs, whatever the hypotheses and theorems. It was a lesson that has served me well throughout my life, in conversations about everything under the sun."

Photo courtesy of Meg Hilton.

Meg, Peter, and son Nicholas (c. 2008).

I was blessed with two great mentors, first George Pólya (for far too short a time) and then Peter Hilton. Pólya introduced me to polyhedral geometry, the wonders of "looking for patterns", and the importance of posing problems. Peter showed me how to generalize and how to see connections between various parts of mathematics. Both Pólya and Peter made the doing of mathematics an adventure constantly filled with excitement. Together Peter and I coauthored 144 papers and 6 books, often giving joint lectures on our current topic of interest.

My family was enriched by our friendship with Peter's family. I was excited to watch Meg perform (as Jim Dale's mother) in the production of the Broadway Tony-Award-winning play, *A Day in the Death of Joe Egg* (right after she won the Clarence Dewent award for her role in *Molly*). I have fond memories of Peter, Meg, Kent (my husband), and me on our land-cruise tour of Alaska. I smile recalling when Kent, Peter, and I walked through some drizzly California redwoods on what Peter called a "lovely London day", and later we played three-handed bridge at our house (where Peter got the bid with an appallingly bad dummy). I will always miss Peter, but I am grateful for the memories and the years of collaboration we shared.

References

[1] Hilton-Pólya DVD, recorded on VHS in 1978, digitized 2011. Available from Media Services, Santa

Photo courtesy of Jean Pedersen.

Kent and Jean Pedersen (on left) with Peter Hilton in the California redwoods (1996).

Clara University, Santa Clara, CA 95053 (US$25.00 + postage and handling).

[2] P. Hilton and J. Pedersen, with illustrations by Sylvie Donmoyer, *A Mathematical Tapestry: Demonstrating the Beautiful Unity of Mathematics*, Cambridge University Press,

Jack Copeland

During 1939–1945 Britain and her allies enjoyed unprecedented access to enemy radio communications. The German military transmitted many thousands of encrypted messages each day, ranging from top-level signals, such as detailed situation reports from generals at the front line, through to the important minutiae of war, such as weather reports and inventories of the contents of supply vessels. Much of this information ended up in Allied hands, often within a few hours of its being transmitted. As a leading codebreaker at Bletchley Park, Peter Hilton played a key role in this incredible operation. In 2002 Peter asked me to introduce a public lecture in which he described his life and work at Bletchley Park. I used much the same words then as here. "It is not often," Peter declared as he took the podium, "that a man has the pleasure of listening to his own obituary."

In October 1941 four of Bletchley Park's most senior codebreakers, Alan Turing, Hugh Alexander, Stuart Milner-Barry, and Gordon Welchman, wrote to Britain's wartime leader, Sir Winston Churchill, emphasizing the urgent need to recruit more staff [1]. "ACTION THIS DAY" was Churchill's famous response to the letter: "Make sure they have all they want on extreme priority and report to me that this had been done," Churchill instructed his chief of staff, General Ismay. A panel toured British universities searching for suitable recruits, and toward the end of 1941, at the University of Oxford, they interviewed a brilliant second-year mathematics undergraduate with a working knowledge of German. Peter had taught himself the language at school in order to read mathematical work. The panel snatched him up.

Jack Copeland is professor of philosophy and director of the Turing Archive for the History of Computing, University of Canterbury, New Zealand. His email is: `jack.copeland@canterbury.ac.nz`.

By exposing the detailed thinking of the enemy, Peter and his fellow codebreakers saved an incalculable number of lives. Occasionally he would even break messages signed by Hitler himself. The significance, horror, and fascination of the work is conveyed by this decrypt, dated December 1944, detailing an order from Hitler to General Guderian regarding the defense of Budapest [2]: "Evacuation without fighting in case of unfavourable development…out of question. Every house to be contested. Measures to prevent troops being endangered by the armed mob of the city to be taken ruthlessly."

Enigma machine and Alan Turing on St. Helena stamp, 2005.

Peter worked at first alongside Turing and Alexander in the organization known simply as "Hut 8", the front line of the attack on Naval Enigma. In 1941 the North Atlantic U-boats were sinking convoys carrying food, oil, and other raw materials from North America—so successfully that Britain was in imminent danger of being starved into surrender. The Bletchley codebreakers were by this time reading large quantities of Enigma traffic being transmitted by the German Air Force and other services, but the highly secure form of Enigma used by the U-boats remained unbroken. If U-boat Enigma could be read, the positions of the submarines would be known, and the convoys could be routed around them; it was not, however, until June 1941, a few months before Peter's arrival in Hut 8, that Turing and his group finally broke into the daily U-boat traffic. The effect was immediate; convoy reroutings based on intelligence from Hut 8 were so successful that the North Atlantic U-boats did not sight a single convoy for twenty-three days following Turing's first break.

It was in the heady period sparked by these early successes that Peter joined Hut 8, on January 12, 1942 [3, Ch. 15]. His talent soon stood out, and he was given the job of breaking "Offizier" messages. Normally, the operators at the sending and receiving ends of the Enigma link would see the plaintext of the transmitted message, but Offiziers were messages so sensitive that only officers, not the machine operators themselves, were allowed to view the contents. Offiziers were encrypted (and decrypted) twice, once by an officer and once by the usual Enigma crew, who saw only ciphertext [4, pp. 14–15]. Alexander (Turing's successor as head of Hut 8) described Offiziers as the hardest of all Naval Enigma messages to break [4, p. 16], but break them Peter did, using "cribs"—guessed words of the message. Peter was a very adept guesser.

Toward the end of 1942 Peter's skills were needed elsewhere, and he was transferred from Hut 8 first to the Research Section and from there to the Testery, a section headed (naturally enough) by Major Ralph Tester. The Testery's single function was to break a new German teleprinter (teletypewriter) cipher codenamed "Tunny" by the British [5]. Tunny was quite unlike Enigma (although the two are often confused in the literature). What the British called the Tunny machine was known to the Germans as the *Schlüsselzusatz SZ40*, a state-of-the-art twelve-wheel cipher machine produced in 1940 by the Lorenz Company. Enigma, on the other hand, had three (or sometimes four) wheels and dated from the early 1920s. The Tunny machine required only a single operator, whereas the clumsier Enigma needed three, including a wireless operator who tapped out the enciphered message in Morse code. The operator of a Tunny machine simply typed plain German at the teleprinter keyboard, and the rest was automatic. Morse was not used; the encrypted output of the Tunny machine went directly to air. The extent to which the Lorenz engineers had succeeded in automating the processes of encryption and decryption was striking; under normal operating conditions, neither the sending nor the receiving operator ever even saw the coded form of the message. The British first intercepted Tunny messages in June 1941, and, in January 1942, the great Bill Tutte single-handedly deduced the fundamental structure of the Tunny machine. The rest of the Research Section joined him in his investigation, and soon the whole machine was laid bare, without any of them ever having set eyes on one. It was the most remarkable feat of cryptanalysis of the war. Tutte's deductions broke open the entire Tunny system.

At first the Testery broke Tunny messages purely by hand, using a method invented by Turing in July 1942 known as "Turingery". The following twelve months saw approximately 1.5 million [6] characters of ciphertext broken in this way. As the Testery's chief mathematician, Peter honed the unit's cryptanalytical methods during this early phase of breaking Tunny. Thereafter, as the volume of Tunny traffic grew and the number of operator errors—manna to the codebreakers—became fewer, the Testery began to receive increasing help from high-speed analytic machinery, most notably the vast Colossus (from February 1944). Designed and built by Tommy Flowers, Colossus was the first large-scale electronic digital computer. By war's end there were ten Colossi operating around the clock in the Newmanry, a section headed by topologist Max Newman. The Newmanry was the world's first electronic computing facility, and Peter took up the role of liaising between it and the Testery. "Life," he said, "was as interesting as it could possibly be" [3, p. 194].

Photo courtesy of Meg Hilton.

A young Peter Hilton.

Using an algorithmic method devised by Tutte and tweaked by Newman and his mathematicians, Colossus mechanically stripped away one layer of encryption from a Tunny message. The result, known as a "de-chi", still carried a second layer of encryption, and the de-chi was passed on to the Testery to be broken by human patience and ingenuity. The two sections, Testery and Newmanry, worked hand in glove. Peter and his fellow breakers in the Testery (Jerry Roberts, Peter Edgerley, Denis Oswald, Peter Ericsson, and others [3, Chs. 18 and 21]) would chip away at the de-chi, using cribs to expose small sections of the plaintext. They extended these short breaks by further guesswork until they had about thirty to eighty or so consecutive characters of "clear"—enough to work out the settings of the wheels, so that the whole message could be deciphered. Some of these short breaks (usually all that the codebreaker saw of the plaintext) were imprinted on Peter's memory for the rest of his life—"*Ich bin so einsam*" (I am so lonely), from an operator on the Leningrad front, and "*Mörderische Hitze*" (murderous heat), from the Italian front. The Testery's "ATS girls" (Auxiliary Territorial Service—the Women's Army) performed the final stage of the decryption. They typed the often lengthy ciphertext into a British replica of the Tunny machine, and plain German would emanate from the printer.

Photo taken by Marge Dodrill, courtesy of Meg Hilton.

Peter, doing mathematics (c. 1980).

Tunny (unlike Enigma) carried only the highest grade of intelligence, messages between the Army High Command and the generals in the field. Tunny decrypts contained intelligence that changed the course of the war, by providing detailed knowledge of German strategy—for example, concerning counter-preparations for the anticipated Allied invasion of Northern France in 1944 (the D-Day landings). The work of the Bletchley codebreakers may have shortened the war by as much as two years, yet Peter and his colleagues received little or no recognition for their massive contribution to the Allied victory. Ludicrously, Turing received nothing more than an OBE (which he kept in his toolbox) and Flowers a cheque for 1,000 pounds—yet if he could have patented or even publicized Colossus, he would have become a wealthy and celebrated engineer. Flowers "had to listen in silence," Peter said [3, p. 201], "as others got the credit for creating the electronic computer." Tutte received no public recognition whatsoever for his priceless work. Men like Peter Hilton and Jerry Roberts deserved knighthoods or at the very least the OBE (an honor now regularly awarded for services to local government, business, or sport). It is a shame that Peter was never adequately thanked by his country.

References

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[2] British message reference number CX/MSS/T400/79 (HP 9622); National Archives document reference DEFE 3/318. Thanks to William Newman for supplying me with a copy of this message.

[3] B. Jack Copeland, et al. *Colossus: The Secrets of Bletchley Park's Codebreaking Computers*, Oxford University Press, 2006; 2nd ed. 2010.

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[5] The full nature and scope of Bletchley's attack on Tunny was not revealed until 2000, when the British Government declassified a 500-page history written in 1945 by Tunny-breakers Jack Good, Donald Michie, and Geoffrey Timms, "General Report on Tunny" (National Archives document reference HW25/4 (vol. 1), HW 25/5 (vol. 2)). A digital facsimile of "General Report on Tunny" is in the Turing Archive for the History of Computing at http://www.AlanTuring.net/tunny_report. The story of Bletchley's attack on Tunny was first told in print in [3]; see also the Colossus website http://www.colossus-computer.com, which contains movies of lectures by Peter Hilton, Jerry Roberts, and others, along with photographs.

[6] Information from Jerry Roberts.

Bill Browder

The name Peter Hilton first came to my attention when, as a graduate student, I read his beautiful paper on loop spaces [1] (which, generalized, became known as the Hilton-Milnor theorem), which played an important role in my thesis.

I first met the man when, in 1957, I visited Cornell (from Rochester) when he was visiting there. His modest charm, his razor-sharp wit, and the elegance of his conversation immediately struck me. I was steeped as a teenager in the Marx Brothers movies and the works of S. J. Perelman, and Hilton resonated with my own attitudes.

I moved to Cornell the following year, and I was absolutely delighted that Peter was visiting for the year. His elegant and beautifully organized lectures that year became the book *Homotopy Theory and Duality* [2], which I came to refer to as the "Hilton Finishing School for Young Topologists". He taught me much about looking for the most general underlying principles and exploiting them.

Meg was not with him much of the time that year, and we hung out quite a bit together. He was more than ten years my senior (almost 50 percent!), but he had a *joie de vivre* and informality that made him a very congenial companion. Many were the anecdotes I heard from him, particularly about Oxford and Henry Whitehead. I decided that I should go to work with Whitehead if I got the

Bill Browder is professor of mathematics at Princeton University. His email is browder@math.princeton.edu.

chance, which came when I got an NSF postdoc the following year.

Peter and I met again the summer of 1960 in Zürich at the Hopf birthday conference. He had recommended to me a hotel run by the *Zürcher Frauenverein für Alkoholfreie Wirtschaft* (the Zürich Ladies Club for Alcohol-Free Hospitality), which was very clean, pleasant, and inexpensive but prohibited alcohol on the premises (obviously). I was newly married, and we decided to stay there to save money and to take our libations elsewhere.

Peter acted as a guide for us at times, and I admired his fluency in German, his deep acquaintance with Zürich, and his general sophistication. My experiment imitating his smoking of the small cigars called *Stumpen* went nowhere, and German remained out of reach, but my admiration of Peter's urbanity only increased.

An inveterate traveler, he soon received the honor of the following joke: An innocent tourist looking for a hotel asks a passing mathematician "Where is the Ithaca Hilton?" and receives the reply "Oh, he's in Zanzibar this week."

Of his many destinations, Africa and Eastern Europe seemed frequent. In Romania, he made the acquaintance of the Bucharest topology group, consisting among others of Tudor Ganea, Israel Berstein, and Valentin Poenaru. Berstein, being Jewish, was allowed to emigrate, ostensibly to Israel; Poenaru was allowed to attend the Stockholm International Congress of Mathematicians and never returned. Berstein came to Cornell, Poenaru went to Paris, but Ganea's case was more difficult. As Peter told me, Ganea had to be "purchased" from the Ceausescu government by a consortium of mathematicians for a large sum of (hard) currency.

After a year in Paris, Ganea accepted a high-salaried job in the United States (at Purdue) in order to "buy himself back" from the consortium. Ganea, a gloomy but very witty person, would call Berstein each evening with his very funny complaints. Berstein was of a very sunny and lighthearted disposition, despite his wooden leg (the result of a wound at age seventeen in the Red army), and he would regale our group the next morning with Ganea's latest observations.

An example: Cities are characterized by fluids: Paris by wine and perfume, West Lafayette, Indiana, by milk and gasoline.

Peter returned to Cornell in 1962. Our topology group at Cornell in those years consisted of, besides Peter and myself, Paul Olum, Roger Livesey, Israel Berstein, Isaac Namioka, Casper Curjel, and David Gillman, as well as a number of graduate students, including Martin Arkowitz and Gerry Porter, who wrote theses in homotopy theory. There were several active seminars, and it was a very congenial and friendly group.

Now Meg was with him, which added dramatically to the social scene. An accomplished professional actress, her charm coupled with Peter's made a great impact.

Peter's mathematical interests overlapped mine a great deal in the early days, and I remember particularly his work with Berstein on the "co-H-spaces" (spaces with Lusternick-Schnirelmann category 2) which were not suspensions [3], which emerged beautifully from the Eckmann-Hilton duality point of view, and later his work with Roitberg producing a new H-space of unexpected homotopy type [4].

The latter earned the nickname the "Hilton-Roitberg criminal" rather than "example" or other standard terminology. The only other similar terminology I know of is the famous "Eilenberg swindle". These are the only "criminals" or "swindles" I know of in mathematics.

I left Cornell in 1963, and Peter left a few years later.

We met again at his sixtieth birthday party in 1983 in St. John's, Newfoundland. He and I were asked to participate in a local television program to discuss mathematics education. We disagreed amicably for most of the program, and then, with exquisite timing at the very end of the program, he came out with a statement that made steam come out of my ears, but left no time for reply. I realized that I was in the presence of a major league debater.

I also attended his eightieth birthday celebration in Binghamton, New York, in 2003. Peter gave a wonderful polished talk about his experiences at Bletchley Park in World War II, which was informative and moving and made a political point.

I noticed that he frequently paused to refer to a very small sheaf of notes in his hand. He left the papers on the rostrum after the talk, and out of curiosity I took a look. They were blank! It was a stage prop.

Needless to say my admiration of Peter was not diminished by these occasions. A person like Peter is a great rarity, and I feel very privileged to have known him.

References

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Ross Geoghegan

The first thing to know about Peter Hilton is that for half his life he was unusually young for whatever he was doing. He was recruited to Bletchley Park for Alan Turing's codebreaking operation at the age of eighteen. On the day of his arrival, Turing posed him a chess problem that he said he had failed to solve, and Peter delivered the solution the next day. From then on, his place as a full participant was secure.

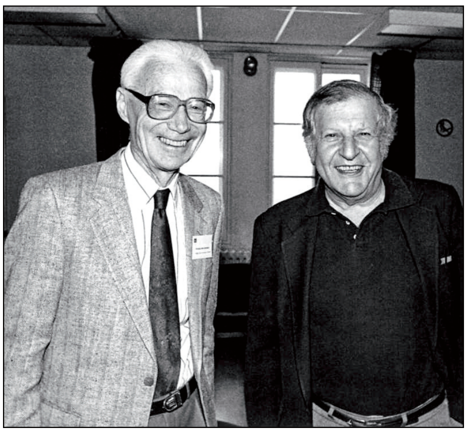


Photo courtesy of Meg Hilton.

Beno Eckmann and Peter in Zürich (c. 1980).

One of his colleagues at Bletchley Park was J. H. C. Whitehead. After the war Whitehead invited Peter to go back with him to Oxford to do a doctorate in topology. At this stage Peter had had only one year of formal university-level mathematical training, and he had doubts about the idea: he told Whitehead that he didn't know what topology was. "Oh don't worry, you'll love it," was the reply. Because of the postwar housing shortage, Peter lived in Whitehead's home, ate dinner each evening with the family, and learned topology, probably other mathematics, too, by osmosis.

But learn he did. In a letter written for Peter's eightieth birthday the topologist Sir Christopher Zeeman, just two years younger than Peter, wrote: "Shaun [Wylie] was a lovely supervisor, who invariably encouraged me and faithfully read everything I wrote. But he didn't know much topology. It was you who taught me topology without which I might never have got started. So I am eternally grateful."

Peter's career moved fast, and at an unusually young age he held the Chair of Mathematics at Birmingham. This was a mixed blessing, for in those days British universities allotted only one Chair to each subject, and the holder was expected to administer the department for life. The salary difference made it impossible to pass this administrative work on to lower-ranking people, and Peter disliked administration—which was the main reason he moved to Cornell. As he used to say: "I resigned to make way for an older man."

I met Peter when I was a graduate student at Cornell and he (when in residence, which first happened in my third year) was the best known topology professor there. In fall 1968 he gave a course on homotopy theory, rather in the style of his elegant book *Homotopy Theory and Duality*. The course was to meet for three hours on Tuesday mornings. On the first day he wrote down a list of those Tuesdays when his mathematical activities would take him out of town. The miracle was that in the remaining weeks he covered a serious amount of mathematics. His handwriting on the blackboard was tiny, and the material was organized to perfection. Peter took much pride in the elegance of his presentations.

His dislike of administration was matched by a distrust of professional administrators. While he was at Cornell, an article in the *Ithaca Journal* mentioned the then-provost of Cornell, Robert Plane, and described him as "Cornell's number two man". Peter wrote a letter to the editor asking "Which of our Nobel prizewinners is Number One?"

While my main thesis work was in infinite-dimensional topology, with David Henderson as my advisor, I had a secondary project on which Peter gave me feedback. And when Henderson was absent in Russia, Peter chaired my defense in 1970. So we knew each other from the late 1960s onward. But our paths rarely crossed until 1982, when Peter joined my department at Binghamton as Distinguished Professor. By then any vestige of the student-professor relationship had gone, and Peter and I became fast friends. By that time he was mainly doing number theory, combinatorics, and polyhedral geometry, which Jean Pedersen describes in her part of this tribute.

That was just after his controversial tour of South African mathematics departments. It was a time when "all right-thinking people" were boycotting that country, but Peter took the view that the mathematics community is worldwide and that shunning rather than influencing is not constructive. For this he made enemies. I can still hear the unprintable comments which a well-known visiting mathematician made about Peter in my living room. (When the singer Paul Simon encountered similar criticism later, I thought of this.)

My favorite Binghamton story about Peter happened not long afterward, when a calculus student came to his office for help with a math problem. Peter asked: "Is it the case that you do not understand what you are being asked to do, or is it the case that you do understand what you are being asked to do but cannot do it?" And (as Peter explained in the Queen's English—it really has to be spoken out loud):

Ross Geoghegan is professor of mathematics at Binghamton University (State University of New York). His email address is ross@math.binghamton.edu.

"She answered: Well... like... kind of... both."

More, the abiding professional lesson I have learned from Peter is about judging the work and worth of mathematicians. I noticed in my years at Cornell how charitable he was in dealing with students and how positive he was in discussing the work of younger people. As a student I sometimes detected a strain of competitive negativity. Standing out against this "destructivism" was Peter, who was a celebrity mathematician and had no need to prove himself.

Later, when he came to Binghamton and I had a chance to observe him as colleague rather than as mentor, I understood his approach better. Peter believed that doing mathematics is intrinsically good and that the work of people, even if their achievements are not fashionable or herculean, should be praised and supported where possible. From time to time I would see surprisingly positive letters of recommendation written by Peter. I finally understood that he saw no good for his beloved subject in our tearing one another down. He had a mantra: "I would rather be a second-class person in a first-class discipline than a first-class person in a second-class discipline," and he was not shy about identifying disciplines he did not consider first class.

In short, the high standards Peter held himself to did not prevent him from emphasizing the positive in others. He was a first-class person in a first-class discipline.

Joe Roitberg

Peter and I first met at New York University in early 1968. Peter was visiting the Courant Institute of Mathematical Sciences at the time, and I had just completed my Ph.D. dissertation there, under Michel Kervaire's supervision. But before I proceed, allow me to backtrack.

Peter was one of J. H. C. Whitehead's most brilliant students, some twenty years earlier, and had long been a leading homotopy theorist. His most famous and most important contribution to homotopy theory was his landmark 1955 paper on the homotopy groups of the finite one-point union of spheres, wherein he proved that any such homotopy group can be expressed as a direct sum of homotopy groups of spheres of various dimensions. Subsequently, John Milnor generalized Peter's result to the case of the finite one-point union of arbitrary suspension spaces. This more general result came to be known as the Hilton-Milnor theorem. A related, frequently cited result in this same paper of Peter's is the "corrected left-distributive law", expressing, for a space X, and elements α in $\pi_n(S^m)$, β, γ in $\pi_m(X)$, an infinite sum expansion for $(\beta + \gamma) \circ \alpha$, in which the first two terms are $\beta \circ \alpha$ and $\gamma \circ \alpha$ and the remaining terms involve Whitehead products and Hilton-Hopf invariants. Among the many other noteworthy research papers published by Peter in this time period, I restrict myself to mentioning two items: a joint paper with J. F. Adams on the chain algebra of a loop space, a forerunner to Adams's work on the cobar construction; and a series of joint papers with Beno Eckmann on duality (Eckmann-Hilton duality).

Joseph Roitberg is a professor in the department of mathematics and statistics at Hunter College and in the Ph.D. program in mathematics at The Graduate School and University Center of the City University of New York. His email address is `jroitber@hunter.cuny.edu`.

Left to right: Emery Thomas, Michael Barratt, Henry Whitehead, Ioan James, and Peter Hilton (c. 1955).

Peter's passion for research was matched by his passion for clear exposition. In fact, he himself regarded the task of making mathematical ideas accessible to students as his highest calling. Among his contributions as an expositor are three early, influential books, which were particularly important for my own mathematical development: *Homology Theory: An Introduction to Algebraic Topology*, coauthored with Shaun Wylie and published in 1960 (predating Edwin Spanier's book *Algebraic Topology* by six years), which was for a long time virtually the only comprehensive, up-to-date text on algebraic topology; *An Introduction to Homotopy Theory*, published in 1953 (predating Sze-Tsen Hu's book *Homotopy Theory* by six years), which included a useful entree to Whitehead's notion of CW-complex and also some of Peter's earliest research, conducted under Whitehead's supervision; and *Homotopy Theory and Duality*, published in 1965, which summarized the recent results of his collaboration with Eckmann. I made good use of these texts after having become hopelessly hooked on algebraic and differential topology in 1964 as a result of exposure to lectures on knot theory by

Kervaire. The Hom-Ext version of the universal coefficient theorem for homotopy groups with coefficients, described in *Homotopy Theory and Duality*, played a role in one of the results in my Ph.D. dissertation, an odd counterexample to the classical Hurewicz conjecture.

Photo courtesy of Niloufer Mackey.

Standing, Niloufer Mackey (Pi Mu Epsilon advisor), along with Peter Hilton, Jean Pedersen, Benjamin Phillips, Jonathan Hodge, and Jay Wood (department chair), during the signing ceremony for Pi Mu Epsilon initiates at Western Michigan University. Peter and Jean had just given a joint mathematical talk (March 21, 2002).

I return now to 1968. After completing my Ph.D. dissertation, I immersed myself in Peter's research at that time, so Peter's arrival at NYU was very fortuitous for me. Peter had demonstrated noncancellation phenomena in the homotopy category by constructing examples of simply connected, finite CW-complexes V, W such that V and W are not homotopy equivalent but, for a suitable sphere S, the one-point unions $V \vee S$ and $W \vee S$ are homotopy equivalent. I introduced myself to Peter, and we proceeded to discuss possible dualizations of his examples. We were not content, however, with a straightforward dualization, which leads to infinite CW-complexes. Eventually, with the aid of the aforementioned "corrected left-distributive law", we were able to construct examples of simply connected, finite CW-complexes X,Y such that X and Y are not homotopy equivalent but $X \times S^3$ and $Y \times S^3$ are homotopy equivalent, S^3 denoting the 3-sphere. In fact, X and Y are the total spaces of principal S^3-bundles over spheres and so are closed, smooth manifolds, and the product spaces are actually diffeomorphic. We were pleased and excited about these examples, but, as an unexpected bonus, our construction tied in with certain cutting-edge developments in homotopy theory: localization theory in the homotopy category of CW-complexes, introduced by Dennis Sullivan for simply connected CW-complexes, then by A. K. Bousfield and D. M. Kan for nilpotent CW-complexes. For our examples X,Y are in the same localization (or Mislin) genus, that is, X and Y are p-equivalent for all primes p; and work of Alexander Zabrodsky on the construction of new H-spaces—for one of our examples, X is the Lie group $Sp(2)$, so that Y is a new H-space. Though this fake Lie group Y cannot be homeomorphic to a topological group, thanks to the solution of Hilbert's Fifth Problem, it was noted by James Stasheff that Y does have the homotopy type of a topological group.

Thus began an intense collaboration on the themes initiated in our discussions at the Courant Institute, with Guido Mislin joining us in several of our joint projects. Our efforts were summarized in the 1975 monograph *Localization of Nilpotent Groups and Spaces*, which we began working on when Guido and I visited Peter at the Battelle Institute Research Center in Seattle for several days in 1974. Homotopy theory, and, more generally, algebraic and differential topology were experiencing a long and spectacular growth period, begun in the middle of the twentieth century—I think of this era as a golden age for topology, if I may be permitted a touch of hyperbole—and it was exhilarating to be (or so I imagined us to be) in the thick of things.

Peter and I continued to collaborate for several more years on various topics in group theory and homotopy theory, our last joint publication appearing in 1987. Of course, Peter's ideas served me well in much of my subsequent research. To give two examples: in 2000, I published a paper which, in particular, settled a question of Peter's on the Lusternik-Schnirelmann category; and in 2007, my former student Huale Huang and I published a paper on the genus of certain connective covering spaces which required revisiting Peter's aforementioned foundational paper of 1955. As for Peter, he continued working in group theory and homotopy theory but also devoted a great deal of his attention to other areas of mathematics and mathematics education, which I will leave to more knowledgeable colleagues to comment on.

Peter's influence on my mathematical career was decisive. However, I was far from the only beneficiary of Peter's mathematical and social genius, which enabled him to carry on successful and long-standing collaborations with many colleagues, too numerous to list here. His published works are, as already noted, models of clarity and eloquence of expression. So too were his lectures—I fondly recall a beautiful talk he delivered to an undergraduate audience on "calculus using infinitesimals", which underscored his highly tuned sensitivity to the audience he was addressing. He had a lifelong commitment to the promotion of excellence in mathematical teaching at all levels. And he was vitally concerned with the development of mathematics world-wide, providing his services wherever his extensive travels brought him.

My social interactions with Peter and his family were always highly pleasurable. Some highlights: the Hiltons hosting my daughter Daphne and me at their home in Binghamton, prior to Daphne's enrolling as an undergraduate at SUNY-Binghamton; hosting Peter at our home; attending an NBA basketball game with Peter's son Tim, while I was visiting in Seattle; attending a New York City theater to see Peter's wife Meg perform in the play *Joe Egg*. My last contact with Peter was in August 2010, when my wife Yael and I visited with Peter, Meg, and their son Nick in Binghamton. While there, we were shown Peter's most recent joint venture with Jean Pedersen, a 2010 book published by Cambridge University Press entitled *A Mathematical Tapestry: Demonstrating the Beautiful Unity of Mathematics*. This was a reminder that Peter was never one to rest on his laurels, that he was ever active and productive. So, while we are deeply saddened by Peter's passing, we take solace in the realization that he lived a full, rich life.

Guido Mislin

When I met Peter Hilton some forty-five years ago in Zürich, I was a student attending one of his classes, a course in homological algebra, with a small group of other graduate students. He was open and approachable, which was an entirely new experience for us, having only been exposed to a Germanic-distant kind of relationship between professors and students. We learned from him how mathematicians think, how they tackle problems, present results, and design the right proof for a theorem.

Later, I had the privilege to collaborate with Peter Hilton for many years, in a time before the convenience of email changed our lives. Peter traveled a lot, so the exchange of letters was an ongoing challenge, as his whereabouts kept constantly changing. His responses were written on thin air-mail stationery, letters one had to open carefully so as not to cut through the text. His handwriting resembled print, his mathematics was crystal clear. Final drafts he completed while crossing one of the oceans. Peter had a unique talent for rendering difficult mathematical concepts transparent and easy to grasp. He applied his gift to further mathematical education on all levels, and the older he got, the more he cared for the very young. We will remember him as a great teacher, mathematician, and friend.

Hilton's mathematical work covers a wide range of topics. The examples which follow are chosen from his work in algebraic topology. They illustrate his clarity of exposition and his artful way of using the interplay between algebra and topology.

The Hilton-Milnor Formula

Let $S^{k_1} \vee \cdots \vee S^{k_m}$ be a 1-connected wedge of spheres. Hilton computed in [7] the homotopy groups of $S^{k_1} \vee \cdots \vee S^{k_m}$, showing that they can be expressed in terms of homotopy groups of spheres,

$$\pi_n(S^{k_1} \vee \cdots \vee S^{k_m}) \cong \bigoplus_{w(\pi)} \pi_n(S^{k_{w(\pi)}}) \, .$$

Here $w(\pi) \in \pi_*(S^{k_1} \vee \cdots \vee S^{k_m})$ runs over all basic Whitehead products, and the summand $\pi_n(S^{k_{w(\pi)}})$ is embedded into the sum via composition with $w(\pi) : S^{k_{w(\pi)}} \to S^{k_1} \vee \cdots \vee S^{k_m}$. The formula was generalized by Milnor [12] to the case of a finite wedge of suspensions of connected CW-complexes. Hilton used it to derive the following result (see [8] and [9]). Denote by $\Sigma\mathbb{P}^1$ the set of homotopy types of 1-connected, finite (pointed) CW-complexes, which are homotopy equivalent to suspensions. Consider this set as a monoid using the wedge operation, and denote by $\mathrm{Gr}(\Sigma\mathbb{P}^1)$ the corresponding Grothendieck group. He proves the following:

Let X and Y be 1-connected finite CW-complexes of the homotopy type of suspensions. If $[X] - [Y]$ is a torsion element in $\mathrm{Gr}(\Sigma\mathbb{P}^1)$, then X and Y have isomorphic homotopy groups.

The Hilton-Hopf Invariants

In [7], Hilton proposed the following generalization of the classical Hopf invariant

$$H : \pi_{2r-1}(S^r) \to \pi_{2r-1}(S^{2r-1}) = \mathbb{Z} \, , r \geq 2.$$

Writing

$$\pi_n(S^r \vee S^r) = \pi_n(S^r) \oplus \pi_n(S^r) \oplus \pi_n(S^{2r-1}) \\ \oplus \pi_n(S^{3r-2}) \oplus \pi_n(S^{3r-2}) \oplus \cdots \, ,$$

he defines homomorphisms H_i, $i \geq 0$, by composing the pinching map $\pi_n(S^r) \to \pi_n(S^r \vee S^r)$ with the projection onto the $(i+3)$rd factor in the sum decomposition of $\pi_n(S^r \vee S^r)$:

$$H_0 : \pi_n(S^r) \to \pi_n(S^{2r-1}) \, ,$$

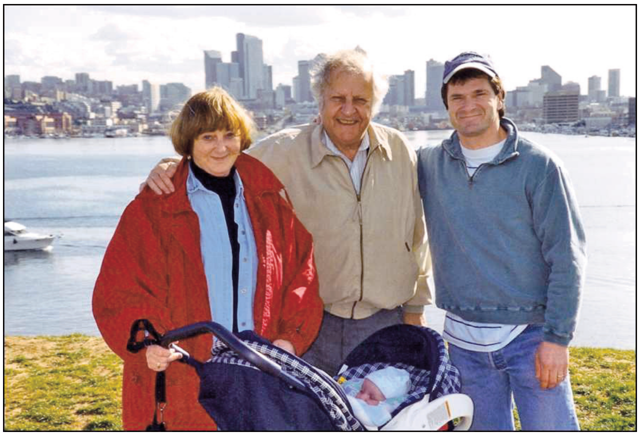
$$H_1, H_2 : \pi_n(S^r) \to \pi_n(S^{3r-2}) \, , \cdots \, .$$

He shows that H_0 agrees with the classical Hopf invariant in case of $n = 2r - 1$. As an application, Hilton obtains the following formula concerning the left distributive law for composition of homotopy classes. If $\alpha, \beta \in \pi_r(X)$ and $\gamma \in \pi_n(S^r)$, then

$$(\alpha + \beta) \circ \gamma = \alpha \circ \gamma + \beta \circ \gamma + [\alpha, \beta] \circ H_0(\gamma) \\ + [\alpha, [\alpha, \beta]] \circ H_1(\gamma) + [\beta, [\alpha, \beta]] \\ \circ H_2(\gamma) + \cdots$$

Guido Mislin is professor emeritus of mathematics at the ETH in Zürich. His email address is `mislin@math.ethz.ch`.

Photo courtesy of Meg Hilton.



Meg and Peter Hilton with son Tim and new grandson Jake, in Seattle, 1999.

measuring the deviation from additivity of the induced map

$$\gamma^* : \pi_r(X) \to \pi_n(X), \quad \alpha \mapsto \alpha \circ \gamma, \quad (r \geq 2).$$

Eckmann-Hilton Duality

In 1955 Beno Eckmann and Peter Hilton worked together on finding a suitable definition of *homotopy* for maps of modules. They came up with two notions, which exhibited interesting internal duality properties; in one notion, a map was considered as nullhomotopic if it can be factored through an injective module, and in the other a map was nullhomotopic when it factored through a projective module. This led naturally to notions such as *cone* and *suspension*, and dually to *path spaces* and *loop spaces* in the module category. They observed that both these homotopy notions correspond just to the ordinary homotopy notion in topology, leading them to consider a framework for an internal duality in the homotopy category of pointed CW-complexes (see [1, 2, 3]). They generalize the pointed homotopy set $[X, Y]$ to

$$\pi_n(X;Y) := [\Sigma^n X, Y] = [X, \Omega^n Y],$$

which, when considered as a functor in the first, resp. second, variable generalizes cohomology groups, resp. homotopy groups. For instance, viewing a pair of spaces $X \subset Y$ as a map $X \to Y$, a triad is a diagram $X \to Y \to Z$, which leads to dual triple sequences in cohomology and homotopy. The concepts of *H-spaces* and *co-H-spaces* are typical examples of dual notions in the sense of Eckmann-Hilton. They can be characterized in the following dual way:

X is an H-space if and only if the canonical map $X \to \Omega\Sigma(X)$ has a left homotopy inverse; Y is a co-H-space if and only if the canonical map $\Sigma\Omega(Y) \to Y$ has a right homotopy inverse.

Eckmann and Hilton went on to internal duality in arbitrary categories in [4, 5, 6].

H-Spaces and Localization

In the 1960s many people worked on understanding finite complexes, which support a group structure up to homotopy. The striking example of Peter Hilton and Joe Roitberg [11] of a 10-dimensional manifold E, not homotopy equivalent to any Lie group but such that $E \times S^3 \cong Sp(2) \times S^3$, was the starting point for many investigations. The example lent itself in a natural way to apply localization techniques in homotopy theory. These techniques, which were then just being developed by several people (including A. K. Bousfield, E. Dror-Farjoun, D. Kan, D. Sullivan, and A. Zabrodsky), opened new doors. An elementary approach to localization in homotopy theory, beginning with a purely algebraic chapter on localization of nilpotent groups and from there passing to localization of nilpotent spaces, is the topic of the book [10]. The localization technique led to a systematic understanding of *noncancellation phenomena* in homotopy theory, of which the Hilton-Roitberg manifold E is an example. The manifold E belongs to the *genus set* $G(Sp(2))$, meaning that for every prime p its p-localization $E_{(p)}$ is homotopy equivalent to the corresponding p-localization $Sp(2)_{(p)}$. Problems in homotopy theory could now be addressed *one prime at a time*, and for the reassembling of the resulting pieces one had to study local-global principles in the context in which one was working. This has all been dealt with in many of Hilton's research articles.

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Urs Stammbach

Peter Hilton was one of the most important people in my life. He was someone with whom I could discuss anything, not just mathematics, but all aspects of life, politics, the state of the world, and the education of our children.

When I was beginning work on my Ph.D. at the ETH in Zürich, Peter was regularly invited by Beno Eckmann to visit the Forschungsinstitut für Mathematics. Peter gave numerous talks during these visits. Every one of impeccable quality. They made a deep impression on me. In those days there was a considerable distance between students and professors in Switzerland. Being a rather shy student at the time, I was particularly affected by this, and it was difficult for me to exchange even some ordinary words, or to ask questions, in this formal environment. Thus it was a great surprise to me when Peter asked me to come to his office and tell him about my work. I was tense before the meeting. But right from the start he put me at ease by addressing me in German. And Peter didn't begin by asking me tough mathematical questions; instead, he asked me to accompany him to buy today's copy of *The Times*. The ice was broken, and we had a long and interesting talk about mathematics afterwards. This encounter was the first of many, many more to come. I learned later that during those years *The Times* was Peter's must-have lifeline to information about the world. He once confessed that after several days without *The Times*, he would have to read the newest edition from cover to cover "to put the internal coordinate system right again."

After I completed my Ph.D., Peter was invited to spend a full year at the ETH, and during this time he gave a course on homological algebra. We got somewhat closer, and during a conversation he suggested that I should think of going to the United States for a year or two and that perhaps Cornell University would be a suitable place. It soon became obvious that this suggestion was actually a joint idea between Beno Eckmann and Peter Hilton (another example of their efficient joint work!).

Thus, in 1967, I had the opportunity to go to Ithaca and to spend two years in Cornell's fine mathematics department. During my first year there Peter was away, visiting the Courant Institute in New York, where he invited me to give a number of talks. The first of these took place only a few weeks after my arrival in the United States. I was terrified; first, my English was rather rudimentary at the time, and, second, I knew that the prospective audience would include many famous people. It was possible that Hyman Bass, Gilbert Baumslag, Michel Kervaire, Wilhelm Magnus, and others of similar standing would be present. Peter completely understood my psychological difficulties. He kindly calmed me down and supported me in a way that instilled confidence.

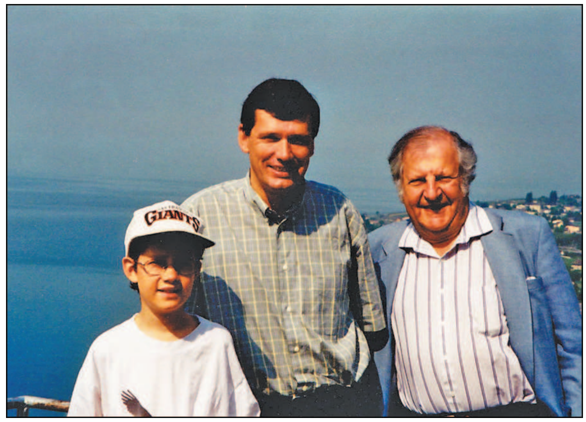
Photo courtesy of Dominique Arlettaz.

Dominique Arlettaz and son Mathieu with Peter in the vineyards near Lausanne in May 1995.

During the first year in Ithaca I received a letter from Peter, written in his characteristic tiny but very clear handwriting, in which he asked me whether I would be willing to write a book with him on homological algebra [1]. I needed to read the letter several times before I comprehended the full impact of his question. Only a couple of years after receiving my Ph.D., I was being asked to coauthor a book with one of the most important people in the field.

Writing the book took us over three years and absorbed a huge amount of my time and energy. This joint work was a unique experience for me. It is unbelievable how much I learned during these years. Each chapter of the book was first written by one of us. The manuscript was then sent to the other and was corrected, criticized, sometimes shortened, sometimes enlarged. With most parts of the book, this process was repeated several times. Some of the drafts were rejected altogether by the other side and had to be completely rewritten. Of course, Peter put my English into an acceptable form; moreover, to my dismay, he regularly spotted instances of mathematical sloppiness on my part. This was a huge learning process as I received, from an expert, firsthand coaching on the art of writing mathematics. Here is an example: Commenting on my writing that such and such a theorem was false, Peter gently but firmly told me that a theorem could never be false, for a theorem, by definition, is true. Only the statement of the

Urs Stammbach is professor emeritus of mathematics at the ETH in Zürich. His email address is `stammb@math.ethz.ch`.

Photo by Gert-Martin Greuel, courtesy of Mathematisches Forschungsinstitut Oberwolfach.

B. Eckmann, P. Hilton, J.-P. Serre, and A. Haefliger at Eckmann's 90th birthday celebration in Zürich in 2007.

theorem can be false. How many instances are there in the literature that ignore this simple fact?

The number of letters we exchanged while writing the book (there was no email at that time) is almost uncountable. It was a great experience to see that many of my suggestions were acceptable to him. At one instance I wrote, in reaction to some early version of a chapter of the book, a rather long essay to Peter with the title "The strategy of using abelian categories" (in a book on homological algebra). I was immensely pleased when most of my suggestions were accepted by Peter. Writing this book was a great experience on many levels!

Only a short time after *A Course in Homological Algebra* came out, I decided to produce some lecture notes on homology in group theory [2], which would be a collection of more specialized material on the applications of homological algebra to group theory proper. I asked Peter whether he would be willing to read through the text and correct my still defective English. No, he answered, he would not do that, but, yes, he would be willing to read the entire manuscript and comment on the mathematics and the English. It is no surprise that this led to many important improvements. Whenever Peter accepted a challenge, he went above and beyond his duty.

During our second year in Ithaca, Peter and his wife, Meg, offered me and my wife, Irene, the opportunity to rent the annex of their house in the Cayuga Heights. The apartment had two small rooms, a tiny kitchen, and an equally tiny bathroom. Despite the cozy living conditions, this year turned out to be one of the happiest years of our lives. We became close friends during that time; after a few days even the Hiltons' dog, Lady, and our cat, Rupert, became friendly with each other! However, when our cat ate one of the Hiltons' pet gerbils, Peter didn't like it, and the relationship between the Stammbachs and the Hiltons became somewhat strained, but only for a few days. Whenever Peter and Meg were away we took care of their teenage boy, Tim. Thus we got some on-the-job training in parenting. I believe Tim would agree that he survived rather well with us.

In 1969 we returned to Zürich. Peter continued to visit the ETH regularly (at least once a year for several weeks). Our deep friendship was continued. We exchanged ideas, mathematical and otherwise; we did mathematical research; and we wrote joint papers. Often Irene and I had Peter as a houseguest. These were always very special days, which we enjoyed enormously, especially when Meg could be with us. With the extended periods Peter spent in Zürich, his German become better and better—so much so that, when speaking German in Germany, he was told that he had developed a distinct Swiss accent. This pleased Peter very much. When in Zürich he developed a liking for a special brand of Swiss cigars, the so-called Stumpen. Peter arranged with the tobacconist to send him a new supply of Stumpen at regular intervals. In order not to have any difficulty with the U.S. Customs, Peter asked that the package be clearly marked as educational material. As far as I know this declaration was always accepted without any problem.

Peter's last visit to Zürich was on Beno Eckmann's ninetieth birthday in 2007. It was difficult for him to walk, but his mind was as sharp as ever. At the dinner he gave a moving speech in honor of Beno Eckmann, his close friend and collaborator of many years. The day after the event, we had Peter and Meg with us for dinner, along with some mutual friends. Early the next morning we drove Peter and Meg to the airport. There he said goodbye to us. None of us expected that it would be the last time.

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Gerald L. Alexanderson

It was in Berkeley, in the mid-1970s, that I first met Peter Hilton at a small conference on mathematics education. Some active and prominent people in curriculum reform were present, and the discussion was lively and sometimes contentious over the several days the group met. At the end of the meetings Peter was looking around for a way to get to the San Francisco Airport to return to Cleveland,

Gerald L. Alexanderson is the Valeriote Professor in Science at Santa Clara University. His email address is `galexand@math.scu.edu`.

where he then held the Louis D. Beaumont Chair at Case Western Reserve. So I volunteered to drop him off at SFO on my way home. On the slow trip across the bridge to the airport in heavy traffic we had a stimulating conversation about the events at the conference, and when I dropped him off I felt that I had a friend in Cleveland.

Those were heady days in mathematics instruction, with the fallout from the 1963 Cambridge Report and the Bourbaki-influenced suggestions for reform of instruction having powerful advocates. The Cambridge Report had many mathematical luminaries as signers—Peter, of course, but also Creighton Buck, Andrew Gleason, Mark Kac, Ted Martin, Ed Moise, Max Schiffer, and Pat Suppes, among many others. It was provocative but in the end probably not very influential, since it came to be viewed as utopian and unrealistic. For example, "drill was to be eliminated from all grades". I recall hearing at about that time a talk by Peter on category theory, for the benefit of teachers! In those days rigorous mathematics in the classroom had many ardent supporters in the mathematical community.

It was about that time, however, that Peter met Jean Pedersen of my department, and Jean brought to the long collaboration he had with her another point of view. This followed from her work with her mentor, George Pólya, who advocated a more participatory approach in the classroom based on his "guess and prove" philosophy. In pedagogy, this seemed a collaboration doomed to failure because their viewpoints were so disparate. It ended up, however, with each shifting more toward a common middle ground, and their joint work—approximately 140 papers and six books—ended up influencing mathematics instruction on at least four continents. Peter was an inveterate traveler, showing up at meetings in the United States, of course, but also all over Europe, as well as New Zealand, Australia, and South Africa. He was always on the move. Whatever Peter talked about, it was fascinating. He brought to any conversation an urbane, sophisticated viewpoint that was wonderfully appealing. He had, as often seen in people educated in Britain, a sharp wit, and, in Peter's case, it could be used to encapsulate a really provocative idea in the form of an ironic quip. For example, I recall his pointing out in a lecture the irony of living in a society that actively supports driver education and teacher training!

Peter was not only a world-class topologist and a spokesman for mathematics education, he was also an erudite and cultivated person who was at ease talking about music, literature, art, politics, just about anything. The fact that his wife Meg is an accomplished actor with a career spanning work in London and New York, regional theater, and festivals opened doors for him to the artistic world that most mathematicians do not have the opportunity to explore. Of course, an education at Oxford and Cambridge opens some doors, too.

Beyond his erudition and his consummate good taste, though, was the basic kindness that he showed to his colleagues and students. He had strong opinions, but he expressed them gently. I saw many instances in which he generously helped colleagues he did not know well, but he provided good advice and a willingness to champion work that they had done by directing them to appropriate journals. Thus he became a mentor to young colleagues who were not his students or even people in his field.

I recall well the last time I had a chance to visit with Peter. He came to see me when I was in the hospital recovering from surgery resulting from a broken leg and hip. At that point we were both walking with comparable difficulty—mine due to an accident, his due to the onset of a decline that resulted eventually in his recent death. But I had a great time visiting with him. It was the old Peter we all loved, and his shuffling walk did not impede his ability to carry on a stimulating conversation with the usual flashes of wit and ability to spin out a good anecdote. And he could still skewer the pompous and the hypocritical with his usual elegant choice of language.

Mathematics has lost one of its most articulate, indeed eloquent, advocates. His kind does not come along very often, and mathematics is the worse for his departure from the scene.