

Notices

of the American Mathematical Society

February 2012

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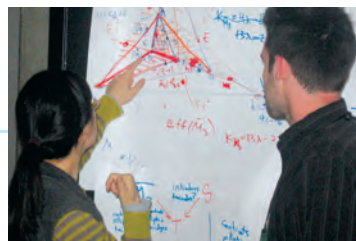
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MathJax
embeds
TEX in
web pages:

$$y = \int f(t) dt.$$

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AMS-Simons Travel Grants



The AMS is accepting applications for the second year of the AMS-Simons Travel Grants program. Each grant provides an early career mathematician with \$2,000 per year for two years to reimburse travel expenses related to research. Sixty new awards will be made in 2012. Individuals who are not more than four years past the completion of the PhD are eligible. The department of the awardee will also receive a small amount of funding to help enhance its research atmosphere.

The deadline for 2012 applications is March 30, 2012.

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www.ams.org/programs/travel-grants/AMS-SimonsTG





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$$t \ln t \frac{dr}{dt} + r = 7te^t$$

$$r = \frac{7e^t + C}{\ln t}$$



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Grants to support collaborations between Chinese and U.S./Canadian researchers are made possible through the generosity of Ky and Yu-Fen Fan.

The Fan China Exchange Program is intended to send eminent mathematicians from the U.S. and Canada to make a positive impact on the mathematical research community in China and to bring Chinese scientists in the early stages of their research to the U.S. and Canada to help further their careers. The program encourages host institutions to provide some type of additional support for the travel or living expenses of the visitor and to ensure a suitable length of stay.

Applications received before March 15 will be considered for the following academic year.

For more information on the Fan China Exchange Program and application process see www.ams.org/employment/chinaexchange.html or contact the AMS Membership and Programs Department by telephone at 800-321-4267, ext. 4170 (U.S. and Canada), or 401-455-4170 (worldwide), or by email at chinaexchange@ams.org.

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of the American Mathematical Society

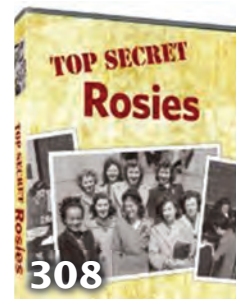
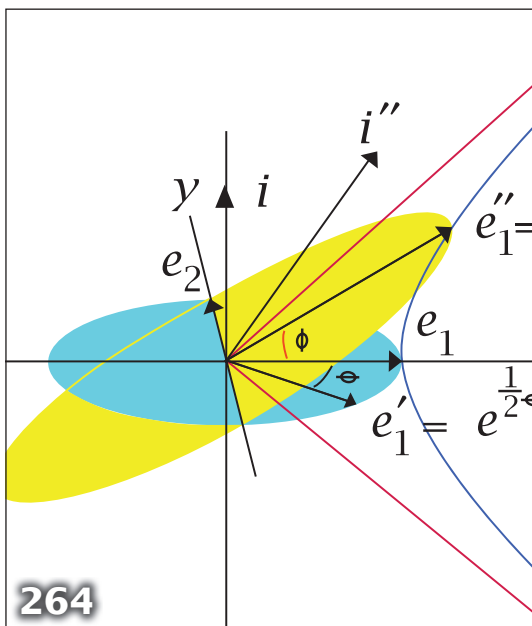
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The February issue showcases an article about stewardship in mathematics, a second about the nature of influential mathematicians, and a third about conformal mappings in geometric algebra. Among our columns are a discussion of sense-making in mathematics teaching and a treatment of issues connected with publishing computational mathematics. Finally, there is a presentation (by its creator) of the exciting new tool MathJax.

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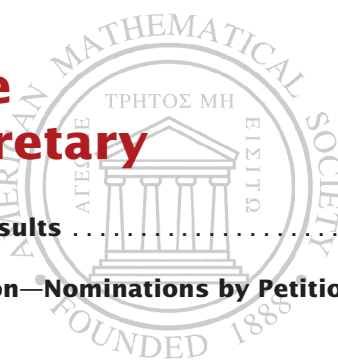
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UNR Mathematics Dodges the Budget Axe—For Now

This past March my department received some chilling news: under the governor's proposed budget, the university would eliminate our mathematics and statistics graduate program and fire half of our tenured faculty. Eight other university programs were also slated for downsizing or outright elimination. For several agonizing months we waited while the budget was debated. Ultimately, an unexpected state court ruling rescued the department from these cuts—at least for now. Here is our story.

In early 2011 the Nevada legislature was debating the governor's proposed budget, which included significant reductions in higher education. Asked how it would meet this budget, my institution, the University of Nevada, Reno (UNR), announced US\$20 million in cuts, including several academic programs (not including mathematics) as well as many administrative positions. The university said it would identify the remaining cuts only after the budget was finalized.

The remaining proposed cuts to UNR amounted to approximately US\$14 million. Some legislators asked the university to identify the remaining programs at stake in order to help them get the votes needed to override the governor's veto of an alternative budget.

In response, UNR listed additional departments that would be cut or downsized. The Department of Mathematics and Statistics was targeted for downsizing. Of our eighteen tenured faculty, fifteen are active in research, and six recently received grants from the National Science Foundation. We have no untenured faculty apart from one Ph.D. lecturer and two MS lecturers. The remaining instructors are not funded through the budget in question. We currently offer BA, BS, and MS degrees. The administration proposed to eliminate the graduate program, cutting US\$1.04 million dollars, just under half our budget, and eliminating eleven state-funded positions. By necessity, that would be at least eight tenured positions.

How does a university facing budget cuts fire tenured faculty? According to guidelines of the American Association of University Professors, tenure can be canceled when the decision to eliminate a program is based on long-term considerations of the institutional mission. Facing severe budget cuts over several years with no prospect of increased revenues for the next five years, UNR has formalized a process of "curricular review", under which program closure proposals by the administration are reviewed by affected departments, colleges, and the Faculty Senate before they are enacted. But, ultimately, faculty face a choice between supporting the proposed cut or proposing alternative program reconfigurations or closures that

result in comparable savings. Several academic departments were already closed under this process last year.

The proposal to cut our graduate program was brief and did not indicate how the downsized department would function. It did not, for example, indicate how we would continue to teach numerous large lecture courses without graduate teaching assistants to staff the recitations. Nor did it identify any metrics by which our program was viewed as underperforming or low-yield. Our program produces more master's degrees, in fact, than any other in the College of Science.

The department was informed that the governor's budget, if approved, would result in all math faculty being given terminal one-year contracts on June 30, 2011. Unless the ensuing curricular review resulted in changes to the proposal, the entire department would be eliminated. A new, smaller department would be formed in its place, and former faculty could then reapply for the smaller number of openings.

Just before the legislative session ended, with no apparent progress on a budget compromise, the governor was handed a state Supreme Court decision that disallowed the diversion of local government funds from their intended purpose to general state use. This meant that some of the creative accounting tricks in his budget were unconstitutional, and hence even the governor's austere budget would exceed the available funds. With the court decision as political cover, the governor hammered out a compromise that included extending some "temporary taxes" from the last legislative session, which had been scheduled to expire. That is, the "no taxes" crowd could actually increase revenue without taking the blame. In short, UNR's remaining cuts were much smaller, and the departments slated to be cut in the last US\$14 million reduction were taken off the table.

With any luck, the legislature's biennium budget will stand, preventing the need for further cuts next year. The following year the process will start again, and the outlook is not promising. The main problem is that Nevada's tax structure hasn't been changed to reflect the state's loss of its monopoly on the gambling industry in the western United States. Gaming taxes used to be the primary source of funding for state services, allowing the state to avoid income tax or significant business taxes. Gaming tax revenue has now fallen below sales tax and is not expected to recover.

For the time being, we are out of the woods. But the impact on the morale of the department's faculty and students has been huge. I don't know whether it will recover before the budget battles begin anew. In the meantime, never has it been more important to highlight the role of math education in producing a well-educated workforce, particularly in a state desperate to diversify its economy.

—Chris Herald
University of Nevada, Reno
herald@unr.edu

Mathematics in Pakistan

We found the article “The Abdus Salam School of Mathematical Sciences in Pakistan” by Loring W. Tu (*Notices*, August 2011) very interesting. It is highly commendable that the efforts made in Pakistan to improve the state of education in general and mathematics, in particular, are acknowledged and appreciated internationally. It is also desirable that these actions get support from mathematicians across the globe as mathematics is essentially an international activity. These include specific forms of support mentioned in this article for SMS as well.

As efforts towards improving the state of education and research in Pakistan do not get much international attention, we have an apprehension that this article might give a wrong impression to the rest of the world that SMS is perhaps the only institution in Pakistan working to achieve these goals for mathematics, although there is no explicit indication to this effect in the article itself. The purpose of this letter is to highlight briefly the efforts made at some other institutions as well, and we mention only four of them here.

Quaid-i-Azam University (QAU) Islamabad was established about fifty years ago. It produced the first Ph.D.’s in mathematics in Pakistan and is regarded as the mother institution in mathematics for all the other institutions to come later. The mathematics graduates from this university were (and are still) the seeds for most of the other institutions to come later. The number of Ph.D.’s and M.Phils. produced here surpasses any other institution in the country and has produced extremely high-quality mathematicians. For a long time this was without rival in terms of research publications.

The Centre for Advanced Mathematics and Physics (CAMP) at the National University of Sciences and Technology, Islamabad, is a relatively new institution (seven years old to be precise), but in terms of the quality and quantity of research output it is performing much better than older

ones on account of its more rigorous requirements for its postgraduate programs and the broad base provided. It has already started producing extremely good Ph.D.’s and M.Phils. in mathematics and physics, many of whom have already received international recognition.

The COMSATS Institute of Information Technology is producing the largest quantum of mathematical research in the country and a very large number of research graduates. As at the SMS and QAU, the graduates are more limited in their backgrounds.

The Lahore University of Management Sciences started a graduate program in mathematics a long time ago. However, it did not become a major producer of graduates or research at the time. Now the university has initiated a Faculty of Science and Engineering that is expected to follow the lines of CAMP in the broader base required.

Other institutions around the country, including the oldest university in Pakistan, the University of the Punjab in Lahore, are also making valuable efforts for the cause of mathematics.

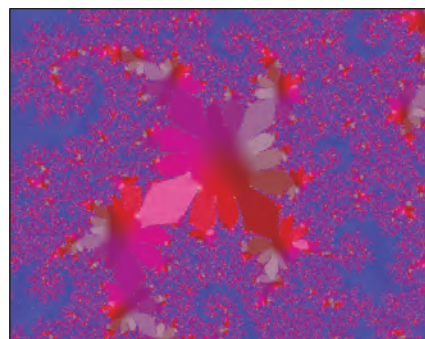
—Asghar Qadir
Centre for Advanced
Mathematics and Physics
National University of Sciences
and Technology
Islamabad, Pakistan
aqadirmath@yahoo.com

—Khalid Saifullah
Department of Mathematics
Quaid-i-Azam University
Islamabad, Pakistan
saifullah@qau.edu.pk

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Phase Plots of Complex Functions

I was very pleased to see the excellent article “Phase plots of complex functions: A journey in illustration”, by Wegert and Semmler in the June/July 2011 *Notices*. A color scheme which I have been using since the mid-1980s shows changes in modulus as



well as phase. A picture using this scheme appeared in the Mathematical Art Exhibit at the Joint Mathematics Meetings in San Antonio in January 2006 [see accompanying graphic]. Visual Basic code for this color mapping can be found at <http://w.american.edu/cas/mathstat/1crone/ComplexPlot.html>.

—Larry Crone
American University
1crone@centurylink.net

(Received November 22, 2011)

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Putting Another Cork in It

A triple cork is a spinning jump in which the snowboarder is parallel to the ground three times while in the air. Such a jump had never been performed in a competition before 2011, which prompted ESPN's Sport Science program to ask math professor Tim Charlier if it could be done under certain conditions. Originally doubtful, he and a recent math major graduate used differential equations, vector analysis, and calculus to discover that yes, a triple cork was indeed possible. A few days later, boarder Torgeir Horgmo landed a successful triple cork at the X-Games (which presumably are named for everyone's favorite variable).



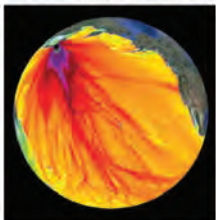
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Sounding the Alarm

Nothing can prevent a tsunami from happening—they are enormously powerful events of nature. But in many cases networks of seismic detectors, sea-level monitors and deep ocean buoys can allow authorities to provide adequate warning to those at risk. Mathematical models constructed from partial differential equations use the generated data to determine estimates of the speed and magnitude of a tsunami and its arrival time on coastlines. These models may predict whether a trough or a crest will be the first to arrive on shore. In only about half the cases (not all) does the trough arrive first, making the water level recede dramatically before the onslaught of the crest.



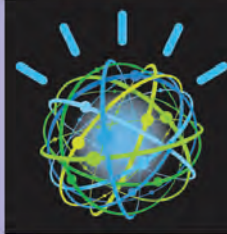
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Answering the Question, and Vice Versa

Experts are adept at answering questions in their fields, but even the most knowledgeable authority can't be expected to keep up with all the data generated today. Computers can handle data, but until now, they were inept at understanding questions posed in conversational language. Watson, the IBM computer that won the Jeopardy! Challenge, is an example of a computer that can answer questions using informal, nuanced, even pun-filled phrases. Graph theory, formal logic, and statistics help create the algorithms used for answering questions in a timely manner—not at all elementary.



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Sustaining the Supply Chain

It's often a challenge to get from Point A to Point B in normal circumstances, but after a disaster it can be almost impossible to transport food, water, and clothing from the usual supply points to the people in desperate need. A new mathematical model employs probability and nonlinear programming to design supply chains that have the best chance of functioning after a disaster. For each region or country, the model generates a robust chain of supply and delivery points that can respond to the combination of disruptions in the network and increased needs of the population.



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Keeping the Beat

The heart's function of pumping blood may seem fairly simple but the underlying mechanisms and electrical impulses that maintain a healthy rhythm are extremely complex. Many areas of mathematics, including differential equations, dynamical systems, and topology help model the electrical behavior of cardiac cells, the connections between those cells and the heart's overall geometry. Researchers aim to gain a better understanding of the normal operation of the heart, as well as learn how to diagnose the onset of abnormalities and correct them.



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Harnessing Wind Power

Mathematics contributes in many ways to the process of converting wind power into usable energy. Large-scale weather models are used to find suitable locations for wind farms, while more narrowly focused models—incorporating interactions arising from factors such as wake effects and turbulence—specify how to situate individual turbines within a farm. In addition, computational fluid dynamics describes air flow and drag around turbines. This helps determine the optimal shapes for the blades, both structurally and aerodynamically, to extract as much energy as possible, and keep noise levels and costs down.



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Conformal Mappings in Geometric Algebra

Garret Sobczyk

In 1878 William Kingdon Clifford wrote down the rules for his *geometric algebra*, also known as *Clifford algebra*. We argue in this paper that in doing so he laid down the groundwork that is profoundly altering the language used by the mathematical community to express geometrical ideas. In the real estate business everyone knows that what is most important is **location**. We demonstrate here that in the business of mathematics what is most important to the clear and concise expression of geometrical ideas is **notation**. In the words of Bertrand Russell,

...A good notation has a subtlety and suggestiveness which at times make it seem almost like a live teacher.

Heinrich Hertz expressed much the same thought when he said,

One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we originally put into them.

The development of the real and complex number systems represents a hard-won milestone in the robust history of mathematics over many centuries and many different civilizations [5], [29]. Without it mathematics could progress only haltingly, as is evident from the history of mathematics and even the terminology that we use today. Negative numbers were referred to by Rene Descartes (1596–1650) as “fictitious”, and “imaginary” numbers were held up to even greater ridicule, though they were first conceived as early as Heron of

Garret Sobczyk is professor of mathematics at Universidad de Las Américas-Puebla. His email address is garret_sobczyk@yahoo.com.

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Herman Gunther Grassmann (1809–1877) was a high school teacher. His far-reaching *Ausdehnungslehre*, “Theory of extension”, laid the groundwork for the development of the exterior or outer product of vectors. William Rowan Hamilton (1805–1865) was an Irish physicist, astronomer, and mathematician. His invention of the quaternions as the natural generalization of the complex numbers of the plane to three-dimensional space, together with the ideas of Grassmann, set the stage for William Kingdon Clifford’s definition of geometric algebra. William Kingdon Clifford (1845–1879) was a professor of mathematics and mechanics at the University College of London. Tragically, he died at the early age of 34 before he could explore his profound ideas.

Alexandria, the illustrious inventor of the windmill and steam engine during the first century AD [18]. What most mathematicians fail to see even today is that geometric algebra represents the grand culmination of that process with the completion of the real number system to include the concept of direction. Geometric algebra combines the two silver currents of mathematics, geometry and algebra, into a single coherent language. As David Hestenes has eloquently stated,

*Algebra without geometry is blind,
geometry without algebra is dumb.*

David Hestenes, a newly hired assistant professor of physics, introduced me to geometric algebra at Arizona State University in 1966. I immediately became captivated by the idea that geometric algebra is the natural completion of the real number system to include the concept of direction. If this idea is correct, it should be possible to formulate all the ideas of multilinear linear algebra, tensor analysis, and differential geometry. I considered it to be a major breakthrough when I succeeded in proving the fundamental Cayley-Hamilton theorem within the framework of geometric algebra. The next task was mastering differential geometry. Five years later, with Hestenes as my thesis director, I completed my doctoral dissertation, "Mappings of Surfaces in Euclidean Space Using Geometric Algebra" [26].

Much work of Hestenes and other theoretical physicists has centered around understanding the geometric meaning of the appearance of imaginary numbers in quantum mechanics. Ed Witten, in his Josiah Willard Gibbs Lecture "Magic, Mystery, and Matrix" says, "Physics without strings is roughly analogous to mathematics without complex numbers" [31]. In a personal communication with the author, Witten points out that "Pierre Ramond, in 1970, generalized the Clifford algebra from field theory (where Dirac had used it) to string theory." Dale Husemoller in his book *Fibre Bundles* includes a chapter on Clifford algebras, recognizing their importance in "topological applications and for giving a concrete description of the group $\text{Spin}(n)$ " [13]. Other mathematicians have developed an extensive new field which has come to be known as "Clifford analysis" [3], [7] and "spin geometry" [2].

During the academic year 1972-73 I continued work at Arizona State University with David on differential geometry as a visiting assistant professor. The following year, at the height of the academic crunch, I was on my own—a young unemployed mathematician, but with a mission: *To convince the world of the importance of geometric algebra*. I travelled to Europe and European universities giving talks when and where I was graciously invited. During the year 1973-74, Professor Roman Duda of the Polish Academy of Sciences offered me a postgraduate research position, which I gratefully accepted.

My collaboration with David culminated a number of years later with the publication of our book *Clifford Algebra to Geometric Calculus: A Unified Language for Mathematics and Physics* [11]. Over the years I have been searching for the best way to handle ideas in linear algebra and differential geometry, particularly with regard to introducing students to geometric algebra at the undergraduate level. To this end, I have been working

to complete a new manuscript *New Foundations in Mathematics: The Geometric Concept of Numbers* which has been accepted for publication by Birkhäuser [24].

Distilling geometric algebra down to its core, it is the extension of the real number system to include new *anticommuting* square roots of ± 1 , which represent mutually orthogonal unit vectors in successively higher dimensions. Whereas everybody is familiar with the idea of the imaginary number $i = \sqrt{-1}$, the idea of a new square root $u = \sqrt{+1} \notin \mathbb{R}$ has taken longer to be assimilated into the main body of mathematics. However, in [22] it is shown that the *hyperbolic numbers*, which have much in common with their more famous brothers the complex numbers, also lead to an efficacious solution of the cubic equation.

The time has come for geometric algebra to become universally recognized for what it is: the completion of the real number system to include the concept of direction. Let twenty-first century mathematics be a century of consolidation and streamlining so that it can bloom with all the renewed vigor that it will need to prosper in the future [10].

Fundamental Concepts

Let $\mathbb{R}^{p,q}$ be the pseudoeuclidean space of the indefinite nondegenerate real quadratic form $q(\mathbf{x})$ with signature $\{p, q\}$, where $\mathbf{x} \in \mathbb{R}^{p,q}$. With this quadratic form, we define the *inner product* $\mathbf{x} \cdot \mathbf{y}$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{p,q}$ to be

$$\mathbf{x} \cdot \mathbf{y} = \frac{1}{2}[q(\mathbf{x} + \mathbf{y}) - q(\mathbf{x}) - q(\mathbf{y})].$$

Let $(\mathbf{e})_{(n)} = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{p+q})$ denote the row of *standard orthonormal anticommuting basis vectors* of $\mathbb{R}^{p,q}$, where $\mathbf{e}_j^2 = q(\mathbf{e}_j)$ for $j = 1, \dots, n$, i.e.,

$$\mathbf{e}_1^2 = \dots = \mathbf{e}_p^2 = 1 \text{ and } \mathbf{e}_{p+1}^2 = \dots = \mathbf{e}_{p+q}^2 = -1$$

and $n = p + q$. Taking the transpose of the row of basis vectors $(\mathbf{e})_{(n)}$ gives the *column* $(\mathbf{e})_{(n)}^T$ of these same basis vectors. The *metric tensor* $[g]$ of $\mathbb{R}^{p,q}$ is defined to be the *Gramian matrix*

$$(1) \quad [g] := (\mathbf{e})_{(n)}^T \cdot (\mathbf{e})_{(n)},$$

which is a diagonal matrix with the first p entries +1 and the remaining entries -1. The *standard reciprocal basis* to $(\mathbf{e})_{(n)}$ is then defined, with raised indices, by

$$(\mathbf{e})^{(n)} = [g](\mathbf{e})_{(n)}^T.$$

In terms of the standard basis $(\mathbf{e})_{(n)}$, and its reciprocal basis $(\mathbf{e})^{(n)}$, a vector $\mathbf{x} \in \mathbb{R}^{p,q}$ is efficiently expressed by

$$\mathbf{x} = (\mathbf{e})_{(n)}(x)^{(n)} = (x)_{(n)}(\mathbf{e})^{(n)}.$$

Both the column vector of components $(x)^{(n)}$ and the row vector of components of $\mathbf{x}$ are, of course,

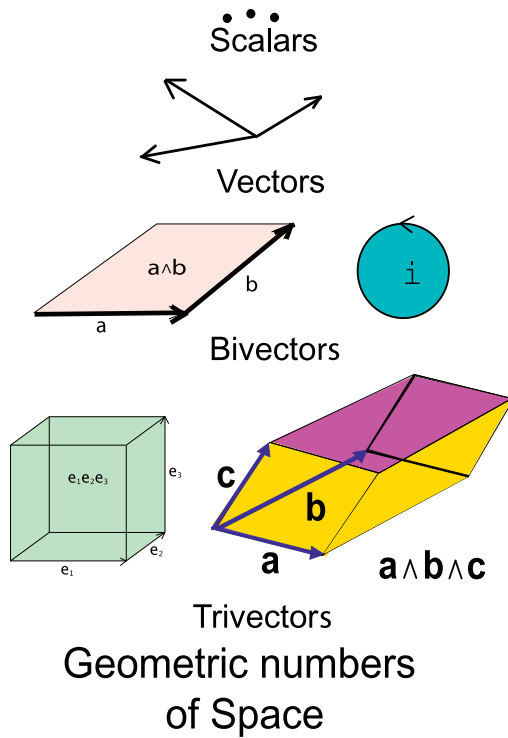


Figure 1.

real numbers. If $\mathbf{y} = (\mathbf{e})_{(n)} (\mathbf{y})^{(n)}$ is a second such vector, then the inner product between them is

$$\begin{aligned} \mathbf{x} \cdot \mathbf{y} &= (\mathbf{x})_{(n)} (\mathbf{e})^{(n)} \cdot (\mathbf{e})_{(n)} (\mathbf{y})^{(n)} \\ &= \sum_{i=1}^n x_i y_i = \sum_{i=1}^p x_i y_i - \sum_{j=p+1}^n x_j y_j, \end{aligned}$$

where $n = p + q$.

Let $(\mathbf{a})_{(n)}$ be a second basis of $\mathbb{R}^{p,q}$. Then

$$(2) \quad (\mathbf{a})_{(n)} = (\mathbf{e})_{(n)} A \quad \text{and} \quad (\mathbf{a})_{(n)}^T = A^T (\mathbf{e})_{(n)}^T,$$

where A is the *matrix of transition* from the standard basis $(\mathbf{e})_{(n)}$ of $\mathbb{R}^{p,q}$ to the basis $(\mathbf{a})_{(n)}$ of $\mathbb{R}^{p,q}$. The Gramian matrix of inner products of the basis vectors $(\mathbf{a})_{(n)}$ is then given by

$$(\mathbf{a})_{(n)}^T \cdot (\mathbf{a})_{(n)} = A^T (\mathbf{e})_{(n)}^T \cdot (\mathbf{e})_{(n)} A = A^T [g] A$$

where $[g]$ is the metric tensor of $\mathbb{R}^{p,q}$ defined in (1).

The real associative *geometric algebra* $\mathbb{G}_{p,q} = \mathbb{G}_{p,q}(\mathbb{R}^{p,q})$ is generated by taking sums of geometric products of the vectors in $\mathbb{R}^{p,q}$, subjected to the rule that $\mathbf{x}^2 = q(\mathbf{x}) = \mathbf{x} \cdot \mathbf{x}$ for all vectors $\mathbf{x} \in \mathbb{R}^{p,q}$. The dimension of $\mathbb{G}_{p,q}$ as a real linear space is 2^n , where $n = p + q$, with the *standard orthonormal basis* of geometric numbers

$$\mathbb{G}_{p,q} = \text{span}\{\mathbf{e}_{\lambda_k}\}_{k=0}^n,$$

where the $\binom{n}{k}$ k -vector basis elements of the form $\mathbf{e}_{\lambda_k}$ are defined by

$$\mathbf{e}_{\lambda_k} = \mathbf{e}_{i_1 \dots i_k} = \mathbf{e}_{i_1} \cdots \mathbf{e}_{i_k}$$

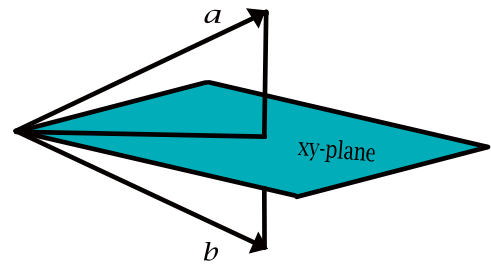


Figure 2. The vector $\mathbf{a}$ is reflected in the xy -plane to give the vector $\mathbf{b} = \mathbf{e}_{12} \mathbf{a} \mathbf{e}_{12}$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors along the x and y axis, respectively, and $\mathbf{e}_{12}$ is the unit bivector of the xy -plane.

for each $\lambda_k = i_1 \dots i_k$ where $1 \leq i_1 < \dots < i_k \leq n$. When $k = 0$, $\lambda_0 = 0$ and $\mathbf{e}_0 = 1$. For example, the 2³-dimensional geometric algebra $\mathbb{G}_3$ of $\mathbb{R}^3$ has the standard basis

$$\begin{aligned} \mathbb{G}_3 &= \text{span}\{\mathbf{e}_{\lambda_k}\}_{k=0}^3 \\ &= \text{span}\{1, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_{12}, \mathbf{e}_{13}, \mathbf{e}_{23}, \mathbf{e}_{123}\}. \end{aligned}$$

The *geometric numbers* of space are pictured in Figure 1.

The great advantage of geometric algebra is that deep geometrical relationships can be expressed directly in terms of the multivectors of the algebra without having to constantly refer to a basis; see Figures 2 and 3. On the other hand, the language gives geometric legs to the powerful matrix formalism that has developed over the last one hundred fifty years. As a real associative algebra, each geometric algebra under geometric multiplication is isomorphic to a corresponding algebra or subalgebra of real matrices, and, indeed, we have advocated elsewhere the need for a uniform approach to both of these structures [23], [27]. Matrices are invaluable for systematizing calculations, but geometric algebra provides deep geometrical insight and powerful algebraic tools for the efficient expression of geometrical properties. Clifford algebras and their relationships to matrix algebra and the classical groups have been thoroughly studied in [15], [21] and elsewhere.

Whereas it is impossible to summarize all of the identities in geometric algebra that we need here, there are many good references that are available, for example, [1], [6], [9], [11], [15]. The most basic identity is the breaking down of the geometric product of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{p,q}$ into the symmetric and skewsymmetric parts, giving

$$(3) \quad \mathbf{x} \mathbf{y} = \frac{1}{2}(\mathbf{x} \mathbf{y} + \mathbf{y} \mathbf{x}) + \frac{1}{2}(\mathbf{x} \mathbf{y} - \mathbf{y} \mathbf{x}) = \mathbf{x} \cdot \mathbf{y} + \mathbf{x} \wedge \mathbf{y},$$

where the inner product $\mathbf{x} \cdot \mathbf{y}$ is the symmetric part and the outer product $\mathbf{x} \wedge \mathbf{y}$ is the skewsymmetric part. For whatever the reason, historical or otherwise, the importance of this identity for mathematics has yet to be fully appreciated. The outer

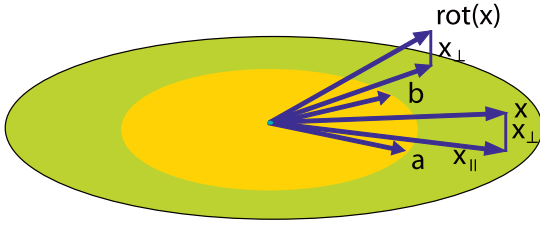


Figure 3. The vector $\mathbf{x} = \mathbf{x}_{||} + \mathbf{x}_{\perp}$ is rotated in the plane of the bivector $\mathbf{a} \wedge \mathbf{b}$ into $\text{rot}(\mathbf{x})$, leaving the perpendicular component unchanged. When $|\mathbf{a}| = |\mathbf{b}| = 1$, $\text{rot}(\mathbf{x}) = e^{-i\frac{\theta}{2}} \mathbf{x} e^{i\frac{\theta}{2}}$, where $e^{i\frac{\theta}{2}} = \frac{\mathbf{a}(\mathbf{a}+\mathbf{b})}{|\mathbf{a}+\mathbf{b}|}$, $e^{-i\frac{\theta}{2}} = \frac{(\mathbf{a}+\mathbf{b})\mathbf{a}}{|\mathbf{a}+\mathbf{b}|}$ and θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. If $\mathbf{a}$ and $\mathbf{b}$ lie in the xy -plane, then $i = \mathbf{e}_{12}$.

product $\mathbf{x} \wedge \mathbf{y}$ has the geometric interpretation of a *bivector* or a *directed plane segment*.

Other, more general but similar, identities are quickly developed. In general, the geometric product between a vector $\mathbf{a}$ and a k -vector $\mathbf{B}_k$, for $k \geq 1$, can always be decomposed into the sum of an *inner product* (contraction) and an *outer product* (extension),

$$(4) \quad \mathbf{a} \mathbf{B}_k = \mathbf{a} \cdot \mathbf{B}_k + \mathbf{a} \wedge \mathbf{B}_k.$$

The inner product $\mathbf{a} \cdot \mathbf{B}_k$ is a $(k-1)$ -vector, defined by

$$\mathbf{a} \cdot \mathbf{B}_k := \frac{1}{2}(\mathbf{a} \mathbf{B}_k + (-1)^{k+1} \mathbf{B}_k \mathbf{a}) = \langle \mathbf{a} \mathbf{B}_k \rangle_{k-1},$$

and the outer product $\mathbf{a} \wedge \mathbf{B}_k$ is a $(k+1)$ -vector, defined by

$$\mathbf{a} \wedge \mathbf{B}_k := \frac{1}{2}(\mathbf{a} \mathbf{B}_k - (-1)^{k+1} \mathbf{B}_k \mathbf{a}) = \langle \mathbf{a} \mathbf{B}_k \rangle_{k+1}.$$

More generally, if $\mathbf{A}_r$ and $\mathbf{B}_s$ are r - and s -vectors of $\mathbb{G}_{p,q}$, where $r > 0, s > 0$, we define

$$\mathbf{A}_r \cdot \mathbf{B}_s = \langle \mathbf{A}_r \mathbf{B}_s \rangle_{|r-s|}$$

and

$$\mathbf{A}_r \wedge \mathbf{B}_s = \langle \mathbf{A}_r \mathbf{B}_s \rangle_{r+s}.$$

In the exceptional cases when $r = 0$ or $s = 0$, we define $\mathbf{A}_r \cdot \mathbf{B}_s = 0$ and $\mathbf{A}_r \wedge \mathbf{B}_s = \mathbf{A}_r \mathbf{B}_s$.

For the vectors $\mathbf{a}$ and $\mathbf{b}$ and the k -vector $\mathbf{C}_k$, where $k \geq 1$, we now show that

$$(5) \quad \mathbf{a} \cdot (\mathbf{b} \wedge \mathbf{C}_k) = (\mathbf{a} \cdot \mathbf{b}) \mathbf{C}_k - \mathbf{b} \wedge (\mathbf{a} \cdot \mathbf{C}_k).$$

Using the above definitions, we decompose the left side of this equation, getting $\mathbf{a} \cdot (\mathbf{b} \wedge \mathbf{C}_k) =$

$$\frac{1}{4}[\mathbf{a} \mathbf{b} \mathbf{C}_k + (-1)^k \mathbf{a} \mathbf{C}_k \mathbf{b} + (-1)^k \mathbf{b} \mathbf{C}_k \mathbf{a} + \mathbf{C}_k \mathbf{b} \mathbf{a}].$$

Decomposing the right-hand side of the equation gives

$$\begin{aligned} & (\mathbf{a} \cdot \mathbf{b}) \mathbf{C}_k - \mathbf{b} \wedge (\mathbf{a} \cdot \mathbf{C}_k) = \\ & \frac{1}{2}[(\mathbf{a} \cdot \mathbf{b}) \mathbf{C}_k + \mathbf{C}_k (\mathbf{a} \cdot \mathbf{b}) - \mathbf{b} (\mathbf{a} \cdot \mathbf{C}_k) + (-1)^k (\mathbf{a} \cdot \mathbf{C}_k) \mathbf{b}] \\ & = \frac{1}{4}[(\mathbf{a} \mathbf{b} + \mathbf{b} \mathbf{a}) \mathbf{C}_k + \mathbf{C}_k (\mathbf{a} \mathbf{b} + \mathbf{b} \mathbf{a}) - \mathbf{b} \mathbf{a} \mathbf{C}_k \\ & \quad + (-1)^k \mathbf{b} \mathbf{C}_k \mathbf{a} + (-1)^k \mathbf{a} \mathbf{C}_k \mathbf{b} - \mathbf{C}_k \mathbf{a} \mathbf{b}]. \end{aligned}$$

After cancellations, we see that the left and right sides of the equations are identical, so the identity (5) is proved.

One very useful identity which follows from (5) is

$$\begin{aligned} \mathbf{a} \cdot (\mathbf{A}_r \wedge \mathbf{B}_s) &= (\mathbf{a} \cdot \mathbf{A}_r) \wedge \mathbf{B}_s + (-1)^r \mathbf{A}_r \wedge (\mathbf{a} \cdot \mathbf{B}_s) \\ (6) \quad &= (-1)^{r+s+1} (\mathbf{A}_r \wedge \mathbf{B}_s) \cdot \mathbf{a}. \end{aligned}$$

It gives the *distributive law* for the inner product of a vector over the outer product of an r - and an s -vector.

The *outer product* of k vectors $\mathbf{a}_i \in \mathbb{R}^{p,q}$ is defined by

$$\mathbf{a}_{(k)} := \wedge (\mathbf{a})_{(k)} = \mathbf{a}_1 \wedge \cdots \wedge \mathbf{a}_k.$$

The outer product can be directly expressed as the completely antisymmetric geometric product of those vectors. We have

$$\mathbf{a}_1 \wedge \cdots \wedge \mathbf{a}_k = \sum_{\pi \in \Pi} \text{sgn}(\pi) \frac{1}{k!} \mathbf{a}_{\pi(1)} \cdots \mathbf{a}_{\pi(k)},$$

where π is a permutation on the indices $(1, 2, \dots, k)$, and $\text{sgn}(\pi) = \pm 1$ according to whether π is an even or odd permutation, respectively.

As an application of our notation and our change of basis formula (2), for $(\mathbf{a})_{(n)} = (\mathbf{e})_{(n)} A$, we have

$$\mathbf{a}_{(n)} = \mathbf{a}_1 \wedge \cdots \wedge \mathbf{a}_n = \wedge (\mathbf{a})_{(n)} = \mathbf{e}_{(n)} \det A.$$

By the *reverse* $A^\dagger$ of the geometric number $A \in \mathbb{G}_{p,q}$, we mean the geometric number obtained by reversing the order of the vectors in all of the geometric products that make up A [15, p. 29]. For example, if $A = \mathbf{a}_1 + \mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3$, then $A^\dagger = \mathbf{a}_1 + \mathbf{a}_3 \mathbf{a}_2 \mathbf{a}_1$. If $A, B \in \mathbb{G}_{p,q}$, then it immediately follows that

$$(A + B)^\dagger = A^\dagger + B^\dagger \quad \text{and} \quad (AB)^\dagger = B^\dagger A^\dagger.$$

By the *magnitude* $|A|$ of a geometric number $A \in \mathbb{G}_{p,q}$, we mean

$$|A| = \sqrt{|\langle A A^\dagger \rangle_0|} \geq 0.$$

In the case of any geometric number in the geometric algebra $\mathbb{G}_n$ of the Euclidean space $\mathbb{R}^n$, the absolute value inside the square root can be removed.

There are many very useful relationships that can be derived and that can be found in the references previously given. The above identities indicate the richness of the geometric algebra $\mathbb{G}_{p,q}$ as a tool for studying properties of the pseudoeuclidean space $\mathbb{R}^{p,q}$. Figure 4 illustrates one application of geometric algebra to physics [24], whereas Figure 5 sets the stage for directed

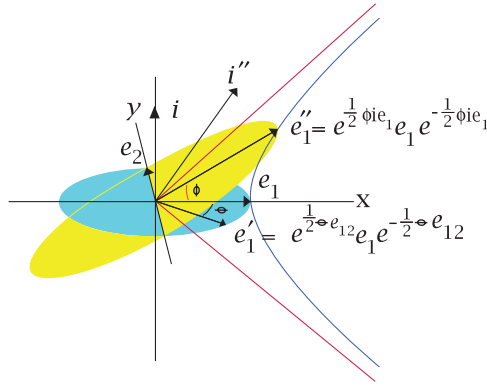


Figure 4. The vector $\mathbf{e}_1$ is rotated in the xy -plane of $\mathbf{e}_{12}$ into the vector $\mathbf{e}'_1$. The vector $\mathbf{e}_1$ is *boosted* into the *relative vector* $\mathbf{e}''_1$ of the *relative plane* of the *relative bivector* $\mathbf{e}''_{12} = \mathbf{e}'_1 \mathbf{e}_2$. The relative plane of $\mathbf{e}''_{12}$ has the relative velocity of $\frac{v}{c} = i\mathbf{e}_1 \tanh \phi$ with respect to the plane of $\mathbf{e}_{12}$, where c is the speed of light.

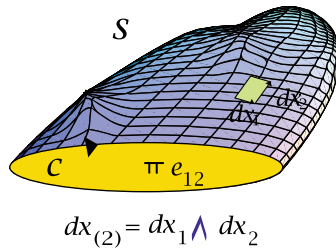


Figure 5. When the element of directed area $d\mathbf{x}_{(2)} = d\mathbf{x}_1 \wedge d\mathbf{x}_2$ is integrated over the smooth oriented surface S bounded by the oriented unit circle C in the xy -plane, we get $\int_S d\mathbf{x}_{(2)} = \pi \mathbf{e}_{12}$. This is the basis for directed integration theory and Clifford analysis.

integration theory and Clifford analysis, [3] and [11].

Differentiation on Vector Manifolds

Let $U \subset \mathbb{R}^{p,q}$ be an open set in the relative topology of $\mathbb{R}^{p,q}$ containing the point $\mathbf{x}$, and let $n = p + q$. A geometric-valued function $F : U \rightarrow \mathbb{G}_{p,q}$ is said to be *differentiable* at $\mathbf{x} \in U$ if and only if there exists a linear map $F_a : \mathbb{R}^{p,q} \rightarrow \mathbb{G}_{p,q}$ such that

$$(7) \quad \lim_{\mathbf{v} \rightarrow 0, \mathbf{v}^2 \neq 0} \frac{|F(\mathbf{x} + \mathbf{v}) - F(\mathbf{x}) - F_a(\mathbf{v})|}{|\mathbf{v}|} = 0.$$

The map $F_a(\mathbf{x})$ is linear in the tangent vector $\mathbf{a} \in \mathbb{R}^{p,q}$ and uniquely determined by the local properties of F at the point $\mathbf{x} \in \mathbb{R}^{p,q}$; see [28, p. 16]. For any $\mathbf{a} \in \mathbb{R}^{p,q}$, we write

$$F_a(\mathbf{x}) = \mathbf{a} \cdot \partial_{\mathbf{x}} F(\mathbf{x})$$

to represent $F_a(\mathbf{x})$ as the more familiar *directional derivative* of the geometric-valued function $F(\mathbf{x})$ in the direction of the vector $\mathbf{a}$ at the point $\mathbf{x}$.

If $\mathbf{a} = (\mathbf{e})_{(n)}(a)^{(n)} \in \mathbb{R}^{p,q}$ and $\mathbf{x} = (\mathbf{e})_{(n)}(x)^{(n)} \in U$, then

$$F_a(\mathbf{x}) = \sum_{i=1}^n a^i \frac{\partial F}{\partial x^i}.$$

In differential geometry books, the directional derivative F_a is sometimes referred to as a *standard connection* on $\mathbb{R}^{p,q}$ [12, p. 18].

A differentiable geometric-valued function $F(\mathbf{x})$ has a *vector derivative* ∂F , defined by

$$(8) \quad \partial F = \partial_{\mathbf{x}} F(\mathbf{x}) = \partial_{\mathbf{v}} F_{\mathbf{v}}(\mathbf{x}).$$

The second equality on the right expresses the vector derivative in terms of the *tangential vector derivative* $\partial_{\mathbf{v}}$ of the directional derivative $F_{\mathbf{v}}(\mathbf{x})$ of $F(\mathbf{x})$. In terms of the standard basis of $\mathbb{R}^{p,q}$,

$$\partial F = \sum_{i=1}^n \mathbf{e}_i \frac{\partial F}{\partial x^i}.$$

Just as in (4), the vector derivative ∂F of a k -vector-valued function F can be decomposed into the sum of a $(k-1)$ -vector part $\partial \cdot F$, called the *divergence*, and a $(k+1)$ -vector part $\partial \wedge F$, called the *curl*. We have

$$\partial F = \partial \cdot F + \partial \wedge F.$$

The important condition of *integrability* has an easy expression in terms of the vector derivative on $\mathbb{R}^{p,q}$,

$$(9) \quad \partial_{\mathbf{x}} \wedge \partial_{\mathbf{x}} = 0,$$

and is simply an expression of the fact that partial derivatives commute and that the standard basis vectors $\mathbf{e}_i$ are constant. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{p,q}$. In terms of directional derivatives, the integrability condition takes the equivalent form

$$(10) \quad (\mathbf{a} \wedge \mathbf{b}) \cdot (\partial_{\mathbf{x}} \wedge \partial_{\mathbf{x}}) = [\mathbf{b} \cdot \partial, \mathbf{a} \cdot \partial] - [\mathbf{b}, \mathbf{a}] \cdot \partial = 0,$$

where $[\mathbf{b} \cdot \partial, \mathbf{a} \cdot \partial] := \mathbf{b} \cdot \partial \mathbf{a} \cdot \partial - \mathbf{a} \cdot \partial \mathbf{b} \cdot \partial$, and $[\mathbf{b}, \mathbf{a}] = \mathbf{b} \cdot \partial \mathbf{a} - \mathbf{a} \cdot \partial \mathbf{b}$ is the *Lie bracket* of the *vector fields* $\mathbf{a}, \mathbf{b}$. Of course, if $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and are not a function of the point $\mathbf{x} \in \mathbb{R}^{p,q}$, then the Lie bracket $[\mathbf{a}, \mathbf{b}] = 0$. In this case, the integrability condition (10) reduces to the condition that the directional derivatives defined by the vectors $\mathbf{a}$ and $\mathbf{b}$ commute.

The definition of the directional and vector derivatives are independent of the signature p, q of the metric involved. A few basic formulas which are valid in any signature and can be easily established, are

$$\mathbf{a} \cdot \partial_{\mathbf{x}} \mathbf{x} = \mathbf{a} = \partial_{\mathbf{x}} \mathbf{x} \cdot \mathbf{a} \implies \partial_{\mathbf{x}} \mathbf{x} = \mathbf{n}.$$

For all integers r ,

$$\mathbf{a} \cdot \partial_{\mathbf{x}} \mathbf{x}^{2r} = 2r(\mathbf{a} \cdot \mathbf{x}) \mathbf{x}^{2r-2} \implies \partial_{\mathbf{x}} \mathbf{x}^{2r} = 2r \mathbf{x}^{2r-1}$$

and

$$\mathbf{a} \cdot \partial_{\mathbf{x}} \mathbf{x}^{2r+1} = (2r(\mathbf{a} \cdot \mathbf{x}) \mathbf{x} + \mathbf{a} \mathbf{x}^2) \mathbf{x}^{2r-2}$$

$$\implies \partial_{\mathbf{x}} \mathbf{x}^{2r+1} = (2r + \mathbf{n}) \mathbf{x}^{2r}.$$

$$\partial_{\mathbf{x}} \mathbf{a} \mathbf{x} = \partial_{\mathbf{x}} (\mathbf{x} \cdot \mathbf{a} - \mathbf{x} \wedge \mathbf{a}) = -(n-2) \mathbf{a}.$$

In light of the Whitney-Nash embedding theorems [4], [19], [20], [32], [30], *little or nothing* is lost by assuming that a general manifold is embedded in $\mathbb{R}^{p,q}$. We call such manifolds *vector manifolds*. By making such an assumption, all the tools of the geometric algebra $\mathbb{G}_{p,q}$ of $\mathbb{R}^{p,q}$ immediately become available for the study of the given vector manifold.

Let $\mathcal{M}$ be a regular k -dimensional manifold in $\mathbb{R}^{p,q}$, with an associated atlas of charts specified with respect to the inherited topology from $\mathbb{R}^{p,q}$. Let U be an open set in $\mathcal{M}$ containing the point $\mathbf{x} \in \mathcal{M}$. In some chart whose range contains the open set U , the point $\mathbf{x} = \mathbf{x}(s^1, \dots, s^k)$ is defined by the local coordinates $s^i \in \mathbb{R}$ for $i = 1, \dots, k$. The beauty of the definitions (7) and (8) is that they can be easily extended to define the *Koszul connection* and *vector derivative* on the embedded manifold $\mathcal{M}$ in $\mathbb{R}^{p,q}$; we simply require in the limit that $\mathbf{v} \rightarrow 0$ in such a way that $\mathbf{x} + \mathbf{v} \in U \subset \mathcal{M}$. See [14, pp. 152–53] and, in particular, Professor Lee’s “confession”. For this reason, we use the same symbols for the directional derivatives $F_{\mathbf{a}}(\mathbf{x})$ and the vector derivative $\partial_{\mathbf{x}}F(\mathbf{x})$ of the geometric-valued function $F(\mathbf{x})$ on $\mathcal{M}$.

The *standard basis* of the *tangent space* $\mathcal{T}_{\mathbf{x}}^1$ of $\mathcal{M}$, with respect to the chart containing U , is $(\mathbf{x})_{(k)} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k)$, where $\mathbf{x}_i = \frac{\partial \mathbf{x}}{\partial s^i}$ and the *geometric algebra* $\mathcal{T}_{\mathbf{x}}$ at the point $\mathbf{x} \in U$ is the subgeometric algebra of $\mathbb{G}_{p,q}$ generated by taking sums of geometric products of the tangent vectors $\mathbf{x}_i$. More care must be taken in writing down the integrability condition for the Koszul derivative $\mathbf{a} \cdot \partial_{\mathbf{x}}$ for $\mathbf{a} \in \mathcal{T}_{\mathbf{x}}$. For an embedded curved manifold $\mathcal{M}$, (9) no longer remains valid, but the alternative condition (10) *does* remain valid.

Projection and Curvature

Two of the most important objects on a vector manifold are the unit pseudoscalar field, defined by

$$(11) \quad I = I_{\mathbf{x}} = \frac{\mathbf{x}_1 \wedge \dots \wedge \mathbf{x}_k}{|\mathbf{x}_1 \wedge \dots \wedge \mathbf{x}_k|},$$

and the projection $P = P_{\mathbf{x}} : \mathbb{G}_{p,q} \rightarrow \mathcal{T}_{\mathbf{x}}$ onto the tangent algebra $\mathcal{T}_{\mathbf{x}}$ at the point $\mathbf{x} \in \mathcal{M}$, defined by

$$(12) \quad P(A_r) = (A_r \cdot I)I^\dagger,$$

where $A_r \in \mathbb{G}_{p,q}^r$ is any r -vector, for $1 \leq r \leq n$. When $r = 0$, the projection acts as the identity, $P_{\mathbf{x}}(\alpha) = \alpha$ for all $\alpha \in \mathbb{R}$. A smooth vector manifold $\mathcal{M}$ is said to be *orientable* if $I_{\mathbf{x}}$ is single valued at all points $\mathbf{x} \in \mathcal{M}$.

Note from (4) that the projection operator can be defined entirely in terms of the geometric product without the use of the inner or outer products. This guarantees that all the rules for the differentiation of geometric-valued functions are the same as for the differentiation of scalar-valued functions. However, the noncommutativity of the geometric

product must always be respected. This means that the projection $P_{\mathbf{x}}$ can be differentiated in the same way as other geometric-valued functions. In particular, we define

$$P_{\mathbf{a}}(A_r) := \mathbf{a} \cdot \partial_{\mathbf{x}}P_{\mathbf{x}}(A_r) - P_{\mathbf{x}}(\mathbf{a} \cdot \partial_{\mathbf{x}}A_r)$$

and

$$P_{\mathbf{a},\mathbf{b}}(A_r) := \mathbf{b} \cdot \partial_{\mathbf{x}}P_{\mathbf{a}}(A_r) - P_{\mathbf{b} \cdot \partial_{\mathbf{x}}}A_r - P_{\mathbf{x}}(\mathbf{b} \cdot \partial_{\mathbf{x}}A_r),$$

thus ensuring the linearity of $P_{\mathbf{a},\mathbf{b}}(A_r)$ in each of its arguments $\mathbf{a}, \mathbf{b}$ and A_r . Note that the integrability condition (10) tells us that

$$P_{\mathbf{a},\mathbf{b}} - P_{\mathbf{b},\mathbf{a}} = (\mathbf{a} \wedge \mathbf{b}) \cdot (\partial \wedge \partial)P_{\mathbf{x}} = 0.$$

The tangent pseudoscalar $I_{\mathbf{x}}$ and the projection $P_{\mathbf{x}}$ defined in terms of it can be used to completely characterize both the *intrinsic* and *extrinsic curvature* of the vector manifold $\mathcal{M}$ as it is situated in the flat pseudo-euclidean space $\mathbb{R}^{p,q}$. The *shape operator* $S : \mathbb{G}_{p,q} \rightarrow \mathbb{G}_{p,q}$ at the point $\mathbf{x} \in U \subset \mathcal{M}$ is defined by

$$S(A) := \partial_{\mathbf{x}}P_{\mathbf{x}}(A) = \partial_{\mathbf{v}}P_{\mathbf{v}}(A),$$

for each $A \in \mathbb{G}_{p,q}$. The shape operator is a generalization of the classical *Weingarten map*, which is defined for hypersurfaces in $\mathbb{R}^{p,q}$ [14, p. 155].

The shape operator S completely determines the classical *Riemann curvature tensor*. By the *Riemann curvature bivector* we mean

$$(13) \quad R(\mathbf{a} \wedge \mathbf{b}) = P_{\mathbf{a}}(S(\mathbf{b})) = \partial_{\mathbf{v}} \wedge P_{\mathbf{a}}P_{\mathbf{b}}(\mathbf{v}),$$

where $\mathbf{a}, \mathbf{b}$ are vectors in the tangent space $\mathcal{T}_{\mathbf{x}}^1$. The contraction of $R(\mathbf{a} \wedge \mathbf{b})$ with respect to the tangent vector $\mathbf{a}$ gives the *Ricci curvature vector*,

$$R(\mathbf{b}) = \partial_{\mathbf{a}} \cdot R(\mathbf{a} \wedge \mathbf{b}) = S^2(\mathbf{b})$$

[11, p. 193]. Taking a second contraction with respect to $\mathbf{b}$ gives the *scalar curvature* $R = \partial_{\mathbf{b}} \cdot R(\mathbf{b})$. The so-called Bianchi identities take the form

$$R(\mathbf{a} \wedge \mathbf{b}) \cdot (\mathbf{c} \wedge \mathbf{d}) = (\mathbf{a} \wedge \mathbf{b}) \cdot R(\mathbf{c} \wedge \mathbf{d}),$$

$$\partial_{\mathbf{a}} \wedge R(\mathbf{a} \wedge \mathbf{b}) = 0, \text{ First Bianchi identity}$$

$$\partial_{\mathbf{v}} \wedge R_{\mathbf{v}}(\mathbf{a} \wedge \mathbf{b}) = 0, \text{ Second Bianchi identity}$$

and can be easily established [11, p. 191].

It is well known that the Riemann curvature tensor is an intrinsic geometric quantity, independent of any extrinsic property that the manifold might inherit because of its embedding in $\mathbb{R}^{p,q}$.

The Levi-Civita Coderivative

There is another intrinsic way of deriving Riemann curvature by utilizing the *Levi-Civita covariant derivative*, or *coderivative*, defined by restricting the range and domain of the Koszul connection $\mathbf{a} \cdot \partial_{\mathbf{x}}$ to the tangent algebra $\mathcal{T}_{\mathbf{x}}$ [14, pp. 165, 503]. Formally, for any differentiable geometric valued function $F = F(\mathbf{x})$ on $\mathcal{M}$ and any $\mathbf{a} \in \mathcal{T}_{\mathbf{x}}^1$, we define

$$(14) \quad F_{\mathbf{a}} = \mathbf{a} \cdot \nabla_{\mathbf{x}}F = P_{\mathbf{x}}(P_{\mathbf{x}}(F))_{\mathbf{a}} = P_{\mathbf{x}}(\mathbf{a} \cdot \partial_{\mathbf{x}}P_{\mathbf{x}}(F(\mathbf{x}))).$$

We then naturally define the *vector coderivative* of $F = F(\mathbf{x})$ by

$$\nabla F = \partial_{\mathbf{v}} F_{\mathbf{v}}.$$

The *Riemann curvature operator*

$$(\mathbf{a} \wedge \mathbf{b}) \cdot (\nabla_{\mathbf{x}} \wedge \nabla_{\mathbf{x}})$$

takes any tangent multivector field $A \in \mathcal{T}_{\mathbf{x}}$ into the tangent multivector field $R(\mathbf{a} \wedge \mathbf{b}) \dot{\times} A$, where

$$R(\mathbf{a} \wedge \mathbf{b}) \dot{\times} A := \frac{1}{2} (R(\mathbf{a} \wedge \mathbf{b})A - AR(\mathbf{a} \wedge \mathbf{b})).$$

We have

$$(\mathbf{a} \wedge \mathbf{b}) \cdot (\nabla_{\mathbf{x}} \wedge \nabla_{\mathbf{x}})A(\mathbf{x}) = ([\mathbf{b} \cdot \nabla, \mathbf{a} \cdot \nabla] - [\mathbf{b}, \mathbf{a}] \cdot \nabla)A$$

$$= (P_{\mathbf{b}}P_{\mathbf{a}} - P_{\mathbf{a}}P_{\mathbf{b}})(A) = R(\mathbf{a} \wedge \mathbf{b}) \dot{\times} A,$$

where $\mathbf{a}, \mathbf{b} \in \mathcal{T}_{\mathbf{x}}^1$ and $A = A(\mathbf{x})$ is any tangent multivector field with values in the tangent algebra $\mathcal{T}_{\mathbf{x}}$. For example, for $\mathbf{c} \in \mathcal{T}_{\mathbf{x}}$, we have, using (13),

$$\begin{aligned} R(\mathbf{a} \wedge \mathbf{b}) \cdot \mathbf{c} &= [\partial_{\mathbf{v}} \wedge P_{\mathbf{a}}P_{\mathbf{b}}(\mathbf{c})] \cdot \mathbf{c} \\ &= \partial_{\mathbf{v}}P_{\mathbf{a}}P_{\mathbf{b}}(\mathbf{v}) \cdot \mathbf{c} - P_{\mathbf{a}}P_{\mathbf{b}}(\mathbf{c}) = (P_{\mathbf{b}}P_{\mathbf{a}} - P_{\mathbf{a}}P_{\mathbf{b}})(\mathbf{c}). \end{aligned}$$

Mappings between Surfaces

We now come to the heart of this article. Let

$$f: \mathcal{M} \rightarrow \mathcal{M}', \quad \mathbf{x}' = f(\mathbf{x}),$$

be a smooth mapping between two regular k -surfaces in $\mathbb{R}^{p,q}$. If $\mathbf{x} = \mathbf{x}(s^1, \dots, s^k)$, then the corresponding point $\mathbf{x}'$ is given by

$$\mathbf{x}' = \mathbf{x}'(s^1, \dots, s^k) := f(\mathbf{x}(s^1, \dots, s^k)).$$

The tangent algebras $\mathcal{T}_{\mathbf{x}}$ and $\mathcal{T}_{\mathbf{x}'}$ are related by the *push forward differential* defined by

$$\mathbf{a}' = \underline{f}(\mathbf{a}) = \mathbf{a} \cdot \partial_{\mathbf{x}} f(\mathbf{x}),$$

extended to an *outermorphism* by the rule

$$\underline{f}(\mathbf{a}_1 \wedge \dots \wedge \mathbf{a}_r) = \underline{f}(\mathbf{a}_1) \wedge \dots \wedge \underline{f}(\mathbf{a}_r).$$

The *adjoint outermorphism* is defined via the inner product to satisfy

$$\underline{f}(\mathbf{a}) \cdot \mathbf{b}' = \mathbf{a} \cdot \overline{f}(\mathbf{b}') \iff \overline{f}(\mathbf{a}') = \partial_{\mathbf{v}} \underline{f}(\mathbf{v}) \cdot \mathbf{a}'$$

for all $\mathbf{a}, \mathbf{b} \in \mathcal{T}_{\mathbf{x}}^1$ and $\mathbf{b}' \in \mathcal{T}_{\mathbf{x}'}^1$.

The *chain rule*

$$\mathbf{a} \cdot \partial_{\mathbf{x}} = \underline{f}(\mathbf{a}) \cdot \partial_{\mathbf{x}'} \iff \overline{f}(\partial_{\mathbf{x}'}) = \partial_{\mathbf{x}}$$

relates differentiation in $\mathcal{M}$ to differentiation in $\mathcal{M}'$.

We now wish to examine the deep relationship between intrinsic codifferentiation in $\mathcal{M}$ and intrinsic codifferentiation in $\mathcal{M}'$. The intrinsic *first* coderivative of the mapping $\underline{f}$ is defined by

$$\underline{f}_{\mathbf{a}} = \mathbf{a} \cdot \nabla \underline{f} = P' \underline{f}_{\mathbf{a}} P,$$

and the *second* coderivative by

$$\underline{f}_{\mathbf{a}, \mathbf{b}} = P'(\underline{f}_{\mathbf{a}})_{\mathbf{b}} P - P' \underline{f}_{\mathbf{b}, \nabla \mathbf{a}} P.$$

Whereas the integrability condition (10) tells us that $\underline{f}_{\mathbf{a}, \mathbf{b}} - \underline{f}_{\mathbf{b}, \mathbf{a}} = 0$ for ordinary differentiation, for codifferentiation we find the fundamental relationship

$$\underline{f}_{\mathbf{a}, \mathbf{b}} - \underline{f}_{\mathbf{b}, \mathbf{a}} = [P'_{\mathbf{b}}, P'_{\mathbf{a}}] \underline{f} - \underline{f}[P_{\mathbf{b}}, P_{\mathbf{a}}].$$

Applying both sides of this relationship to the tangent vector $\mathbf{c} \in \mathcal{T}_{\mathbf{x}}^1$ gives

$$(15) \quad (\underline{f}_{\mathbf{a}, \mathbf{b}} - \underline{f}_{\mathbf{b}, \mathbf{a}})(\mathbf{c}) = \mathbf{R}'(\mathbf{a}' \wedge \mathbf{b}') \cdot \mathbf{c}' - \underline{f}(\mathbf{R}(\mathbf{a} \wedge \mathbf{b}) \cdot \mathbf{c}),$$

where $\mathbf{a}' \wedge \mathbf{b}' = \underline{f}(\mathbf{a} \wedge \mathbf{b})$. This tells us how the curvature tensors on $\mathcal{M}$ and $\mathcal{M}'$ are related by the coderivatives of the differential $\underline{f}$ of the mapping $f(\mathbf{x})$ between them.

Conformal Mapping

In order to justify this tour de force introduction to the methods of geometric algebra in differential geometry, we will show how deep relationships regarding conformal mappings are readily at hand. We say that $\mathbf{x}' = f(\mathbf{x})$ is a *sense-preserving conformal mapping* between the surfaces $\mathcal{M}$ and $\mathcal{M}'$ if for all $\mathbf{a}, \mathbf{b} \in \mathcal{T}_{\mathbf{x}}$ and corresponding $\mathbf{a}' = \underline{f}(\mathbf{a})$ and $\mathbf{b}' = \underline{f}(\mathbf{b})$ in $\mathcal{T}_{\mathbf{x}'}$,

$$(16) \quad \underline{f}(\mathbf{a}) \cdot \underline{f}(\mathbf{b}) = e^{2\phi} \mathbf{a} \cdot \mathbf{b} \iff \underline{f}(\mathbf{a}) = \psi \mathbf{a} \psi^{\dagger},$$

where $\phi = \phi(\mathbf{x})$ is the real-valued *dilation factor*, and the *spinor* $\psi = e^{\frac{\phi}{2}} U$ and $UU^{\dagger} = 1$. The factor U determines the *rotation part* of the conformal mapping $f(\mathbf{x})$.

Whereas we cannot give here all the explicit calculations, let us summarize the crucial results of the analysis. It turns out that for the general conformal mapping $f(\mathbf{x})$ satisfying (16) all quantities are completely determined by the dilation factor ϕ and its derivatives. Letting $\mathbf{w} = \partial_{\mathbf{x}} \phi$, we find that

$$\mathbf{a} \cdot \mathbf{w} = \frac{1}{k} \partial_{\mathbf{v}'} \cdot \underline{f}_{\mathbf{v}'}(\mathbf{a}), \quad U_{\mathbf{a}} = U \mathbf{a} \wedge \mathbf{w},$$

$$\psi_{\mathbf{a}} = \frac{1}{2} \psi (\mathbf{a} \cdot \mathbf{w} + \mathbf{a} \wedge \mathbf{w}) = \frac{1}{2} \psi \mathbf{a} \mathbf{w},$$

$$\underline{f}_{\mathbf{b}}(\mathbf{a}) = \frac{1}{2} \underline{f}(\mathbf{a} \mathbf{w} \mathbf{b} + \mathbf{b} \mathbf{w} \mathbf{a}),$$

and

$$(17) \quad (\underline{f}_{\mathbf{a}, \mathbf{b}} - \underline{f}_{\mathbf{b}, \mathbf{a}})(\mathbf{c}) = \underline{f}(\Omega \cdot \mathbf{c})$$

where

$$\Omega = (\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{w}) \mathbf{w} + (\mathbf{a} \wedge \mathbf{b}) \cdot \nabla \mathbf{w}.$$

Using (15) and taking the outer product of both sides of (17) with $\underline{f}(\partial_{\mathbf{c}}) = e^{2\phi} \partial_{\mathbf{c}'}$ gives the important relationship

$$(18) \quad \underline{f}(\Omega) = e^{2\phi} \mathbf{R}'(\mathbf{a}' \wedge \mathbf{b}') - \underline{f}(\mathbf{R}(\mathbf{a} \wedge \mathbf{b})).$$

We call

$$\mathcal{W}_4(\mathbf{a} \wedge \mathbf{b}) = \mathbf{R}(\mathbf{a} \wedge \mathbf{b}) \wedge (\mathbf{a} \wedge \mathbf{b})$$

the *Weyl 4-vector* because it is closely related to the classical conformal Weyl tensor, which is important in Einstein's general relativity [17], [25].

Notice that the Weyl 4-vector vanishes identically if the dimension of the k -manifold is less than 4. Taking the outer product of both sides of (18) with $\underline{f}(\mathbf{a} \wedge \mathbf{b}) = \mathbf{a}' \wedge \mathbf{b}'$, and noting that $\Omega \wedge \mathbf{a} \wedge \mathbf{b} = 0$, gives the relationship

$$(19) \quad \underline{f}(\mathcal{W}_4(\mathbf{a} \wedge \mathbf{b})) = e^{2\phi} \mathcal{W}'_4(\mathbf{a}' \wedge \mathbf{b}').$$

The classical *conformal Weyl tensor* can be identified with

$$\begin{aligned} \mathcal{W}_C(\mathbf{a} \wedge \mathbf{b}) &= \frac{1}{(k-1)(k-2)} (\partial_{\mathbf{v}} \wedge \partial_{\mathbf{u}}) \cdot \mathcal{W}_4(\mathbf{u} \wedge \mathbf{v}) \\ &= R(\mathbf{a} \wedge \mathbf{b}) - \frac{1}{k-2} [R(\mathbf{a}) \wedge \mathbf{b} + \mathbf{a} \wedge R(\mathbf{b})] \\ &\quad + \frac{R\mathbf{a} \wedge \mathbf{b}}{(k-1)(k-2)}. \end{aligned}$$

Taking the contraction of both sides of (19) with $\partial_{\mathbf{b}} \wedge \partial_{\mathbf{a}}$ gives the relation

$$\underline{f}(\mathcal{W}_C(\mathbf{a} \wedge \mathbf{b})) = e^{2\phi} \mathcal{W}'_C(\mathbf{a}' \wedge \mathbf{b}'),$$

which is equivalent to

$$\underline{f}(\mathcal{W}_C(\mathbf{a} \wedge \mathbf{b}) \cdot \mathbf{c}) = \mathcal{W}'_C(\mathbf{a}' \wedge \mathbf{b}') \cdot \mathbf{c}'$$

where $\mathbf{c}' = \underline{f}(\mathbf{c})$.

When we make the assumption that our conformal mapping $\mathbf{x}' = f(\mathbf{x})$ is between the flat space $\mathcal{M} = \mathbb{R}_{p,q} = \mathcal{M}'$ and itself, for which the curvature bivectors $R(\mathbf{a} \wedge \mathbf{b})$ and $R'(\mathbf{a}' \wedge \mathbf{b}')$ vanish, (18) simplifies to the simple equation that $\Omega \equiv 0$. Taking the contraction of this equation with respect to the bivector variable $\mathbf{B} = \mathbf{a} \wedge \mathbf{b}$ gives the relationship

$$\nabla \cdot \mathbf{w} = -\frac{k-2}{2} \mathbf{w}^2$$

for all values of $k = n > 2$. Calculating $\partial_{\mathbf{b}} \cdot \Omega = 0$ and eliminating $\nabla \cdot \mathbf{w}$ from this equation leads to the surprisingly simple differential equation

$$(20) \quad \mathbf{w}_{\mathbf{a}} = \mathbf{a} \cdot \nabla \mathbf{w} = \frac{1}{2} \mathbf{w} \mathbf{a} \mathbf{w}.$$

The equation (20) specifies the extra condition that $\mathbf{w} = \nabla \phi$ must satisfy in order for $f(\mathbf{x})$ to be a nondegenerate conformal mapping in the pseudoeuclidean space $\mathbb{R}^{p,q}$ onto itself, where $n = p + q > 2$.

Trivial solutions of (20) satisfying (16) consist of (i) $\nabla \phi = 0$ so that ψ is a constant dilation factor and (ii) $\psi = U$ where U is a constant rotation in the plane of some constant bivector $\mathbf{B}$.

Let $\mathbf{c}$ be a constant nonnull vector in $\mathbb{R}^{p,q}$. A nontrivial solution to (20) is

$$(21) \quad f(\mathbf{x}) = \psi \mathbf{x} = \mathbf{x}(1 - \mathbf{c}\mathbf{x})^{-1} = \frac{1}{2} \mathbf{c}^{-1} \mathbf{w} \mathbf{x}$$

where $\mathbf{w} = \nabla \phi = 2(1 - \mathbf{c}\mathbf{x})^{-1} \mathbf{c}$ and

$$e^{-\phi} = (1 - \mathbf{x}\mathbf{c})(1 - \mathbf{c}\mathbf{x}) = 4\mathbf{c}^2 \mathbf{w}^2.$$

Equivalently, we can write (21) in the form

$$f(\mathbf{x}) = \frac{\mathbf{x} - \mathbf{x}^2 \mathbf{c}}{1 - 2\mathbf{c} \cdot \mathbf{x} + \mathbf{c}^2 \mathbf{x}^2}.$$

The mapping $f(\mathbf{x})$ is called a *transversion* in the identity component of the conformal or *Möbius group* of $\mathbb{R}^{p,q}$. In addition to transversions, the sense-preserving conformal group is generated by rotations, translations, and dilations. A related, more detailed, derivation of the sense-preserving conformal group can be found in [11, pp. 210–19].

Möbius Transformations and Vahlen Matrices

We must take a step back to see the whole picture of the structure of conformal transformations in $\mathbb{R}^{p,q}$. Following [15, pp. 248–51], by the *Vahlen matrix* $[f]$ of a conformal transformation of the form $f(\mathbf{x}) = (a\mathbf{x} + b)(c\mathbf{x} + d)^{-1}$ where $a, b, c, d \in \mathbb{G}_{p,q}$, we mean

$$[f] = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

The Vahlen matrix $[f]$ of the identity transformation $f(\mathbf{x}) = \mathbf{x}$ is easily seen to be the identity matrix $[f] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The Vahlen matrices of conformal transformations greatly simplify the study of the conformal group, because the group action of composition is reduced to the matrix product of the corresponding Vahlen matrices. If $f(\mathbf{x})$ and $g(\mathbf{x})$ are conformal transformations, then $h(\mathbf{x}) = g(f(\mathbf{x}))$ is also a conformal transformation of $\mathbb{R}^{p,q}$ and their Vahlen matrices satisfy the rule

$$[h] = [g \circ f] = [g][f].$$

By the *pseudodeterminant* of a Vahlen matrix, we mean

$$p\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad^\dagger - bc^\dagger.$$

A conformal transformation f is *sense-preserving* (rotation) if $p\det[f] > 0$ and *sense reversing* (reflection) if $p\det[f] < 0$. By the *normalized identity component* of the conformal group, we mean all conformal transformations f with the property that $p\det[f] = 1$.

We can now talk about the conformal Möbius transformations shown in Figures 6–10. Figures 6, 7, and 8 all have Vahlen matrices of the form $\begin{pmatrix} 1 & 0 \\ -\mathbf{c} & 1 \end{pmatrix}$ with the pseudodeterminant

$$p\det \begin{pmatrix} 1 & 0 \\ -\mathbf{c} & 1 \end{pmatrix} = 1.$$

This means that these transformations are sense-preserving conformal transformations of $\mathbb{R}^3$ onto itself which are continuously connected to the identity component represented by the identity 2×2 matrix.

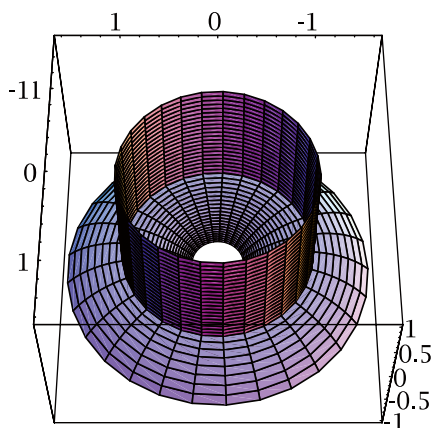


Figure 6. The transversion $f(\mathbf{x}) = \mathbf{x}(1 - \mathbf{c}\mathbf{x})^{-1}$, with $\mathbf{c}=(0,0,3/4)$, maps the cylinder in $\mathbb{R}^3$ conformally onto the figure shown.

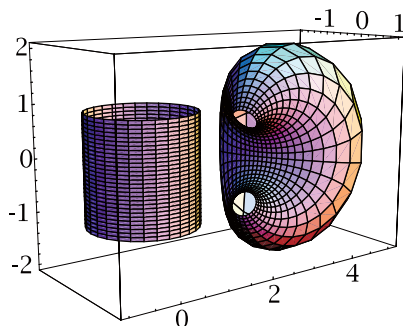


Figure 7. The transversion $f(\mathbf{x}) = \mathbf{x}(1 - \mathbf{c}\mathbf{x})^{-1}$ with $\mathbf{c} = (-1/4, 1/2, 0)$ maps the cylinder in $\mathbb{R}^3$ conformally onto the figure shown. The image is translated by 3 units on the y -axis for clarity.

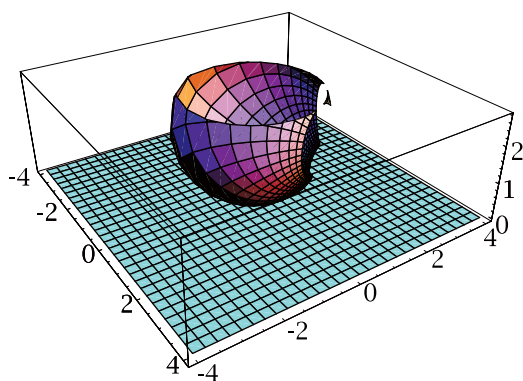


Figure 8. The transversion $f(\mathbf{x}) = \mathbf{x}(1 - \mathbf{c}\mathbf{x})^{-1}$ with $\mathbf{c} = (-1/4, -1/4, -1/4)$ maps the xy -plane conformally onto the unit sphere in $\mathbb{R}^3$ above the plane as shown.

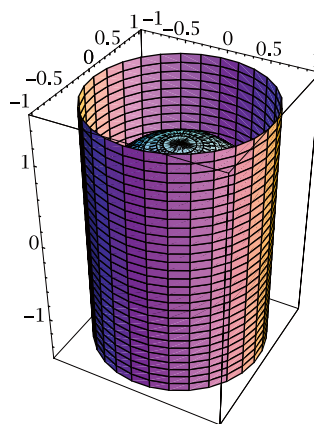


Figure 9. The stereographic projection $f(\mathbf{x}) = (\mathbf{c}\mathbf{x} + 1)(\mathbf{x} - \mathbf{c})^{-1}$ in $\mathbb{R}^3$, with $\mathbf{c} = (0, 0, 1)$, wraps the unit cylinder conformally around the unit sphere as shown. For a cylinder 4 times as long as the radius of the sphere, the sphere is covered twice.

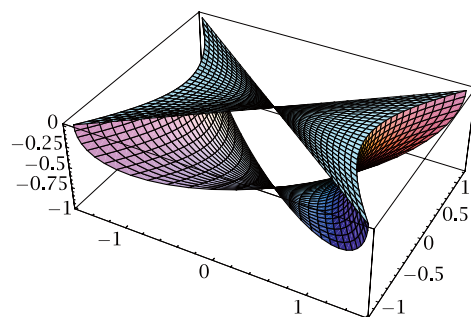


Figure 10. The hyperbolic stereographic projection $f(\mathbf{x}) = (\mathbf{x} - \mathbf{c})(1 + \mathbf{c}\mathbf{x})^{-1}$ with $\mathbf{c} = (0, 0, 1)$ maps the hyperbolic xy -plane conformally onto the unit hyperboloid. The metric g has the Lorentzian signature $(+, -, +)$.

Figures 9 and 10 have Vahlen matrices of the respective forms

$$\begin{pmatrix} \mathbf{c} & 1 \\ 1 & -\mathbf{c} \end{pmatrix} \text{ and } \begin{pmatrix} 1 & -\mathbf{c} \\ \mathbf{c} & 1 \end{pmatrix}$$

with pseudodeterminants -2 and 2 , respectively. These *global conformal transformations* on $\mathbb{R}^3$ and $\mathbb{R}^{2,1}$ are the extensions of the standard *stereographic projection* from the Euclidean plane in $\mathbb{R}^2$ to the unit sphere in $\mathbb{R}^3$ in the case of Figure 9, and from the hyperbolic plane in $\mathbb{R}^{1,1}$ to the unit hyperboloid in $\mathbb{R}^{2,1}$ in the case of Figure 10. Whereas Figure 10 is continuously connected to the identity component as $\mathbf{c} \rightarrow 0$, this is not the case for Figure 9.

We wish to call one peculiarity to the attention of the reader. The standard stereographic transformation from the Euclidean plane in $\mathbb{R}^2$ to the unit sphere in $\mathbb{R}^3$, with the north pole at

the unit vector $\mathbf{e}_3$ on the z -axis, can be represented either by $f(\mathbf{x}) = (\mathbf{e}_3\mathbf{x} + 1)(\mathbf{x} - \mathbf{e}_3)^{-1}$ or by $g(\mathbf{x}) = (\mathbf{x} - \mathbf{e}_3)(\mathbf{e}_3\mathbf{x} + 1)^{-1}$. Both of these transformations are identical when restricted to the xy -plane, but are globally distinct on $\mathbb{R}^3$. One of these conformal transformations is sense preserving and continuously connected to the identity, while the other one is not. How is this possible?

For the reader who wants to experiment with these ideas but is not so familiar with geometric algebra, I highly recommend the Clifford algebra calculator software [16], which can be downloaded. The reader may find it interesting to compare our methods and results to those found in [8, pp. 106–118].

At the website <http://www.garretstar.com/algebra> can be found supplementary material, including the **Links** referred to in this article and to other related websites, a discussion of the horosphere explaining the deep relationship between the Vahlen matrices of elements in $G_{p,q}$ to the orthogonal group of $R^{p+1,q+1}$, and figures further illustrating the richness of conformal mappings. At present, the username: **garretams** and password: **garretams** are required to view this material. Also included is a list of additional references to the literature to give the reader a better idea of the many different applications that Clifford algebras have found in mathematics, theoretical physics, and in the computer science and engineering communities.

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Influential Mathematicians: Birth, Education, and Affiliation

John Panaretos and Chrisovalandis Malesios

There is currently a surge of interest in comparing research impact and performance to produce league tables. These may be done at various levels, ranking countries, universities, departments, programs, journals, or even individual scientists, and they are typically based on certain simple bibliometric measures, such as impact factors, the *h*-index, and so forth.

This interest is not purely academic: these rankings have caught the attention of policy makers and have caused serious concern especially within European policy making due to the apparent lagging performance of Europe as compared with the United States. This has been documented by several indicators and reports commissioned by the European Union (see, e.g., [8], [5], [6]) but perhaps is best exemplified by the French president's public declaring as an aim (in January 2008) the amelioration of the position of French universities in the international rankings. If rankings can affect educational policy at such a high level, it is natural to revisit the question of how accurately they represent the truth, research quality being so difficult to quantify—which is especially true in the field of mathematics.

Criticisms focus on the appropriateness of different measures, their sensitivity/robustness, and their interpretability (see, e.g., [1], [8], [4]). For a detailed critical review of such indices, see [7].

John Panaretos is professor of probability and statistics, Athens University of Economics and Business, Greece. He also served as Deputy Minister of Education, Lifelong Learning, and Religious Affairs for the Greek government from 2009 to 2011. Part of the work on this article was done while he was on leave at the Department of Statistics at the University of California, Berkeley. His email address is jpan@aueb.gr.

Chrisovalandis Malesios is a Ph.D. student in the Department of Statistics, Athens University of Economics and Business. His email address is valadism@hotmail.com.

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Quantitative analyses based on a wide spectrum of indices indicate a clear advantage of U.S. institutions as compared with institutions in Europe and the rest of the world. However, a different aspect that has not received attention is the static character of several of the indices employed, which fail to capture the “liquidity” of the modern academic landscape, in which high mobility of scientists is the rule rather than the exception. The measures used to quantify research performance are mostly static: even though research output is the result of a process that extends in time as well as in space, indices often take into account only the current affiliation when assigning influential research to institutions. This is manifested as a sort of Markovian property: the past is irrelevant given the present. But aside from the most recent affiliations of the scientists considered, is it reasonable to ignore the movement of scientists at various stages of their careers?

To take an example from the field of mathematics: should the credit of the achievements of Jong-Shi Pang, a highly cited mathematician (<http://www.iese.uiuc.edu/research/faculty/pang.html>), be attributed to a country or institution? Jong-Shi Pang was born in Vietnam, obtained his first degree at the National University of Taiwan, completed his Ph.D. at Stanford University, and has been affiliated with the University of Texas at Dallas, Carnegie Mellon University, the University of Wisconsin-Madison, Johns Hopkins University, and the Rensselaer Polytechnic Institute before moving to the University of Illinois at Urbana-Champaign in 2007. Although his present affiliation obviously deserves a lot of the credit stemming from his high citations, should we not take into account the fact that the scientist has been “nurtured” and “grown scientifically” in many places?

The purpose of this article is to attempt a first probe of the “movement effect” to see how this might influence a concrete question, such as the comparison between the United States and Europe

in the field of mathematics. We focus on highly cited mathematicians, since citations are often taken as a strong indicator of research impact, and track their countries of birth, education, and current affiliation.

In general, comparable data on researchers' movement between Europe, Asia, or Africa and the United States are incomplete. A database on highly cited researchers (HCRs) is compiled by the Institute of Scientific Information (ISI) covering twenty-one disciplines and 6,103 researchers.¹ These data are freely available from Thomson Scientific (<http://hcr3.isiknowledge.com/>) and cover the time period between 1981 and 1999.

With regard to mathematics, the Thomson database lists 343 highly cited mathematicians from 152 institutions. While the Thomson database may provide the list of HCRs and their present affiliations, we had to conduct a personalized case-by-case search in order to obtain data on the countries in which they obtained their first degrees, and their Ph.D.'s, as well as their birthplaces, either by searching through their webpages or by contacting them directly.

Table A3 summarizes the data on HCRs in the field of mathematics according to the countries of their present affiliations. One easily sees that the United States—as in all disciplines—gets the lion's share of HCRs. The United Kingdom and France are far behind the United States, but well ahead of the rest of the countries.

By bringing in the additional background data, we can immediately observe that intercontinental movement is indeed a very common practice. Specifically, based on the data collected, only 46.9 percent of HCRs were born and educated and are working in the same continent, while a significant 42.6 percent of them have completed at least one of their degrees or are working in a continent other than the one they were born in (due to missing information we cannot answer this question for 10.5 percent of HCRs). Our findings are presented in more detail in the following sections.

¹Table A1 in the Appendix provides information on the numbers of HCRs according to the countries of their present affiliations. A further breakdown by scientific discipline of the numbers of HCRs according to country of present affiliation (United States, Europe, and the rest of the world) is given in Table A4. As one can observe, U.S. institutes dominate the list—in terms of HCRs—in the fields of social sciences (93.1%), economics (86.2%), psychology-psychiatry (86.1%), clinical medicine (75.8%), and computer science (73.9%). On the other hand, European institutions have the highest concentration of HCRs in the field of pharmacology (46.8%). In fact, this is the only instance in which Europe outperforms the United States in terms of HCRs (123 HCRs in comparison to 94 HCRs working in the United States). The highest percentage of HCRs working in non-U.S. and E.U. countries is observed in the agricultural sciences field (26.2%).

The Educational Background of HCRs in the Field of Mathematics

In this section, we examine the geographical breakdown of the numbers of HCRs in the field of mathematics, taking into consideration the countries of their birth and the countries in which their first degrees and their Ph.D. degrees were obtained.

Current Affiliations of HCRs

Table 1 presents the percentages of HCRs in the field of mathematics according to their current affiliations. The majority of researchers are working in the United States (68.2 percent), while 22.7 percent work in Europe.² Only 9 percent work in countries outside the United States and Europe. (Countries with more than one HCR outside the United States and Europe are Israel, Australia, Canada, Japan, and China). The percentages in the mathematics discipline are quite analogous to the percentages of all twenty-one disciplines (see Table A2).

Evidently, when looking only at current affiliations, the United States most emphatically dominates Europe, which in turn is well ahead of the rest of the world. Will this pattern persist when bringing in more background information?

Ph.D. Studies of HCRs

When focusing on the countries in which HCRs completed their Ph.D. education, the United States maintains an advantage over Europe and the rest of the world, but not nearly as strong as when compared with respect to current affiliations of the HCRs (Table 2). In particular, 57.7 percent of HCRs in mathematics have acquired their Ph.D. degrees in U.S. universities, 32.1 percent in Europe, and 8.5 percent in the rest of the world: the difference between the United States and Europe drops by approximately twenty percentage points.

The distribution provided in Table 3 reveals that a stunning one in three HCRs who completed their doctorates in Europe are now affiliated with U.S. institutions. Even more extreme is the situation when looking at HCRs with Ph.D.'s from outside the United States or Europe, one in two of whom have eventually settled in the United States. The above findings outline an

²The majority of European institutions with HCRs are based in E.U. countries. Three HCRs are working in Switzerland. In some places we use the term EU with this in mind.

		FREQ	(%)
Valid	US	234	68.2
	Europe	78	22.7
	Israel	8	2.3
	Australia	6	1.7
	Canada	6	1.7
	Japan	5	1.5
	China/Taiwan	3	0.9
	India	1	0.3
	Singapore	1	0.3
	Turkey	1	0.3
TOTAL		343	100.0

Table 1. Frequencies and percentages of HCRs according to the country of present affiliation.

		FREQ	(%)
Valid	US	198	57.7
	Europe	110	32.1
	Israel	7	2.0
	Canada	6	1.7
	Russia	5	1.5
	Japan	5	1.5
	India	2	0.6
	Australia	2	0.6
	Argentina	1	0.3
	South Africa	1	0.3
	Total	337	98.3
	Missing	6	1.7
TOTAL		343	100.0

Table 2. Frequencies and percentages of HCRs according to the country in which Ph.D. studies were completed.

			Country of Present Affiliation of the HCRs			TOTAL
			US	EU	Rest of the world	
Country in which the Ph.D. Degree of the HCRs was obtained	US	Count	180	6	12	198
		%	90.9%	3.0%	6.1%	100.0%
	EU	Count	37	65	8	110
		%	33.6%	59.1%	7.3%	100.0%
Rest of the world		Count	16	2	11	29
		%	55.2%	6.9%	37.9%	100.0%
TOTAL		Count	233	73	31	337
		%	69.1%	21.7%	9.2%	100.0%

Table 3. Contingency table between country of present affiliation of HCRs and country of Ph.D. degree.

overflow of outstanding mathematicians to the United States (a phenomenon known as “the brain drain”), which is confirmed to be a significant factor contributing to the global dominance of U.S. institutions.

The opposite type of movement is very rare, since only 3 percent and 6.1 percent of those who have completed their Ph.D. studies in the United States have moved to Europe and to non-European countries, respectively. In particular, the percentage of “E.U. doctors” moving to the United States is over ten times higher than the percentage of “U.S. doctors” moving to Europe: it seems that Europe is failing not only to retain its top talent but is also failing to attract top talent (a more detailed contingency table (A6) is presented in the Appendix).

BSc Studies of HCRs

		FREQ	(%)
Valid	EU	114	33.2
	US	112	32.7
	China/Taiwan	18	5.2
	Canada	14	4.1
	Australia	11	3.2
	India	9	2.6
	Russia	7	2.0
	Israel	6	1.7
	Hong Kong	4	1.2
	Japan	4	1.2
	South Africa	4	1.2
	rest of the world (*)	10	2.9
Total		313	91.3
Missing		30	8.7
TOTAL		343	100.0

Table 4. Frequencies and percentages of HCRs according to the country in which first degree completed.

(*) 1 HCR for each of Argentina, Peru, Egypt, Brazil, Mexico, New Zealand, Venezuela, Algeria, Turkey, and Chile.

Examination of the countries in which the HCRs in mathematics earned their first degrees reveals further interesting facts (Table 4). Only 32.7 percent of the HCRs completed their B.Sc. degree studies in the United States, while 33.2 percent completed their first degrees in Europe, and a quite significant number (25.4 percent) have completed their B.Sc. studies in countries outside the United States and Europe. The distribution of HCRs between the three different “regions” seems close to uniform at this stage. As we go further into the background of the HCRs, the distribution of HCRs among countries becomes more and more diffuse. This could be an indication that “promising” undergraduate mathematics students are found equally in Europe and in the United States and also in other countries outside the

			Country in which the B.Sc. Degree of the HCRs was obtained			TOTAL
			US	EU	Rest of the world	
Country of Present Affiliation of the HCRs	US	Count	107	50	61	218
		%	49.1%	22.9%	28.0%	100.0%
	EU	Count	3	62	2	67
		%	4.5%	92.5%	3.0%	100.0%
Rest of the world		Count	2	2	24	28
		%	7.1%	7.1%	85.7%	100.0%
TOTAL		Count	112	114	87	313
		%	35.8%	36.4%	27.8%	100.0%

Table 5. Contingency table between country of present affiliation of HCRs and country in which first degree completed.

		FREQ	(%)
Valid	EU	129	37.6
	US	108	31.5
	China/Taiwan	19	5.5
	Canada	11	3.2
	Australia	11	3.2
	Israel	9	2.6
	India	9	2.6
	Russia	8	2.3
	Japan	5	1.5
	Hong Kong	4	1.2
	South Africa	3	0.9
	Argentina	2	0.6
	New Zealand	2	0.6
	rest of the world (*)	12	3.5
Total		332	96.8
Missing		11	3.2
TOTAL		343	100.0

Table 6. Frequencies and percentages of HCRs according to the country of birth.

(*) 1 HCR for each of Peru, Egypt, Brazil, Mexico, Venezuela, Algeria, Turkey, Chile, Tunisia, Vietnam, Pakistan, and Republic of Congo.

first degree is taken into account (from a total of 218 HCRs affiliated with U.S. institutions, 50 and 61, respectively, have acquired their first degrees in Europe and the rest of the world). Notice how diffuse the distribution of HCRs affiliated with U.S. institutions is with respect to the countries of their alma maters: only one in two were undergraduates in U.S. universities. The contrast with Europe is stark, as its respective distribution is acutely concentrated: nine out of ten HCRs affiliated with European institutions also received their bachelor's degrees from within Europe.

A more detailed version of the contingency table is presented in the Appendix (Table A5). The majority of highly cited researchers affiliated with U.S. institutions with B.Sc. studies outside the United States and Europe are coming from China,

United States and Europe.

Table 5 provides a contingency table between the country in which the first degree was completed and the country of present affiliation and allows more detailed comparisons.

The results indicate a significant transfer of mathematics researchers to the United States from the rest of the world when the

Canada, and India (sixteen, eleven and seven, respectively). On the other hand, only five HCRs are affiliated with European institutions, having acquired their B.Sc. degrees outside European countries (three HCRs working in Europe obtained their first degrees in the United States; however, only one of them was born in the United States).

Birthplace of HCRs

Finally, we focus on the data regarding the birthplaces of the HCRs (Table 6), which show that the majority of HCRs were born in Europe (37.6 percent), while 31.5 percent came from the United States, and the remaining 27.7 percent were born in countries in other parts of the world.

In Table 7, a classification of the HCRs with respect to the countries of current affiliation and the countries of birth is presented. The results are quite similar to the previous results. It is obvious that for the HCRs currently working in the United States, less than half were native born (46.5 percent), while the vast majority of researchers working in Europe or the rest of the world are native-born citizens (94.7 percent and 83.3 percent, respectively). We also see that the movement from Europe to the United States (23.9 percent) heavily outnumbers the opposite movement (1.3 percent). A more detailed breakdown of the percentages is given in Table A7 in the Appendix. As observed, the majority of HCRs affiliated with U.S. institutions and born outside the United States and Europe come from China (7.5 percent), followed by Canada (4 percent). Although the status of a scientist as being highly cited is influenced by his or her whole career, if we are to accept that these scientists have achieved a potential they had all along, it is clear that the United States is doing best in harnessing this potential.

Generally, the majority of HCRs working in U.S. universities and institutions were born elsewhere (121 out of 226 researchers), while exactly the opposite holds true for the rest of the world, where the vast majority of researchers are native-born citizens (see Figure 1).

In relation to the movement of HCRs in the early stages of their lives, we observe from Table 8 that moving between the United States, Europe, and the rest of the world is minimal. Indeed, the vast majority of HCRs complete their B.Sc. studies in their native countries (96 percent, 91.5 percent and 90 percent, for the United States, Europe, and the rest of the world, respectively). Still, though, the number of HCRs who left Europe (and the rest of the world) in order to study for an undergraduate degree is larger than the number of those who leave the United States to go abroad for the same reason.

Finally, Table 9 relates the countries of undergraduate and Ph.D. studies of the highly cited mathematicians. As we observe, almost all of the researchers who obtained their B.Sc. degrees in the

United States continued their studies there (99.1 percent). In contrast, a highly significant number of European researchers (20.2 percent) left Europe to continue their Ph.D. studies in the United States, while the majority of the researchers from other countries (59.8 percent) continued their Ph.D. studies in the United States. In total, of the 186 HC researchers who acquired their Ph.D.'s in the United States, 75 came from European universities and from the rest of the world. A further breakdown can be found in Table A8 of the Appendix. By inspection of Table A8, it becomes evident that a significant percentage of the HCRs who completed their Ph.D. studies in the United States had done their undergraduate studies elsewhere, in particular in Europe (12.4 percent), China (9.7 percent), Canada (4.8 percent), India (3.8 percent) and Hong Kong (2.2 percent). It is worth observing that none of the HCRs who did their undergraduate studies in Europe or the United States chose to go to another continent for their Ph.D. studies.

HCRs and Top Institutions

We now turn to a more detailed investigation and include the specific university of current affiliation. Table A9 in the Appendix lists the institutions (24 in all) that employ almost half of the HCRs (45.22 percent) in a total number of 161 institutions/universities. It

			Country of Birth of the HCRs			TOTAL
			US	EU	Rest of the world	
Country of Present Affiliation of the HCRs	US	Count	105	54	67	226
		%	46.5%	23.9%	29.6%	100.0%
	EU	Count	1	72	3	76
		%	1.3%	94.7%	3.9%	100.0%
Rest of the world	Count	2	3	25	30	
	%	6.7%	10.0%	83.3%	100.0%	
TOTAL		Count	108	125	95	332
		%	32.5%	38.9%	28.6%	100.0%

Table 7. Contingency table between the country of present affiliation and the country of birth of HCRs.

			Country in which the B.Sc. Degree of the HCRs was obtained			TOTAL
			US	EU	Rest of the world	
Country of Birth of the HCRs	US	Count	96	3	1	100
		%	96.0%	3.0%	1.0%	100.0%
	EU	Count	7	107	3	117
		%	6.0%	91.5%	2.6%	100.0%
Rest of the world	Count	6	3	81	90	
	%	6.7%	3.3%	90.0%	100.0%	
TOTAL		Count	109	113	85	307
		%	35.5%	36.8%	27.7%	100.0%

Table 8. Contingency table between the country of birth of the HCRs and the country where the first degree of the HCRs was completed.

			Country in which the Ph.D. Degree of the HCRs was obtained			TOTAL
			US	EU	Rest of the world	
Country in which the B.Sc. Degree of the HCRs was obtained	US	Count	111	1	0	112
		%	99.1%	0.9%	0.0%	100.0%
	EU	Count	23	91	0	114
		%	20.2%	79.8%	0.0%	100.0%
Rest of the world	Count	52	9	26	87	
	%	59.8%	10.3%	29.9%	100.0%	
TOTAL		Count	186	101	26	313
		%	59.4%	32.3%	8.3%	100.0%

Table 9. Contingency table between the country of B.S. degree and the country of Ph.D. degree of the HCRs.

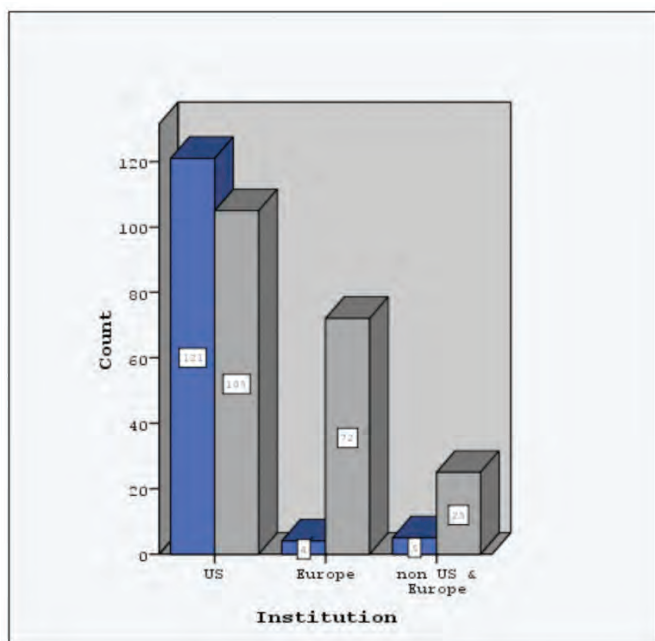


Figure 1. Counts of HCRs for U.S., European, and non-U.S. and European institutions. (Blue: non-native-born, gray: native-born.)

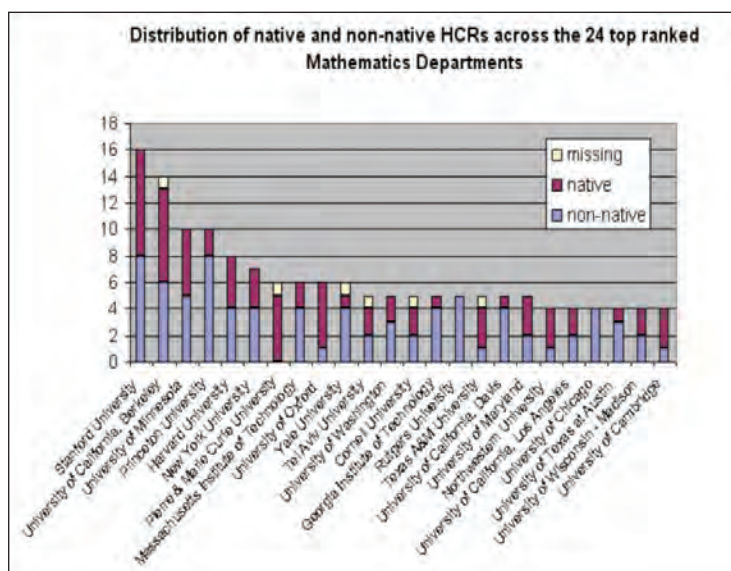


Figure 2. Distribution of native and non-native HCRs across the twenty-four top ranked mathematics departments.

has been reported elsewhere [2] that 30.1 percent of all HCRs in all fields work in the twenty-five top institutions. Our findings indicate a much higher concentration of HCRs in top mathematics institutions than in other scientific fields (one might attempt to attribute this to the fact that hiring a top mathematician is less “expensive” for institutions than hiring an experimental scientist). As one may observe, twenty of the top twenty-four institutions in mathematics ranked from the point of view of HCRs are in the United States, while only three are

in Europe (University of Oxford, Pierre and Marie Curie University, and University of Cambridge) and one is located in Israel (Tel Aviv University).

Observing, however, the percentages of native and non-native HCRs in each one of the top universities, it is obvious that for the majority of the U.S. universities their HCRs come mostly from countries outside the United States. For instance, at Princeton University eight out of the ten HCRs come from countries outside the United States, while at Rutgers University, all five of the HCRs were born outside of the United States (see Figure 2).

On the other hand, we observe the exact opposite effect for the three European institutions that complete the table. For example, in Pierre and Marie Curie University and the University of Cambridge, the majority of the HCRs are native-born citizens (five and three, respectively), while for the University of Oxford only one out of five was born elsewhere. One may argue that the top European institutions have difficulties in attracting and retaining non-European-born HCRs.

We conclude with more general observations regarding the affiliations of the HCRs. In Table 11 we present the number of HCRs in mathematics and in all scientific fields in the top-ranking institutions.

The table indicates that the majority of top institutions in overall performance in terms of number of HCRs also have high numbers of HCRs in mathematics. Specifically, sixteen out of the twenty-seven top institutions in all disciplines also appear in the top list of the HCRs in mathematics. Stanford University and the University of California, Berkeley, are well ahead of the rest when we look at the number of HCRs in mathematics (4.66 percent and 4.08 percent of HCRs in the top ranking Institutions, respectively).³

To further investigate the impact of HCRs in mathematics on their institutions/universities, we present in Table 10 the proportion of mathematician HCRs to the overall number of highly cited researchers in the institutions. It is evident that the proportion of HCRs in mathematics is higher in institutions that are mainly (or solely) focused on science, such as the Georgia Institute of Technology or the Pierre and Marie Curie University. It is also of interest to note that in Tel Aviv University there are five HCRs in mathematics and twelve HCRs in all departments.

³In cases of ties we have ranked the institution with fewer faculty members higher. Data on the number of faculty members associated with departments of mathematics/statistics have been collected from each department's webpage (data on the number of faculty members of universities has been collected from Wikipedia, The Free Encyclopedia, <http://en.wikipedia.org>).

Conclusions

Research output and impact is currently the focus of a serious debate worldwide. In this article, we focused on the field of mathematics and investigated whether the image that emerges from static indices persists when bringing in more dynamic information through the study of the “trajectories” of highly cited mathematicians: birthplace, country of first degree, country of Ph.D., and current affiliation. While the dominance of the United States remains noticeable, some interesting patterns—that perhaps explain this dominance—emerged.

In particular, the results of the current study verify the widely held belief of a brain drain in mathematics from Europe and the rest of the world to the United States, at least among those mathematicians who have become highly cited. Moreover, it provides evidence supporting the view that this brain drain becomes more acute as the careers of the HCRs evolve. Focusing within this influential group of mathematicians we see that while only 6 percent of Europeans moved to the United States for their undergraduate studies, 20 percent of Europeans with bachelor's degrees did their Ph.D. work in the United States. At the next level, 33.6 percent of European Ph.D.'s were attracted to faculty or research positions in the United States.

The situation is worse for the HCRs born outside the United States and Europe. 59.8 percent of non-Europeans with foreign bachelor's degrees did their Ph.D. work in the United States, while 55.2 percent of non-European foreign Ph.D.'s were attracted to faculty positions in the United States. On the other hand, the retention level of the HCRs in mathematics is high at every level in the United States. The United States has managed to retain 99 percent of their bachelor's degrees for Ph.D. work and 90 percent of their Ph.D.'s as faculty members in U.S. institutions.

These results, combined with other findings in this article, reveal that a significant number of HCRs working in the United States have been scientifically “nurtured” elsewhere. The United States is able to attract some of the best minds in mathematics from all over the world and has found the means and conditions to keep them there.

One could think of a series of causes for this flow of human capital from the European Union and the rest of the world towards the United States. The United States has become the main pole of attraction for highly qualified scientists in general (and HCRs in particular), and various reasons, such as the higher wages offered by the U.S. institutions and the heavy taxes and inflexible labor legislation, combined with the lack of research opportunities and/or lack of research funding in the European Union and the rest of the world, could be accounted as responsible for attracting highly skilled foreign researchers to

Rank	Institution of Affiliation (Mathematics/Statistics)	HCRs in math Departs	HCRs in the University	% of HCRs in math Departs	no of students	Country
1	Pierre & Marie Curie University	6	11	54.55%	30.000	France
2	Georgia Institute of Technology	5	12	41.67%	18.747	USA
3	Tel Aviv University	5	12	41.67%	29.000	Israel
4	Texas A&M University	5	22	22.73%	46.540	USA
5	New York University	7	31	22.58%	40.870	USA
6	University of Minnesota	10	47	21.28%	50.402	USA
7	Rutgers University	5	30	16.67%	49.760	USA
8	Princeton University	10	68	14.71%	7.334	USA
9	University of Oxford	6	45	13.33%	19.486	UK
10	University of California, Davis	5	40	12.50%	30.475	USA
11	University of Maryland	5	44	11.36%	36.014	USA
12	Northwestern University	4	40	10.00%	15.129	USA
13	University of Texas at Austin	4	40	10.00%	49.696	USA
14	University of California, Berkeley	14	142	9.86%	34.953	USA
15	Yale University	6	61	9.84%	16.714	USA
16	University of Washington	5	53	9.43%	42.974	USA
17	Cornell University	5	54	9.26%	19.800	USA
18	Stanford University	16	187	8.56%	14.945	USA
19	University of Chicago	4	48	8.33%	14.721	USA
20	Massachusetts Institute of Technology	6	76	7.89%	10.220	USA
21	University of Cambridge	4	52	7.69%	18.396	UK
22	University of Wisconsin - Madison	4	52	7.69%	42.041	USA
23	University of California, Los Angeles	4	59	6.78%	36.611	USA
24	Harvard University	8	187	4.28%	19.139	USA

Table 10. Percentages of HCRs in mathematics at the top institutions.

the United States. As a sign of the demand for immigration of scientists to the United States, it is worth mentioning the change in policy by the U.S. Congress in 2000, that was manifested in the raising of the number of temporary work visas granted to highly skilled foreign professionals to 195,000 annually (from about 115,000).

However, the phenomenon of highly qualified scientists' being attracted by the United States cannot—and should not—be tucked into a narrow economic framework and is not just about salaries. It has to do with broader concepts, such as the prestige and the overall quality of institutions, the opportunities offered by each institution for recognition, and more generally the opportunities for the researchers to use their competencies and expertise.

If Europe wants to compete with the United States, at least in mathematics, it should follow the example of the United States and find ways

Rank	Institution of Affiliation (Mathematics/Statistics)	HCRs	% of HCRs	Country	Institution of Affiliation (All 21 disciplines)	HCRs	% of HCRs	Country	Rank
1	Stanford University	16	4.66%	USA	Harvard University	187	3.06%	USA	1
2	University of California, Berkeley	14	4.08%	USA	Stanford University	142	2.33%	USA	2
3	Princeton University	10	2.92%	USA	National Institutes of Health	136	2.23%	USA	3
4	University of Minnesota	10	2.92%	USA	University of California, Berkeley	87	1.43%	USA	4
5	Harvard University	8	2.33%	USA	Massachusetts Institute of Technology	76	1.25%	USA	6
6	New York University	7	2.04%	USA	Max-Planck-Institute	76	1.25%	Germany	5
7	University of Oxford	6	1.75%	UK	Princeton University	68	1.11%	USA	8
8	Yale University	6	1.75%	USA	University of Michigan	68	1.11%	USA	7
9	Massachusetts Institute of Technology	6	1.75%	USA	University of California, San Diego	66	1.08%	USA	9
10	Pierre & Marie Curie University	6	1.75%	France	University of Pennsylvania	64	1.05%	USA	10
11	Cornell University	5	1.46%	USA	California Institute of Technology	61	1.00%	USA	12
12	University of California, Davis	5	1.46%	USA	Yale University	61	1.00%	USA	11
13	University of Maryland	5	1.46%	USA	University of California, Los Angeles	59	0.97%	USA	13
14	University of Washington	5	1.46%	USA	University of California, San Francisco	54	0.88%	USA	14
15	Georgia Institute of Technology	5	1.46%	USA	Cornell University	54	0.88%	USA	15
16	Rutgers University	5	1.46%	USA	University of Washington	53	0.87%	USA	16
17	Tel Aviv University	5	1.46%	Israel	University of Wisconsin - Madison	52	0.85%	USA	17
18	Texas A&M University	5	1.46%	USA	Columbia University	52	0.85%	USA	18
19	University of Cambridge	4	1.17%	UK	University of Cambridge	51	0.84%	UK	19
20	University of Chicago	4	1.17%	USA	University of Chicago	48	0.79%	USA	20
21	Northwestern University	4	1.17%	USA	University of Minnesota	47	0.77%	USA	21
22	University of Wisconsin - Madison	4	1.17%	USA	University of Oxford	45	0.74%	UK	22
23	University of California, Los Angeles	4	1.17%	USA	University of Maryland	44	0.72%	USA	23
24	University of Texas at Austin	4	1.17%	USA	NASA	43	0.70%	USA	24
					Duke University	41	0.67%	USA	25
					University of California, Davis	40	0.66%	USA	26
					Northwestern University	40	0.66%	USA	27

Table 11: Comparing percentages of HCRs in mathematics and in all 21 disciplines at the top.

not only of retaining its best scientists but also of attracting more from other parts of the world, including the United States. For this to happen, significant changes in research policy are necessary.

There is an opportunity these days for this to happen, as there are already voices in the United States talking about a weakness of the United States in retaining their skilled foreign professionals and terms such as “reverse brain drain” are more frequently used (http://www.businessweek.com/smallbiz/content/aug2007/sb20070821_920025.htm).

Possibilities in this direction could be the development of large research centers (such as CERN in the field of physics) that could attract highly skilled researchers from abroad and at the same time prevent movement of young and promising native-born researchers towards the United States. China, for example, recently launched a large-scale project to transform 100 universities into world-class institutions [3].

There are already examples of similar efforts in Europe, looking to improve the attractiveness of European research institutions. The European Research Council (ERC) established recently and the Starting and Advanced Research Grants awarded are certainly steps in this direction.

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Appendix

Country of present affiliation	Number of HCRs	Percentage of HCRs
United States	4007	65.66%
United Kingdom	464	7.60%
Germany	262	4.29%
Japan	256	4.19%
Canada	185	3.03%
France	163	2.67%
Switzerland	113	1.85%
Australia	109	1.79%
Netherlands	97	1.59%
Italy	81	1.33%
Sweden	62	1.02%
Israel	48	0.79%
Belgium	39	0.64%
Denmark	31	0.51%
Spain	22	0.36%
People's Rep. China	20	0.33%
New Zealand	18	0.29%
Finland	17	0.28%
Austria	13	0.21%
Norway	13	0.21%
India	11	0.18%
Taiwan	9	0.15%
Ireland	8	0.13%
South Africa	7	0.11%
Hungary	6	0.10%
Russia	6	0.10%
Brazil	5	0.08%
Greece	5	0.08%
Chile	4	0.07%
Singapore	4	0.07%
Mexico	3	0.05%
Republic of Korea	3	0.05%
Panama	2	0.03%
Poland	2	0.03%
Algeria	1	0.02%
Hong Kong	1	0.02%
Iran	1	0.02%
Pakistan	1	0.02%
Philippines	1	0.02%
Portugal	1	0.02%
Romania	1	0.02%
Turkey	1	0.02%
TOTAL	6103	100%

Table A1 (left): Numbers of HCRs in all 21 disciplines according to their present affiliation.

Country of present affiliation	Number of HCRs	Percentage of HCRs
United States	4007	65.66%
EU	1400	22.94%
Rest of the world	696	11.40%
TOTAL	6103	100%

Table A2: Numbers of HCRs in all 21 disciplines according to their present affiliation.

Country of present affiliation	Number of HCRs	Percentage of HCRs
United States	234	68.22%
United Kingdom	24	7.00%
France	22	6.41%
Germany	9	2.62%
Israel	8	2.33%
Australia	6	1.75%
Canada	6	1.75%
Japan	5	1.46%
Denmark	4	1.17%
Italy	4	1.17%
Netherlands	4	1.17%
Spain	4	1.17%
Switzerland	3	0.87%
Hungary	2	0.58%
People's Rep. of China	2	0.58%
Belgium	1	0.29%
India	1	0.29%
Singapore	1	0.29%
Sweden	1	0.29%
Taiwan	1	0.29%
Turkey	1	0.29%
TOTAL	343	100.00%

Table A3: Numbers of HCRs in the field of mathematics according to their present affiliation.

Discipline	Country of present affiliation			TOTAL
	US	EU	Rest of the world	
Agricultural Sciences	118 42.3%	88 31.5%	73 26.2%	279 100.0%
Biology and Biochemistry	141 62.7%	43 19.1%	41 18.2%	225 100.0%
Chemistry	143 57.2%	72 28.8%	35 14.0%	250 100.0%
Clinical Medicine	166 75.8%	41 18.7%	12 5.5%	219 100.0%
Computer Science	241 73.9%	46 14.1%	39 12.0%	326 100.0%
Ecology-Environment	201 64.4%	75 24.0%	36 11.5%	312 100.0%
Economics-Business	268 86.2%	26 8.4%	17 5.5%	311 100.0%
Engineering	142 67.3%	39 18.5%	30 14.2%	211 100.0%
Geosciences	219 69.3%	73 23.1%	24 7.6%	316 100.0%
Immunology	209 63.7%	84 25.6%	35 10.7%	328 100.0%
Materials Science	163 59.7%	55 20.1%	55 20.1%	273 100.0%
Mathematics	225 67.4%	78 23.4%	31 9.3%	334 100.0%
Microbiology	215 64.2%	96 28.7%	24 7.2%	335 100.0%
Molecular Biology and Genetics	215 71.4%	65 21.6%	21 7.0%	301 100.0%
Neuroscience	190 64.0%	85 28.6%	22 7.4%	297 100.0%
Pharmacology	94 35.7%	123 46.8%	46 17.5%	263 100.0%
Physics	160 55.6%	91 31.6%	37 12.8%	288 100.0%
Plant and Animal Science	148 48.5%	101 33.1%	56 18.4%	305 100.0%
Psychology-Psychiatry	229 86.1%	24 9.0%	13 4.9%	266 100.0%
General	296 93.1%	12 3.8%	10 3.1%	318 100.0%
Space Sciences	224 64.7%	83 24.0%	39 11.3%	346 100.0%
TOTAL	4,007 65.7%	1,400 22.9%	696 11.4%	6,103 100.0%

Table A4: Distribution of HCRs in all 21 disciplines according to present affiliation and discipline.

		Country in which the B.Sc. Degree was obtained										
		US	EU	India	Canada	Russia	Israel	China-Taiwan	Australia	Japan	Turkey	Argentina
Country of Present Affiliation	US	107 49.1%	50 22.9%	7 3.2%	11 5.0%	4 1.8%	2 0.9%	16 7.3%	5 2.3%	0 0.0%	0 0.0%	1 0.5%
	EU	3 4.5%	62 92.5%	0 0.0%	0 0.0%	2 3.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	India	0 0.0%	0 0.0%	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Canada	1 16.7%	0 0.0%	1 16.7%	3 50.0%	0 0.0%	0 0.0%	0 0.0%	1 16.7%	0 0.0%	0 0.0%	0 0.0%
	Israel	1 16.7%	0 0.0%	0 0.0%	0 0.0%	1 16.7%	4 66.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	China-Taiwan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	2 66.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Australia	0 0.0%	1 16.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 83.3%	0 0.0%	0 0.0%	0 0.0%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	4 100.0%	0 0.0%	0 0.0%
	Singapore	0 0.0%	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Turkey	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%	0 0.0%
TOTAL		112 35.8%	114 36.4%	9 2.9%	14 4.5%	7 2.2%	6 1.9%	18 5.8%	11 3.5%	4 1.3%	1 0.3%	1 0.3%
		Country in which the B.Sc. Degree was obtained										
		Hong Kong	Peru	South Africa	Egypt	Brazil	Mexico	New Zealand	Venezuela	Algeria	Chile	TOTAL
Country of Present Affiliation	US	3 1.4%	1 0.5%	3 1.4%	1 0.5%	1 0.5%	1 0.5%	2 0.9%	1 0.5%	1 0.5%	1 0.5%	218 100.0%
	EU	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	67 100.0%
	India	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	Canada	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	Israel	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	China-Taiwan	1 33.3%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	3 100.0%
	Australia	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	4 100.0%
	Singapore	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	Turkey	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
TOTAL		4 1.3%	1 0.3%	3 1.0%	1 0.3%	1 0.3%	1 0.3%	2 0.6%	1 0.3%	1 0.3%	1 0.3%	313 100.0%

Table A5: Contingency table between the country of present affiliation and the country in which first degree completed in mathematics.

		Country of Present Affiliation										TOTAL
		US	EU	India	Canada	Israel	China-Taiwan	Australia	Japan	Singapore	Turkey	
Country in which the Ph.D. Degree was obtained	US	180 90.9%	6 3.0%	1 0.5%	3 1.5%	3 1.5%	3 1.5%	1 0.5%	0 0.0%	0 0.0%	1 0.5%	198 100.0%
	EU	37 33.6%	65 59.1%	0 0.0%	3 2.7%	0 0.0%	0 0.0%	4 3.6%	0 0.0%	1 0.9%	0 0.0%	110 100.0%
	India	2 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	2 100.0%
	Canada	6 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	Russia	2 40.0%	2 40.0%	0 0.0%	0 0.0%	1 20.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 100.0%
	Israel	3 42.9%	0 0.0%	0 0.0%	0 0.0%	4 57.1%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	7 100.0%
	Australia	1 50.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 50.0%	0 0.0%	0 0.0%	0 0.0%	2 100.0%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 100.0%	0 0.0%	0 0.0%	5 100.0%
	Argentina	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	South Africa	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	TOTAL	233 69.1%	73 21.7%	1 0.3%	6 1.8%	8 2.4%	3 0.9%	6 1.8%	5 1.5%	1 0.3%	1 0.3%	337 100.0%

Table A6: Contingency table between the country of present affiliation and the country of Ph.D. degree in the field of mathematics.

		Country of Birth												
		US	EU	India	Canada	Russia	Israel	China-Taiwan	Australia	Japan	Turkey	Argentina	Hong Kong	Peru
Country of Present Affiliation	US	105 46.5%	54 23.9%	7 3.1%	9 4.0%	5 2.2%	4 1.8%	17 7.5%	5 2.2%	0 0.0%	0 0.0%	2 0.9%	3 1.3%	1 0.4%
	EU	1 1.3%	72 94.7%	0 0.0%	0 0.0%	2 2.6%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	India	0 0.0%	0 0.0%	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Canada	1 16.7%	1 16.7%	1 16.7%	2 33.3%	0 0.0%	0 0.0%	0 0.0%	1 16.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Israel	1 14.3%	0 0.0%	0 0.0%	0 0.0%	1 14.3%	5 71.4%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	China-Taiwan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	2 66.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 33.3%	0 0.0%
	Australia	0 0.0%	1 16.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 83.3%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Singapore	0 0.0%	1 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%
	Turkey	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%	0 0.0%	0 0.0%	0 0.0%
TOTAL		108 32.5%	129 38.9%	9 2.7%	11 3.3%	8 2.4%	9 2.7%	19 5.7%	11 3.3%	5 1.5%	1 0.3%	2 0.6%	4 1.2%	1 0.3%

		Country of Birth												
		South Africa	Egypt	Brazil	Mexico	New Zealand	Venezuela	Algeria	Chile	Tunisia	Vietnam	Pakistan	Rep of Congo	TOTAL
Country of Present Affiliation	US	3 1.3%	1 0.4%	1 0.4%	1 0.4%	2 0.9%	1 0.4%	1 0.4%	1 0.4%	1 0.4%	1 0.4%	1 0.4%	0 0.0%	226 100.0%
	EU	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 1.3%	76 100.0%
	India	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	Canada	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	Israel	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	7 100.0%
	China-Taiwan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	3 100.0%
	Australia	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	5 100.0%
	Singapore	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
	Turkey	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%
TOTAL		3 0.9%	1 0.3%	1 0.3%	1 0.3%	2 0.6%	1 0.3%	1 0.3%	1 0.3%	1 0.3%	1 0.3%	1 0.3%	1 0.3%	332 100.0%

Table A7: Contingency table between the country of present affiliation and the country of birth in the field of mathematics.

		Country in which the Ph.D. Degree was obtained										TOTAL
		US	EU	India	Canada	Russia	Israel	Australia	Japan	Argentina	South Africa	
Country in which the B.Sc. Degree was obtained	US	111 59.7%	1 1.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	112 35.8%
	EU	23 12.4%	91 90.1%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	114 36.4%
	India	7 3.8%	0 0.0%	2 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	9 2.9%
	Canada	9 4.8%	2 2.0%	0 0.0%	3 60.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	14 4.5%
	Russia	2 1.1%	0 0.0%	0 0.0%	0 0.0%	5 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	7 2.2%
	Israel	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 100.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	6 1.9%
	China-Taiwan	18 9.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	18 5.8%
	Australia	3 1.6%	5 5.0%	0 0.0%	1 20.0%	0 0.0%	0 0.0%	2 100.0%	0 0.0%	0 0.0%	0 0.0%	11 3.5%
	Japan	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	4 100.0%	0 0.0%	0 0.0%	4 1.3%
	Turkey	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	Argentina	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%	0 0.0%	1 0.3%
	Hong Kong	4 2.2%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	4 1.3%
	Peru	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	South Africa	2 1.1%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 100.0%	3 1.0%
	Egypt	0 0.0%	0 0.0%	0 0.0%	1 20.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	Brazil	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	Mexico	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	New Zealand	1 0.5%	1 1.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	2 0.6%
	Venezuela	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	Algeria	0 0.0%	1 1.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
	Chile	1 0.5%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	1 0.3%
TOTAL		186 100.0%	101 100.0%	2 100.0%	5 100.0%	5 100.0%	6 100.0%	2 100.0%	4 100.0%	1 100.0%	1 100.0%	313 100.0%

Table A8: Contingency table between the country of B.S. degree and the country of Ph.D. degree in the field of mathematics.

Institution of Affiliation	HCRs	% of HCRs	non-native HCRs	% of non-native HCRs	native HCRs	% of native HCRs	BSs acquired in same country	% of BSs acquired in same country	BSs acquired elsewhere	% of BSs acquired elsewhere	PhDs acquired in same country	% of PhDs acquired in same country	PhDs acquired elsewhere	% of PhDs acquired elsewhere	Country
Stanford University	16	4.66%	8	50.0%	8	50.0%	8	50.0%	8	50.0%	16	100.0%	0	0.0%	USA
University of California. Berkeley (*)	14	4.08%	6	42.9%	7	50.0%	7	50.0%	5	35.7%	11	78.6%	3	21.4%	USA
University of Minnesota	10	2.92%	5	50.0%	5	50.0%	6	60.0%	3	30.0%	8	80.0%	2	20.0%	USA
Princeton University	10	2.92%	8	80.0%	2	20.0%	3	30.0%	7	70.0%	5	50.0%	5	50.0%	USA
Harvard University	8	2.33%	4	50.0%	4	50.0%	4	50.0%	4	50.0%	8	100.0%	0	0.0%	USA
New York University	7	2.04%	4	57.1%	3	42.9%	4	57.1%	3	42.9%	6	85.7%	1	14.3%	USA
Pierre & Marie Curie University (*)	6	1.75%	0	0.0%	5	83.3%	4	66.7%	0	0.0%	3	50.0%	2	33.3%	France
Massachusetts Institute of Technology	6	1.75%	4	66.7%	2	33.3%	1	16.7%	5	83.3%	5	83.3%	1	16.7%	USA
University of Oxford	6	1.75%	1	16.7%	5	83.3%	4	66.7%	2	33.3%	4	66.7%	2	33.3%	UK
Yale University (*)	6	1.75%	4	66.7%	1	16.7%	2	33.3%	3	50.0%	4	66.7%	2	33.3%	USA
Tel Aviv University	5	1.46%	2	40.0%	2	40.0%	2	40.0%	2	40.0%	2	40.0%	3	60.0%	Israel
University of Washington	5	1.46%	3	60.0%	2	40.0%	2	40.0%	3	60.0%	3	60.0%	2	40.0%	USA
Cornell University (*)	5	1.46%	2	40.0%	2	40.0%	1	20.0%	3	60.0%	3	60.0%	2	40.0%	USA
Georgia Institute of Technology	5	1.46%	4	80.0%	1	20.0%	1	20.0%	3	60.0%	3	60.0%	2	40.0%	USA
Rutgers University	5	1.46%	5	100.0%	0	0.0%	0	0.0%	5	100.0%	2	40.0%	3	60.0%	USA
Texas A&M University (*)	5	1.46%	1	20.0%	3	60.0%	4	80.0%	1	20.0%	5	100.0%	0	0.0%	USA
University of California. Davis	5	1.46%	4	80.0%	1	20.0%	1	20.0%	4	80.0%	3	60.0%	2	40.0%	USA
University of Maryland	5	1.46%	2	40.0%	3	60.0%	3	60.0%	2	40.0%	4	80.0%	1	20.0%	USA
Northwestern University	4	1.17%	1	25.0%	3	75.0%	3	75.0%	1	25.0%	4	100.0%	0	0.0%	USA
University of California. Los Angeles	4	1.17%	2	50.0%	2	50.0%	2	50.0%	2	50.0%	3	75.0%	1	25.0%	USA
University of Chicago	4	1.17%	4	100.0%	0	0.0%	2	50.0%	2	50.0%	3	75.0%	1	25.0%	USA
University of Texas at Austin	4	1.17%	3	75.0%	1	25.0%	1	25.0%	3	75.0%	2	50.0%	2	50.0%	USA
University of Wisconsin - Madison	4	1.17%	2	50.0%	2	50.0%	2	50.0%	2	50.0%	3	75.0%	1	25.0%	USA
University of Cambridge	4	1.17%	1	25.0%	3	75.0%	4	100.0%	0	0.0%	2	50.0%	2	50.0%	UK

Table A9: Top institutions in the field of mathematics with reference to HCRs.
 (*) One missing value as concerns the birthplace.

MATHEMATICAL CONGRESS OF THE AMERICAS

Guanajuato, Mexico, August 5-9, 2013



CALL FOR NOMINATIONS

FOR PRIZES TO BE AWARDED AT THE MCA2013

The following prizes winners will be chosen by the MCA Awards Committee which is appointed by the MCA2013 Steering Committee. In making their decisions the Awards Committee will be guided by the information in the nominating material and detailed insights about the nominees' professional accomplishments. It is important that the nominees most significant contributions and their impact be part of the nominating material. The Awards Committee may, if it chooses to do so, make nominations.

● THE MCA PRIZE:

Five prizes of \$1000 each will be awarded to mathematicians who are no more than 12 years past their PhD in August 2013.

Eligibility for consideration of nominees requires that they either received their graduate education or that they currently hold a position in one or more countries in the Americas.

The choice of the prize winners will be based on the documented mathematical achievements of the nominees.

The nominations must include a justifying statement of up to 2 pages, the CV of the nominee and one additional letter of support.

● THE AMERICAS PRIZE:

One prize of \$5000 will be awarded to an individual or a group in recognition of their work to enhance collaboration and the development of research that links mathematicians in several countries in the Americas.

The nominations must include a description (up to 4 pages) of the work and any relevant citations that justify the award of the Americas Prize.

CVs of the nominees should be provided.

There must be four co-nominators from at least two different countries.

● THE SOLOMON LEFSCHETZ MEDAL

Two medals with an award of \$5000 will be given to mathematicians in recognition of their excellence in research and their contributions to the development of mathematics in a country or countries in the Americas.

Nominations must include a justifying statement (up to 2 pages), and a brief paragraph that can be used in the announcement of the Medal, the CV of the nominee, and two additional supporting letters.

An individual may make up to two nominations in each category.

Self nominations will not be accepted in any category.

Nominations and requests for information concerning the nominating process should be sent by e-mail to mca2013.prizes@gmail.com

The deadline for nominations is January 31, 2013.



CALL FOR SESSION PROPOSALS

Proposals to organize a special session at the MCA 2013 are welcomed by the Steering Committee. A proposal should include

- the names, affiliations and contact information of all the organizers
- a brief presentation of the topic and scope (up to one page)
- a preliminary list of the expected speakers

The topics should be broad and fairly well represented throughout the Americas. The list of organizers must include at least two mathematicians from different countries in the Americas. Preference will be given to proposals whose list of suggested speakers represents diversity in all aspects.

Each special session will consist of two 4-hour periods. We recommend that the organizers base their session on a total of 16 half hour time slots for their speakers.

Proposals should be sent before July 31, 2012 to mca2013.sessions@gmail.com.

Supporting the Next Generation of “Stewards” in Mathematics Education

Barbara and Robert Reys

The purpose of doctoral education, taken broadly, is to educate and prepare those to whom we can entrust the vigor, quality, and integrity of the field... Someone who will creatively generate new knowledge, critically conserve valuable and useful ideas, and responsibly transform those understandings through writing, teaching, and application. We call such a person a “steward of the discipline”.

Chris M. Golde
The Carnegie Foundation
for the Advancement of Teaching

Supporting Research, Service and Teaching in Mathematics Education, or STaR, is a program designed to support new faculty who have taken a tenure-track mathematics education position in a U.S. institution of higher education. It is modeled after Project NeXT, a successful induction program for early-career mathematicians. Together, these programs strive to meet the goals articulated by Golde in the passage quoted above, albeit with different audiences in mind.

The STaR program was funded in 2009 by the National Science Foundation (NSF) and provides support for three cohorts of STaR Fellows.

Barbara Reys is Curators’ Professor of Mathematics at the University of Missouri–Columbia. Her email address is reysb@missouri.edu.

Robert Reys is Curators’ Professor of Mathematics at the University of Missouri–Columbia. His email address is reysr@missouri.edu.

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DOI: <http://dx.doi.org/10.1090/noti796>

Christine Stephens, St. Louis University, who was instrumental in establishing and maintaining Project NeXT, provided important advice to the program coordinators. The program was also influenced by the NSF Centers for Learning and Teaching (CLT) program that supported doctoral development in mathematics education.¹

A primary goal of the CLTs was to recruit and prepare doctoral students in mathematics education, addressing the shortage of doctorates in the area that has existed for more than two decades [1], [2], [3], and [4]). In addition, each CLT promoted cross-institutional research collaboration that helped establish professional networks for early-career mathematics educators. One example of this collaboration was initiated by the Center for the Study of Mathematics Curriculum (CSMC), which analyzed state-level mathematics standards in 2006. Faculty members and doctoral students from multiple institutions engaged in work that required reviewing and synthesizing state mathematics standards. One result of this work was the publication of *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?* [5]. This research documented the variability of grade-level learning goals in mathematics and promoted discussions that led to the development of the Common Core State Standards Initiative. Similar collaborative work was initiated by other CLTs and helped establish new models for doctoral program development.

¹The CLTs that focused on mathematics education include the Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM); the Mid-Atlantic Center for Mathematics Teaching and Learning; the Center for the Mathematics Education of Latinos (CEMELA); the Center for the Study of Mathematics Curriculum (CSMC); and the Center for Proficiency in Teaching Mathematics.

The transition from doctoral student to contributing faculty member is often underestimated. New faculty immediately assume a variety of responsibilities, including collegiate teaching, student advising, service activities with local schools, advancement of a research agenda, and engagement in scholarly writing [6], [7], [8], and [9]). They often do so in an environment where they are alone or among a small number of mathematics educators at their institution. In many cases, their work is different from other colleagues in their home department, which may be mathematics or education. Like Project NeXT, the STaR Program provides an opportunity for early-career mathematics educators to discuss challenges they are facing, learn how their colleagues at other institutions are dealing with these challenges, and establish professional and collaborative relationships to advance their work.

More specifically, the goals of the STaR Program are to establish a network of future leaders of mathematics education in order to:

- Provide a support structure for advancing the scholarship of recent doctoral graduates in mathematics education;
- Expand the networking of recent graduates/advanced graduate students initiated by CLTs to graduates/advanced graduate students from other institutions;
- Showcase research priorities for the field and facilitate the establishment and development of research groups involving young mathematics education scholars from different institutions.

Two specific experiences constitute the nucleus of the STaR Program: a one-week summer institute of STaR Fellows held in conjunction with the Park City Mathematics Institute (PCMI), and a follow-up meeting in conjunction with the annual conference of the Association of Mathematics Teacher Educators (AMTE) the following January or February.

The STaR Summer Institute provides an opportunity for fellows to gather and work together, and it also provides an opportunity to interact with mathematicians and secondary mathematics teachers attending other PCMI programs. Table 1 provides some information about the first and second cadre of STaR Fellows. It also

points out one difference between STaR and NeXT Fellows. While Project NeXT Fellows must be employed in a mathematics department, STaR Fellows might have their academic home in a department of mathematics or in a college/department of education.

The first two cohorts of STaR Fellows have reflected on their experiences and the program's impact on their emerging careers. For example, two STaR Fellows commented on how the experience assisted them in transitioning to a faculty position:

I know that my department has invested many resources in hiring me and helping set me on a path toward success in and for the department, but rarely is that investment “matched” by an outside source.... the STaRs program has invested in my growth as a professional, researcher, and hopefully successful tenure-track professor.

Having the opportunity to attend the STaR program has given me an irreplaceable gift as a new academic—a professional learning community in which I can interact and grow. Transitioning from graduate student to professor sometimes seems as if you are traveling the sea without a detailed map, sending out beacons as you navigate between your own personal shift to academia while attending to expectations with which you have little understanding, knowledge or prior experience.

Another STaR Fellow noted how the program gave her a broader perspective of her research agenda:

I feel that the STaR institute provided me with a shift in mindset regarding my short- and long-term goals. Up to this time, I feel like my research has been varied and influenced by the interests of my peers and graduate faculty. Now I see the importance to develop a true research agenda that builds on my prior

	1 st Cadre	2 nd Cadre
Number of STaR Fellows	44	37
Number of different institutions awarding their doctorate	28	24
Number of different institutions where they are employed	42	34
Number employed in doctoral-granting institutions	22	18
Number with appointments in departments of mathematics	24	21
Number of females	32	25

Table 1. Background information on the first two cadres of STaR Fellows.

work. I feel like I am still in the process of developing this, but I am starting to respect the importance of it, and this acknowledgement is a direct result of participating in the STaR program.

Yet another STaR Fellow commented on the long-term impact of the program:

I believe that, twenty years from now, when I look back over my career, I will point to the STaR Program as the source of my focused research agenda, the source of much of my pedagogy, the source of my inspiration, and the source of many enduring relationships.

In closing, we recognize that professionals continue to grow and adapt throughout their careers. It is unreasonable to expect that a Ph.D. program will adequately prepare mathematics educators for the wide range of challenges and expectations they will confront. These inherent limitations of graduate education were conveyed in a summary statement during a recent national conference on doctoral programs in mathematics education:

The U.S. system of mathematics education doctoral studies severely underestimates the depth of training required to do the work of a mathematics educator. If the system is to improve, it must acknowledge the extent of the problem and generate realistic approaches to address it. [10]

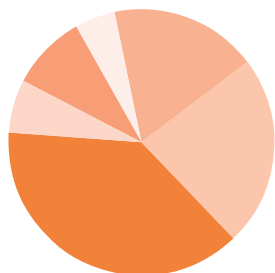
The STaR Program is one effort to support the transition path for early-career mathematics educators.² While we are preparing for the third cadre of STaR Fellows, we are also exploring ways of sustaining this effort as the MAA has done for Project NExT. We believe the STaR program, like Project NExT, will have a significant and long-term impact on the profession. As one STaR Fellow put it:

The STaR faculty clearly “paid it forward” during this experience. It was easy to see and understand their love for the field of mathematics education—each imparting pieces of their expertise, knowing that we as junior faculty will one day lead the field of mathematics education and hopefully mentor those individuals that will one day take our place.

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²For information about the 2012 STaR application procedures see <http://matheddb.missouri.edu/star/>.



2009-2010 Departmental Profile Report

Richard Cleary, James W. Maxwell, and Colleen Rose

This report presents a profile of mathematical sciences departments at four-year colleges and universities in the United States, as of fall 2010. The information presented includes the number of faculty in various categories, undergraduate and graduate course enrollments, number of bachelor's and master's degrees awarded during the preceding year, and the number of graduate students.

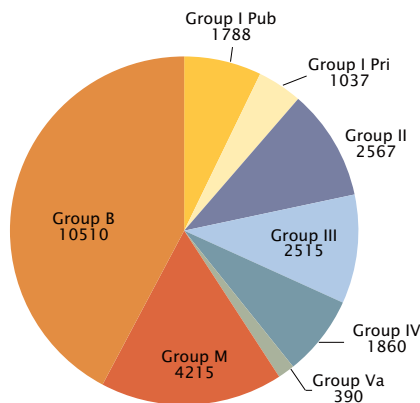
Data collected earlier from these departments on recruitment and hiring and faculty salaries were presented in the Report on 2009-2010 Academic Recruitment and Hiring (pages 693-696 of the May 2011 issue of *Notices of the AMS*) and the 2010-2011 Faculty Salaries Report (pages 438-443 of the March 2011 issue of *Notices of the AMS*).

Detailed information, including tables which traditionally appeared in this report, is available on the AMS website at www.ams.org/annual-survey/survey-reports.

Faculty Size

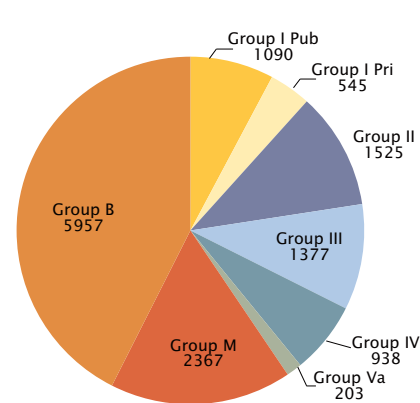
Changes in the numbers of faculty from 2009 to 2010 were modest. The estimated number of full-time faculty in all departments is 24,882 with 23,023 of these in all mathematics departments combined (Groups I, II, III, Va, M, and B), up 2% from 22,463 last year. The majority of this increase is the result of the 6% increase in estimated full-time faculty in Group B, up 635 to 10,510 (with a standard error of 384.) Full-time faculty among the doctoral mathematics departments combined (Groups I-III & Va) increased slightly from last year, to 8,297 from 8,260. In the mathematics departments combined the number of nondoctoral full-time faculty is 3,817 (with a standard error of 187), down 4% from 3,969 last year. The total part-time faculty in all mathematics departments combined is estimated to be 6,067 (with a standard error of 301), down 7% from 6,570 last year.

Figure F.1: All Full-time Faculty by Department Groupings



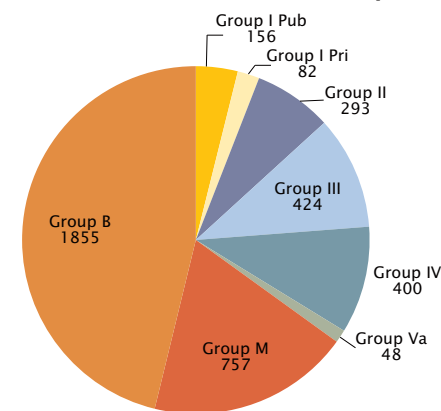
Total: 24,882

Figure F.2: Full-time Tenured Doctoral Faculty



Total: 14,002

Figure F.3: Full-time Untenured, Tenure-track Doctoral Faculty



Total: 4,016

Richard Cleary is a professor in the Department of Mathematical Sciences at Bentley University. James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Doctoral Faculty

The estimated number of full-time doctoral faculty in all mathematics departments combined (Groups I-III, Va, M and B) is 19,206 (with a standard error of 345), up 4% from last year's number of 18,493. For these same groups combined, total doctoral tenured faculty increased 7% to 13,063. Essentially all of the increase is due to a reported increase of just under 900 for Group B; the standard error of this estimate is 341.

Figure D.1: Gender of Full-time Doctoral Faculty
Total: 20,969

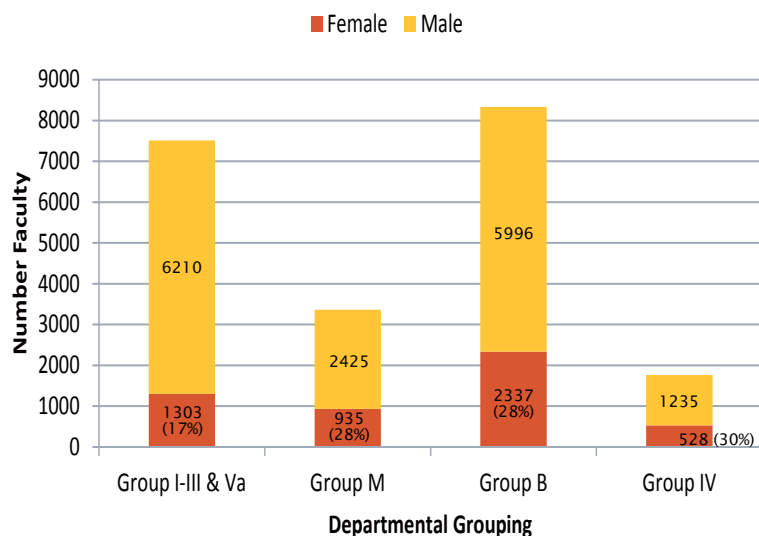
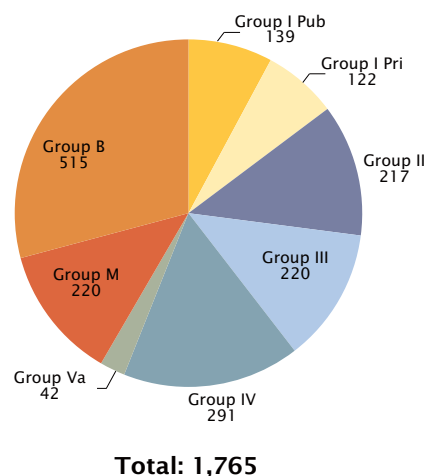


Figure D.2: Non-tenure-track Doctoral Faculty (excluding Postdocs)



Postdoctoral appointments continue to climb among the doctoral mathematics departments reaching a high for fall 2010 of 1,031. This is a 4% increase from last year and 14% of the total full-time doctoral faculty in these departments. Females hold 22% of all postdoctoral appointments. Since 2003 total postdoctoral appointments have increased 45% and females holding postdocs increased 60% to 229 from 143. Postdoctoral appointments as a percentage of total full-time doctoral faculty, which held steady at 11% from 2005 to 2007, have been steadily increasing by about one percentage point each year to 14% this year.

Figure D.3: Full-time Postdoctoral Faculty

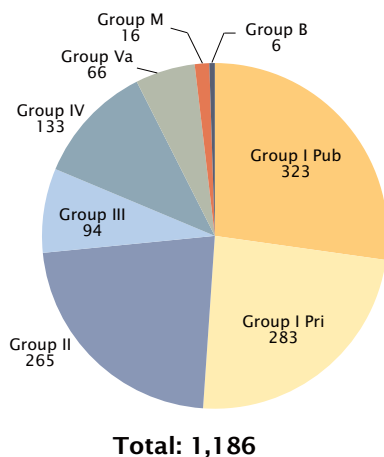
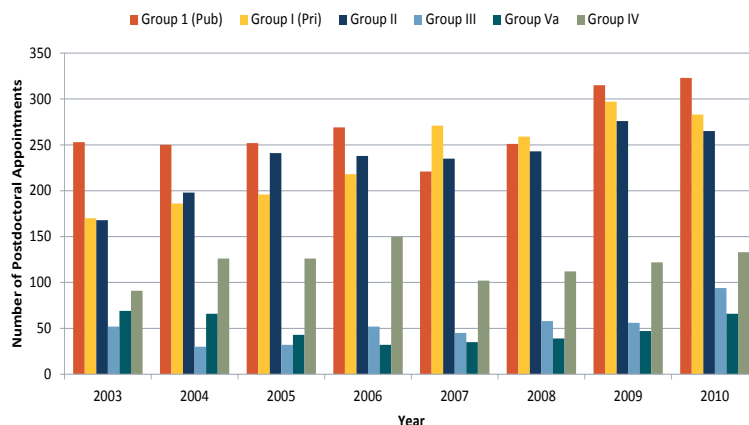


Figure D.4: Postdoctoral Faculty, by Year and Departmental Grouping, Fall 2003 to Fall 2010



*Information on faculty holding postdoctoral appointments was not collected prior to 2003.

Nondoctoral Faculty

The estimated number of nondoctoral full-time faculty in all mathematics departments combined (Groups I-III, Va, M and B) is 3,817. This is down 4% from last year and is 17% of all full-time faculty. In all mathematics departments combined nondoctoral tenured faculty decreased 21% from last year, with all groups reporting decreases. 183 of the nondoctoral faculty in all mathematics departments are untenured, tenure-track faculty, 5% of all untenured tenure-track faculty in these groups. Nondoctoral full-time non-tenure-track faculty (including postdocs) increased to 3,002; this is 79% of all nondoctoral mathematics faculty.

Figure ND.1: Full-time Nondoctoral Faculty by Departmental Grouping

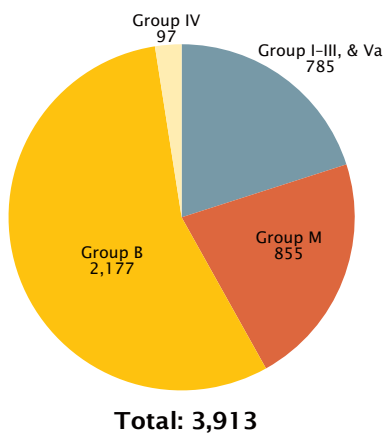


Figure ND.2: Full-time Nondoctoral Tenured Faculty

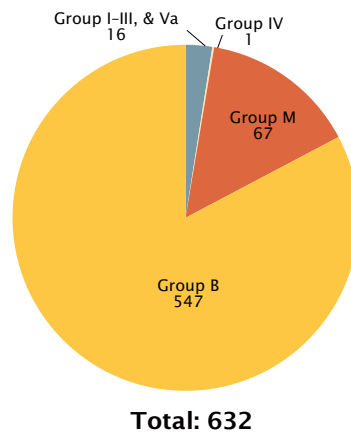


Figure ND.4: Full-time Nondoctoral Untenured, Tenure-track Faculty

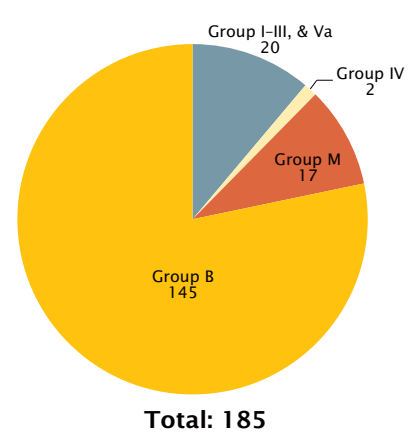
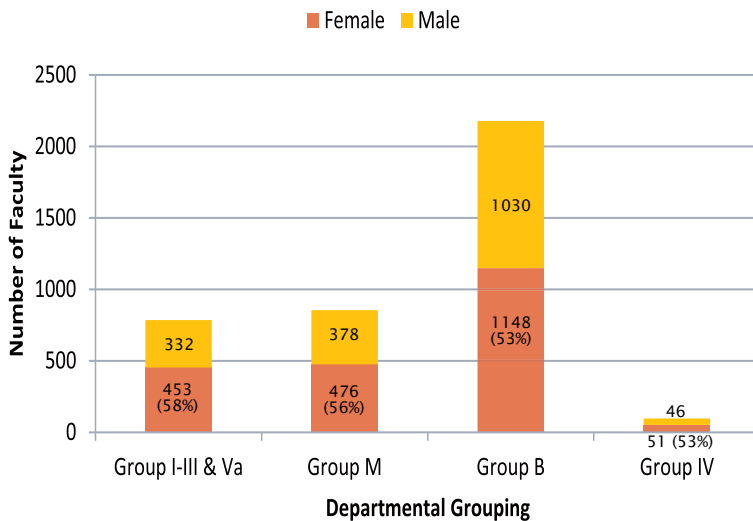


Figure ND.4: Gender of Full-time Nondoctoral Faculty
Total: 3,913



- Females account for 54% of full-time nondoctoral faculty in all mathematics groups combined (up from 53% last year).
- Total part-time nondoctoral faculty in all doctoral mathematics departments combined (Groups I-III, and Va) is 4,728, 78% of all part-time faculty in these groups.

Female Faculty

For the combined mathematics departments (Groups I-III, Va, M and B), women comprised 29% (6,651 with a standard error of 184) of the full-time faculty (23,023) in fall 2010. For the doctoral mathematics departments combined (Groups I-III, and Va), women comprised 14% of the combined doctoral-holding tenured and tenure-track faculty and 28% of the doctoral-holding non-tenure-track (including postdocs) faculty in fall 2010. For Group M faculty these same percentages are 27 and 38, and for Group B faculty they are 28 and 23, respectively. Among the nondoctoral full-time faculty in all math departments combined, women comprise 54%. Females account for 44% of all part-time faculty in mathematics departments combined.

Figure FF.1: Tenured Female Doctoral Faculty

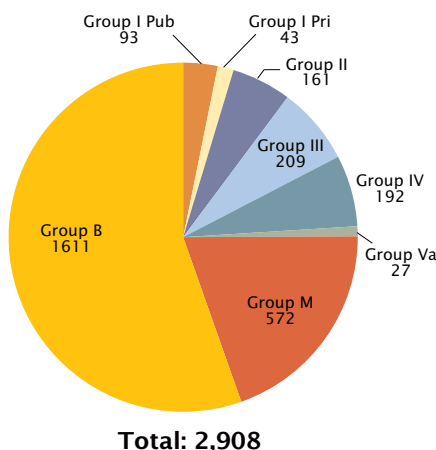


Figure FF.2: Untenured, Tenure-track Female Doctoral Faculty

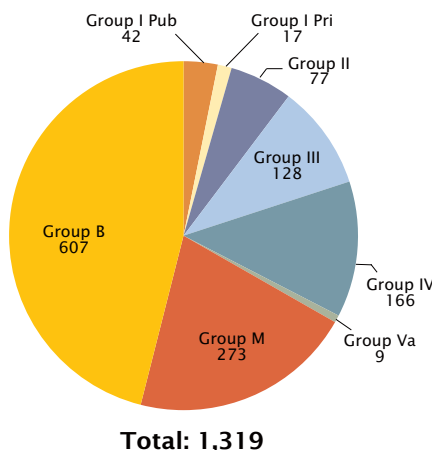


Figure FF.3: Postdoctoral Female Faculty

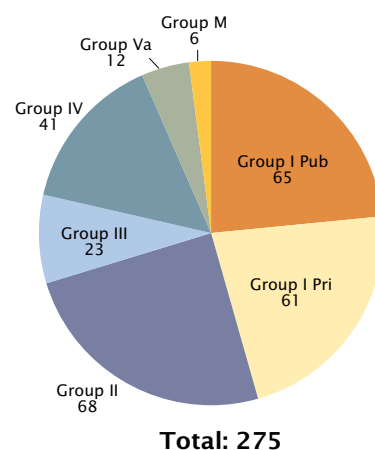
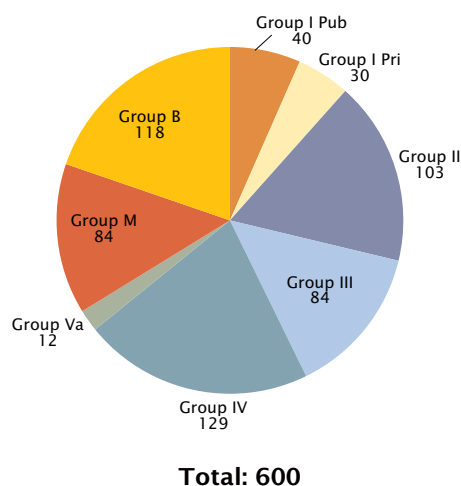


Figure FF.4: Female Doctoral Non-tenure-track Faculty (excluding Postdocs)

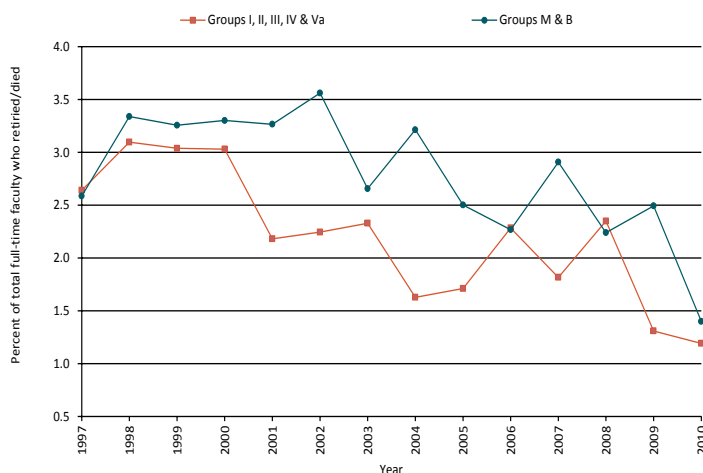


- Groups M & B combined reported the highest percentages of full-time female faculty (33%), while Group I (Pri) reported the lowest (15%).
- Doctoral mathematics departments reported small increases in full-time female faculty in all categories except doctoral untenured, tenure-track which dropped 2%.
- Females hold 22% of all postdoctoral appointments; the number of female postdocs increased slightly in all doctoral departments except Group Va.
- 30% of all female postdocs in doctoral mathematics departments combined are found in Group II. This group also has the highest percentage (26%) of female postdocs.
- 47% of all part-time female faculty among the mathematics departments combined are found in Group B, a 4 percentage point increase from last year for this group.

Faculty Attrition

Figure A.1 shows the trends in attrition from deaths and retirements among the full-time faculty between 1997 and 2010. In the late 1990s attrition leveled off, then began dropping after 2000, reaching the smallest rate of attrition this year.

Figure A.1: Full-time Faculty Retired/Died

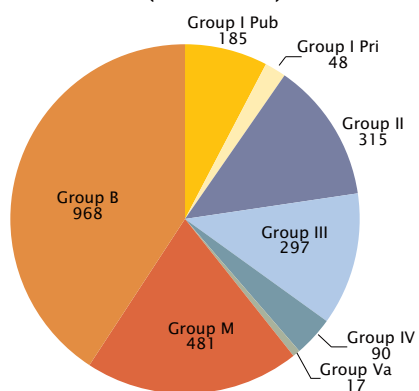


- Group III reported the smallest rate of attrition, 0.6%
- Group M reported highest rate of attrition, 2.1%.
- Group I Pri reported the largest percentage increase in attrition at 1.8% up from 0.7% last year.

Undergraduate Course Enrollments

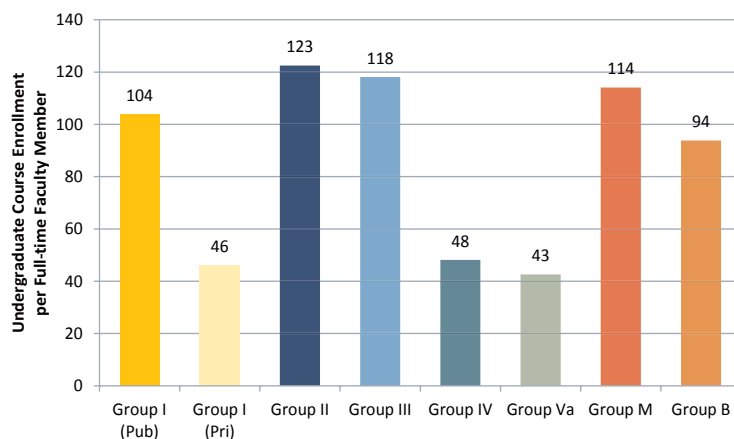
Total undergraduate enrollments for all groups combined increased by 6% (143,000) to 2,418,000 (with a standard error of 50,000); most of this increase came from Group B which increased 16% (134,000) to 986,000 (with a standard error of 41,000). With fall 2010 we see a slight increase in the number of undergraduate course enrollments per full-time faculty member in all groups except Groups III, Va, and M.

Figure UE.1: Undergraduate Course Enrollments by Department Groupings (Thousands)



Total Undergraduate Enrollments (thousands): 2,418

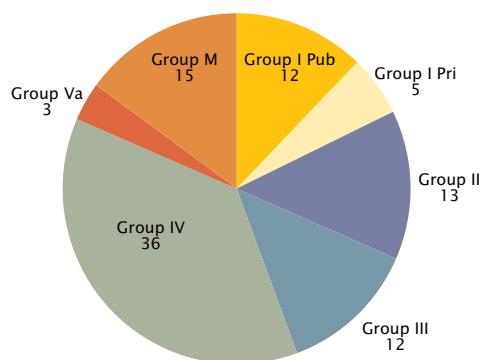
Figure UE.2: Undergraduate Course Enrollment per Full-Time Faculty Members, Fall 2010



Graduate Course Enrollments

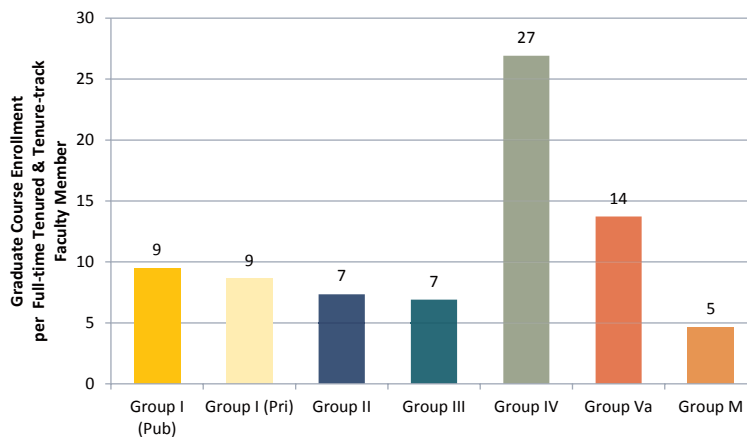
Total graduate course enrollments have remained flat at 97,000 (with a standard error of 2,000). However, increases in the number of graduate course enrollments per full-time tenured/tenure-track faculty member occurred in Groups I (Pri), III, and Va.

Figure GE.1: Graduate Course Enrollments by Department Groupings (Thousands)



Total Graduate Enrollments (thousands): 97

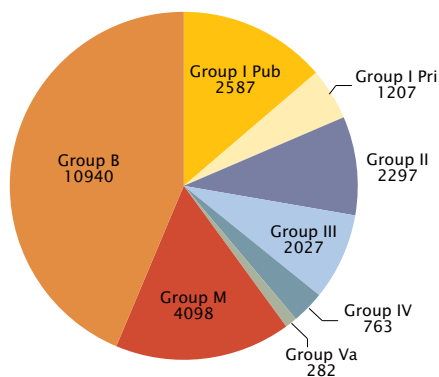
Figure GE.2: Graduate Course Enrollment per Full-Time Tenured and Tenure-track Faculty Member, Fall 2010



Undergraduate Degrees Awarded

The estimated number of undergraduate degrees awarded during 2009-2010 by all mathematics departments combined (Groups I-III, Va, M and B) is 23,438 (with a standard error of 744), a 4% drop from last year's estimate of 24,328. Females accounted for 43% (10,118) of these degrees, a slight increase over last year. This year's estimated number of undergraduate degrees awarded included 490 statistics-only and 1,902 computer-science only.

Figure UD.1: Undergraduate Degrees Awarded by Department Groupings

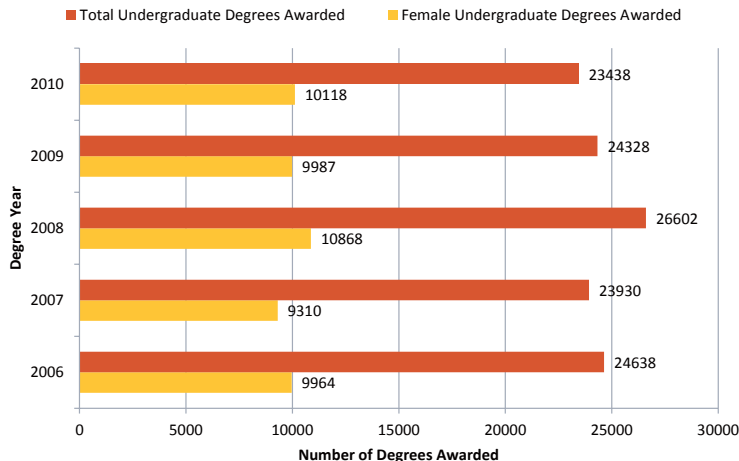


Total Degrees Awarded: 24,201

- All groups reported a decrease in the number of degrees awarded except for Groups I-II. Group 1 (Pri) reported the largest increase, up 271 from last year.
- Group B awarded 47% of all the degrees, down from 49% last year in all mathematics departments combined.
- Group IV reported a 2% increase in degrees awarded.
- Total statistics-only degrees dropped in all mathematics departments combined by 34% to 490.
- Males were more likely to receive combined statistics-only or computer science-only degrees. About 13% of males earned such degrees compared to just 6% of females.

Undergraduate Degrees Awarded

**Figure UD.2: Undergraduate Degrees Awarded
Groups I, II, III, Va, M & B Combined**



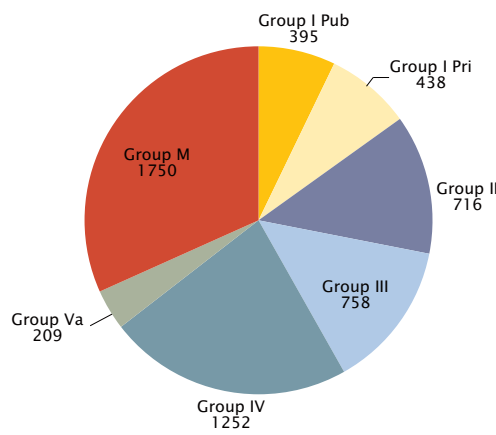
Comparing undergraduate degrees awarded this year with those awarded in 2006:

- Degrees awarded have decreased 5% overall.
- Degrees awarded to females increased by 2%.
- The percentage of total degrees awarded to females increased from 40% to 43%.

Master's Degrees Awarded

The estimated number of master's degrees awarded during 2009-2010 in all mathematics departments combined (Groups I-III, Va, and M) is 4,265, a 5% increase from last year's estimate of 4,060. This year's estimated graduate degrees included 456 statistics-only and 162 computer science-only degrees. Departments reported a 6% increase in the number of degrees awarded to females, 1,723.

**Figure MD.1: Master's Degrees Awarded
by Department Groupings**

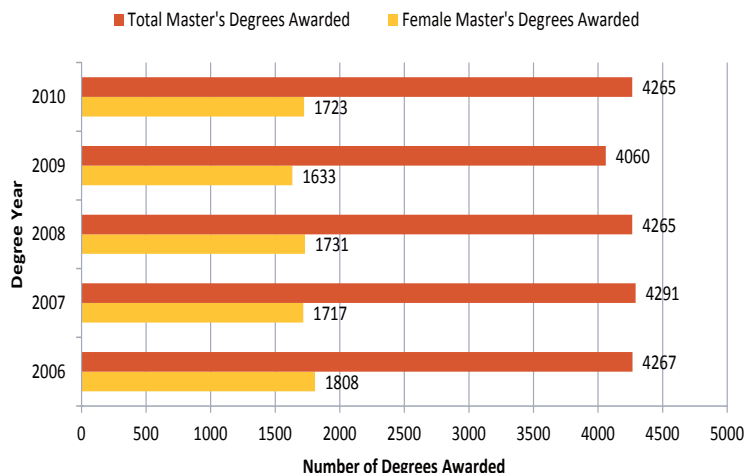


Total Degrees Awarded: 5,517

- Group M awarded the highest percentage of degrees (41%, up from 39% last year) and the largest percentage to females (49%),
- Group Va awarded the fewest degrees, 5% of all degrees, the same as last year.
- Group I (Pri) reported the largest percentage increase in degrees awarded; up 27% to 438 from 346 reported last year.
- Females received 40% of all degrees awarded, the same as last year, among all the mathematics departments combined.
- 16% of degrees awarded to females in all mathematics departments combined were in statistics-only or computer science-only, compared to 13% for males.
- Group IV awarded 1,252 degrees, a decrease of 12% from last year; females received 47% of these degrees.

Master's Degrees Awarded

Figure MD.2: Master's Degrees Awarded
Groups I, II, III, Va, M & B Combined



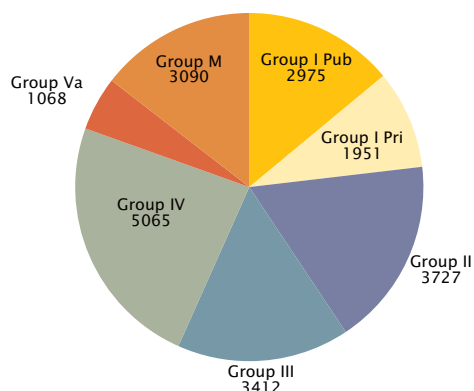
Comparing master's degrees awarded this year with those awarded in 2006:

- Total degrees awarded is essentially unchanged.
- Total degrees awarded to females dropped by 2 percentage points to 40%.

Graduate Students

The total number of full-time graduate students in all mathematics departments combined is 16,223, up from 13,954 in fall 2009. The total number of full-time graduate students in doctoral mathematics departments combined (Groups I-III, & Va) is 13,133 (up from 11,286). The number of U.S. citizens among the doctoral mathematics departments combined increased 19% to 7,511 and first-year students increased 10% to 3,335. For Group M, full-time graduate students increased 16% to 3,090, the number of U.S. citizens is 2,428 (up from 1,919), and the number of first-year students is 1,266 (up from 1,206). Group IV reported full-time graduate students as 5,065, up from 4,892. Group IV is the only group to report decreases in the number of full-time first-year graduate students and full-time U.S. citizens graduate students.

Figure GS.1: Graduate Students by Department Groupings



Total Graduate Students: 21,288

- Females account for 35% (7,410) of the full-time graduate students; all mathematics groups (I-Va, M, B & Va) reported increases.
- Group Va had the largest percentage increase in graduate students with 39%, while Group III had the largest number increase—up 861 from 2,551 to 3,412.
- First-year graduate students increased in all groups except Group IV which dropped slightly from 1,545 to 1,532.
- U.S. citizen graduate students increased 18% across the doctoral mathematics departments.
- Total part-time graduate students in all doctoral mathematics departments combined increased 5%, while Groups M and IV decreased by 15% and 29% respectively.

Graduate Students

Table GS.2: Full-Time Graduate Students in Groups I, II, III, & Va by Gender and Citizenship, Fall 2005–2010

	2005	2006	2007	2008	2009	2010
Total full-time graduate students	10565	10984	10937	10883	11286	13133
Female	3111	3279	3249	3193	3248	3839
% Female	29%	30%	30%	29%	29%	29%
% U.S. Citizen	56%	56%	56%	55%	56%	57%
% Underrepresented minorities ¹	10.0%	9.0%	9.0%	9.0%	9.0%	9.0%
Total first-year graduate students	2832	2960	2964	2924	3040	3335
Female	851	961	950	870	904	1019
% Female	30%	32%	32%	30%	30%	31%
% U.S. Citizen	59%	55%	56%	56%	55%	51%
% Underrepresented minorities	10.0%	10.0%	10.0%	10.0%	10.0%	9.0%

Looking at Table GS.2 we see that although the numbers and percentages have fluctuated somewhat among the categories, the numbers of full-time, female, first-year, and female first-year graduate students have all reached a six-year high, as has the percentage of U.S. citizens. The number of full-time and full-time first-year graduate students have increased 24% and 18%, respectively, since 2005.

Remarks on Statistical Procedures

The questionnaire on which this report is based, “*Departmental Profile*”, is sent to all doctoral and master’s departments. It is sent to a stratified random sample of Group B departments, the stratifying variable being the undergraduate enrollment at the institution.

The response rates vary substantially across the different department groups. For most of the data collected on the Departmental Profile form, the year-to-year changes in a given department’s data are very small when compared to the variations among the departments within a given group. As a result of this, the most recent prior year’s response is used (imputed) if deemed suitable. After the inclusion of prior responses, standard adjustments for the remaining nonresponse are then made to arrive at the estimates reported for the entire groups.

Standard errors were calculated for some of the key estimates for Groups I, II, III, and Va combined, for Groups M and B, and for Group IV. Standard errors are calculated using the variability in the data and can be used to measure how close our estimate is to the true

value for the population. As an example, the number of full-time faculty in Group M is estimated at 4,215 with a standard error of 66. This means the actual number of full-time faculty in Group M is most likely between 4,215 plus or minus two standard errors, or between 4,083 and 4,347. This is much more informative than simply giving the estimate of 4,215.

Estimates are also given for parameters that are totals from all groups, such as the total number of full-time faculty. For example, an estimate of the total number of full-time faculty in all groups but group IV is 23,023, with a standard error of 385.

The careful reader will note that a row or column total may differ slightly from the sum of the individual entries. All table entries are the rounded values of the individual projections associated with each entry, and the differences are the result of this rounding (as the sum of rounded numbers is not always the same as the rounded sum).

Survey Response Rates

Departmental Profile
Department Response Rates

pDepartment Group	Number	Percent	Imputed ¹
Group I (Public)	21 of 25	84%	2
Group I (Private)	22 of 23	96%	1
Group II	51 of 56	91%	5
Group III	70 of 81	86%	7
Group IV (Statistics)	44 of 57	77%	8
Group IV (Biostatistics)	21 of 35	60%	7
Group Va	16 of 18 ²	89%	0
Group M	93 of 178	52%	40
Group B	117 of 276 ³	42%	34

¹ See paragraph two under 'Remarks on Statistical Procedures.'

² The population for Group Va is slightly less than for the Doctorates Granted Survey because four programs do no formally "house" faculty, teach undergraduate courses, or award undergraduate degrees.

³ This is the sampled population, the total population for Group B is 1,013.

Other Sources of Data

Visit the AMS website at www.ams.org/annual-survey/other-sources for a listing of additional sources of data on the Mathematical Sciences.

Group Descriptions

The data in this report is presented for departments divided into groups according to several characteristics, the principal one being the highest degree offered in the mathematical sciences. Doctoral-granting departments of mathematics are further subdivided according to their ranking of "scholarly quality of program faculty" as reported in the 1995 publication *Research-Doctorate Programs in the United States: Continuity and Change*.

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions, respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science which report a doctoral program.

Group Va is applied mathematics/applied science; Group Vb, which was no longer surveyed as of 1998–99, was operations research and management science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at www.ams.org/annual-survey/groups_des.



WHAT IS . . .

a Pseudoconvex Domain?

R. Michael Range

Pseudoconvexity is a most central concept in modern complex analysis. However, if your training in that area is limited to functions of just *one* complex variable, you probably have never heard of it, since every open subset of the complex plane $\mathbb{C}$ is pseudoconvex. Pseudoconvexity or, better, *nonpseudoconvexity*, is a higher-dimensional phenomenon. Incidentally, this is true also for *Euclidean* convexity: every open connected subset of $\mathbb{R}$ is convex. Life is definitely more interesting in higher dimensions!

Pseudoconvexity is so central because it relates to the very core of holomorphic (i.e., complex analytic) functions, which is intimately intertwined with power series, the identity theorem, and analytic continuation. This concept has its roots in Friedrich Hartogs's surprising discovery in 1906 of a simple domain H in $\mathbb{C}^2$ with the property that every function that is holomorphic¹ on H has a holomorphic extension to a *strictly larger* open set $\hat{H}$. In dimension one there is no such thing! In fact, if P is a boundary point of a domain $D \subset \mathbb{C}$, the function $f_P(z) = 1/(z - P)$ is clearly holomorphic

on D , but surely it has no holomorphic extension to any neighborhood of P .²

Hartogs's example is so amazing and historically significant, and yet completely elementary, that it deserves to be presented in any exposition of the subject. Consider the domain $H \subset \mathbb{C}^2 = \{(z, w) : z, w \in \mathbb{C}\}$ defined by

$$H = \{(z, w) : |z| < 1, 1/2 < |w| < 1\} \cup \{|z| < 1/2, |w| < 1\}.$$

Let $f : H \rightarrow \mathbb{C}$ be holomorphic and fix r with $1/2 < r < 1$. The function

$$F(z, w) = \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{f(z, \zeta) d\zeta}{\zeta - w}$$

is easily seen to be holomorphic on $G = \{(z, w) : |z| < 1, |w| < r\}$. Observe that for fixed z_0 with $|z_0| < 1/2$ the function $w \mapsto f(z_0, w)$ is holomorphic on the disc $\{|w| < 1\}$, and hence, by the Cauchy integral formula, $f(z_0, w) = F(z_0, w)$ for $|w| < r$. Thus $f \equiv F$ on $\{(z, w) : |z| < 1/2, |w| < r\}$, which implies $f \equiv F$ on $H \cap G$ by the identity theorem, so that F provides the holomorphic extension of f from H to $\hat{H} = H \cup G$.

Hartogs's discovery immediately raises the fundamental problem of characterizing those domains $D \subset \mathbb{C}^n$ for which holomorphic extension of *all* holomorphic functions on D does NOT hold. Such domains are called *domains of holomorphy*. More precisely, according to this point of view, D is a domain of holomorphy, if for each boundary point $P \in \partial D$ there exists a function f_P holomorphic on D which does not extend

R. Michael Range is professor of mathematics at the State University of New York at Albany. His email address is range@math.albany.edu.

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¹Holomorphic functions in several complex variables are those continuous functions on an open set $D \subset \mathbb{C}^n$ which are holomorphic in each variable separately. Iteration of the Cauchy integral formula for discs readily implies that such functions are then C^∞ in all underlying real variables and have local power series expansions in the complex variables.

²This sort of simple construction does not extend to more than one variable, as the zeroes and singularities of holomorphic functions are not isolated in the case of two or more variables. The reader may find more details in [3].

holomorphically to any neighborhood of P .³ As mentioned earlier, every domain in the complex plane is trivially a domain of holomorphy, and this easily implies that every product domain $D = D_1 \times D_2 \times \cdots \times D_n$ with $D_j \subset \mathbb{C}$, $j = 1, \dots, n$, is also a domain of holomorphy. Furthermore, it is elementary to show that every *Euclidean convex* domain $D \subset \mathbb{C}^n$ is a domain of holomorphy. Hartogs, of course, produced the first example of a domain which is NOT a domain of holomorphy. The reader may consult [2] for other surprising consequences of Hartogs's discovery.

The essence of *pseudoconvexity* is now captured by the following statement.

Pseudoconvexity is that local analytic/geometric property of the boundary of a domain D in $\mathbb{C}^n$ which characterizes domains of holomorphy.

Note that it is not at all clear that the *global* property of being a domain of holomorphy should allow a purely *local* characterization, i.e., something that can be recognized by just looking near each boundary point. In fact, the validation of this statement was the culmination of major efforts over a period of more than forty years.

Just a few years after Hartogs's discovery, E. E. Levi studied domains of holomorphy with *differentiable* boundaries. He found the following simple differential condition, which is remarkably similar to the familiar differential characterization of Euclidean convexity. We assume that $D \cap U = \{z \in U : r(z) < 0\}$, where r is a C^2 real-valued function with $dr \neq 0$ on a neighborhood U of $P \in bD$.

Theorem (Levi, 1910/11). *A) If there exists a holomorphic function on $D \cap U$ which does not extend holomorphically to P (in particular, if D is a domain of holomorphy), then*

$$L_P(r; t) = \sum_{j,k=1}^n \frac{\partial^2 r}{\partial z_j \partial \bar{z}_k}(P) t_j \bar{t}_k \geq 0 \text{ for all } t \in \mathbb{C}^n$$

$$\text{with } \sum_{j=1}^n \frac{\partial r}{\partial z_j}(P) t_j = 0.$$

*B) If $L_P(r; t) > 0$ for all $t \neq 0$ which satisfy $\sum_{j=1}^n \frac{\partial r}{\partial z_j}(P) t_j = 0$, then the neighborhood U can be chosen so that $U \cap D$ is a domain of holomorphy.*⁴

³This definition is formally weaker than the one commonly found in the literature; namely, a domain of holomorphy is a domain on which there exists a single holomorphic function which cannot be extended holomorphically to any of its boundary points. However, by a 1932 fundamental theorem of H. Cartan and P. Thullen, the two notions are in fact equivalent.

⁴With $z_j = x_j + i y_j$, $j = 1, \dots, n$, the complex partial differential operators $\partial/\partial z_j$ are defined by $\partial/\partial z_j = (1/2)(\partial/\partial x_j - i \partial/\partial y_j)$; an analogous definition holds for their conjugates $\partial/\partial \bar{z}_j$.

Note that if $D \subset \mathbb{C}$, the restriction on t is satisfied only for $t = 0$, so the conditions in A) and B) trivially hold in this case.

Levi's results made it clear that the "complex Hessian" $L_P(r; t)$ —now universally called the *Levi form*—plays a fundamental role in the characterization of domains of holomorphy. The term "pseudoconvex" was introduced in this context in the influential 1934 "Ergebnisbericht" of H. Behnke and P. Thullen, which summarized the status and principal open questions in multidimensional complex analysis at that time. To distinguish Levi's differential conditions from other formulations of pseudoconvexity, one refers to the condition in A) as *Levi pseudoconvexity*. If the stronger version in B) holds, one says that D is *strictly* or *strongly* pseudoconvex at P .

By Levi's result, if D is strictly pseudoconvex at every boundary point, then D is **locally** a domain of holomorphy. The emphasis on "locally" is critical. Levi himself recognized that his result was far from yielding the wished-for corresponding global version. For many years it remained a central open problem—known as the *Levi problem*—to show that a strictly pseudoconvex domain is indeed a domain of holomorphy. Solutions were finally obtained in the early 1950s by K. Oka, H. Bremermann, and F. Norguet, thereby vindicating the central role of pseudoconvexity.

The extension to arbitrary domains requires an appropriate definition of pseudoconvexity. Many equivalent versions have been introduced over the years. Perhaps most elegant is a formulation that involves the notion of *plurisubharmonic* function introduced by Oka and P. Lelong in the 1940s.⁵ Suffice it to say that a C^2 function r on D is plurisubharmonic precisely when its Levi form satisfies $L_z(r; t) \geq 0$ for all $t \in \mathbb{C}^n$ and $z \in D$ and that general plurisubharmonic functions can be well approximated from above by C^2 or even C^∞ plurisubharmonic functions. Let us denote by $\text{dist}(z, bD)$ the Euclidean distance from z to bD .

Definition. A domain $D \subset \mathbb{C}^n$ is said to be *pseudoconvex* (or Hartogs pseudoconvex) if the function $\varphi(z) = -\log \text{dist}(z, bD)$ is plurisubharmonic on D .

Note that φ is a continuous function which tends to ∞ as $z \rightarrow bD$. One verifies that convex domains are pseudoconvex and that a domain with C^2 boundary is pseudoconvex according to this definition if and only if it is Levi pseudoconvex. Also, any pseudoconvex domain is the increasing

⁵Subharmonic functions were first introduced in one complex variable by F. Hartogs in 1906. That concept was generalized in the obvious way to n real variables in the 1920s. In contrast, plurisubharmonic functions are those functions of n complex variables which are subharmonic on each complex line where defined.

union of strictly (Levi) pseudoconvex domains with C^∞ boundaries. Incidentally, it is known that a domain D is (Euclidean) convex if and only if $-\log \text{dist}(z, bD)$ is a convex function.

The general version of the solution of Levi's problem is then stated as:

A domain in $\mathbb{C}^n$ is a domain of holomorphy if and only if it is pseudoconvex.

To conclude, let me briefly mention two topics involving pseudoconvexity which continue to stimulate important research work.

The first deals with studying the boundary behavior of analytic objects, such as special classes of holomorphic functions, biholomorphic maps between open sets in $\mathbb{C}^n$, and solutions of the inhomogeneous Cauchy-Riemann equations. Many such questions are pretty well understood in case the boundary of the domain is *strictly* pseudoconvex. (See [3] for some references.) A natural goal then is to extend such results to the general Levi pseudoconvex case, say with C^∞ boundary. Let us emphasize that the problems are genuinely higher dimensional, since in dimension one all smoothly bounded domains are automatically *strictly* pseudoconvex. The situation is quite complicated and very technical. Some results are known to fail in the general case, as evidenced, for example, by the so-called “worm domain” discovered by K. Diederich and J. E. Fornaess in 1976. (See [1].) Other results have been verified assuming additional conditions such as Euclidean convexity and/or “finite type”—an important generalization of strict pseudoconvexity that was introduced by J. J. Kohn in the early 1970s. And other questions still remain unsolved. For example, Charles Fefferman proved in the mid-1970s that every biholomorphic mapping between smoothly bounded strictly pseudoconvex domains in $\mathbb{C}^n$ extends smoothly to the boundary. This result has been extended to the case of pseudoconvex domains of finite type and to some other special situations, but to my knowledge—in spite of many efforts—the problem remains open for arbitrary Levi pseudoconvex domains.

Another natural question centers around our basic understanding of pseudoconvexity and its relationship to Euclidean convexity. The explicit formulations of pseudoconvexity mentioned in this article clearly are complex analogues of corresponding characterizations of convexity. In particular, convexity implies pseudoconvexity. Furthermore, it is elementary, but nontrivial, to show that a domain is *strictly pseudoconvex* near P if and only if it is strictly Euclidean convex (i.e., the relevant matrix of second-order partial derivatives is positive definite) with respect to suitable local holomorphic coordinates centered at P . Stated differently, strict pseudoconvexity is—locally—simply the biholomorphically invariant version of

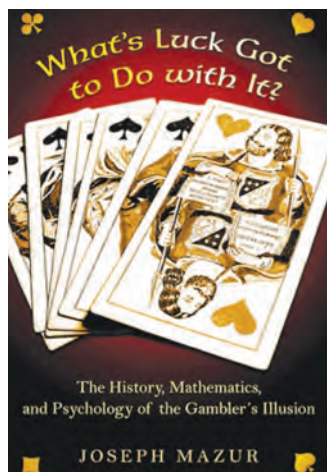
strict convexity. Unfortunately, this neat characterization breaks down already in the case of simple pseudoconvex domains of finite type, as shown by an example discovered by J. J. Kohn and L. Nirenberg in 1972. However, if one drops all regularity conditions of the coordinates on the boundary, one is left with the following tantalizing question, whose answer is still unknown.⁶

Given a smoothly bounded domain $D \subset \mathbb{C}^n$ and a point $P \in bD$ such that $D \cap U$ is pseudoconvex for some neighborhood U of P , can U be chosen so that $D \cap U$ is biholomorphically equivalent to a Euclidean convex domain $W \subset \mathbb{C}^n$?

Further Reading

- [1] H. P. BOAS, WHAT IS... a Worm? *Notices Amer. Math. Soc.* **50** (2003), 554–555.
- [2] R. M. RANGE, Extension phenomena in multidimensional complex analysis: Correction of the historical record, *Math. Intell.* **24** (2002), 4–12.
- [3] ———, Complex analysis: A brief tour into higher dimensions, *Amer. Math. Monthly* **110** (2003), 89–108. (Includes references to systematic introductions to multidimensional complex analysis.)

⁶The question is purely local. At the global level, it is known that the answer is negative. For example, in 1986 N. Sibony produced a smoothly bounded pseudoconvex domain in $\mathbb{C}^2$ which cannot even be properly embedded in a convex domain in some $\mathbb{C}^N$.



What's Luck Got to Do with It?

Reviewed by Andrew I. Dale

What's Luck Got to Do with It?: The History, Mathematics, and Psychology behind the Gambler's Illusion

Joseph Mazur

Princeton University Press, 2010

US\$29.95, 296 pages

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The origin of the study of mathematical probability is often, though incorrectly, seen as arising in an exchange of letters between Antoine Gombauld (the Chevalier de Méré), Blaise Pascal, and Pierre Fermat in the mid-seventeenth century. This “origin” was rooted in gambling, yet probability theory itself has had little, if any, effect on gamblers’ behavior. In *What's Luck Got to Do with It?*, a book enlivened by numerous literary and personal anecdotes, Mazur explores various facets of gambling and luck in a manner that will appeal not only to the general reader but also to those who relish little-known facts and tidbits.

Divided into three parts, *What's Luck Got to Do with It?* leads the reader through historical, mathematical, and psychological aspects of matters relating to gambling. The reader must draw his own conclusions about the wisdom of indulging in such a pastime, for Mazur does not sermonize. Although he no more preaches against gambling than he advocates it, one gets a distinct sense of the unreasonableness of gambling and of its obsessive and destructive nature.

Authors of earlier centuries were less restrained in their opinions of gamblers. For instance, in 1785 Samuel Johnson, in his usual forthright and inimitable style, defined a *gambler* as “A knave

whose practice it is to invite the unwary to game and cheat them,” while a *gamester* was “one who is vitiously addicted to play; a merry, frolicksome person” or a prostitute.

The ancient gods and goddesses were thought to be bearers of luck (good or bad), and even in these more enlightened times Dame Fortune is believed to be influenced by things like a rabbit’s foot or a stepladder. So much for free will! A belief in one’s luck may of course result in a warm inner glow, and while not denigrating the psychological value of such a benefit, Mazur notes that “this book concentrates on the mathematics behind gambling to empower the reader who knew—all along—that the powerful illusion of luck is not some acquired supernatural essence but something that can be cogently explained by rules of probability” [p. xvii].

Mazur begins his first chapter with a most descriptive passage that invites one to picture the brute Neanderthals “reflexively gambling every day against the impending extinction of their race” [p. 3] as preparations are made for a sabre-toothed tiger hunt. This fierce scene is contrasted with the image of the proto-human child, innocently indulging in a gentle game with astragali (huckle bones). Mazur then goes on to consider topics ranging from rock painting to the Tudors in England.

Games and gambling were the subject of tight control in England from early times, though the reasons for such control seem perhaps silly today. Act 33, Henry VIII, c. 9, prohibited unlawful games because they interfered with other activities more useful to the kingdom: for example, the maintenance of archery, which was considered more important than the social evils of things like crime and neglect of divine service. In practice, of course, gambling was allowed (or at any rate winked at) to the rich but forbidden to the poor: Mazur records that Henry VIII (1491–1547) and

Andrew I. Dale is professor emeritus at the University of Kwazulu-Natal in Durban, South Africa. His email address is dale@ukzn.ac.za.

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his courtiers indulged themselves in the privacy of the royal quarters whenever they wished. By the end of the nineteenth century various statutes had clearly defined which games were lawful and which were not. The former included backgammon, billiards, bowls, chess, cricket, football, golf, rowing, tennis, whist, and wrestling; among the unlawful were dice (excluding backgammon) and lotteries, while boxing was doubtful.

Coin tossing is discussed in Chapter 2: the naive reader may have his faith in things like “the probability of a *head* when a coin is tossed is $1/2$ ” shaken on reading [3]. Attention is also paid to Pascal’s Triangle (I missed a reference to [5] here) and Jacob (Jacques, Jakob, James) Bernoulli’s *Ars Conjectandi*. (Incidentally, it is not clear from Mazur’s index that James and Jacob were the same person.)

The solution of the *problem of points*—concerned with the division of stakes when a game is called off before the agreed-on concluding stage is reached—unfortunately receives little attention from Mazur (see his p. 29). I would also like to have seen more on the St. Petersburg Paradox.

It is a pity that more is not made here of the gambler’s ruin problem. For example, suppose a gambler decides to bet on the n th trial only when he has seen many more *heads* than *tails* in the preceding $n - 1$ outcomes. Or, after such a sequence of observations, he may decide to bet on *tails* because of the “maturity of chances” (the Monte Carlo fallacy). Unfortunately all such gambling systems are futile.

Feller provides a precise definition of a gambling system, giving it as “a set of fixed rules which for every trial uniquely determine whether or not the bettor is to bet ... the rules must be such as to ensure an indefinite continuation of the game. ... The importance of this statement was first recognized by von Mises, who introduced the impossibility of a successful gambling system as a fundamental axiom” [6, p. 199]. Shafer and Vovk [9] also take the hypothesis of the impossibility of a gambling system as a fundamental idea, an assumption that, together with something similar on dynamic hedging, serves as a starting point for their development of probability.

Mazur’s reader is next taken across the Atlantic to North America and is first of all treated to a discussion of riverboat gamblers. Figure 4.1 in the book showing such a person sharking a businessman recalls Ambrose Bierce’s definition in *The Devil’s Dictionary*: “The gambling known as business looks with austere disfavor upon the business known as gambling.” Things like shipwrecks, the stock exchange, hedge funds, and insurance are discussed, preparing the way for Chapter 5, in which is detailed the general, almost global, economic collapse that started in 2008.

The purchase of an annuity is perhaps one of the soundest gambles in which one can indulge, yet even here there may be pitfalls. Bierce took note of this in defining insurance as “An ingenious modern game of chance in which the player is permitted to enjoy the comfortable conviction that he is beating the man who keeps the table” (op. cit.).

In the anecdote with which Chapter 5 starts, Mazur talks of greed, risk, and reckless behaviour. He details some of the recent bank and financial scandals, often caused by those supposedly “in the know” gambling that some event or other would or would not occur and then finding that they were wrong. Mazur goes so far as to write, “The banking industry’s extensive risks ... were reckless ventures goaded by unrestrained greed” [p. 61]: Wall Street and Las Vegas are but sisters under the skin. There is discussion of the reasons for the fall of the world financial equities, a fall that one may perhaps uncharitably say came at exactly the right time to be featured in Mazur’s book.

In Chapter 7 Mazur notes that, when properties of a long sequence of events are noticed, one should not believe that these properties necessarily hold for shorter subsequences. Ville [10] in fact showed that there exist collectives—i.e., sequences, in which every sub-sequence selected in advance has the same proportion of “successes”, say—taking values in $\{0,1\}$ that have limiting frequency $1/2$ yet are such that in every initial segment the relative frequency of 1’s, say, exceeds $1/2$. This phenomenon occurs in connection with a long run of reds (say) in roulette: those who would firmly bet on a black on the next throw (because of some belief in a “balancing effect”) should bear in mind that there is also a possibility, however unsavory one might find it, that the roulette wheel is unfair.

Mazur provides a reasonable discussion of Galton’s quincunx (a device intended to illustrate the tendency of a number of small accidental causes to approximate a normal distribution), leading to an investigation of coin tossing in a “double or nothing” game. Here the matter of *risk* arises, leading in turn to *utility* and *value*.

Consideration is given to Daniel Bernoulli and his 1738 paper on utility [1]. The view expressed here, Lopes has noted, was that “For Bernoulli, utility was a *psychological* construct capturing the common intuition” [8, p. 482], and she illustrates this by citing the following passage: “any increase in wealth, no matter how insignificant, will always result in an increase in utility which is inversely proportionate to the quantity of goods already possessed” [1, p. 25] (Sommer’s translation). Lopes also notes, however, that the interpretation of *utility* explored in von Neumann and Morgenstern’s

masterly work of 1944 was not Bernoulli's, the former now being only for money under risk.

Starting with the binomial distribution in Chapter 8, Mazur leads on in the usual way to the normal. I suspect that the general reader will be perplexed by the continuity correction that appears in the formula on p. 115. (Incidentally, it is worth remarking that the different descriptions "bell-shaped" and "gend'arme's hat", the latter attributed to "a lively French statistician" by Edgeworth [4, p. 600], merely reflect the change in shape of the Normal distribution under various changes of scale.)

That "Truly Astonishing Result", the weak law of large numbers, receives careful attention and illustration in Chapter 9. It is important in considering this law to bear in mind Jacob Bernoulli's own preamble to his result:

What cannot be ascertained a priori, may at least be found out a posteriori from the results many times observed in similar situations, since it should be presumed that something can happen or not happen in the future in as many cases as it was observed to happen or not to happen in similar circumstances in the past.

[Sylla's translation, 2006, p. 327]. That is, a probability can be "learned" from observation.

It is a pity that there are a number of slips that detract from the usefulness of Mazur's presentation here. For example, in (1) on p. 120 kp should be Np (perhaps occasioned by too slavish a following of [12]?); (2) the symbols μ and σ appearing in Chebyshev's Inequality on p. 121 are not defined; and (3) at the top of p. 122 "at least $1/4$ " should be "at most $1/4$ ". Once again Mazur scores, however, in exploring the connection between mathematics and the random winds of fortune.

In the last chapter in his second section, "The Skill/Luck Spectrum", Mazur investigates the essential eight gambling games: roulette, craps, slot machines (almost the worst betting values), lotteries, blackjack, poker, horse racing (pari-mutuel betting in America), and sports. He presents the odds on various hands in poker and considers the "pure luck" gambles of lotteries. Here we find the basic gambling strategy: "maximize expectation while minimizing risk" [p. 136].

There is apparently an old French proverb to the effect that "There are two great pleasures in gambling: that of winning and that of losing," and the investigation of these pleasures is explored in the final section.

Theories of gambling addiction, Mazur suggests, have resulted from "tensions between the demands of conscience and the performances of the ego" [p. 155]. Further, it seems that there

may well be those in whom a hidden gambling tendency can be aroused by any one of a number of environmental factors. "It is still a mystery," writes Mazur, "why some habitual social gamblers can manage their gambling pleasures while others lose all judgment of rational gambling behavior in thrill-seeking flirtation with jeopardy" [p. 179]. To some degree one must ask whether one is investigating how people *ought* rationally to act or how they actually *do* act.

Chapter 11, beginning with personal reminiscences, is concerned with the "house money" effect, observed when a gambler freely risks money he has gained from the house. "That fantasy of controlling chance—the overconfident belief in one's personal luck—is the gambler's illusion. It is the daring that confuses chance with skill" [p. 166].

Knowing when to stop playing is perhaps one of the most difficult aspects of gambling. Is it, for instance, greed that makes contestants on live television shows continue even though their chances of a large win are decreasing? Mazur also notes the importance of the *house effect*, viz. "under some circumstances, an earlier gain can increase a subject's eagerness to gamble and an earlier loss can decrease his or her willingness to take risks" [p. 172], and further, "Behavior toward risk depends not only on how that risk is formulated but also on the risk taker's view of gains and losses. For example, a venture may be presented in terms of a risk-aversion or a risk-seeking experience" [p. 178]. Here it might well be the case that one averse to risk may prefer a game having a lower expected value (utility?) to one in which the expected value is higher if the possible losses in the first instance are smaller than those in the second.

Psychopathological theories of the twentieth-century distinguishing between social and neurotic gamblers are explored in Chapter 13. Mazur differentiates between *pathological* and *problem* gamblers: briefly, in the case of the former there is manifested a preoccupation with gambling, irrational behavior, and continuation of such behavior even in the face of adverse consequences, while in the latter the gambler's behavior has a harmful effect not only on himself but also on his family, friends, etc. Freud's ideas are of course considered, a consequence of which is "that the gambler's true motivation may not be his conscious will to win but an unconscious desire caused by some internal conflict, possibly even an unconscious desire to lose" [p. 183]. Or could belief in luck be more a desire to control than a wish to win? After exploring such things Mazur is reluctantly led to conclude that current thinking "is still all theoretical and inconclusive" [p. 186].

There is also discussion of behavioral theories of psychology, where it is concluded that

“Reinforcement and conditioning ... motivate the gambler’s decisions” [p. 191]. Early wins may also encourage a player to continue: *luck is on my side*. Other behaviorists believe that gambling is driven by boredom or even the euphoria and the action—perhaps even like that induced by drugs (interestingly, Mazur notes on p. 264 that some casinos in the United States have successfully fought smoking bans, the argument being that smoking and gambling go hand in hand for many people). Other psychologists would see the workings of an irrational mind or even the influence of a mixture of “pure” theories.

Mazur concludes this chapter by considering what makes a person a gambler. Neuroscience has shown heightened levels of dopamine during the gambling process. However, “dopamine transmission does not differentiate the activities of extensive gambling, obsessive drinking, and so forth” [p. 200]. Can one in fact conclude anything more than *quot homines tot sententiæ*?

Chapter 14 describes the “hot hands” phenomenon, in which a gambler is enticed back to the gaming table even though losing. Is it caused by a need for excitement? Does one feel “hot” after a winning streak? Does one argue that luck is on one’s side or that one has a certain amount of luck in one’s account, so to speak, that may be drawn? If the latter, then perhaps one’s chances of winning decrease.

The final chapter is mainly concerned with gambling on slot machines, and Mazur notes the importance of being aware of the machine paybacks, often actually less than naively expected. Drawing a comparison with entropy, Mazur writes that “In the long run the chips will drift uniformly in the direction of the house’s baited treasury” [p. 214].

The main text ends as follows:

I would argue that some—if not most—gambling behavior is primarily connected to an intrinsic desire to manipulate luck in order to validate life, to test the forces of uncertainty under a fantasy of knowing something unknowable or to experiment with the new. ... Gambling is confirmation that *someone* is in control; it is as natural as belief in God. [p. 216]

While the five appendices are useful, it is unfortunate that they contain some serious errors. For example, in Appendix C [p. 227] it is stated that in a binomial experiment with k successes in N trials “the expected ratio of successes is k/N ”. What is in fact required is $E[k/N] = p$. Appendix D shows some confusion between conjunction and disjunction: for instance, we find “the probability of A or

B happening is the product of the probability of A and the probability of B ” [p. 234].

The reading of *What’s Luck Got to Do with It?* has led me to find out more about many of the topics Mazur discusses. Despite some of the shortcomings I have mentioned here, I have no hesitation in recommending it to the interested layman, who will find the treatment and style fascinating and will be grateful to Mazur for having set his feet on a path “to fresh woods and pastures new”.

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Film Review: *Top Secret Rosies*

Reviewed by Judy Green

Top Secret Rosies: The Female Computers of World War II

High-definition video documentary, running time 56:40

Producer/Director: LeAnn Erickson

Website: <http://www.topsecretrosies.com>

Top Secret Rosies: The Female Computers of World War II is a documentary that focuses on four women who worked as “human computers”, computing individual ballistic trajectories for the Army at the University of Pennsylvania’s Moore School of Electrical Engineering. These trajectories were then compiled into tables at the Army’s Aberdeen Proving Ground (APG). Three men are also featured: two who were members of the Army Air Corps and one, Joseph Chapline, who worked with John Mauchly at the university. Several historians also appear giving commentary.

The film gives a flavor of the wartime experiences of the seven principal interviewees and explores how they felt about their work. We learn from the women something of what it was like to work on the tables, and we learn from the men in the planes something of what it was like to use the tables for dropping bombs. We also learn that some of the calculations were done by hand, some were done using calculating machines, and some were done using the university’s differential analyzer, an analog electromechanical computing machine used to solve differential equations. Unfortunately, but understandably, we do not learn precisely what went into creating the tables or what sort of calculations the women were doing. Other technical details that would interest



mathematicians are also not included. Nonetheless, the film is interesting and informative. It is particularly suitable for an audience that might not be aware of the pervasiveness of mathematics, in military applications and elsewhere. However, by mislabeling the women in the film as mathematicians, it does somewhat distort the role

of women in the mathematical sciences in the mid-twentieth century. Furthermore, while *Top Secret Rosies* shows a piece of history not usually seen, it does not show anything of the history of mathematics or the history of women in mathematics, as is claimed in some reviews of the film.

The title of the film is clearly meant to evoke a comparison with “Rosie the Riveter”, a World War II symbol of women who worked in shipyards and aircraft factories and did other jobs previously done mainly by men. This comparison is not really appropriate since it was not unusual for women to do computations before the war. Furthermore, in the 1930s about 15 percent of all the American Ph.D.’s in mathematics were granted to women, and there were at least two hundred American women with Ph.D.’s in mathematics at the start of World War II.¹ Many of these women, most notably

Judy Green is emeritus professor of mathematics at Marymount University, Arlington, VA. Her email address is judy.green@marymount.edu.

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¹*See Judy Green and Jeanne LaDuke, Pioneering Women in American Mathematics: The Pre-1940 Ph.D.’s (Providence, RI: American Mathematical Society and London Mathematical Society, 2009).*

Grace Hopper² (Ph.D., Yale, 1934) and Mina Rees³ (Ph.D., Chicago, 1931), were involved in the war effort. In 1938 another mathematician, Gertrude Blanch (Ph.D., Cornell, 1935), was hired by the WPA's Mathematical Tables Project to train both women and men, many without high school diplomas, to do the computations necessary to compile mathematical tables using only paper and pencil.⁴ Furthermore, in referring to the human computers, a male employee of the Ballistic Research Laboratory at Aberdeen noted that "women were regarded as capable of doing the work more rapidly and accurately than men."⁵

Of the four Rosies of the film, only three joined the project in 1942. That spring the "Blumberg Twins", Doris Blumberg (later Polsky) and Shirley Blumberg (later Melvin), graduated from the Philadelphia High School for Girls and were encouraged by their high school principal to apply to work in the newly forming group at the Moore School. Marlyn Wescoff (later Meltzer) had just received a degree from Temple University in secondary education with a minor in business. Although these three women are referred to as mathematicians, when they started working at the Moore School, none of them had taken any college mathematics courses. The film's website reports that Wescoff's degree was in mathematics education⁶ despite the fact that in 1995 Marlyn Wescoff Meltzer reported that her major was "called social studies and English" and that she was hired because she "could operate an adding machine."⁷ While the Blumberg twins and Meltzer clearly had significant mathematical talent, labeling them as mathematicians echoes a common misconception that anyone who does computations must be a mathematician.

²Grace Hopper joined the U.S. Naval Reserve in December 1943 and worked on the Mark I computer at the Bureau of Ships Computation Project at Harvard University. She is best known as the creator and developer of the first compilers, as an advocate for making computers accessible to large numbers of people, and as the oldest officer on duty in the armed services when she retired as a rear admiral in 1986.

³During the war Mina Rees worked for the Applied Mathematics Panel, which enlisted civilians to assist the military with war-related work. Later she served as the head of the mathematics branch of the Office of Naval Research and was the first president of the Graduate School of the City University of New York.

⁴See David Alan Grier, "Gertrude Blanch of the Mathematical Tables Project", *IEEE Ann. Hist. Comput.* **19** (1997), no. 4, 18–27.

⁵W. Barkley Fritz, "The women of ENIAC", *IEEE Ann. Hist. Comput.* **18** (1996), no. 3, 13.

⁶Short backgrounds of the seven people featured in the film can be found at http://www.topsecretrosies.com/Top_Secret_Rosies/Project_Background.html.

⁷*Ibid.*, Fritz, "The women of ENIAC", 22.

The film does not include anything about the mathematical training the women of the project received. However, in 1995 Marlyn Wescoff Meltzer indicated that at some point she attended classes that included trigonometry and calculus.⁸ One of the other women who came to the project without any college-level mathematics training was Betty Snyder (later Holberton), a journalism major who later became a well-known computer programmer. In a 1973 interview for the Smithsonian Institution's Computer Oral History Project, Betty Snyder Holberton indicated that she learned about the program from an ad saying "they needed girls to do mathematics and they would train you."⁹ As part of her participation in the ballistics project, Holberton spent three months studying "mathematics eight hours a day, six days a week."¹⁰ One of the three months was spent on calculus.

Although it is not clear if the women without college degrees participated in any mathematics training, it is clear that for the ballistic tables project the Blumberg twins and Meltzer executed algorithms that other people had formulated. Thus they were part of a project in which individuals with varying levels of mathematical training produced mathematical tables in a team setting. The WPA's Mathematical Tables Project and the Army's ballistic tables project were far from being the first projects of that kind. In 1977 Uta C. Merzbach described the phenomenon as presented by Charles Babbage in the chapter "On the division of mental labour" of his book *On the Economy of Machinery and Manufacture*:

Babbage...related the story of the French mathematician Baron de Prony (1755–1839), who was charged by the French government with the production of the logarithmic and trigonometric tables necessitated by the French attempt to extend use of the decimal system to the division of the circle into 100 parts. While pondering the organization of this massive undertaking, Prony is said to have chanced upon a copy of Adam Smith's *Wealth of Nations*. Scanning the introductory chapter, on the division of labor, it occurred to Prony to divide the "manufacture" of the mathematical tables in a fashion analogous to that

⁸*Ibid.*, 23.

⁹Jean J. Bartik and Frances E. (Betty) Snyder Holberton interview. Interview by Henry Tropp, April 27, 1973, Washington, DC. Computer Oral History Collection, Archives Center, National Museum of American History, Smithsonian Institution. Transcript at http://invention.smithsonian.org/downloads/fa_cohc_tr_bart730427.pdf, 2.

¹⁰*Ibid.*, 5.

which Smith described for the manufacture of pins. Prony...established three sections of work. To the first section he assigned five or six distinguished mathematicians. Their sole function was to select, from numerous available analytic expressions for a certain function, that formula most easily computed by a large number of individuals working simultaneously. To the second section he assigned seven or eight competent mathematicians charged with giving numerical values to the formulas selected by the first section, on which the actual computations would be based. The members of the second section also verified subsequent calculations by analytic means. To the third section Prony assigned 60 to 80 individuals who needed no mathematical knowledge beyond the ability to add and subtract. This section carried out the required computations.¹¹

What was different about the ballistic tables project was that the group that carried out the computations used two different methods of computing. Doris Blumberg was one of six computers who used the Moore School's differential analyzer that could compute in fifteen minutes what Shirley Blumberg and Marlyn Wescoff's group needed sixty hours to compute.

The second half of *Top Secret Rosies* introduces another Army project that was located at the University of Pennsylvania's Moore School—the development of the ENIAC, the first fully functional electronic general purpose digital computer. Once the ENIAC became close to operational, there was a new call for women mathematics majors to join the group at the Moore School. One of those answering that call was the fourth Rosie of the film, Betty Jean Jennings (later Bartik). Jennings had studied mathematics at Northwest Missouri State Teachers College and joined the project in April 1945. A few months later, she and Marlyn Wescoff became part of a team of six women who were to set up problems for the first test of the ENIAC.¹² This team was drawn from those who carried out the computations needed to produce

the tables because the task of setting up problems was viewed as clerical.

It is in the discussion of programming the ENIAC that *Top Secret Rosies* is most likely to present mathematicians with new insights into the work that was done by the women. It is made abundantly clear that their work was much more than clerical and that computer programming in 1945 was not at all like computer programming today. Rather, we learn that the ENIAC consisted of many separate units, and programming it involved connecting one unit with another to get a path that would solve a specific problem.¹³ Soon after the end of the war five of the six programmers went to Aberdeen to be trained.¹⁴ Jean Jennings Bartik later wrote:

We spent much of our time at APG learning how to wire the control boards for the various punch card machines: tabulator, sorter, reader, reproducer, and punch. As part of our training, we took apart and attempted to fully understand a fourth-order difference board that the APG people had developed for the tabulator.¹⁵

They also learned “how each unit of the ENIAC worked, what the switch settings meant, how the different units operated together to perform the operations of multiplication and division, how the information was read into the ENIAC from the card reader, and so on.”¹⁶

At about the same time that the six women went to Aberdeen to learn about the ENIAC, two physicists from Los Alamos came east to see if the ENIAC could help perform a calculation needed for the development of the hydrogen bomb. The film includes a wonderful description by Jean Jennings Bartik of the installation of that program, the first to be installed on the ENIAC. She tells us that Herman Goldstine¹⁷ stood in front of the ENIAC and gestured like an orchestra conductor calling out commands to the women who were throwing

¹¹Uta C. Merzbach, Georg Scheutz and the First Printing Calculator (Washington, DC: Smithsonian Institution Press, 1977), 6–7. http://www.sil.si.edu/smithsoniancontributions/HistoryTechnology/pdf_10/SSHT-0036.pdf.

¹²Three of the other four women programmers had been mathematics majors [Frances Bilas (later Spence), Ruth Lichterman (later Teitelbaum), and Kathleen McNulty (later Mauchly then Antonelli)] and one [Frances Elizabeth Snyder (later Holberton)] had been a journalism major.

¹³A short description of the ENIAC written by Adele Goldstine can be found in the first chapter of a 1946 technical report, “A Report on the ENIAC (Electronic Numerical Integrator and Computer)”, issued by the University of Pennsylvania. <http://ftp.arl.army.mil/~mike/comphist/46eniac-report/chap1.html>.

¹⁴Fran Bilas did not attend the original training.

¹⁵Fritz, “The women of ENIAC”, IEEE Ann. Hist. Comput. 18 (1996), no. 3, 19–20.

¹⁶W. Barkley Fritz, “ENIAC—A problem solver”, IEEE Ann. Hist. Comput. 16 (1994), no. 1, 28.

¹⁷Herman Goldstine was the mathematician (Ph.D., Chicago, 1936) who, while on active duty at the Army's Ballistic Research Laboratory in Aberdeen, was appointed liaison between the Army and the Moore School.

switches and plugging cables into the units they had been assigned.

The six women programmers were working on the same sort of problem they had been doing earlier, i.e., calculating a trajectory. They were divided into three groups of two: Jean Jennings and Betty Snyder were to figure out how to get the ENIAC to do the calculation, Marlyn Wescoff and Ruth Lichterman were to calculate the trajectory using calculating machines, and Fran Bilas and Kay McNulty would do the calculations using the differential analyzer. Despite this division of labor, all the women discussed how to program the ENIAC, and it was during these discussions that the idea of repeating parts of the program first surfaced. In her 1973 interview Jean Jennings Bartik explained:

When we first started everybody was trying to program a trajectory. ...We didn't know how to do it at all, so everybody was trying, and we were interchanging ideas of how to do it. But, Kay McNulty was the first person that taught me the concept of repeating sections of program. So it was very practical in terms of doing this trajectory problem because with the idea of not having to repeat a whole program, you could just repeat pieces of it and set up the master programmer to do this. And I do remember that that was crucial because we were running out of switches.¹⁸

The first public demonstration showed that the ENIAC could calculate the trajectory of a shell faster than the shell traveled along that trajectory. Although this successful demonstration was the result of the work of six women, none was invited to the celebratory dinner, nor were they mentioned in any of the press releases about the demonstration.

While things today are better for women, mathematicians and others, than they were in the 1940s, it took awhile for the women of this film to be recognized. Nonetheless, many may come away from the film thinking that before World War II women not only did not work as computers or mathematicians but they also thought themselves incapable of doing such work. Listening to the four Rosies, one has the impression that such thoughts never entered their minds.

¹⁸Jean J. Bartik and Frances E. (Betty) Holberton interview transcript, 38. In 1997 all the ENIAC programmers were inducted into the *Women in Technology International (WITI) Hall of Fame*.

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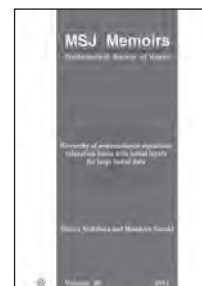
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MathJax: A Platform for Mathematics on the Web

Davide Cervone

Over the past three years, a project has been quietly evolving that has important implications for those interested in using mathematical notation within webpages in a way that not only displays that mathematics beautifully but allows it to interact with other applications and environments. That project is MathJax [1], and it is an attempt to provide a universal, industrial-strength, math-on-the-web solution that is standards-based and applicable to a diverse range of audiences. Current users include publishers of large-scale scientific websites, bloggers and social networkers, users of course-management systems, and individual faculty members who just want to post their homework assignments easily online. Under the sponsorship of the American Mathematical Society (AMS), the Society for Industrial and Applied Mathematics (SIAM), and Design Science, Inc. (DSI), with the support of StackExchange, the American Physical Society (APS), Elsevier, the Optical Society of America (OSA), Project Euclid, WebAssign, and others, MathJax is an open-source project, drawing on the talents of a variety of individuals. MathJax builds on the techniques developed by the author as part of his earlier jsMath package [7] and can be considered the “next generation” of that software.

Anyone who has tried to include mathematical notation in a webpage knows that this is not an easy task. The traditional solution is to use images of the equations and link those into the page to represent the mathematics. This is a cumbersome approach that has a number of drawbacks (it is hard to get the images to match the surrounding text, they don’t scale or print well, they cannot be easily copied, and so on). The Mathematical Markup Language (MathML) was intended to solve this problem (see [19] and [20]), but for a variety of reasons, more than a decade after its specification was released, most of the major browsers still

don’t support it.¹ The MathJax project plugs the gap left by a lack of browser support for MathML, making it possible for mathematicians—and the scientific community at large—finally to take advantage of the MathML standard and all it implies.

In the past, images were the only reliable cross-platform way to present mathematical equations within webpages, despite their faults. Recently, however, several web technologies have come together that make it possible to use a different approach that can resolve a number of these issues. The increased speed of JavaScript engines, the standardization of Cascading Style Sheets (CSS) implementations across browsers, the support for unicode fonts, and the ability to obtain fonts over the Web when they are not installed on the user’s computer can be harnessed to provide a means of including mathematics in webpages that overcomes many of the deficiencies inherent with the use of images.

MathJax is being developed as a platform for mathematics in webpages that works across all the major browsers (including mobile devices such as the iPad, iPhone, and Android phones). It allows authors to write their equations using several formats, including MathML and T_EX, and displays the results using MathML in those browsers that support it or HTML-with-CSS in those that don’t. A Scalable Vector Graphics (SVG) output mode is nearing completion, and other input and output formats are possible. For example, a user-written input processor for the ASCIIMathML format [8] is scheduled for inclusion in the next release of MathJax.

¹Historically, only Firefox has had native support for MathML, and Internet Explorer has a plug-in that handles it, but these have required users to install extra components before being able to view the mathematics. Opera has limited MathML capability through CSS styling. Apple’s WebKit recently has added support for MathML (though it is not as full-featured an implementation as Firefox’s). It is included in Safari 5.1, and eventually we should see it in Chrome and other WebKit-based browsers.

*Davide Cervone is professor of mathematics at Union College. His email address is dpvc@union.edu.
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Lai, Hsin-Hong(1-BRND)

Gromov-Witten invariants of blow-ups along submanifolds with convex normal bundles. (English summary)*Geom. Topol.* 13 (2009), no. 1, 1–48.

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In this paper the author compares the Gromov-Witten invariants of a projective manifold X with those of the blow-up $\pi: \tilde{X} \rightarrow X$ of X along a submanifold Z . A connected submanifold $Z \subset X$ is said to be of type I if the normal bundle $N_{Z/X}$ is convex (i.e. $H^1(\mathbb{P}^1, f^* N_{Z/X}) = 0$ for any holomorphic map $f: \mathbb{P}^1 \rightarrow Z$) and there exists a subbundle $\mathcal{F}$ in $N_{Z/X}$ of rank $\text{rk}(\mathcal{F}) \geq 2$ which is generated by global sections. A connected submanifold $Z \subset X$ is said to be of type II if every holomorphic map $f: \mathbb{P}^1 \rightarrow Z$ is constant. The first result of the paper (Theorem 1.4) states that if $Z \subset X$ is a submanifold such that each connected component is of type I or of type II then the following equality between the twisted n -pointed genus zero Gromov-Witten invariants of X and of $\tilde{X}$ holds:

$$\langle \alpha_1, \dots, \alpha_n \rangle_{0,n,\beta}^{X,c,V} = \langle \pi^* \alpha_1, \dots, \pi^* \alpha_n \rangle_{0,n,\pi^* \beta}^{\tilde{X},c,\pi^* V},$$

where $\alpha_i \in H^*(X)$, $\beta \in H_2(X, \mathbb{Z})$, V is a vector bundle on X and c is an invertible multiplicative characteristic class. The previous formula is also generalized to include descendant insertions (Corollary 4.8).

Figure 1. MathJax output as it appears in an entry in MathSciNet.

MathJax does not require the viewer to download any software (though it will take advantage of certain locally installed fonts when they are present), and since it uses actual fonts, its output scales and prints better than math presented as images. Because the pages include the original $\text{T}_\text{E}\text{X}$ or MathML markup, search engines can index the mathematics within them. Since there are no images to create, the mathematics on the page can be dynamically generated and can include links and other interactive content. The internal format for MathJax is essentially MathML, and the user can obtain either a MathML or a $\text{T}_\text{E}\text{X}$ version of the equations on the page, which can be copied and pasted into other documents or programs.

For a user viewing a page that uses MathJax, nothing special needs to be done (provided JavaScript and page-specific fonts are enabled). After the document is opened, MathJax will look through the page for mathematics to typeset. Users can control some settings for MathJax by using the MathJax contextual menu, which can be obtained by right-clicking (in Windows and Linux) or control-clicking (in MacOS) on any typeset equation.² The menu allows the user to view the MathML or $\text{T}_\text{E}\text{X}$ version of the mathematics, so that it can be copied and pasted into another document, for example. It also allows users to enable a zooming feature for the visually impaired or to select whether to

use the browser's native MathML support (if any) for displaying mathematics, rather than MathJax's HTML-based rendering.

It is possible for users to take advantage of MathJax even on pages where the author has not included MathJax explicitly. Using Greasemonkey [9] (a plug-in for Firefox) or one of its work-alikes for other browsers, one can arrange for MathJax to be inserted into existing webpages to render the mathematics. This can be used, for example, to make pages with MathML work in browsers that don't have native support for it, or in some cases, to replace math displayed as images by font-based mathematics generated by MathJax. This has implications particularly for visually impaired users who need assistance reading the mathematics on the screen. With MathJax's zoom and scaling features, these users can get larger versions of the mathematics easily (without the artifacts of enlarged images).

Those who use screen readers to read their webpages can also take advantage of MathJax, as MathJax works together with programs like MathPlayer [10] to turn the mathematics on the page into a form that can be passed to screen readers. When combined with Greasemonkey, this can make a wide range of existing pages accessible to the visually challenged. For example, this process can be used to make much of the mathematics in Wikipedia accessible (see [9]).

If you are a page author and want to use MathJax to typeset the mathematics in your webpages,

²Note that sites can disable the menu if they choose to. This is the case with MathSciNet, for example, so the contextual menu is not available there.

where κ_g is the geodesic curvature of the curve C . If C at a point P makes the angle θ with the coordinate curve $v = \text{constant}$ and if the coordinate curves are orthogonal, then, according to Liouville's formula,

$$\kappa_g ds = d\theta + \kappa_1(\cos \theta) ds + \kappa_2(\sin \theta) ds.$$

Here, κ_1 and κ_2 are the geodesic curvatures of the curves $v = \text{constant}$ and $u = \text{constant}$ respectively. Since

$$\cos \theta ds = \sqrt{E} du, \quad \sin \theta ds = \sqrt{G} dv,$$

we find by application of Green's theorem:

Figure 2. An equation zoomed by MathJax for greater readability.

```
<!DOCTYPE html>
<html>
  <head>
    <title>MathJax LaTeX Example Page</title>
    <script type="text/javascript"
      src="http://cdn.mathjax.org/mathjax/latest/MathJax.js?config=TeX-AMS_HTML">
    </script>
  </head>
  <body>
    <p>
      When  $(a \neq 0)$ , there are two solutions
      to  $(ax^2 + bx + c = 0)$  and they are
       $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .
    </p>
  </body>
</html>
```

Figure 3. A complete example that uses MathJax to typeset L^AT_EX mathematics within a webpage. The output can be seen at [3].

you need only include a single tag in your HTML file that loads MathJax into the page. MathJax can be configured either within the HTML file itself or through the use of a configuration file on the server, and many of its features are customizable. Since early 2011 MathJax has been available as a web service through a “cloud” distributed network. That means you don’t even have to install MathJax on your own server; simply link your page to the MathJax web service and you can start including mathematics in your pages right away. Because the service resides on a collection of servers around the world, access to MathJax should be fast wherever your readers are located, and they may already have MathJax cached in their browser from viewing previous pages that use it.

You can enter equations into your pages either using T_EX and L^AT_EX notation or as MathML (other input formats may be available in the future). For T_EX notation, MathJax will look through the page for math delimiters like $(\dots)$ and $\$ \$ \dots \$ \$$ to identify the mathematics to be typeset. Note that MathJax implements only L^AT_EX’s math macros and environments, not all the text-mode commands, so you can’t simply place a complete T_EX file on the Web and have MathJax render it. MathJax expects that the textual formatting will be performed

through HTML, not T_EX, though there are preprocessors that can convert a T_EX file to HTML with the mathematics handled by MathJax (e.g., [11]). On the other hand, MathJax does implement a large subset of the L^AT_EX commands and environments, including the amsmath symbols and environments. See the MathJax homepage [1] for more details on using it in your own pages and [2] for a list of the L^AT_EX macros and environments supported by MathJax.

MathJax already has been adopted by a number of important mathematics sites. You may have noticed that it is now being used in MathSciNet [12] and in recent AMS Feature Columns [13], but it is also in use by the *Annals of Mathematics* [14], the MathDL library [15], Project Euclid [16], and in online forums such as MathOverflow [17] and StackExchange [18]. There are plug-ins for a number of popular content-management systems to allow them to use MathJax for their mathematics, and MathJax is ideally suited for use in blogs and wikis, where the content is dynamically generated by the users. A more complete list is available on the MathJax website [5].

As MathJax is an open-source project, you can lend your expertise to it whether you are a programmer, a writer, a tester, or just a user who

can help others learn about MathJax; see the community page [6] for details. MathJax continues to develop and improve, and as more groups and individuals support and adopt it, it should provide a rich platform for handling math on the Web, both now and into the future.

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Notices Interview with Davide Cervone, Chief Developer of MathJax

Notices: In your article you mention that MathML on its own never took off. Can you say more about what went wrong?

Cervone: Actually, I think I said that it did not have wide support in Web browsers, which is somewhat different. It turns out that MathML is used quite heavily in publishing workflows and as an interchange format for mathematical content. Even with image-based websites, the data behind the scenes might have been MathML originally. It would not make it directly into webpages, however, because you could not rely on your viewers having MathML access in their browsers and because until recently you had to use XHTML (which is far less forgiving than HTML) in order for it to work.

Making matters worse, the two major MathML viewers (Firefox and MathPlayer in Internet Explorer) required the content to be delivered in incompatible ways, so servers had to be set up to detect the browsers and customize the pages to suit them. This was cumbersome and unreliable and required too much technical expertise for general use.

It appears that the future looks much brighter for MathML than the past, however. The recent incorporation of native MathML support in WebKit (available in Safari 5.1 and above) suggests that MathML will become much more widely implemented, as WebKit underlies many popular Web browsers including those used on mobile devices. Moreover, MathML has been incorporated into HTML5 (making it much easier to use it in webpages) and in the latest EPUB specification for electronic books. This has caused renewed interest in MathML for display purposes. Apple's iBook application can process eBooks that include MathJax to render mathematics, as can Calibre, which is available on Mac, Windows, and Linux. I expect to see others in the near future that can do the same. In fact, I know of one new eBook reader that includes MathJax within its own code in order to implement the MathML portion of the EPUB3 specification.

After fifteen years of waiting, it seems that the scene is finally set for MathML to begin to fulfill its potential as the means of representing and viewing mathematics in a wide range of settings. I like to think that MathJax contributes to that, as it makes MathML available today, in all modern browsers, allowing mathematical content in MathML to be developed now so that we can take advantage of the new document formats and applications as they become available.

Notices: You say also that certain technical innovations made MathJax feasible. Can you say more about what went right?

Cervone: It may sound trite, but one of the most important steps was the development of standards that the browser manufacturers actually implemented (largely the work of the W3C and its working groups). In the early days of Web development, there was a proliferation of innovations and ideas about how the Web should evolve, and each browser had its own set of extensions, incompatible with other browsers. Anyone doing Web development in those days will tell you how difficult that made things. In recent years, there has been a coalescing of the best of these ideas, and while there are still differences to contend with, you can now rely on a uniformity of results that was not possible in the past.

Great strides have been made by the browser manufacturers in standardizing their implementations of Cascading Style Sheets (CSS) and in the Document Object Model (DOM) and the methods for manipulating it via JavaScript programs. These

are two key ingredients that make MathJax possible. Browser vendors have also been hard at work optimizing their JavaScript engines, and that makes them fast enough and efficient enough to handle the heavy load that MathJax puts on them.

Another critical piece is font support. Mathematics involves a wide range of specialized symbols, and traditionally there was no uniform way of specifying such symbols. The development of Unicode, which provides a standardized means of identifying an enormous range of characters (from musical notation to Canadian aboriginal syllabics), including hundreds of mathematical and technical symbols, makes it possible to specify mathematical notation in a reliable, font-independent way. The adoption of this standard by all modern browsers means MathJax can refer unambiguously to the mathematical symbols it needs.

Being able to indicate a symbol is not quite enough, however. No font includes all the hundreds of thousands of characters in the Unicode specification, and it turns out that very few include the mathematical symbols, so there is no guarantee that an appropriate font will be available to MathJax on a reader's computer. In the past, this would have required users to download and install fonts, and that would have made MathJax impractical. In the last two years, however, one final technical advance has become available widely enough to solve this problem. Browsers now have the ability to download a font over the Web when it is not available on the user's computer already, and MathJax takes advantage of this by providing a set of mathematical fonts (based on TeX's Computer Modern fonts) that are loaded on demand when required. This makes it possible for MathJax to work without any installation on the part of the user, as all its resources can be obtained via the Web as they are needed. This last step is probably what made it possible for MathJax to be as successful as it is.

Notices: *How was the development of MathJax apportioned among various people and/or groups? Along what time line?*

Cervone: So far, MathJax has been nearly four years in the making, and it is based on my experience with an earlier program called jsMath that I developed initially in 2004; but MathJax is a complete rewrite from the ground up. As far as the programming goes, that is pretty much entirely mine, but as an open source project, anyone has the potential to contribute.

MathJax has had major support from groups like the AMS, SIAM, the APS, and the other sponsors mentioned in the article. MathJax would never have happened without their contributions and their readiness to incorporate MathJax into their own websites. Of course, special recognition is due to Design Science, Inc., who administers the MathJax project. Robert Miner has been indispensable in his role as both project leader and a collaborator in the

design decisions and the vision for MathJax. Sean Hogan has done important work in performance analysis, and Frédéric Wang has developed the test suite for MathJax (now comprising thousands of individual tests), which has been crucial to the reliability and quality of MathJax. Hylke Koers has been tireless in his promotion of MathJax within the publication industry and in recruiting sponsors for the project.

There are a number of groups that are building applications around MathJax (like equation editors, eBook readers, and e-learning sites), taking advantage of MathJax's programmer's interface. As MathJax gains wider use, we hope to receive contributions from those types of users back to the MathJax project. Indeed, we are beginning to see this happening with several major extensions being developed by the user community and being made available for others to use. The beauty of an open source application is that it grows and matures based on the experiences of its users.

Robert Miner, the director of the MathJax project at Design Science, Inc., passed away on December 6th from liver cancer. He is survived by his wife, Emily, and his son, Bill. Robert was one of the guiding lights for mathematics on the Web. His contributions to the field began in the mid 1990s with his WebEq software, originally developed at the Geometry Center in Minneapolis, MN, and later acquired by Design Science, Inc. At Design Science he oversaw the MathType and MathPlayer products, and most recently the MathJax initiative. Robert was co-chair of the World Wide Web Consortium (W3C) working group that developed the MathML standard, and was tireless in promoting its use in mathematical publications, both electronic and in print. He spoke often at conferences, and wrote extensively on the use of MathML to enhance mathematical communication.

Without Robert Miner, MathJax likely would not exist. He helped put together the consortium that funded the initial development of MathJax, and his vision and design insight were critical to the success of the project. Robert was one of those who could take all the arguments surrounding a contentious issue and synthesize them into a coherent whole that focused everyone on the right direction. He had a keen sense of humor, a sharp intellect, a deep sense of honor, and a personal philosophy that took great joy in living. His leadership, his energy, and his camaraderie will be missed; we are impoverished by his loss.

—Davide Cervone



A Modest Proposal

Alan H. Schoenfeld

This essay is about mathematical sense-making. From kindergarten through college, precious little of it is found in our mathematics classrooms. We—by which I mean the community of professional mathematicians and society at large—would all benefit from dramatic change.

I begin with some horror stories. These are documented in the literature, and I'm sure *Notices* readers have their own to match. The stories confirm that we are teaching our students not to think or analyze and that we are in fact encouraging them to forego common sense. At a recent conference,¹ for example, Lieven Verschaffel reported that upper elementary school students, trained by their years of school experience, ignore

Alan H. Schoenfeld is the Elizabeth and Edward Conner Professor of Education and affiliated professor of mathematics at the University of California, Berkeley. His email address is alans@berkeley.edu.

Author note: I have borrowed my title from Jonathan Swift, whose Modest Proposal (for Preventing the Children of Poor People in Ireland from Being a Burden to Their Parents or Country, and for Making Them Beneficial to the Public), written in 1729, was that the Irish poor should sell their 1-year-old children to the wealthy for use as food. The full text of his proposal can be found at <http://art-bin.com/art/omodest.html>. Were Swift alive today, he might say that he is gratified to observe (given data on attrition rates, demographics, etc.) that mathematicians have taken his advice—we are eating our young. But the title is the only tongue-in-cheek part of this note. I'm dead serious about the rest.

¹*The Future of Mathematics Education in Europe, sponsored by the Academia Europaea, Lisbon, Portugal, December 16–18, 2007.*

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the “real world” constraints in the following problem:

How many two-foot boards can be cut from two five-foot boards?

Failing to note that one of the resulting “two-foot boards” actually consists of two one-foot pieces, they simply divide 10 (the combined length of the two boards) by two. We might laugh at the silliness here, but the example is hardly unique and it points to more serious issues. For example, Reusser [2] asked ninety-seven first- and second-grade students the following question:

There are 26 sheep and 10 goats on a ship. How old is the captain?

More than 3/4 of the students “solved” the problem, obtaining their answers by combining the integers 26 and 10. Reusser taped students working on the following problem:

There are 125 sheep and 5 dogs in a flock. How old is the shepherd?

A typical “solution” was

$125 + 5 = 130$, this is too big...and
 $125 - 5 = 120$, this is still too big...while
 $125/5 = 25$.

That works.

I think the shepherd is 25 years old.

He also gave 101 fourth and fifth graders the following nonsense problem:

Yesterday 33 boats sailed into port and 54 boats left it. Yesterday at noon there were 40 boats still in the port. How many boats were still in the port yesterday evening?

One hundred of the 101 students produced a numerical answer to the problem, and only five of the students indicated they thought the task statement was in any way unusual or problematic.

Kilpatrick [1, p. 140] reports similar phenomena:

Recently, some [German] children from kindergarten to grade 6 were confronted with “problems” in which no question was posed:

Mr. Lorenz and 3 colleagues started at Bielefeld at 9 AM and drove the 360 km to Frankfurt, with a rest stop of 30 minutes.

These stories were inserted into a set of ordinary word problems. The higher the grade level, the more likely the children were to attempt a calculation to solve the problems.

Plainly, students are learning to tolerate nonsensical problems as they go through school! (I assume you’ve heard the ritual chant, “Ours is not to reason why; just invert and multiply!”)

This trend needs to be undone, and we have the tools and experience to fix the problem. The above stories contrast dramatically, for example, with my own experiences as a mathematician and, occasionally, as a student. The wonderful thing about mathematics is that it coheres: when you understand a mathematical idea, everything fits in place beautifully. That beauty is a large part of my attraction to mathematics.

An experience I had as an undergraduate crystallized this understanding for me. In a probability course, the professor was about to write the statement of the binomial theorem up on the board. She paused and said,

You can get confused if you try to write the statement of the theorem from memory. But you don’t have to memorize it, because the theorem is so easy to derive. Let the statement come at the end.

Consider the product of n terms of the form $(x + y)$. It will, ultimately, be a collection of terms of the form $x^k y^{n-k}$, since each term in the product contains either an x or a y from each of the n $(x + y)$ terms. So, how many $x^k y^{n-k}$ terms are there? It’s the number of ways you can choose k x ’s out of the n $(x + y)$ ’s, or $\binom{n}{k}$. Thus, $(x + y)^n =$

$$\sum_{k=0}^{k=n} \binom{n}{k} x^k y^{n-k}.$$

Having derived the result, she wrote the statement of the theorem in its “proper place” at the beginning of the theorem.

That example has stayed with me for more than forty years, because it captures what I believe about

the nature of mathematics. Things may, when one first encounters them, seem strange—when you first look at the formula, it’s not immediately apparent

why $\binom{n}{k}$ is there at all or why it should be the coefficient of $x^k y^{n-k}$. But, ultimately, there is a good reason—and once one sees it, then what may have seemed arbitrary now seems natural and inevitable. That is, all of elementary mathematics really does make sense. The rules for adding fractions, solving equations, or any other topic our students are likely to encounter in school are, once one understands them, natural and inevitable. But if they are not understood as such, then they appear as arbitrary rules to be memorized and applied mechanically. The results of exposure to such mathematics are the stories that started this essay.

From my perspective, then, the first moral imperative of mathematics instruction is that mathematics must be seen, and taught, as an act of sense-making. Students must be led to see that mathematics is not arbitrary but natural and inevitable—and that they can, with the right experiences, come to grips with it in ways that provide powerful tools for thinking. Whether it is in pure mathematics, where one is rewarded with elegance such as that of the binomial theorem, or in applications and modeling, where the real world phenomena that one models can and should be reflected in the symbols used to represent them, mathematics is and should be experienced in ways that cohere naturally.

Here’s a case in point. Many years ago, when I first took a position at the University of Rochester, I had the standard “what are you going to teach” conversation with my department chair. He was happy to have me teach my problem-solving course, but he said I’d have to pay for the privilege: since I got to teach a course of my own design and choosing, I’d have to teach another course that he picked. The course he chose was pre-calculus, because it was universally despised—nobody (including TA’s!) wanted to teach such a low-level course, to students who’d obviously had problematic mathematical histories.

The following story is typical of how things went during the course. It was time for us to study the arc length formula $s = r\theta$. I asked the students to read the text overnight and to try to do the homework. The next day I asked how many of them had read the text. All the hands went up. I asked how many had understood it. All the hands went down. So, I said, let’s look at some examples. Suppose I had a circle of radius 1. What’s the length of the part of the circumference cut off (a.k.a. “subtended”) by a central angle of 90° ? No problem, they said: that’s $1/4$ of the way around the circle, or $1/4$ of 2π . How about an angle of 180° ? Easy. 60° ? No problem: it’s $1/6$ of the way around. What

if we measured in radians? Same thing, you're just saying $[(\pi/3)/(2\pi)]$ of the way around, instead of $(60/360)$ of the way around. What if the angle was θ radians? Again, no problem: it's $(\theta/2\pi)$ of the way around the circle.

So, I said, we can do circles of radius 1. What if the radius of the circle was 7? Once again, what about a right angle? Same thing, they said: it's $1/4$ of the circumference, which is 14π . A subset of the same sequence of examples led to $(\theta/2\pi)(14\pi)$ for a central angle of θ radians. After that it was a small step to go from radii of 1 and 7 to a radius of r and the formula $s = r\theta$. What had been mystical now made sense. Moreover, my students began to believe that math can (and should) make sense and that they were capable of doing that kind of sense-making. My problem-solving courses are an advanced version of the same principle.

I truly believe that all of the mathematics in the K-16 curriculum—or at least all of the mathematics that should be in the K-16 curriculum!—can be seen as a set of sensible answers to a set of reasonable questions. My immodest proposal is that we revise the entire curriculum so that all students experience it as such, so that they come to see mathematics as a domain that not only makes sense, but as one that they can make sense of. On a more truly modest scale, I propose that we all, each time we teach, stop to think about how and why the mathematics fits together the way it does and how we can help our students to see it that way. We owe our students no less. Approaching instruction this way will make mathematics easier to learn and will make more accessible to students some of the pleasures of the discipline that we find so appealing. Indeed, if we emphasize sense-making, I predict that more students will take to mathematics and that the rest will have a much better appreciation of its power and uses.

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数学科学中心
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Applications are welcome at any time. The review process starts in December 2010, and closes by April 30, 2012. Applicants are encouraged to submit their applications before November 30th.

graduate programs

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ASU SCHOOL OF MATHEMATICAL
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ARIZONA STATE UNIVERSITY

Publishing Computational Mathematics

Tim Daly

Computational mathematics is a new science born of the collision of mathematics with computers. It is a new way of thinking about mathematics. The existing large computer algebra systems are the “Newton’s notebooks” of this new field. They contain implementations of algorithms to compute exact symbolic results. For example, most have an implementation of the Risch algorithm for exact definite integration. This algorithm is a decision procedure that will compute the exact antiderivative of a function if it exists and can be expressed in elementary terms. If it cannot, then the failure of the algorithm is a proof that the antiderivative does not exist.

This note is not about the mathematics of the algorithm but about the implementation. This is equivalent to the difference between computability and complexity. The mathematics may be right, but the implementation may be wrong in many ways. Future results will depend on the implementation, as that is the heart of these computer algebra systems. Some areas of mathematics need to rely on machine results because the algorithms cannot be done by hand. But are they correct?

Suppose your research relies on one of these systems for a result. What are the standards of citation? If you have implemented an algorithm to solve a problem, what are the standards of documentation?

Tim Daly is the lead developer for the Axiom software project. His email address is daly@axiom-developer.org.

Members of the Editorial Board for Scripta Manent are Jon Borwein, Thierry Bouche, John Ewing, Andrew Odlyzko, Ann Okerson.

DOI: <http://dx.doi.org/10.1090/noti797>

What does it mean to publish a “proof” of an algorithm in computational mathematics? This is a vital point that is critical for placing computational mathematics on a solid foundation.

Currently, algorithms are described using mathematical notation and, if necessary, pseudocode to illustrate the basic idea. We are told that the algorithm was “implemented” in some system; that it is “better” by some measure of space, time, or breadth; and presented with tables of measurements as “proof”. In my opinion, this is pretty but it is not mathematics. It fits the “hand waving” standards of the nineteenth century but not the rigor of the twentieth century.

A viable standard of publication has several requirements. One requirement is the ability to fully understand and reproduce the results. Another requirement is developing a body of reference standards. A third requirement is a paper trail of citations.

Consider the requirement about reproducing the work. Mathematics depends on reviewers to examine publications. Often these reviews uncover mistakes or missing elements in a proof, keeping invalid results out of the literature. What standards should apply to a review of an algorithm?

Clearly, the details of the algorithm are important. No implementation stands alone, as it uses the facilities of existing systems. A journal reviewer needs access to the actual code that claims to implement the algorithm. There is no other way to review the results.

Consider the requirement of reference standards. The Digital Library of Mathematical Functions (DLMF) publishes equations that define functions, like the gamma function, in many different

forms. One can find series expansions, or continued fraction expansions, or many others. Try to find a reference standard for an implementation of the gamma function. There is only a table listing the existing computer algebra systems. It seems reasonable to expect that a “digital” reference standard ought to include a reference implementation. Some of these systems are forty years old. Which of these twenty-one implementations is “the reference”?

Consider the requirement of a paper trail of citations. When using algorithms to, say, factor polynomials, what should be the citation? Is it acceptable to “hand wave” at one of these existing systems? Can the reviewer follow the citation, open that algorithm, find the supporting publication, and verify that it handles the cases presented? Clearly not.

This is not an acceptable basis for computational mathematics.

Computational mathematics rests on software. In order to reproduce the results, make reference standards, and create a citation trail, we need to establish ever higher requirements for publication. This is not general computer science; this is mathematics in a new age.

It is technically possible to create a publication format that includes not only the mathematics but the actual implementation. Such a format, called a “literate program”, was proposed by Donald Knuth. A literate program contains the detailed documentation and the source code. It is possible to extract the actual, runnable code from the “paper”.

Literate programs could change the field. It would be possible to attend a lecture, download the paper as a literate document, and execute it during the talk. Imagine validating the claimed performance by reproducing the results in real time. Imagine testing boundary conditions while the talk is in progress. Imagine following a citation, downloading the literate document, and finding that the citation does not apply. New, more general algorithms with details of limitations and boundary conditions could be collected in the “Software Index” of the DLMF.

This is an acceptable basis for computational mathematics.

There are many objections that could be raised.

Mathematicians do not write code. But computational mathematicians do, and, as I claim, this is a new science. Code is the “coin of the realm”.

Current journals would not accept a 300-page paper. This is based on limitations of paper publication, but we live in a digital age. There are no limits to the size of an electronic publication. Perhaps we need a *Literate Journal for Computational Mathematics*.

Current citations which contain source code do not exist. We have to begin to create this body of work. It will take time, and early papers will suffer

from rising standards, just as early “proofs” would not meet the standards of today.

There are no language standards for computational mathematics. This is a sign of immaturity in the field. If you read mathematics papers from prior centuries, they are difficult to follow, as they make up notation “on the fly” to suit the problem. Newton and Leibniz had this same discussion, and we still accept either notation.

Implementations include details that have nothing to do with the theory. This is misguided. The implementation details matter a lot. Numerical software shows that details can make or break an algorithm. Whole areas of mathematics, such as numerical analysis, depend on the details.

Algorithm implementations are proprietary. This needs to end. Science is not done behind a curtain. At least, it has not been hidden since Tartaglia and Cardano fought over solving the cubic.

A firm foundation of science rests on fully understandable and reproducible results, reference standards, and citations. We must develop a “proof” standard for publication that goes to the heart of the subject, namely the implementation.

Computational mathematics needs literate software.

As a point of full disclosure, I am one of the original IBM authors of Axiom as well as the lead developer of the open source version. My interest in literate programming stems from the fact that I have personal experience trying to maintain and document hundreds of thousands of lines of computational mathematics. A survey paper discussing issues in greater detail will be posted on the Axiom website <http://axiom-developer.org>.

Mathematics People

Budaghyan Awarded Artin Junior Prize

LILYA BUDAGHYAN of the University of Bergen, Norway, has been awarded the 2011 Emil Artin Junior Prize in Mathematics. She was chosen for her joint paper with Tor Helleseeth, “New commutative semifields defined by new PN multinomials”, published in *Cryptography and Communications* 3 (2011), 1–16. Established in 2001, the Emil Artin Junior Prize in Mathematics carries a cash award of US\$1,000 and is presented usually every year to a student or former student of an Armenian university under the age of thirty-five for outstanding contributions to algebra, geometry, topology, and number theory—the fields in which Emil Artin made major contributions. The prize committee consisted of A. Basmajian, Y. Movsisyan, and V. Pambuccian.

—Victor Pambuccian, New College,
Arizona State University

Rapoport Awarded Hopf Prize

MICHAEL RAPOPORT of the University of Bonn has been awarded the 2011 Heinz Hopf Prize of the ETH Zürich. He was honored “for his broad and extraordinarily deep scientific works.” Rapoport uses algebraic geometry to establish higher reciprocity laws, which serve as a bridge between the field of arithmetic and the theory of automorphic forms. He is interested in Shimura varieties and their local variants from the point of view of constructing interesting Galois representations, of identifying algebraic cycles on them, and of studying their deformations. He delivered the Heinz Hopf Lectures, titled “How Geometry Meets Arithmetic”, in October 2011.

Born in 1948, Michael Rapoport grew up in the former East Germany. After initial studies in Berlin, he emigrated and continued his studies in Paris and at Princeton and Harvard. He received his Ph.D. in 1976 from the Université

de Paris-Sud under the direction of Pierre Deligne. He has held positions in Heidelberg, Bonn, Wuppertal, and Köln. In 2003 he was appointed to his current position as professor of arithmetic algebraic geometry at the University of Bonn. Rapoport received the Leibniz Prize in 1992 for his work on Shimura varieties and on the proof of the local Langlands conjecture for the general linear group $GL_n(K)$ for local fields K of positive characteristic, work done jointly with Gérard Laumon and Ulrich Stuhler. Rapoport has had a great impact on the arithmetic of elliptic curves.

The Hopf Prize is awarded every two years on the occasion of the Heinz Hopf Lectures, which are given by the prizewinners. The prize carries a cash award of 30,000 Swiss francs (approximately US\$32,700).

—From an ETH announcement

Maharaj and Sarkar Awarded Bhatnagar Prize

MAHAN MAHARAJ of Ramakrishna Mission Vivekananda University and PALASH SARKAR of the Indian Statistical Institute have been awarded the 2011 Shanti Swarup Bhatnagar Prize for Science and Technology in the mathematical sciences. The prize is awarded by the Council of Scientific Research and Industrial Development to recognize outstanding Indian work in science and technology. Shanti Swarup Bhatnagar was the founding director of the Council. It is the highest award for science in India. The prize carries a cash award of 500,000 rupees (approximately US\$10,000).

—Council of Scientific Research and Industrial
Development, India

Soundararajan Awarded Infosys Prize 2011

KANNAN SOUNDARARAJAN of Stanford University has been awarded the Infosys Prize in Mathematical Sciences for 2011. According to the prize citation, "Soundararajan has made fundamental contributions to analytic number theory. These include numerous brilliant breakthroughs in well-known and difficult problems, as well as the resolution of some that have been open for a long time. In particular, his recent development of new, unexpected techniques to study the critical values of general zeta functions has led to the proof of the quantum unique ergodicity conjecture for classical holomorphic modular forms. Many of the analytic and combinatorial tools that Soundararajan and his collaborators have developed, in works ranging from prime numbers and sieve methods to character sums and zeta functions, have become standard tools for researchers in these fields."

The Infosys Prize recognizes outstanding contributions to research in engineering and computer science, life sciences, mathematical sciences, physical sciences, and social sciences. The prize carries a cash award of 5 million rupees (approximately US\$97,000).

—From an Infosys Science Foundation announcement

Brandenberger Awarded CAP-CRM Prize

ROBERT BRANDENBERGER of McGill University has been awarded the 2011 CAP-CRM Prize in Theoretical and Mathematical Physics by the Canadian Association of Physicists (CAP) and the Centre de recherches mathématiques (CRM). He was honored "for his pioneering contributions in the area of theoretical cosmology, especially the interplay of particle physics and cosmology." The prize is intended to recognize research excellence in the fields of theoretical and mathematical physics.

—From a CAP-CRM announcement

Hinrichs Awarded 2011 Information-Based Complexity Prize

AICKE HINRICHS of the Mathematisches Institut, University of Jena, Germany, has been awarded the 2011 Information-Based Complexity (IBC) Young Researcher Prize. The prize consists of US\$3,000 and a plaque. This annual prize is given for outstanding contributions to information-based complexity.

—Joseph Traub, Columbia University

CAREER Awards Presented

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has honored thirty-one young mathematicians in fiscal year 2011 with Faculty Early Career Development (CAREER) awards. The NSF established the awards to support promising scientists, mathematicians, and engineers who are committed to the integration of research and education. The grants provide funding of at least US\$400,000 over a five-year period. The 2011 CAREER grant awardees and the titles of their grant projects follow.

MARK BEHRENS, Massachusetts Institute of Technology, Arithmetic Structure of Homotopy Theory; HONGJIE DONG, Brown University, Problems in Regularity Theory for Linear and Nonlinear Partial Differential Equations; KIRSTEN EISENTRAEGER, Pennsylvania State University, Undecidability in Number Theory and Applications of Arithmetic Geometry; VOLKER ELLING, University of Michigan, Ann Arbor, Non-Uniqueness in Inviscid Flow and Algebraic Vortex Spirals; MARIA EMELIANENKO, George Mason University, Developing Mathematical Tools for Modeling Complex Materials Systems; IBRAHIM FATKULLIN, University of Arizona, Defects and Singularities in Liquid Crystalline and Bacterial Systems; ILSE C. F. IPSEN, North Carolina State University, Early Career and Student Support for the XVIII Householder Symposium; MAX LIEBLICH, University of Washington, The Brauer Group in Algebraic and Formal Geometry; LEK-HENG LIM, University of Chicago, Numerical Multilinear Algebra and Its Applications: From Matrices to Tensors; PING MA, University of Illinois, Urbana-Champaign, Subsampling Methods in Statistical Modeling of Ultra-Large Sample Geophysics; MOKSHAY MADIMAN, Yale University, An Integrated Probabilistic Approach to Discrete and Continuous Extremal Problems via Information Theory; SVITLANA MAYBORODA, Purdue University, Analysis of Partial Differential Equations in Non-Smooth Media; JASON METCALFE, University of North Carolina, Chapel Hill, The Wave Equation on Black Hole Backgrounds; IRINA MITREA, Temple University, Spectral Theory for Singular Integrals, Validated Numerics and Elliptic Problems in Non-Lipschitz Polyhedra: Research and Outreach; ADILSON MOTTER, Northwestern University, Rescue and Control of Complex Networks of Dynamical Systems: Nonlinear Dynamics Approaches and Applications to Biological and Physical Networks; ALVARO PELAYO, Washington University, Symplectic and Spectral Theory of Integrable Systems; ZHIGUANG QIAN, University of Wisconsin, Madison, A Flexible Design and Modeling Framework for Computer Experiments and Beyond; PHILIPPE RIGOLLET, Princeton University, Large-Scale Stochastic Optimization and Statistics; DAVID SAVITT, University of Arizona, p -adic and mod p Galois Representations; NATASA SESUM, Rutgers University, Singularities and Singularity Models in Curvature Flows; ERIC SHEA-BROWN, University of Washington, Bridging Dynamical and Statistical Models of Neural Circuits: A Mechanistic Approach to Multispike Synchrony; MAGGY TOMOVA, University of Iowa, New Approaches to Classical Knot Invariants; IGNACIO URIARTE-TUERO, Michigan State

University, Weighted Inequalities and Their Applications to Quasiconformal Maps; BENEDEK VALKO, University of Wisconsin, Madison, Random Eigenvalue Problems and Fluctuations of Large Stochastic Systems; MARIEL VAZQUEZ, San Francisco State University, Topological Mechanism of DNA Unlinking by the XerCD-FtsK System; STEFAN WENGER, University of Illinois, Chicago, Geometric Inequalities, Asymptotic Geometry, and Geometric Measure Theory; LAUREN WILLIAMS, University of California, Berkeley, Cluster Algebras, Total Positivity, and Physical Combinatorics; YICHAO WU, North Carolina State University, New Statistical Methods for Classification and Analysis of High Dimensional and Functional Data; DAPENG ZHAN, Michigan State University, Analysis of the Geometric Properties of the SLE Curves; QING ZHOU, University of California, Los Angeles, Sparse Modeling Driven by Large-Scale Genomic Data; ANDREJ ZLATOS, University of Wisconsin, Madison, Reactive Processes and Turbulent Flows.

—Elaine Kehoe

Wetzel Chosen Professor of the Year

KATHRYN C. WETZEL of Amarillo College in Texas has been chosen as Professor of the Year for community colleges. A professor of mathematics and engineering and department chair of mathematics, sciences, and engineering, she is one of four selected for the honor by the Council for Advancement and Support of Education (CASE) and the Carnegie Foundation for the Advancement of Teaching. A former nuclear engineer, Wetzel began teaching mathematics part-time at Amarillo after the birth of her daughter. Since then she has become department head of the math and engineering departments, reestablished the engineering program, and created a robust math tutoring center. She takes pride in helping students who have no confidence in their math skills. The math tutoring center she created is now in its sixth year and was used for a total of more than 22,000 visits in the past year.

—Elaine Kehoe

Rhodes Scholarships Awarded

Three students in the mathematical sciences are among thirty-two American men and women who have been chosen as Rhodes Scholars by the Rhodes Scholarship Trust. The Rhodes Scholars were chosen from among 830 students at 299 colleges and universities.

ISHAN NATH of Atlanta, Georgia, is a senior at Stanford University, where he will receive bachelor's degrees in economics and earth systems with a minor in mathematics. His senior thesis relates to clean energy and a national cap-and-trade emissions trading system. He has also interned at the Office of Economic Policy at the White House and served as a consultant to the U.S. Department of Energy. A Truman Scholar and a Udall Scholar, he has also been

an editorial writer for the *Stanford Daily* and a political columnist. A marathon runner, he will do the M.Sc. in economics for development at Oxford.

NABEEL N. GILLANI of Glen Allen, Virginia, is a senior at Brown University majoring in applied mathematics and computer science. He has also served as a research assistant on a biotechnology project and as a Microsoft project manager, and he is working now at Brown's optimization lab on electricity restoration for disaster relief. He founded a Providence-based microfinance organization, as well as an outreach program in the Providence, Rhode Island, public schools to help younger students learn math. At Oxford he plans to do the M.Sc. in computer science and the M.Sc. in education.

MOHIT AGRAWAL of West Lafayette, Indiana, received his B.A. in mathematics at Princeton University last year and is currently doing a master's degree in economic policy evaluation at the National University of Ireland. Elected early to Phi Beta Kappa and the winner of a Mitchell Scholarship, he was copresident of Engineers Without Borders and proposed the Ghana School Library Initiative to construct a library in Ashaiman, Ghana. Mohit also spent a semester at the National University of Singapore and developed tools for anticryptology systems for the National Security Agency. He plans to do the D.Phil. in financial economics at Oxford.

—From a Rhodes Scholarship Trust announcement

Mathematics Opportunities

MfA Fellowship Program

The Math for America Foundation (MfA) sponsors the MfA Fellowship Program, which trains mathematically talented individuals to become high school mathematics teachers in New York City, Boston, Los Angeles, Washington DC, San Diego, or Utah. The fellowship provides an aggregate stipend of up to US\$100,000 over five years, a full-tuition scholarship for a master's-level teaching or teacher credentialing program at one of MfA's partner universities, and ongoing support mechanisms, including mentoring and professional development.

Candidates should hold a bachelor's degree with substantial coursework in mathematics and should be new to teaching and able to demonstrate a strong interest in teaching. Candidates must be U.S. citizens or permanent residents of the United States. The deadline for applications is **January 20, 2012**; for the San Diego site only, the deadline is **April 9, 2012**. For more detailed information, see the website at <http://www.mathforamerica.org/>.

—From an MfA announcement

Summer Program for Women Undergraduates

The 2012 Summer Program for Women in Mathematics (SPWM) will take place at George Washington University in Washington DC, from June 30 to August 4, 2012. This is a five-week intensive program for mathematically talented undergraduate women who are completing their junior years and may be contemplating graduate study in mathematical sciences. The goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school. A number of seminars will be offered, led by active research mathematicians with the assistance of graduate students. The seminars will be organized to enable the students to obtain a deep understanding of basic concepts in several areas of mathematics, to learn how to do independent work, and to gain experience in expressing mathematical ideas orally and in writing. There will be panel discussions on graduate schools, career opportunities, and the job market. Weekly field trips will be organized to facilities of mathematical interest around the Washington area.

Applicants must be U.S. citizens or permanent residents studying at a U.S. university or college who are completing their junior years or the equivalent and have mathematical experience beyond the typical first courses in calculus and

linear algebra. Sixteen women will be selected. Each will receive a travel allowance, campus room and board, and a stipend of US\$1,750. The deadline for applications is **March 1, 2012**. Early applications are encouraged. Applications are accepted only by mail. For further information, please contact the director, Murli M. Gupta, email: mmg@gwu.edu, telephone: 202-994-4857; or visit the program's website at <http://www.gwu.edu/~spwm/>. Application material is available on the website.

—From an SPWM announcement

NSF Support for Undergraduate Training in Biological and Mathematical Sciences

The National Science Foundation (NSF) offers opportunities for support through its Undergraduate Biology and Mathematics (UBM) program. The goal of the program is to enhance undergraduate education and training at the intersection of the biological and mathematical sciences and to better prepare undergraduate biology or mathematics students to pursue graduate study and careers in fields that integrate the mathematical and biological sciences.

The program will provide support for jointly conducted long-term research experiences for interdisciplinary balanced teams of at least two undergraduates from departments in the biological and the mathematical sciences. Projects should focus on research at the intersection of the mathematical and biological sciences and should provide students exposure to contemporary mathematics and biology addressed with modern research tools and methods. Projects must involve students from both areas in collaborative research experiences and include joint mentorship by faculty in both fields.

Between six and nine awards are expected to be made in 2012. The deadline for full proposals is **February 9, 2012**. For more information, see <http://www.nsf.gov/pubs/2008/nsf08510/nsf08510.htm>. The UBM program is a joint effort of the Education and Human Resources (EHR), Biological Sciences (BIO), and Mathematical and Physical Sciences (MPS) directorates of the NSF.

—From an NSF announcement

News from the IMA

Thematic Program. The Institute for Mathematics and its Applications (IMA), located on the University of Minnesota

campus, recently launched its new Annual Thematic Program, running from September 2011 to June 2012. The timely topic, the Mathematics of Information, will gather researchers from mathematical and computer sciences, engineering, and other fields to address the core issues facing extraction of useful information from massive data sets. Seven workshops on rapidly growing research areas are planned for the year, and each workshop will be preceded by tutorials. Each workshop is designed to be truly interdisciplinary, involving researchers from mathematical sciences, computer sciences, engineering, and other fields. More information about this year's workshops and the thematic program are available online at www.ima.umn.edu/2011-2012. Details on the 2012-2013 thematic program on Infinite Dimensional and Stochastic Dynamical Systems and Their Applications are also available at www.ima.umn.edu/2012-2013.

Proposals for Hot Topics Workshops. The IMA's Hot Topics Workshops cover rapidly developing areas of interest, focusing on a specific problem or area of exceptional contemporary significance. These workshops are often cosponsored by a participating institution, corporation, government funding agency, or an NSF-focused research group. More information on submitting a workshop proposal is available online at www.ima.umn.edu/solicit/hot-topics-guidelines.html.

New Directions Research Professorships. New Directions Research Professorships provide an extraordinary opportunity for established mathematicians—typically mid-career faculty at U.S. universities—to branch into new directions and increase the impact of their research by spending an academic year immersed in the thematic program at the IMA, where they learn new mathematics and applications, connect their research with important problems, and establish new contacts and collaborations. Applications for professorships during the 2012-2013 program year, Infinite Dimensional and Stochastic Dynamical Systems and Their Applications, will be reviewed beginning **January 15, 2012**.

—IMA announcement

EDGE for Women Summer Program

The Enhancing Diversity in Graduate Education (EDGE) Program is a postbaccalaureate summer enrichment program designed to strengthen the ability of women and minority students to successfully complete graduate programs in the mathematical sciences.

The summer program consists of two core courses in analysis and algebra/linear algebra. There will also be minicourses in vital areas of mathematical research in pure and applied mathematics, short-term visitors from academia and industry, guest lectures, graduate student mentors, and problem sessions. In addition, a follow-up mentoring program and support network will be established with the participants' respective graduate programs.

Applicants to the program should be women who are (1) graduating seniors who have applied to graduate programs in the mathematical sciences, (2) recent recipients of undergraduate degrees who are now entering graduate programs, or (3) first-year graduate students. All applicants should have completed standard junior- or senior-level undergraduate courses in analysis and abstract algebra and have a desire to earn the doctorate degree. Women from minority groups who fit one of the above three categories are especially encouraged to apply. A stipend of US\$3,000 plus travel, room, and board will be provided to participants. Final acceptance to the program is contingent on acceptance to a graduate program in the mathematical sciences.

The EDGE Program will be held in the summer of 2012 at Pomona College in Claremont, CA. The application deadline is **February 27, 2012**. See the website <http://www.edgeforwomen.org/> for further information as it becomes available.

—EDGE Program announcement

CRM-PISA Junior Visiting Program

The Centro di Ricerca Matematica Ennio De Giorgi (CRM) invites applications for three one-year Junior Visiting positions for the Academic Year 2012-2013.

Successful candidates will be new or recent Ph.D.'s in mathematics with exceptional research potential. The monthly Junior Visitors' grant is approximately 2,000 Euros (approximately US\$2,700) plus a research allowance of 1,000 Euros (approximately US\$1,300) that Junior Visitors can use to invite other researchers to CRM. Junior Visitors are expected to start their research activity at CRM no later than October 2012.

CRM is devoted to promoting excellence in a vast spectrum of research fields, from pure mathematics to mathematics applied to the natural and social sciences. As a consequence, CRM provides a thriving international and interdisciplinary research environment. Junior Visitors can take part in a great variety of scientific activities, including intensive research periods, workshops, and seminars. Moreover, Junior Visitors have a unique opportunity to interact with top-class scientists who visit the CRM as part of our Senior Visiting Program.

The deadline for applications is **January 16, 2012**. Full details about how to apply are available at <http://www.crm.sns.it/grant/19/>.

Further information may be found at <http://www.crm.sns.it>.

—From a CRM-PISA announcement

For Your Information

2012 Mathematics Awareness Month: “Mathematics, Statistics, and the Data Deluge”

Massive amounts of data are collected every day, often from services we use regularly but never think about. Scientific data comes in huge amounts from sensor networks, astronomical instruments, biometric devices, and so forth and needs to be sorted out and understood. Personal data from our Google searches, our Facebook or Twitter activities, our credit card purchases, our travel habits, and so on are being mined to provide information and insight. These data sets provide great opportunities and pose dangers as well. Mathematics and statistics are providing the tools to understand this data and to help prevent misuse.

Original short essays (1500 words or less) related to the theme are invited, as well as contributions of previously published papers that are accessible to talented undergraduates, to be posted on the 2012 Mathematics Awareness Month website at www.mathaware.org. Please contact Ron Wasserstein directly at ron@amstat.org.

Mathematics Awareness Month is sponsored each year by the Joint Policy Board for Mathematics (American Mathematical Society, American Statistical Society, Mathematical Association of America, and Society for Industrial and Applied Mathematics) to recognize the importance of mathematics through written materials and an accompanying poster that highlight mathematical developments and applications in a particular area.

—American Statistical Association

IMU and ICIAM Request Input on Journal Rankings Report

A working group (N. Joshi, D. N. Arnold, C. Hutchins, J. D. S. Jones, M. MacCallum, P. Michor, S. Mueller, and

T. Tang) charged with considering whether or not a joint ICIAM/IMU method of ranking mathematical journals should be instituted has published a report at <http://www.mathunion.org/publications/reports-recommendations>. Before taking any further action, the International Mathematical Union (IMU) and the International Council for Industrial and Applied Mathematics (ICIAM) are soliciting opinions on a larger scale from the mathematical community. Individuals are invited to submit opinions on the “Blog on Mathematical Journals” (moderated by D. Arnold, C. Hutchins, N. Joshi, P. Olver (chair), F. Planchon, and T. Tang) at <http://www.mathunion.org/journals> by posting an article (email to: journal.blog@mathunion.org) and/or forwarding a comment (add a comment to a posted article by typing in the Comments window or sending email to journal.blog@mathunion.org).

—From an IMU announcement

Inside the AMS

Math in Moscow Scholarships Awarded

The AMS has made awards to five mathematics students to attend the Math in Moscow program in the spring of 2012. Following are the names of the undergraduate students and their institutions: PING NGAI (BRIAN) CHUNG, Massachusetts Institute of Technology; JOSEPH W. FERRARA, University of California, Berkeley; JORDAN J. FRANKS, Rice University; JEREMY LEE GARCIA, University of Northern Colorado; and ETHAN J. MCCARTHY, Michigan State University.

Math in Moscow is a program of the Independent University of Moscow that offers foreign students (undergraduate or graduate students specializing in mathematics and/or computer science) the opportunity to spend a semester in Moscow studying mathematics. All instruction is given in English. The fifteen-week program is similar to the Research Experiences for Undergraduates programs that are held each summer across the United States.

The AMS awards several scholarships for U.S. students to attend the Math in Moscow program. The scholarships are made possible through a grant from the National Science Foundation. For more information about Math in Moscow, consult <http://www.mccme.ru/mathinmoscow> and the article "Bringing Eastern European mathematical traditions to North American students", *Notices*, November 2003, pages 1250–4.

—Elaine Kehoe

From the AMS Public Awareness Office

AMS at the 2011 SACNAS National Conference. The AMS hosted an exhibit and a version of the *Who Wants to Be a Mathematician* game for undergraduate students at the 2011 SACNAS National Conference in San Jose, California, October 26–29, 2011. The conference had record-breaking attendance (approximately 4,000) and a record-breaking number of student poster presentations in the mathematical sciences; the game drew approximately 3,000. See a report by the Public Awareness Office on all the mathematics at the conference, as well as photographs and video clips, at <http://www.ams.org/meetings/sacnas2011-mtg>.

Math in the Media. Recent media coverage of mathematics and mathematicians includes reviews of the "Lost and Found: The Secrets of Archimedes" exhibition, the 2011 Nobel Prize for quasicrystals, mathematics of finance, and the Museum of Mathematics. View the most recent issue of Math in the Media and explore the archive of Tony Phillips's Take and Math Digest summaries at <http://www.ams.org/mathmedia/>.

This Mathematical Month: February. Read about events that occurred in the month of February, such as AMS President Thomas S. Fiske's presidential address, "Mathematical Progress in America", published in the February 1905 issue of the *Bulletin of the AMS*, and the birth of John W. Milnor in February 1931, at <http://www.ams.org/samplings/this-math-month/thismathmonth-feb>.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
paoffice@ams.org

AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions. The following is a list of those addresses together with a description of the types of inquiries that should be made through each address.

abs-coord@ams.org for questions regarding a particular abstract or abstracts questions in general.

acquisitions@ams.org to contact the AMS Acquisitions Department.

ams@ams.org to contact the Society's headquarters in Providence, Rhode Island.

amsdc@ams.org to contact the Society's office in Washington, D.C.

amsmem@ams.org to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.

ams-survey@ams.org for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.

bookstore@ams.org for inquiries related to the online AMS Bookstore.

classads@ams.org to submit classified advertising for the *Notices*.

cust-serv@ams.org for general information about AMS products (including electronic products), to send address changes, place credit card orders for AMS products, to correspond regarding a balance due shown on a monthly statement, or conduct any general correspondence with the Society's Customer Services Department.

development@ams.org for information about giving to the AMS, including the Epsilon Fund.

eims-info@ams.org to request information about Employment Information in the Mathematical Sciences (EIMS). For ad rates and to submit ads go to <http://www.eims.ams.org>.

emp-info@ams.org for information regarding AMS employment and career services.

eprod-support@ams.org for technical questions regarding AMS electronic products and services.

mathcal@ams.org to send information to be included in the "Mathematics Calendar" section of the *Notices*.

mathjobs@ams.org for questions about the online job application service for Mathjobs.org.

mathrev@ams.org to submit reviews to *Mathematical Reviews* and to send correspondence related to reviews or other editorial questions.

meet@ams.org to request general information about Society meetings and conferences.

mmsb@ams.org for information or questions about registration and housing for the Joint Mathematics Meetings (Mathematics Meetings Service Bureau).

msn-support@ams.org for technical questions regarding MathSciNet.

notices@ams.org to send correspondence to the managing editor of the *Notices*, including items for the news columns. The editor (notices@math.wustl.edu) is the person to whom to send articles and letters. Requests for permission to reprint from the *Notices* should be sent to reprint-permission@ams.org (see below).

notices-ads@ams.org to submit electronically paid display ads for the *Notices*.

notices-booklist@ams.org to submit suggestions for books to be included in the "Book List" in the *Notices*.

notices-letters@ams.org to submit letters and opinion pieces to the *Notices*.

notices-whatis@ams.org to comment on or send suggestions for topics for the "WHAT IS...?" column in the *Notices*.

paoffice@ams.org to contact the AMS Public Awareness Office.

president@ams.org to contact the president of the American Mathematical Society.

prof-serv@ams.org to send correspondence about AMS professional programs and services.

publications@ams.org to send correspondence to the AMS Publication Division.

pub-submit@ams.org to submit accepted electronic manuscripts to AMS publications (other than Abstracts). See <http://www.ams.org/submit-book-journal> to electronically submit accepted manuscripts to the AMS book and journal programs.

reprint-permission@ams.org to request permission to reprint material from Society publications.

sales@ams.org to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales Department.

secretary@ams.org to contact the secretary of the Society.

student-serv@ams.org for questions about AMS programs and services for students.

tech-support@ams.org to contact the Society's typesetting Technical Support Group.

textbooks@ams.org to request examination copies or inquire about using AMS publications as course texts.

webmaster@ams.org for general information or for assistance in accessing and using the AMS website.

Deaths of AMS Members

PAO-HSIEN FANG, of Belmont, Massachusetts, died on October 21, 2011. Born on August 18, 1923, he was a member of the Society for 1 year.

STOUGHTON BELL, professor, University of New Mexico, died on November 18, 2010. Born on December 20, 1923, he was a member of the Society for 56 years.

BORIS V. DEKSTER, of New Brunswick, Canada, died on September 21, 2008. Born on October 8, 1938, he was a member of the Society for 32 years.

EUGENE M. FRIEDMAN, of Los Angeles, California, died on July 13, 2011. Born on July 1, 1927, he was a member of the Society for 47 years.

DAVE FURTH, of the Netherlands, died on February 11, 2011. Born on February 11, 1941, he was a member of the Society for 31 years.

FRANK E. GERTH III, of Austin, Texas, died on May 23, 2006. Born on October 8, 1945, he was a member of the Society for 38 years.

HERBERT AARON HAUPTMAN, of Buffalo, New York, died on October 23, 2011. Born on February 14, 1917, he was a member of the Society for 56 years.

MICHAŁ KRYNICKI, professor, University of Warsaw, died on October 12, 2011. Born on February 3, 1950, he was a member of the Society for 22 years.

JOHN MCCARTHY, of Stanford, California, died on October 23, 2011. Born on September 4, 1927, he was a member of the Society for 61 years.

JAMES E. SIMPSON, of Corvallis, Oregon, died on April 2, 2011. Born on July 6, 1931, he was a member of the Society for 51 years.

G. RALPH STROHL, of Annapolis, Maryland, died on May 28, 2010. Born on October 19, 1919, he was a member of the Society for 62 years.

ROBERT H. SZCZARBA, professor, Yale University, died on October 18, 2011. Born on November 27, 1932, he was a member of the Society for 54 years.

ROBERT C. WHITTON, of Davidson, North Carolina, died on November 11, 2011. Born on February 1, 1944, he was a member of the Society for 42 years.

MAX A. WOODBURY, of Birmingham, Alabama, died on January 30, 2010. Born on April 30, 1917, he was a member of the Society for 68 years.

Reference and Book List

The *Reference* section of the *Notices* is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wustl.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

December 16, 2011: Applications for NDSEG Fellowships. See "Mathematics Opportunities" in this issue. For application forms see http://ndseg.asee.org/apply_online; for further information, see <http://ndseg.asee.org/>.

January 10, 2012: Full proposals for NSF Algorithms for Threat Detection program. See <http://www.nsf.gov/pubs/2012/nsf12502/nsf12502.htm>.

January 10, 2012: Applications for American Association of University Women (AAUW) Selected Professions Fellowships. See http://www.aauw.org/fga/fellowships_grants/selected.cfm; or contact the AAUW Fellowships and Grants, 101 ACT Drive, P. O. Box 4030, Iowa City, IA 52243-4030; 319-337-1716, ext. 60; aauw@act.org.

January 13, 2012: Applications for Jefferson Science Fellows (JSF) program. See <http://sites.nationalacademies.org/PGA/>

Jefferson/PGA_046612; or contact jfsf@nas.edu; 202-334-2643.

January 16, 2012: Applications for CRM-PISA Junior Visiting Program. See "Mathematics Opportunities" in this issue.

January 20, 2012: Applications for Math for America Fellowship Program. See "Mathematics Opportunities" in this issue.

January 23, 2012: Proposals for NSF Program in Computational and Data-Enabled Science and Engineering in Mathematical and Statistical Sciences. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=504687&WT.mc_id=USNSF_25&WT.mc_ev=click.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—November 2009, p. 1320

AMS Email Addresses—February 2012, p. 328

AMS Ethical Guidelines—June/July 2006, p. 701

AMS Officers 2010 and 2011 Updates—May 2011, p. 735

AMS Officers and Committee Members—October 2011, p. 1311

Conference Board of the Mathematical Sciences—September 2011, p. 1142

IMU Executive Committee—December 2011, p. 1606

Information for *Notices* Authors—June/July 2011, p. 845

Mathematics Research Institutes Contact Information—August 2011, p. 973

National Science Board—January 2011, p. 70

New Journals for 2008—June/July 2009, p. 751

NRC Board on Mathematical Sciences and Their Applications—March 2011, p. 482

NRC Mathematical Sciences Education Board—April 2011, p. 619

NSF Mathematical and Physical Sciences Advisory Committee—February 2011, p. 329

Program Officers for Federal Funding Agencies—October 2011, p. 1306 (DoD, DoE); December 2011, page 1606 (NSF Mathematics Education)

Program Officers for NSF Division of Mathematical Sciences—November 2011, p. 1472

January 26, 2012: Proposals for NSF Major Research Instrumentation Program. See <http://www.nsf.gov/pubs/2011/nsf11503/nsf11503.htm>.

January 31, 2012: Nominations for CAIMS/PIMS Early Career Award. See <http://www.pims.math.ca/pims-glance/prizes-awards>.

January 31, 2012: Entries for the Association for Women in Mathematics (AWM) essay contest. See <http://www.awm-math.org/biographies/contest.html>.

February 1, 2012: Applications for Clay Mathematics Institute Summer School. See <http://www.claymath.org/summerschool>.

February 1, 2012: Applications for AWM Travel Grants and Mentoring Travel Grants. See <http://www.awm-math.org/travelgrants.html#standard>; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; awm@awm-math.org.

February 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

February 9, 2012: Full proposals for NSF Undergraduate Biology and Mathematics (UBM) program. See "Mathematics Opportunities" in this issue.

February 12, 2012: Applications for IPAM summer program, Research in Industrial Projects for Students (RIPS). See www.ipam.ucla.edu.

February 15, 2012: Applications for AMS Congressional Fellowship. See <http://www.ams.org/programs/ams-fellowships/ams-aas/ams-aas-congressional-fellowship> or contact the AMS Washington Office at 202-588-1100; amsdc@ams.org.

February 27, 2012: Applications for EDGE for Women Summer Program. See "Mathematics Opportunities" in this issue.

March 1, 2012: Applications for Summer Program for Women in

Mathematics (SPWM). See "Mathematics Opportunities" in this issue.

April 9, 2012: Applications for Math for America San Diego site. See "Mathematics Opportunities" in this issue.

May 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

May 1, 2012: Applications for National Academies Christine Mirzayan Graduate Fellowship Program for fall 2012. See the website <http://sites.nationalacademies.org/PGA/policyfellows/index.htm> or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

May 1, 2012: Applications for AWM Travel Grants. See <http://www.awm-math.org/travelgrants.html#standard>; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; awm@awm-math.org.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click.

August 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

October 1, 2012: Applications for AWM Travel Grants. See <http://www.awm-math.org/travelgrants.html#standard>; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200,

Fairfax, VA 22030; 703-934-0163; awm@awm-math.org.

November 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900-1940, by Paolo Mancosu. Oxford University Press, January 2011. ISBN-13: 978-01995-465-34.

At Home with André and Simone Weil, by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN-13: 978-08101-270-43. (Reviewed May 2011.)

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55.

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed November 2011.)

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13: 978-18491-624-01.

The Black Swan: The Impact of the Highly Improbable, by Nassim Nicholas Taleb. Random House Trade Paperbacks, second edition, May 2010. ISBN-13: 978-08129-738-15. (First edition reviewed March 2011.)

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13: 978-06911-468-43.

The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse, by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN-13: 978-01431-173-77.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53. (Reviewed January 2012.)

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10.

The Clockwork Universe: Isaac Newton, the Royal Society, and the Birth of the Modern World, by Edward Dolnick. Harper, February 2011. ISBN-13: 978-00617-195-16. (Reviewed April 2011.)

Crafting by Concepts: Fiber Arts and Mathematics, by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

Divine Machines: Leibniz and the Sciences of Life, by Justin E. H. Smith. Princeton University Press, May 2011. ISBN-13: 978-06911-417-87.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

Fascinating Mathematical People: Interviews and Memoirs, edited by Donald J. Albers and Gerald L. Alexanderson. Princeton University Press,

October 2011. ISBN: 978-06911-482-98.

Gottfried Wilhelm Leibniz: The Polymath Who Brought Us Calculus, by M. B. W. Tent. AK Peters/CRC Press, October 2011. ISBN: 978-14398-922-20.

The Grand Design, by Stephen Hawking and Leonard Mlodinow. Bantam, September 2010. ISBN-13: 978-05538-053-76.

Hidden Harmonies (The Lives and Times of the Pythagorean Theorem), by Robert and Ellen Kaplan. Bloomsbury Press, January 2011. ISBN-13: 978-15969-152-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

Hot X: Algebra Exposed, by Danica McKellar. Hudson Street Press, August 2010. ISBN-13: 978-15946-307-05.

**The Information: A History, a Theory, a Flood*, by James Gleick. Pantheon, March 2011. ISBN-13: 978-03754-237-27.

Knots Unravelling: From String to Mathematics, by Meike Akveld and Andrew Jobbings. Arbelos, October 2011. ISBN: 978-09555-477-20.

Le Operazioni del Calcolo Logico, by Ernst Schröder. Original German version of Operationskreis des Logikkalküls and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN-13: 978-88-7916-474-0.

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life, by Reuben Hersh and Vera John-Steiner. Princeton University Press, January 2011. ISBN-13: 978-06911-424-70.

Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks, by Persi Diaconis and Ron Graham. Princeton University Press, November 2011. ISBN: 978-06911-516-49.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

Mathematics Education for a New Era: Video Games as a Medium for Learning, by Keith Devlin. A K Peters/CRC Press, February 2011. ISBN-13: 978-1-56881-431-5. (Reviewed December 2011.)

The Mathematics of Life, by Ian Stewart. Basic Books, June 2011. ISBN-13: 978-04650-223-80. (Reviewed in this issue.)

Mathematics, Religion and Ethics: An Epistemological Study, by Salilesh Mukhopadhyay. Feasible Solution LLC, September 2010. ISBN: 978-1-4507-3558-2.

Mysteries of the Equilateral Triangle, by Brian J. McCartin. Hikari, August 2010. ISBN-13: 978-954-91999-5-6. Electronic copies available for free at <http://www.m-hikari.com/mccartin-2.pdf>.

NIST Handbook of Mathematical Functions, Cambridge University Press, Edited by Frank W. J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Cambridge University Press, May 2010. ISBN-13: 978-05211-922-55 (hardback plus CD-ROM); ISBN-13: 978-05211-406-38 (paperback plus CD-ROM). (Reviewed September 2011.)

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN 978-0-19-958405-5. (Reviewed January 2012.)

One, Two, Three: Absolutely Elementary Mathematics [Hardcover] David Berlinski. Pantheon, May 2011. ISBN-13: 978-03754-233-38.

Origami Inspirations, by Meenakshi Mukerji. A K Peters, September 2010. ISBN-13: 978-1568815848.

The Perfect Swarm: The Science of Complexity in Everyday Life, by Len Fisher. Basic Books, March 2011 (paperback). ISBN-13: 978-04650-202-49.

Problem-Solving and Selected Topics in Number Theory in the Spirit of the Mathematical Olympiads, by Michael Th. Rassias. Springer, 2011. ISBN-13: 978-1-4419-0494-2.

The Proof is in the Pudding: A Look at the Changing Nature of Mathematical Proof, by Steven G. Krantz. Springer, May 2011. ISBN: 978-03874-890-87.

Proofiness: The Dark Arts of Mathematical Deception, by Charles Seife. Viking, September 2010. ISBN-13: 978-06700-221-68.

The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It, by Scott Patterson. Crown Business, January 2011. ISBN-13: 978-03074-533-89. (Reviewed May 2011.)

Roads to Infinity: The Mathematics of Truth and Proof, by John C. Stillwell. A K Peters/CRC Press, July 2010. ISBN-13: 978-15688-146-67.

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0-262-51429-3. (Reviewed August 2011.)

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90.

**Top Secret Rosies: The Female Computers of World War II*. Video documentary, produced and directed by LeAnn Erickson. September 2010. Website: <http://www.topsecretrosies.com>. (Reviewed in this issue.)

Towards a Philosophy of Real Mathematics, by David Corfield. Oxford University Press, April 2003. ISBN-13: 0-521-81722-6. (Reviewed November 2011.)

Train Your Brain: A Year's Worth of Puzzles, by George Grätzer. A K Peters/CRC Press, April 2011. ISBN-13: 978-15688-171-01.

Viewpoints: Mathematical Perspective and Fractal Geometry in Art, by Marc Frantz and Annalisa Crannell. Princeton University Press, August 2011. ISBN-13: 978-06911-259-23.

Visual Thinking in Mathematics, by Marcus Giaquinto. Oxford University Press, July 2011. ISBN-13: 978-01995-755-34.

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN: 978-0-691-13890-9. (Reviewed in this issue.)

Why Beliefs Matter: Reflections on the Nature of Science, by E. Brian Davies. Oxford University Press, June 2010. ISBN13: 978-01995-862-02.

Lecturer (Assistant Professor) in Pure Mathematics

£33,734 - £44,016

We seek an outstanding candidate for a Lectureship in Pure Mathematics. You will join the Pure Mathematics Group within the School of Mathematics at the University of Bristol and will be expected to demonstrate outstanding research potential in an area within pure mathematics.

The Pure Mathematics Group at the University of Bristol is internationally recognised with interests spanning pure mathematics, including number theory, arithmetic geometry, arithmetic combinatorics, ergodic theory, random matrix theory, representation theory, complex analysis, partial differential equations, set theory and logic. The Group has strong links with the Heilbronn Institute for Mathematical Research, which focuses on discrete mathematics and funds about 20 postdoctoral fellows in the School of Mathematics.

We seek applicants whose research further broadens and expands upon the strengths of the existing group. We particularly welcome suitably qualified female applicants, as women are currently under-represented in the School of Mathematics. You should arrange for three referees to send letters of recommendation to Joanne Malcolm at recruit-spr@bristol.ac.uk by the closing date. Please note that it is your responsibility to ensure that the reference letters are received by the closing date. Please also note that your referees do not have to include your current or most recent Head of Department or Line Manager as stated on the application form.

For informal enquiries please contact Dr Lynne Walling on +44 (0) 117 331 5245 or e-mail l.walling@bristol.ac.uk

For further details and an application form please visit www.bristol.ac.uk/jobs Alternatively you can e-mail recruitment@bristol.ac.uk or telephone +44 (0) 117 954 6947, quoting the reference number 16800.

The closing date for applications is 9.00am, 20 February 2012.

Interviews are expected to be held during March 2012.

EXCELLENCE THROUGH DIVERSITY



University of
BRISTOL

2011 Election Results

In the elections of 2011 the Society elected a president, a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee.

President

Elected as the new president is **David Vogan** from the Massachusetts Institute of Technology. Term is one year as president elect (1 February 2012—31 January 2013), two years as president (1 February 2013—31 January 2015), and one year as immediate past president (1 February 2015—31 January 2016).

Vice President

Elected as the new vice president is **Andrew M. Odlyzko** from the University of Minnesota. Term is three years (1 February 2012—31 January 2015).

Trustee

Elected as trustee is **Ruth Charney** from Brandeis University. Term is five years (1 February 2012—31 January 2017).

Members at Large of the Council

Elected as new members at large of the Council are:

Dan Abramovich from Brown University
Hélène Barcelo from Arizona State University
Arthur T. Benjamin from Harvey Mudd College
James Carlson from the Clay Mathematics Institute
Victoria Powers from Emory University
Terms are three years (1 February 2012—31 January 2015).

Nominating Committee

Elected as new members of the Nominating Committee are:

Frederick R. Cohen from the University of Rochester
Susan Friedlander from the University of Southern California
Fan Chung Graham from the University of California, San Diego

Terms are three years (1 January 2012—31 December 2014).

Editorial Boards Committee

Elected as new members of the Editorial Boards Committee are:

Ralph Greenberg from the University of Washington
Dana Randall from the Georgia Institute of Technology
Terms are three years (1 February 2012—31 January 2015).

Proposal for a Fellows Program of the AMS

The proposal was approved.

2012 AMS Election

Nominations by Petition

Vice President or Member at Large

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2012. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations. The Council of 23 January 1979 stated the intent of the Council of nominating all persons on whose behalf there were valid petitions.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and operational considerations, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice president, member at large, or member of the Nominating and Editorial Boards Committees.

1. To be considered, petitions must be addressed to Robert J. Daverman, Secretary, American Mathematical Society, Department of Mathematics, 1403 Circle Drive, University of Tennessee, 1534 Cumberland Avenue, Knoxville, TN 37996-1320, USA, and must arrive by 24 February 2012.
2. The name of the candidate must be given as it appears in the *Combined Membership List* (www.ams.org/cm1). If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate's mailing label or by the candidate contacting the AMS headquarters in Providence (amsmem@ams.org).
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the *Combined Membership List* and the mailing lists. No attempt will be made to match variants of names with the form of name in the *CML*. A name neither in the *CML* nor on the mailing lists is not that of a member. (Example: The name Robert J. Daverman is that of a member. The name R. Daverman appears not to be.)
7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

Nomination Petition

for 2012 Election

The undersigned members of the American Mathematical Society propose the name of

as a candidate for the position of (check one):

- ☐ **Vice President**
- ☐ **Member at Large of the Council**
- ☐ **Member of the Nominating Committee**
- ☐ **Member of the Editorial Boards Committee**

of the American Mathematical Society for a term beginning 1 February, 2013

Return petitions by 24 February 2012 to:

Secretary, AMS, Department of Mathematics, 1403 Circle Drive, University of Tennessee, Knoxville, TN 37996-1320 USA

Name and address (printed or typed)

	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at <http://www.ams.org/cgi-bin/mathcal-submit.pl>.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at <http://www.ams.org/mathcal/>.

March 2012

* 1–3 **Trends in Mathematical Analysis**, Dipartimento di Matematica “F.Brioschi” Politecnico di Milano, Milan, Italy.

Description: Trends in Mathematical Analysis is the first of a series of meetings organized by equadif@polimi.it, the recently founded research team in differential equations of Politecnico di Milano. The main aim is to put young researchers in contact with leading experts in the fields of calculus of variations and PDEs. This first workshop will cover a wide range of important and challenging topics, such as geometric measure theory, dynamical systems, local and non local optimization, nonlinear phenomena.

Information: <http://www.mate.polimi.it/TMA2012/>.

* 13–16 **Computational Methods in High Energy Density Plasmas: Tutorials**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: “Computational Methods in High Energy Density Plasmas” opens with four days of tutorials that will provide an introduction to major themes of the entire program and the four workshops.

Goal: To build a foundation for the participants of this program who have diverse scientific backgrounds that range from physics to mathematics and computational sciences.

Registration: A registration form and application for funding is available online. Registration for tutorials is free, to encourage broad participation.

Information: <http://www.ipam.ucla.edu/programs/plutut/>.

* 22–24 **46th Annual Spring Topology and Dynamics Conference**, Universidad Nacional Autonoma de Mexico, Mexico City, Mexico.

Description: The Spring Topology and Dynamics Conference 2012 is being organized by the Universidad Nacional Autonoma de Mexico and by Rhodes College and will be held in the Centro Historico in Mexico City, March 22–24, 2012. It will feature six plenary and 12 semi-plenary speakers, as well as special sessions in continuum theory, dynamical systems, general/set theoretic topology, geometric group theory/geometric topology and low dimensional topology.

Information: <http://www.matem.unam.mx/STDC2012/>.

* 26–30 **Computational Challenges in Hot Dense Plasmas**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: The goals of this workshop are to identify key physics components in integrated simulations of hot, dense plasmas; to identify critical computation issues at the different length and time scales; to propose reduced model descriptions that can be experimentally validated; to define experiments critical to an integrated simulation of hot, dense plasmas; to promote new collaborations, and to engage young scientists.

Register/Deadline: The application form is for those requesting financial support to attend the workshop. Applications received by Monday, January 30, 2012 will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM’s mission and we welcome

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences

in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence eight months prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: <http://www.ams.org/>.

their applications. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/plws1/>.

April 2012

- * 4–6 **Advances in Scientific Computing, Imaging Science and Optimization: Stan Osher's 70th Birthday Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Description: Efficient algorithms for scientific computing, imaging science and optimization have developed rapidly over the past few decades. Stan Osher has been at the forefront of this progress with ground-breaking contributions on high resolution shock capturing schemes, the level set method, applications to multi-phase flows, computer vision, TV (total variation) based image restoration and optimization. In honor of Stan's 70th birthday, this will be a forward looking conference covering recent progress and new directions in several important aspects of scientific computing, imaging science and optimization. It will emphasize the crucial role of significant mathematics in the design of advanced algorithms and methodologies applicable to real world applications.

Deadline/Registration: Applications received by Wednesday, February 08, 2012 will receive fullest consideration. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/sjo2012/>.

- * 16–17 **2nd International Conference on E-Learning & Knowledge Management Technology (ICEKMT 2012)**, Hotel Corus, Kuala Lumpur, Malaysia.

Description: 2nd ICEKMT aims at bringing together researchers and practitioners who are interested in e-Learning Technology and Knowledge Management Technology and its current applications.

Information: <http://www.icekm.com>.

- * 16–20 **Computational Challenges in Magnetized Plasma**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: The workshop will focus on accurate, effective simulation of critical science elements that elucidate grand scientific challenges in magnetized plasmas.

Goal: The goal of the workshop is to identify promising approaches for achieving solution of grand challenge problems in magnetized laboratory and astrophysical/space plasmas and also cross-cutting areas with HEDP in both science, mathematics, and computing, and for defining related validation requirements that are needed to test the simulation predictions.

Applications: Applications are due Monday, February 20, 2012. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/plws2/>.

- * 18–21 **Fourth Discrete Geometry and Algebraic Combinatorics Conference**, South Padre Island, Texas.

Description: The event is partially supported by the National Security Agency. This conference is a unique opportunity for Geometry and Combinatorics researchers to meet, share their specialized knowledge and learn from others.

Organizer: Department of Mathematics, University of Texas at Brownsville (UTB).

Information: <http://blue.utb.edu/dg2012/>.

- * 19–21 **XIV International Scientific Conference Devoted to the memory of Academician M. Kravchuk (1892–1942)**, National Technical University of Ukraine, KPI, Institute of Mathematics of NASU, National Shevchenko University, National Drahomanov Pedagogical University Kyiv, Ukraine.

Description: The Programme of the Conference includes the following 4 sections: 1. Differential and integral equations, its applications. 2. Algebra, geometry. Mathematical and numerical analysis. 3. Theory of probability and mathematical statistics. 4. History, methods of teaching of Mathematics.

Opening ceremony: April 19 at 2:30 p.m.

Organizing Committee: Cordially invites you to participate in the work of the Conference.

Thesis: The Thesis (1 page) can be submitted by February 1, 2012 at the address: kravchukconf@yandex.ru, only pdf format. The abstracts have a left margin of 3cm and right margin of 1sm. Title, Authors name, Author's affiliation (Department, University, address, e-mail).

Registration fee: US\$50.00 (you can pay, when you arrive to Kyiv). All registration fees include a book of abstracts, daily coffee breaks and excursion.

- * 27–28 **International Conference on Web Information System and Computing Education**, Bangkok, Thailand.

Description: ICWISCE 2012 focuses on real world applications; therefore authors should highlight the benefits of Web Information Systems and Computing Education for industry and services, in addition to academic applications. Ideas on how to solve business problems, using web based information systems and technologies, will arise from the conference.

Information: <http://www.icwisce.com>.

May 2012

- * 7–11 **Mathematical and Computer Science Approaches to High Energy Density Physics**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: This workshop is intended to be a cross-cutting session that will focus on the fundamental modeling challenges that arise when simulating high energy density plasmas. We will consider methods for the simulation of the governing integro-differential equations that scale with problem size and are suitable for high-performance computing, including exa-scale and peta-scale platforms. One goal of the workshop is to make the modeling challenges accessible to applied mathematicians and computational scientists with little or no prior experience in plasma physics.

Deadline/Registration: Applications received by Monday, March 12, 2012 will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/plws3/>.

- * 10–12 **International Conference on Functional Equations and Geometric Functions and Applications (ICFGA 2012)**, Department of Mathematics, Payame Noor University, Tabriz, Iran.

Description: We have the pleasure to invite all researchers and scientists to participate in an international conference under the name of (ICFGA) "International Conference on Functional Equations and Geometric Functions and Applications".

Scope: The scope of the conference covers, Mathematical analysis, functional analysis, operator theory, numerical analysis, ODE, PDE, general functional equations and its applications.

Aim: To bring together researchers and scientists working in the desired fields. After the conference, some selected papers will be published in International Journal of Nonlinear Analysis and Applications (IJNAA, <http://www.ijnaa.com>).

Information: <http://www.icfga2012pnu.ir/>.

- * 21–25 **Computational Challenges in Warm Dense Matter**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: In this workshop, we will explore established and promising approaches to the modeling of WDM, foundational issues concerning the correct theoretical description of WDM, and the challenging practical issues of numerically modeling strongly coupled systems with many degrees of freedom. The list of computational frameworks and methods to be discussed includes: Finite-temperature density functional theory, including orbital-free methods; quantum Monte Carlo, including path integral Monte Carlo; wave-packet molecular dynamics; and classical and semi-classical molecular dynamics.

Deadline/Registration: Applications received by Monday, March 26, 2012 will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/plws4/>.

- * 29–June 2 **The 2012 NSF-CBMS Conference on Mathematical Methods of Computed Tomography**, Department of Mathematics, University of Texas at Arlington, Arlington, Texas.

Description: Sponsored by the National Science Foundation. This conference will feature Distinguished Professor Peter Kuchment of Texas A&M University, who will deliver ten lectures on the mathematics of tomographic imaging. Established researchers, postdoctoral fellows, and graduate students are all welcome to attend.

Support: Financial support for about 25–30 participants will be provided. The noncommuting participants are expected to arrive on May 28, 2012 and depart in the afternoon of June 2, 2012 (a flight departure of 2:00 pm or later). To apply, please fill out the online application form at the conference web page.

Contacts: The contact persons for this NSF-CBMS conference are Professor Tuncay Aktosun; email: aktosun@uta.edu, Professor Gaik Ambartsoumian; email: gambarts@uta.edu, and Professor Juliann Chung; email: chungj@uta.edu.

Information: <http://omega.uta.edu/~aktosun/cbms2012>.

June 2012

- * 8–10 **The International Conference on the Frontier of Computational and Applied Mathematics: Tony Chan's 60th Birthday Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: The aim of this conference is to provide a forum to discuss recent developments and future directions in applied and computational mathematics. It will also provide an opportunity to get leading experts as well as junior researchers together to exchange and stimulate new ideas from a wide spectrum of disciplines. The program of the conference includes invited talks, panel discussion and poster session.

Theme: Will include theory, numerics and applications with a focus on the following topics: 1. Fundamental theory and numerical analysis. 2. Numerics: Multiscale and stochastic methods, optimizations, and high performance computing. 3. Applications: compressed sensing, imaging, optimal engineering design and networking.

Deadline/Registration: Applications received by Friday, April 13, 2012 will receive fullest consideration. If you do not need or want to apply for funding, you may simply register.

Information: <http://www.ipam.ucla.edu/programs/chan2012/>.

- * 11–15 **Arrangements in Pyrenees: School and Workshop on “Hyperplane arrangements and related topics”**, Université de Pau et des Pays de l'Adour, Pau, France.

Description: The school's lectures will be mainly aimed at Ph. D. students and young researchers, and should be of interest for all researchers in this area of mathematics, as it should cover many of the combinatorial, topological, algebraic and geometric aspects of the topic. Participants are encouraged to propose contributed talks

of 30 minutes. The workshop will include these and a small number of invited talks on recent advances in the field.

Information: <http://lma.univ-pau.fr/equipes/algebre-geometrie/arrangements/>.

- * 18–August 10 **SummerICERM: Geometry and Dynamics**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island.

Description: The SummerICERM program is designed for a select group of 10–12 undergraduate scholars. Students will work in small groups of two or three, supervised by a faculty advisor and aided by a teaching assistant. The faculty advisors will describe a variety of enticing open questions in geometry and in dynamical systems of geometric origin. Topics discussed will include Euclidean, hyperbolic and projective geometry, iteration of geometric constructions, and mathematical billiards. A variety of activities around these research themes will allow participants to engage in collaborative research, communicate and examine their findings in formal and informal settings, and report-out their findings with a finished product.

Applications: Will open on Mathprograms.org on December 1, 2011. All application materials are due by February 10, 2012.

Information: <http://icerm.brown.edu/summerug>.

- * 19–24 **FRG Workshop and Conference on Characters, Liftings, and Types**, American University, Washington, District of Columbia.

Description: This conference centers on three main themes in the representation theory of p -adic groups: characters, liftings, and types. The first portion (from June 19 to June 22) will be a workshop aimed at presenting selected topics from these themes to new researchers and graduate students. The second portion (from June 22 to June 24) will be a research conference in which experts will present important new advances in these areas.

Information: <http://www.american.edu/cas/FRG2012/>.

- * 25–28 **5th Podlasie Conference on Mathematics**, Białystok University of Technology, Białystok, Poland.

Description: The conference is organized by the Białystok Branch of the Polish Mathematical Society. It will consist of Plenary Session and special sessions. Special sessions will be organized in two categories: Four Main Special Sessions: Algebra, control theory and dynamical systems, mathematical foundations of computer science, and applications of mathematics in economy and Finance, which reflect the main research areas of mathematicians belonging to the Białystok Branch of the Polish Mathematical Society; Contributed Special Sessions, devoted to any topic related to mathematics or its applications. The conference has a status of a satellite conference of the 6th European Congress of Mathematics to be held in Kraków in July 2012. Białystok is the capital of the Podlasie province, which, besides unspoiled nature in four National Parks, offers rich culture with ethnic and religious diversity.

Language: All sessions will be in English.

Information: <http://katmat.pb.bialystok.pl/pcm12/>.

July 2012

- * 1–16 **Special Case for Interpolation by Using Bézier Curve Numerically**, Al-Muthana University, College of Science, Department of Mathematics & Computer Applications, Muthana, Iraq.

Description: To study The main aim of this thesis is to find new method to interpolation by using Bézier method numerically. The problem studied here is the way of determine the control points. There are many studies in this field, the researchers are state the leading methods and formulate programmed examples to show the details of these studies. Compare this method with known methods by test examples and find the merits and drawbacks of each study.

Information: email: mustaffa8@hotmail.com.

- * 9–27 **Graduate Summer School: Deep Learning, Feature Learning**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Description: One of the challenges for machine learning, AI, and computational neuroscience is the problem of learning representations of the perceptual world. This summer school will review recent developments in feature learning and learning representations, with a particular emphasis on “deep learning” methods, which can learn multi-layer hierarchies of representations.

Topics: Will include unsupervised learning methods such as stacked restricted Boltzmann machines, sparse coding, denoising auto-encoders, and methods for learning over-complete representations; supervised methods for deep architectures, metric learning criteria for vector-space embeddings; deep convolutional architectures and their applications to images, video, audio, and text; compositional hierarchies and latent-variable models. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM’s mission and we welcome their applications.

Deadline: Applications are due March 15, 2012.

Information: <http://www.ipam.ucla.edu/programs/gss2012/>.

* 10–14 **3Quantum: Algebra, Geometry, Information**, Tallinn University of Technology, Tallinn, Estonia.

Description: The main purpose of this conference is to stimulate and promote interactions between three major research areas: (quantum) algebra, (quantum) geometry, and (quantum) information theory.

Information: <http://www.agmp.eu/qqq/>.

August 2012

* 27–31 **8th International Symposium on Geometric Function Theory and Applications**, Ohrid, Republic of Macedonia.

Description: Geometric Function Theory and Applications (GFTA) is a continuation of the series of the other annual international GFTA symposia: Canada (2005), Romania (2006), Turkey (2007), Malaysia (2008), Romania (2009), Bulgaria (2010) and Romania (2011).

Main topics: Univalent and multivalent functions; special functions and series, differential subordinations, conformal and quasiconformal mappings, geometric function theory in several complex variables, potential analysis and applications and other areas related to GFTA.

Dedication: The conference is dedicated to the retirement of Professor Shigeyoshi Owa.

Information: <http://www.uab.ro/gfta-2012/>.

September 2012

* 10–12 **3rd IMA Conference on Numerical Linear Algebra and Optimisation**, University of Birmingham, United Kingdom.

Scope: The success of modern codes for large-scale optimisation is heavily dependent on the use of effective tools of numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimisation problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest.

Topics: Include any subject that could be of interest to both communities, such as: Direct and iterative methods for large sparse linear systems, eigenvalue computation and optimisation, large-scale nonlinear and semidefinite programming, effect of round-off errors, stopping criteria, embedded iterative procedures, optimisation issues for matrix polynomials, fast matrix computations, compressed/sparse sensing, PDE-constrained optimisation, applications and real time optimisation.

Information: http://www.ima.org.uk/conferences/conferences_calendar/numerical_linear_algebra_and_optimisation.cfm.

October 2012

* 1–April 30 **Semester on Curves, Codes, Cryptography**, Sabanci University, Istanbul, Turkey.

Description: A series of events (workshops, conferences, visits, talks) are organized at Sabanci University with the purpose of providing a platform to present state-of-the-art results as well as to discuss open problems on various topics including: Algebraic curves over finite fields (towers of function fields, modular towers, maximal curves), coding theory (list decoding, polar codes, algebraic coding), sequences over finite fields, pseudo random sequences (measures of randomness, generation of pseudorandom numbers, complexity), functions over finite fields (polynomials, permutations, polynomial systems, rational functions), exponential sums, cryptographically significant functions.

Supporter: This semester is supported by Sabanci University and TUBITAK.

Information: <http://sccc.sabanciuniv.edu>.

November 2012

* 9–10 **Blackwell Tapia Conference 2012**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island.

Description: This is the seventh in a series of biannual conferences honoring David Blackwell and Richard Tapia, two seminal figures who inspired a generation of African-American, Native American and Latino/Latina students to pursue careers in mathematics. Carrying forward their work, this one and a half day conference will: Recognize and showcase mathematical excellence by minority researchers, recognize and disseminate successful efforts to address under-representation, inform students and mathematicians about career opportunities in mathematics, especially outside academia, provide networking opportunities for mathematical researchers at all points in the higher education/career trajectory. The conference will include a mix of activities including scientific talks; poster presentations; panel discussions; and ample opportunities for discussion and interaction.

Information: <http://icerm.brown.edu/blackwell-tapia-2012>.

December 2012

* 10–14 **36ACCMCC—The 36th Australasian Conference on Combinatorial Mathematics and Combinatorial Computing**, University of New South Wales, Sydney, Australia.

Description: Annual conference of the Combinatorial Mathematics Society of Australia.

Information: <http://conferences.science.unsw.edu.au/36accmcc/>.

* 17–21 **International Conference on The Theory, Methods and Applications of Nonlinear Equations**, Department of Mathematics, Texas A&M University-Kingsville, Kingsville, Texas.

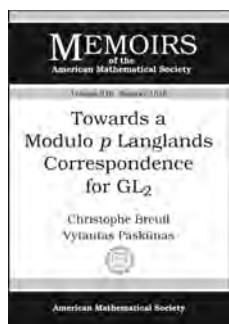
Description: The main aim of the conference is to promote, encourage, and bring together researchers in the fields of ordinary differential equations, partial differential equations, functional equations, fractional differential equations, Delay-differential equations, integral equations, integrodifferential equations, stochastic differential equations, impulsive differential equations, operator equations, difference equations and dynamic equations on time scales. A special emphasis will be on the applications of Nonlinear Equations. It is anticipated that the conference will attract about 400 participants with 9 plenary speakers (50 minutes), 50 main speakers (40 minutes), and 300 lectures (20 minutes). It will be a mathematically enriching and socially exciting event.

Information: http://www.tamuk.edu/artsci/math/conference_2012/about.html.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications,
please go to <http://www.ams.org/bookstore-email1>.

Algebra and Algebraic Geometry



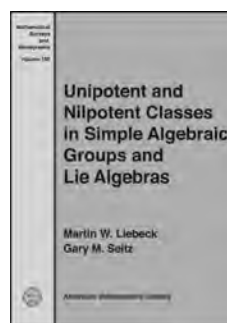
Towards a Modulo p Langlands Correspondence for GL_2

Christophe Breuil, *CNRS, Bures-sur-Yvette, France, and IHES, Bures-sur-Yvette, France,* and **Vytautas Paškūnas**, *Universität Bielefeld, Germany*

Contents: Introduction; Representation theory of Γ over $\mathbb{F}_p I$; Representation theory of Γ over $\mathbb{F}_p II$; Representation theory of Γ over $\mathbb{F}_p III$; Results on K -extensions; Hecke algebra; Computation of $\mathbb{R}^1 I$ for principal series; Extensions of principal series; General theory of diagrams and representations of GL_2 ; Examples of diagrams; Generic Diamond weights; The unicity lemma; Generic Diamond diagrams; The representations $D_0(\rho)$ and $D_1(\rho)$; Decomposition of generic Diamond diagrams; Generic Diamond diagrams for $f \in \{1, 2\}$; The representation $R(\sigma)$; The extension Lemma; Generic Diamond diagrams and representations of GL_2 ; The case $F = \mathbb{Q}_p$; References.

Memoirs of the American Mathematical Society, Volume 216, Number 1016

March 2012, 114 pages, Softcover, ISBN: 978-0-8218-5227-9, 2010 *Mathematics Subject Classification*: 22E50, 11F80, 11F70, **Individual member US\$42**, List US\$70, Institutional member US\$56, Order code MEMO/216/1016



Unipotent and Nilpotent Classes in Simple Algebraic Groups and Lie Algebras

Martin W. Liebeck, *Imperial College of London, United Kingdom,* and **Gary M. Seitz**, *University of Oregon, Eugene, OR*

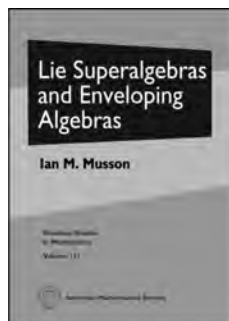
This book concerns the theory of unipotent elements in simple algebraic groups over algebraically closed or finite fields, and nilpotent elements in the corresponding simple Lie algebras. These topics have been an important area of study for decades, with applications to representation theory, character theory, the subgroup structure of algebraic groups and finite groups, and the classification of the finite simple groups.

The main focus is on obtaining full information on class representatives and centralizers of unipotent and nilpotent elements. Although there is a substantial literature on this topic, this book is the first single source where such information is presented completely in all characteristics. In addition, many of the results are new—for example, those concerning centralizers of nilpotent elements in small characteristics. Indeed, the whole approach, while using some ideas from the literature, is novel, and yields many new general and specific facts concerning the structure and embeddings of centralizers.

Contents: Introduction; Preliminaries; Classical groups in good characteristic; Classical groups in bad characteristic: Statement of results; Nilpotent elements: The symplectic and orthogonal cases, $p = 2$; Unipotent elements in symplectic and orthogonal groups, $p = 2$; Finite classical groups; Tables of examples in low dimensions; Exceptional groups: Statement of results for nilpotent elements; Parabolic subgroups and labellings; Reductive subgroups; Annihilator spaces of nilpotent elements; Standard distinguished nilpotent elements; Exceptional distinguished nilpotent elements; Nilpotent classes and centralizers in E_8 ; Nilpotent elements in the other exceptional types; Exceptional groups: Statement of results for unipotent elements; Corresponding unipotent and nilpotent elements; Distinguished unipotent elements; Non-distinguished unipotent classes; Proofs of theorems 1, 2 and corollaries 3–8; Tables of nilpotent and unipotent classes in the exceptional groups; Bibliography; Glossary of symbols; Index.

Mathematical Surveys and Monographs, Volume 180

February 2012, 380 pages, Hardcover, ISBN: 978-0-8218-6920-8, LC 2011043518, 2010 *Mathematics Subject Classification*: 20G15, 20G40, 20G41, 20E45, 20E32, 17B45, 17B08, **AMS members US\$76.80**, List US\$96, Order code SURV/180



Lie Superalgebras and Enveloping Algebras

Ian M. Musson, *University of Wisconsin, Milwaukee, WI*

Lie superalgebras are a natural generalization of Lie algebras, having applications in geometry, number theory, gauge field theory, and string theory. This book develops the theory of Lie superalgebras, their enveloping algebras, and their representations.

The book begins with five chapters on the basic properties of Lie superalgebras, including explicit constructions for all the classical simple Lie superalgebras. Borel subalgebras, which are more subtle in this setting, are studied and described. Contragredient Lie superalgebras are introduced, allowing a unified approach to several results, in particular to the existence of an invariant bilinear form on $\mathfrak{g}$.

The enveloping algebra of a finite dimensional Lie superalgebra is studied as an extension of the enveloping algebra of the even part of the superalgebra. By developing general methods for studying such extensions, important information on the algebraic structure is obtained, particularly with regard to primitive ideals. Fundamental results, such as the Poincaré-Birkhoff-Witt Theorem, are established.

Representations of Lie superalgebras provide valuable tools for understanding the algebras themselves, as well as being of primary interest in applications to other fields. Two important classes of representations are the Verma modules and the finite dimensional representations. The fundamental results here include the Jantzen filtration, the Harish-Chandra homomorphism, the Šapovalov determinant, supersymmetric polynomials, and Schur-Weyl duality. Using these tools, the center can be explicitly described in the general linear and orthosymplectic cases.

In an effort to make the presentation as self-contained as possible, some background material is included on Lie theory, ring theory, Hopf algebras, and combinatorics.

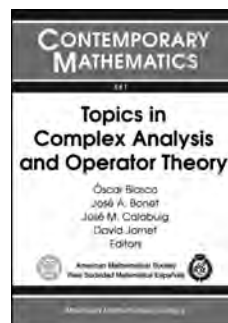
This item will also be of interest to those working in mathematical physics.

Contents: Introduction; The classical simple Lie superalgebras. I; Borel subalgebras and Dynkin-Kac diagrams; The classical simple Lie superalgebras. II; Contragredient Lie superalgebras; The PBW Theorem and filtrations on enveloping algebras; Methods from ring theory; Enveloping algebras of classical simple Lie superalgebras; Verma modules. I; Verma modules. II; Schur-Weyl duality; Supersymmetric polynomials; The center and related topics; Finite dimensional representations of classical Lie superalgebras; Prime and primitive ideals in enveloping algebras; Cohomology of Lie superalgebras; Zero divisors in enveloping algebras; Affine Lie superalgebras and number theory; Appendix A; Appendix B; Bibliography; Index.

Graduate Studies in Mathematics, Volume 131

April 2012, approximately 500 pages, Hardcover, ISBN: 978-0-8218-6867-6, 2010 *Mathematics Subject Classification*: 17B35; 17B10, 17B20, 17B22, 17B25, 17B30, 17B40, 17B55, 17B56, 16S30, **AMS members US\$69.60**, List US\$87, Order code GSM/131

Analysis



Topics in Complex Analysis and Operator Theory

Óscar Blasco, *Universidad de Valencia, Spain*, **José A. Bonet**, *Universidad Politénica de Valencia, Spain*, and **José M. Calabuig and David Jornet**, *Universidad Politénica de Valencia, Spain*, Editors

This book contains the lecture notes as well as some invited papers presented at the Third Winter School in Complex Analysis, Operator Theory and Applications held February 2–5, 2010, in Valencia, Spain.

The book is divided into two parts. The first is an extended self-contained version of the mini-courses taught at the School. The papers in this first part are: *Notes on real analytic functions and classical operators*, by Paweł Domański; *Shining a Hilbertian lamp on the bidisk*, by John E. McCarthy; *Selected problems in perturbation theory*, by Vladimir V. Peller; and *Composition operators on Hardy-Orlicz spaces*, by Luis Rodríguez-Piazza.

The second part consists of several research papers on recent advances in the area and some survey articles of an expository character. The articles in this second part are: *Remarks on weighted mixed norm spaces*, by O. Blasco; *Interpolation subspaces of L^1 of a vector measure and norm inequalities for the integration operator*, by J. M. Calabuig, J. Rodríguez, and E.A. Sánchez-Pérez; *On the spectra of algebras of analytic functions*, by D. Carando, D. García, M. Maestre, and P. Sevilla-Peris; *Holomorphic self-maps of the disk intertwining two linear fractional maps*, by M.D. Contreras, S. Díaz-Madrigal, M.J. Martín, and D. Vukotić; *ABC-type estimates via Garsia-type norms*, by K.M. Dyakonov; and *Volterra type operators on Bergman spaces with exponential weights*, by J. Pau and J. A. Peláez.

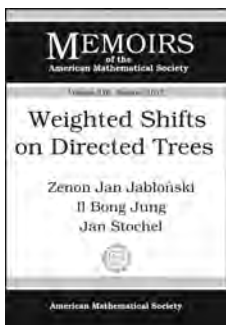
The topics selected for the mini-courses cover several aspects of complex analysis and operator theory that play important roles in understanding connections between different areas that are considered in fashion these days. This part is aimed at graduate students and young researchers. The courses are self-contained, focusing on those aspects that are basic and that can lead the readers to a quick understanding of the theories presented in each topic. They start with the classical results and reach a selection of open problems in each case. The research and survey articles are aimed at young researchers in the area, as well as post-doc and senior researchers interested in complex analysis and operator theory.

Contents: *Mini-courses:* **P. Domański**, Notes on real analytic functions and classical operators; **J. E. McCarthy**, Shining a Hilbertian lamp on the bidisk; **V. V. Peller**, Selected problems in

perturbation theory; **L. Rodríguez-Piazza**, Composition operators on Hardy-Orlicz spaces; *Articles*: **O. Blasco**, Remarks on weighted mixed norm spaces; **J. M. Calabuig, J. Rodríguez, and E. A. Sánchez-Pérez**, Interpolation subspaces of L^1 of a vector measure and norm inequalities for the integration operator; **D. Carando, D. García, M. Maestre, and P. Sevilla-Peris**, On the spectra of algebras of analytic functions; **M. D. Contreras, S. Díaz-Madriral, M. J. Martín, and D. Vukotić**, Holomorphic self-maps of the disk intertwining two linear fractional maps; **K. M. Dyakonov**, *ABC*-type estimates via Garsia-type norms; **J. Pau and J. A. Peláez**, Volterra type operators on Bergman spaces with exponential weights.

Contemporary Mathematics, Volume 561

February 2012, approximately 256 pages, Softcover, ISBN: 978-0-8218-5275-0, 2010 *Mathematics Subject Classification*: 30Cxx, 30Hxx, 30Jxx, 47Bxx, **AMS members US\$71.20**, List US\$89, Order code CONM/561



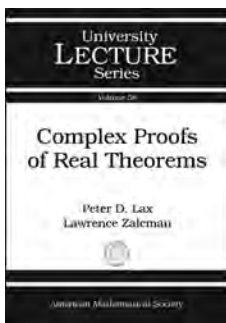
Weighted Shifts on Directed Trees

Zenon Jan Jabłoński, *Uniwersytet Jagielloński, Krakow, Poland*, **Il Bong Jung**, *Kyungpook National University, Daegu, South Korea*, and **Jan Stochel**, *Uniwersytet Jagielloński, Krakow, Poland*

Contents: Introduction; Prerequisites; Fundamental properties; Inclusions of domains; Hyponormality and cohyponormality; Subnormality; Complete hyperexpansivity; Miscellanea; Bibliography; List of symbols.

Memoirs of the American Mathematical Society, Volume 216, Number 1017

March 2012, 107 pages, Softcover, ISBN: 978-0-8218-6868-3, 2010 *Mathematics Subject Classification*: 47B37, 47B20; 47A05, 44A60, **Individual member US\$42**, List US\$70, Institutional member US\$56, Order code MEMO/216/1017



Complex Proofs of Real Theorems

Peter D. Lax, *Courant Institute, New York, NY*, and **Lawrence Zalcman**, *Bar-Ilan University, Ramat Gan, Israel*

Complex Proofs of Real Theorems is an extended meditation on Hadamard's famous dictum, "The shortest and best way between two truths of the real domain

often passes through the imaginary one." Directed at an audience acquainted with analysis at the first year graduate level, it aims at illustrating how complex variables can be used to provide quick and efficient proofs of a wide variety of important results in such areas of analysis as approximation theory, operator theory, harmonic analysis, and complex dynamics.

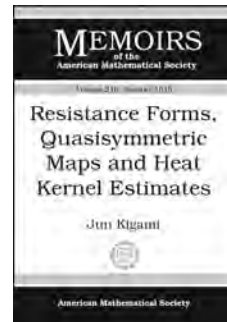
Topics discussed include weighted approximation on the line, Müntz's theorem, Toeplitz operators, Beurling's theorem on the invariant spaces of the shift operator, prediction theory, the Riesz convexity theorem, the Paley-Wiener theorem, the Titchmarsh convolution theorem, the Gleason-Kahane-Żelazko theorem, and the Fatou-Julia-Baker theorem. The discussion begins with the world's shortest proof of the fundamental theorem of algebra and concludes with Newman's almost effortless proof of the prime number theorem. Four brief appendices provide all necessary background in complex analysis beyond the standard first year graduate course. Lovers of analysis and beautiful proofs will read and reread this slim volume with pleasure and profit.

Contents: Early triumphs; Approximation; Operator theory; Harmonic analysis; Banach algebras: The Gleason-Kahane-Żelazko theorem; Complex dynamics: The Fatou-Julia-Baker theorem; The prime number theorem; Coda: Transonic airfoils and SLE; Liouville's theorem in Banach spaces; The Borel-Carathéodory inequality; Phragmén-Lindelöf theorems; Normal families.

University Lecture Series, Volume 58

January 2012, 90 pages, Softcover, ISBN: 978-0-8218-7559-9, 2010 *Mathematics Subject Classification*: 30-XX, 41-XX, 47-XX, 42-XX, 46-XX, 26-XX, 11-XX, 60-XX, **AMS members US\$23.20**, List US\$29, Order code ULECT/58

Differential Equations



Resistance Forms, Quasisymmetric Maps and Heat Kernel Estimates

Jun Kigami, *Kyoto University, Japan*

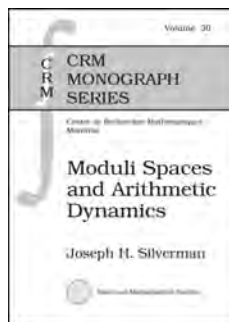
Contents: Introduction; *Part 1. Resistance forms and heat kernels*: Topology associated with a subspace of functions;

Basics on resistance forms; The Green function; Topologies associated with resistance forms; Regularity of resistance forms; Annulus comparable condition and local property; Trace of resistance form; Resistance forms as Dirichlet forms; Transition density; *Part 2. Quasisymmetric metrics and volume doubling measures*: Semi-quasisymmetric metrics; Quasisymmetric metrics; Relations of measures and metrics; Construction of quasisymmetric metrics; *Part 3. Volume doubling measures and heat kernel estimates*: Main results on heat kernel estimates; Example: the α -stable process on $\mathbb{R}$; Basic tools in heat kernel estimates; Proof of Theorem 15.6; Proof of Theorems 15.10, 15.11, and 15.13; *Part 4. Random Sierpinski gaskets*: Generalized Sierpinski gasket; Random Sierpinski gasket; Resistance forms on Random Sierpinski gaskets; Volume doubling property; Homogeneous case; Introducing randomness; Bibliography; Assumptions, conditions and properties in parentheses; List of notations; Index.

Memoirs of the American Mathematical Society, Volume 216, Number 1015

March 2012, 132 pages, Softcover, ISBN: 978-0-8218-5299-6, 2010 *Mathematics Subject Classification*: 30L10, 31E05, 60J35;

28A80, 43A99, 60G52, **Individual member US\$42.60**, List US\$71, Institutional member US\$56.80, Order code MEMO/216/1015



Moduli Spaces and Arithmetic Dynamics

Joseph H. Silverman, *Brown University, Providence, RI*

This monograph studies moduli problems associated to algebraic dynamical systems. It is an expanded version of the notes for a series of lectures delivered at a workshop on *Moduli Spaces and the Arithmetic of Dynamical Systems* at the Bellairs Research Institute, Barbados, in 2010.

The author's goal is to provide an overview, with enough details and pointers to the existing literature, to give the reader an entry into this exciting area of current research. Topics covered include:

- (1) Construction and properties of dynamical moduli spaces for self-maps of projective space.
- (2) Dynatomic modular curves for the space of quadratic polynomials.
- (3) The theory of canonical heights associated to dynamical systems.
- (4) Special loci in dynamical moduli spaces, in particular the locus of post-critically finite maps.
- (5) Field of moduli and fields of definition for dynamical systems.

This item will also be of interest to those working in algebra and algebraic geometry.

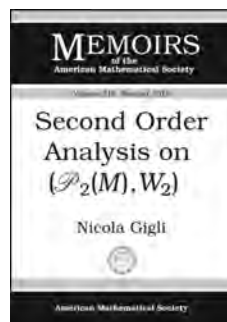
Titles in this series are co-published with the Centre de Recherches Mathématiques.

Contents: Moduli spaces associated to dynamical systems; The geometry of dynamical moduli spaces; Dynamical moduli spaces—Further topics; Dynatomic polynomials and dynamical modular curves; Canonical heights; Postcritically finite maps; Field of moduli and field of definition; Schedule of talks at the Bellairs workshop; Glossary; Bibliography; Index.

CRM Monograph Series, Volume 30

March 2012, 140 pages, Hardcover, ISBN: 978-0-8218-7582-7, 2010 *Mathematics Subject Classification*: 37P45; 37A45, 37F45, 37P30, 37Pxx, 14D20, 14D22, **AMS members US\$47.20**, List US\$59, Order code CRMM/30

Geometry and Topology



Second Order Analysis on $(\mathcal{P}_2(M), W_2)$

Nicola Gigli, *J. A. Dieudonné Université, Nice, France, and University of Bordeaux, Talence, France*

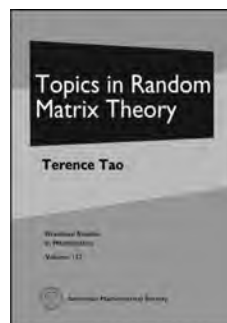
Contents: Introduction; Preliminaries and notation; Regular curves; Absolutely continuous vector fields; Parallel transport; Covariant derivative; Curvature;

Differentiability of the exponential map; Jacobi fields; Appendix A. Density of regular curves; Appendix B. C^1 curves; Appendix C. On the definition of exponential map; Appendix D. A weak notion of absolute continuity of vector fields; Bibliography.

Memoirs of the American Mathematical Society, Volume 216, Number 1018

March 2012, 154 pages, Softcover, ISBN: 978-0-8218-5309-2, 2010 *Mathematics Subject Classification*: 49Q20, 51F99, 58B10, **Individual member US\$46.20**, List US\$77, Institutional member US\$61.60, Order code MEMO/216/1018

Probability and Statistics



Topics in Random Matrix Theory

Terence Tao, *University of California, Los Angeles, CA*

The field of random matrix theory has seen an explosion of activity in recent years, with connections to many areas of mathematics and physics. However, this makes the current state of the field almost too large to survey in a single book. In this

graduate text, we focus on one specific sector of the field, namely the spectral distribution of random Wigner matrix ensembles (such as the Gaussian Unitary Ensemble), as well as iid matrix ensembles. The text is largely self-contained and starts with a review of relevant aspects of probability theory and linear algebra. With over 200 exercises, the book is suitable as an introductory text for beginning graduate students seeking to enter the field.

This item will also be of interest to those working in analysis.

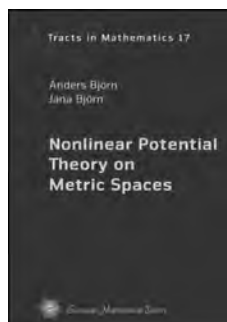
Contents: Preparatory material; Random matrices; Related articles; Bibliography; Index.

Graduate Studies in Mathematics, Volume 132

April 2012, approximately 291 pages, Hardcover, ISBN: 978-0-8218-7430-1, 2010 *Mathematics Subject Classification*: 60B20, 15B52, 15B35, **AMS members US\$51.20**, List US\$64, Order code GSM/132

New AMS-Distributed Publications

Analysis



Nonlinear Potential Theory on Metric Spaces

Anders Björn and Jana Björn,
Linköping University, Sweden

The p -Laplace equation is the main prototype for nonlinear elliptic problems and forms a basis for various applications, such as injection moulding of plastics, nonlinear elasticity theory, and image

processing. Its solutions, called p -harmonic functions, have been studied in various contexts since the 1960s, first on Euclidean spaces and later on Riemannian manifolds, graphs, and Heisenberg groups. Nonlinear potential theory of p -harmonic functions on metric spaces has been developing since the 1990s and generalizes and unites these earlier theories.

This monograph gives a unified treatment of the subject and covers most of the available results in the field, so far scattered over a large number of research papers. The aim is to serve both as an introduction to the area for interested readers and as a reference text for active researchers. The presentation is rather self contained, but it is assumed that readers know measure theory and functional analysis.

The first half of the book deals with Sobolev type spaces, so-called Newtonian spaces, based on upper gradients on general metric spaces. In the second half, these spaces are used to study p -harmonic functions on metric spaces, and a nonlinear potential theory is developed under some additional, but natural, assumptions on the underlying metric space. Each chapter contains historical notes with relevant references, and an extensive index is provided at the end of the book.

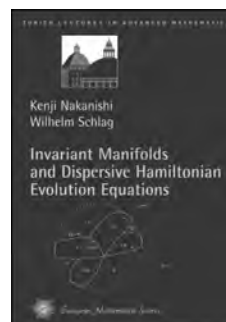
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Newtonian spaces; Minimal p -weak upper gradients; Doubling measures; Poincaré inequalities; Properties of Newtonian functions; Capacities; Superminimizers; Interior regularity; Superharmonic functions; The Dirichlet problem for p -harmonic functions; Boundary regularity; Removable singularities; Irregular boundary points; Regular sets and applications thereof; Appendices; Bibliography; Index.

EMS Tracts in Mathematics, Volume 17

November 2011, 415 pages, Hardcover, ISBN: 978-3-03719-099-9, 2010 *Mathematics Subject Classification*: 31-02, 31E05, **AMS members US\$67.20**, List US\$84, Order code EMSTM/17

Differential Equations



Invariant Manifolds and Dispersive Hamiltonian Evolution Equations

Kenji Nakanishi, *Kyoto University, Japan,* and **Wilhelm Schlag,** *University of Chicago, IL*

The notion of an invariant manifold arises naturally in the asymptotic stability

analysis of stationary or standing wave solutions of unstable dispersive Hamiltonian evolution equations such as the focusing semilinear Klein–Gordon and Schrödinger equations. This is due to the fact that the linearized operators about such special solutions typically exhibit negative eigenvalues (a single one for the ground state), which lead to exponential instability of the linearized flow and allows for ideas from hyperbolic dynamics to enter.

One of the main results proved here for energy subcritical equations is that the center-stable manifold associated with the ground state appears as a hyper-surface which separates a region of finite-time blowup in forward time from one which exhibits global existence and scattering to zero in forward time. The authors' entire analysis takes place in the energy topology, and the conserved energy can exceed the ground state energy only by a small amount.

This monograph is based on recent research by the authors. The proofs rely on an interplay between the variational structure of the ground states and the nonlinear hyperbolic dynamics near these states. A key element in the proof is a virial-type argument excluding almost homoclinic orbits originating near the ground states, and returning to them, possibly after a long excursion.

These lectures are suitable for graduate students and researchers in partial differential equations and mathematical physics. For the cubic Klein–Gordon equation in three dimensions all details are provided, including the derivation of Strichartz estimates for the free equation and the concentration-compactness argument leading to scattering due to Kenig and Merle.

This item will also be of interest to those working in analysis.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; The Klein–Gordon equation below the ground state energy; Above the ground state energy I: Near Q ; Above the ground state energy II: Moving away from Q ; Above the ground state energy III: Global NLKG dynamics; Further developments of the theory; References; Index.

Zurich Lectures in Advanced Mathematics, Volume 14

September 2011, 258 pages, Softcover, ISBN: 978-3-03719-095-1, 2010 *Mathematics Subject Classification*: 35L70, 35Q55, 37D10, 37K40, 37K45, **AMS members US\$41.60**, List US\$52, Order code EMSZLC/14

About the Cover

MathJax

The article by Davide Cervone in this issue (pp. 312–316) says it all.

—Bill Casselman
Graphics Editor
(notices-covers@ams.org)

New AMS-Distributed Publications

Mathematical Physics

Exploring New Structures and Natural Constructions in Mathematical Physics

Koji Hasegawa, *Tohoku University, Sendai, Japan*, **Takahiro Hayashi**, *Nagoya University, Japan*, **Shinobu Hosono**, *University of Tokyo, Japan*, and **Yasuhiko Yamada**, *Kobe University, Hyogo, Japan*,
Editors

2007, Professor Akihiro Tsuchiya of Nagoya University reached the retirement age of sixty-three. He has played a significant role in mathematical physics over the decades, most particularly in the foundation of conformal field theory, which was the first nontrivial example of a mathematically rigorous quantum field theory.

This volume contains the proceedings of the international conference on the occasion of his retirement. Included are conformal field theories and related topics such as solvable statistical models, representation theory of affine algebras, monodromy preserving deformations, and string theories. Readers interested in these subjects will find exciting and stimulating insights and questions from these articles.

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Contents: **K. Nagatomo** and **A. Tsuchiya**, The triplet vertex operator algebra $W(p)$ and the restricted quantum group $\bar{U}_q(sl_2)$ at $q = e^{\pi i/p}$; **T. Arakawa**, Representation theory of W -algebras, II; **V. V. Bazhanov**, Chiral Potts model and the discrete Sine-Gordon model at roots of unity; **T. Eguchi**, **Y. Sugawara**, and **A. Taormina**, Modular forms and elliptic genera for ALE spaces; **B. Feigin**, **E. Feigin**, and **I. Tipunin**, Fermionic formulas for characters of $(1, p)$ logarithmic model in $\Phi_{2,1}$ quasiparticle realisation; **B. Feigin** and **E. Frenkel**, Quantization of soliton systems and Langlands duality; **K. Hasegawa**, Quantizing the Bäcklund transformations of Painlevé equations and the quantum discrete Painlevé VI equation; **G. Kuroki**, Quantum groups and quantization of Weyl group symmetries of Painlevé systems; **W. Nakai** and **T. Nakanishi**, On Frenkel-Mukhin algorithm for q -character of quantum affine algebras; **H. Nakajima** and **K. Yoshioka**, Perverse coherent sheaves on blow-up. I. A quiver description; **K. Takasaki**, Differential Fay identities and auxiliary linear problem of integrable hierarchies.

Advanced Studies in Pure Mathematics, Volume 61

October 2011, 441 pages, Hardcover, ISBN: 978-4-931469-64-8, 2010 *Mathematics Subject Classification*: 81R10; 05E10, 05E15, 11F23, 12E15, 12H10, 14D21, 14D24, 14J81, 14M15, 16G20, 17Bxx, 22E57, 32G34, 33B30, 34M55, 34M56, 35Q51, 35Q53, 39A13, 81Txx, 82B20, 82B23, 83E30, **AMS members US\$62.40**, List US\$78, Order code ASPM/61

Classified Advertisements

Positions available, items for sale, services available, and more

CALIFORNIA

MATHEMATICAL SCIENCES RESEARCH INSTITUTE

Director Search Announcement

Applications are invited for the position of Director at the Mathematical Sciences Research Institute (MSRI), an independent research organization located on the campus of the University of California in Berkeley. The appointment will be for a five-year term beginning August 1, 2013.

The Institute is dedicated to the advancement and communication of fundamental knowledge in mathematics, to the development of human capital for the growth and use of that knowledge, and to the cultivation in the larger society of awareness and appreciation of the beauty, power and importance of mathematical ideas and ways of understanding the world.

A full job description can be viewed at <http://www.msri.org/directorsearch>. The attributes of a successful candidate for Director will include: 1) Outstanding mathematical accomplishments and visibility within and respect of the mathematical community; 2) Strong managerial, administrative and implementation skills; 3) A knowledge of and interest in furthering the programs of MSRI; and 4) Strong interpersonal skills with a variety of constituencies.

Please communicate interest in this position to the Director Search Committee, MSRI, by writing to directorsearch@msri.org. While applications will be considered until the position is filled, the

committee will start discussions at the end of February 2012.

A completed application will include a CV and a statement of views about how MSRI should continue to develop. Letters of recommendation solicited by the candidate are not required, but will be accepted, and should be addressed to the Director Search Committee, 17 Gauss Way, Berkeley, CA 94720-5070.

MSRI is an Equal Opportunity Employer/Affirmative Action Employer.

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FLORIDA

FLORIDA INTERNATIONAL UNIVERSITY (FIU)

Department of Mathematics and Statistics

FIU is a multi-campus public research university located in Miami, a vibrant, international city. FIU offers more than 180 baccalaureate, masters, and professional and doctoral degree programs to over 44,000 students. The Department of Mathematics and Statistics at FIU invites applications for two positions in Mathematics, beginning fall 2012:

1. Tenure-Track Assistant Professor or higher rank
2. Instructor

Qualifications for the first position include a Ph.D. in mathematics and outstanding record in research, appropriate to rank, and teaching. Established record of funded research and successful Ph.D. student supervisions is a plus.

Duties for the second position include curriculum development and teaching

innovation in undergraduate courses. Experience and specialization in computer assisted instruction in college algebra is preferred. A master's degree or higher in mathematics is required for the position.

To apply, send an application letter, curriculum vita and at least three letters of references to: <https://www.fiujobs.org>. The deadline of application is February 25, 2012. For more information, please visit <http://www.fiu.edu> and <http://casgroup.fiu.edu/mathstatistics>.

FIU is a member of the State University System of Florida and is an Equal Opportunity, Equal Access, Affirmative Action Employer.

000022

GEORGIA

GEORGIA STATE UNIVERSITY Department of Mathematics and Statistics

The Department of Mathematics and Statistics at Georgia State University invites applications for an anticipated lecturer position in mathematics beginning August 2012 pending budgetary approval. A Ph.D. in Mathematics, Bioinformatics, Physics, Neuroscience or related area. One year of college/university teaching experiences and interdisciplinary experience. Submit directly to: <http://www.mathjobs.org>. If unable to submit electronically, send to Chair of Lecturer Hiring Committee, Department of Mathematics and Statistics, Georgia State University, P.O. Box 4110, Atlanta, GA, 30303-3083, USA. Applications review begins January 2012. An offer of employment will be conditional upon background verification. Georgia State University is a Research University of the

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2011 rate is \$3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: March 2012 issue–December 28, 2011; April 2012 issue–January 30, 2012; May 2012

issue–February 28, 2012; June/July 2012 issue–April 30, 2012; August 2012 issue–May 29, 2012; September 2012 issue–June 28, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classifieds@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

University System of Georgia and an EEO/AAA institution that values the diversity of its faculty, staff, and student body. Visit <http://www2.gsu.edu/~wwwmat/> for further information.

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NEW JERSEY

RUTGERS UNIVERSITY-NEW BRUNSWICK Mathematics Department

The Mathematics Department of Rutgers University-New Brunswick invites applications for the following positions which may be available September 2012.

HILL ASSISTANT PROFESSORSHIPS and NON-TENURE-TRACK ASSISTANT PROFESSORSHIPS: These are both three-year nonrenewable positions. Subject to availability of funding, the Department may have one or more open positions of these types. The Hill Assistant Professorship carries a reduced teaching load of 2-1 for research; candidates for it should have received the Ph.D., show outstanding promise of research ability in pure or applied mathematics, and have concern for teaching. The Non-Tenure-Track Assistant Professorship carries a teaching load of 2-2; candidates for it should have received the Ph.D., show evidence of superior teaching accomplishments and promise of research ability. Review of applications begins January 1, 2012.

Furthermore, subject to the availability of funding there may be positions at other levels. No such positions are, however, authorized at this point.

Applicants for the above position(s) should submit a curriculum vitae (including a publication list) and arrange for four letters of reference to be submitted, one of which evaluates teaching.

Applicants should first go to the website <https://www.mathjobs.org/jobs> and fill out the AMS Cover Sheet electronically. It is essential to fill out the cover sheet completely, including naming the positions being applied for (TP, TTAP, HILL, NTTAP, respectively) giving the AMS Subject Classification number(s) of area(s) of specialization, and answering the question about how materials are being submitted. The strongly preferred way to submit the CV, references, and any other application materials is online at <https://www.mathjobs.org/jobs>. If necessary, however, application materials may instead be mailed to: Search Committee, Dept. of Math-Hill Center, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019.

Review of applications will begin on the date indicated above, and will continue until openings are filled. Updates on these positions will appear on the Rutgers Mathematics Department webpage at: <http://www.math.rutgers.edu>.

Rutgers is an Affirmative Action/Equal Opportunity Employer and encourages

applications from women and minority-group members.

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SOUTH CAROLINA

UNIVERSITY OF SOUTH CAROLINA UPSTATE Assistant Professor of Mathematics Requisition #004183

The Division of Mathematics and Computer Science has a tenure-track Assistant Professor of Mathematics position beginning August 16, 2012 to teach undergraduate courses in mathematics, develop curriculum, engage in scholarly activities and advise students. Ph.D. in mathematics required before August 15, 2012. Consideration given to candidates with research interest in algebra. Review of application begins immediately and continues until the position is filled. For complete information, requirements and online application submission process, go to <http://www.uscupstate.edu/jobs>. Contact: Dr. Jerome Lewis, chair, jlewis@uscupstate.edu. USC Upstate is an Equal Opportunity Institution.

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ENGLAND

IMPERIAL COLLEGE, LONDON Department of Mathematics Two Chapman Fellowships in Mathematics South Kensington Campus Salary range \$31,300-\$39,920

Applications are invited for two Chapman Fellowships in Mathematics, tenable for two years, starting October 1, 2012 or as soon as possible thereafter. One of the positions will be in the field of Applied Partial Differential Equations, and one will be in any area of Pure Mathematics, Statistics or Mathematical Finance.

This research post provides an excellent opportunity for those seeking to pursue an academic career in mathematics.

Candidates for a Chapman Fellowship are expected to have proven research ability in mathematics. Candidates will demonstrate the potential for strong leadership qualities in the subject, as illustrated, for example, through showing initiative on research projects. Candidates will hold a good honours degree and a Ph.D. (or equivalent) in mathematics or a closely related subject. Candidates will show exceptional ability in their chosen research areas, will have an outstanding research record commensurate with their level of experience, as demonstrated, for example, through an outstanding thesis, publications and conference presentations, etc. Candidates will be able to contribute significantly to the research environment

of the Department of Mathematics, shown through, for example, through proven research interaction.

In addition to pursuing research, Fellows will be expected to undertake a limited range of teaching duties in the Department of Mathematics. The ability to teach mathematics at undergraduate and postgraduate level is essential, as demonstrated by excellent written and verbal communication skills with the ability to give effective presentations.

Further particulars, together with information about Imperial College London, the Department of Mathematics can be found at: <http://www.ma.imperial.ac.uk>. Our preferred method is online via the Imperial College website: <http://www.imperial.ac.uk/employment>. Further particulars for the post are available on the website searching with the reference number XXXXXX. Please state on the application form which area you are applying to.

Alternatively, if you are unable to apply online or have any queries about the application process please contact Ms. Magdalena Myc, m.myc@imperial.ac.uk quoting the reference number above to request an application form.

Closing date: ASAP

Committed to Equality and Valuing Diversity. We are an Athena Silver SWAN Award winner and a Stonewall Diversity Champion.

000017

HOME RENTALS AND SWAPS

HOME RENTALS AND SWAPS

SabbaticalHomes.com. Do you have a home to rent? Are you looking for housing while on sabbatical? We are the academic community's resource for home rentals and home swaps worldwide.

000019

NEW PUBLICATIONS

NEW PUBLICATIONS Books from Sampling Publishing

Sampling Publishing announces the publication of *Introduction to Wavelets* by A. J. Jerri with Solutions Manual; *Advances in The Gibbs Phenomenon*, A. J. Jerri, ed.; and *Intro to Integral Equations* with Solutions Manual by A. J. Jerri. For more information visit <http://www.stsip.org>; see Published Books. To order, contact email: bethat73@yahoo.com.

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See <http://www.ams.org/meetings/>. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Honolulu, Hawaii

University of Hawaii at Manoa

March 3–4, 2012

Saturday – Sunday

Meeting #1078

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: December 2011

Program first available on AMS website: January 26, 2012

Program issue of electronic *Notices*: March 2012

Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: Expired

For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Zhiqin Lu, University of California Irvine, *Geometry of Calabi-Yau moduli*.

Peter Schröder, California Institute of Technology, *Conformal editing of surface meshes*.

Pham Tiep, University of Arizona, Tucson, *Representations of finite groups: Conjectures, reductions, and applications*.

Lauren Williams, University of California Berkeley, *Title to be announced*.

Special Sessions

Algebraic Combinatorics, **Federico Ardila**, San Francisco State University, **Sara Billey**, University of Washing-

ton, and **Kelli Talaska** and **Lauren Williams**, University of California, Berkeley.

Algebraic Geometry: Singularities and Moduli, **Jim Bryan**, University of British Columbia, and **Jonathan Wise**, Stanford University.

Algebraic Number Theory, Diophantine Equations and Related Topics, **Claude Levesque**, Université de Laval, Quebec, Canada.

Applications of Nonstandard Analysis, **Tom Lindstrom**, University of Oslo, Norway, **Peter Loeb**, University of Illinois at Urbana-Champaign, and **David Ross**, University of Hawaii, Honolulu.

Arithmetic Geometry, **Xander Faber**, **Michelle Manes**, and **Gretel Sia**, University of Hawaii.

Asymptotic Group Theory, **Tara Davis**, Hawaii Pacific University, **Erik Guentner**, University of Hawaii, and **Michael Hull** and **Mark Sapir**, Vanderbilt University.

Automorphic and Modular Forms, **Pavel Guerzhoy**, University of Hawaii, and **Zachary A. Kent**, Emory University.

C-algebras and Index Theory*, **Erik Guentner**, University of Hawaii at Manoa, **Efren Ruiz**, University of Hawaii at Hilo, and **Erik Van Erp** and **Rufus Willett**, University of Hawaii at Manoa.

Computability and Complexity, **Cameron E. Freer**, Massachusetts Institute of Technology, and **Bjorn Kjos-Hanssen**, University of Hawaii at Manoa.

Geometry and Analysis on Fractal Spaces, **Michel Lapidus**, University of California, Riverside, **Hung Lu**, Hawaii Pacific University, **John A. Rock**, California State Polytechnic University, Pomona, and **Machiel van Frankenhuijsen**, Utah Valley University.

Holomorphic Spaces, **Hyungwoon Koo**, Korea University, and **Wayne Smith**, University of Hawaii.

Kaehler Geometry and Its Applications, **Zhiqin Lu**, University of California Irvine, **Jeff Streets**, Princeton University, **Li-Sheng Tseng**, Harvard University, and **Ben Weinkove**, University of California San Diego.

Kernel Methods for Applications on the Sphere and Other Manifolds, **Thomas Hangelbroek**, University of Hawaii at Manoa.

Knotting in Linear and Ring Polymer Models, **Tetsuo Deguchi**, Ochanomizu University, **Kenneth Millett**, University of California, Santa Barbara, **Eric Rawdon**, University of St. Thomas, and **Mariel Vazquez**, San Francisco State University.

Linear and Permutation Representations, **Robert Guralnick**, University of Southern California, and **Pham Huu Tiep**, University of Arizona.

Mathematical Coding Theory and its Industrial Applications, **J. B. Nation**, University of Hawaii, and **Manabu Hagiwara**, National Institute of Advanced Industrial Science and Technology, Japan.

Mathematical Teacher Preparation, **Diane Barrett** and **Roberto Pelayo**, University of Hawaii at Hilo.

Model Theory, **Isaac Goldbring**, University of California Los Angeles, and **Alice Medvedev**, University of California Berkeley.

New Techniques and Results in Integrable and Near-Integrable Nonlinear Waves, **Jeffrey DiFranco**, Seattle University, and **Peter Miller**, University of Michigan.

Noncommutative Algebra and Geometry, **Jason Bell**, Simon Fraser University, and **James Zhang**, University of Washington.

Nonlinear Partial Differential Equations at the Common Interface of Waves and Fluids, **Ioan Bejenaru** and **Vlad Vicol**, University of Chicago.

Nonlinear Partial Differential Equations of Fluid and Gas Dynamics, **Elaine Cozzi**, Oregon State University, and **Juhi Jang** and **Jim Kelliher**, University of California Riverside.

Singularities, Stratifications and Their Applications, **Terence Gaffney**, Northeastern University, **David Trotman**, Université de Provence, and **Leslie Charles Wilson**, University of Hawaii at Manoa.

Transformation Groups in Topology, **Karl Heinz Dovermann**, University of Hawaii at Manoa, and **Daniel Ramras**, New Mexico State University.

Universal Algebra and Lattice Theory, **Ralph Freese**, **William Lampe**, and **J. B. Nation**, University of Hawaii.

Registration and Meeting Information

Registration and the AMS Book Exhibit will be held in Keller Hall. Invited Addresses and other sessions will be held in several buildings on the University of Hawaii campus. Please refer to the AMS Sectional Meeting website at http://www.ams.org/meetings/sectional/2190_program.html or the University of Hawaii website <http://math.hawaii.edu/ams/> for specific locations. The registration desk will be open on Friday, March 3, 2012, from 7:00 a.m. to 4:00 p.m. and Saturday, March 4, from 7:30 a.m. to 12:00 p.m. Fees are US\$52 for AMS members, US\$72 for nonmembers; and US\$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check or credit card.

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday

Meeting #1079

Southeastern Section

Associate secretary: Matthew Miller

Announcement issue of *Notices*: January 2012

Program first available on AMS website: February 2, 2012

Program issue of electronic *Notices*: March 2012

Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: Expired

For abstracts: January 18, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anne Condon, University of British Columbia, *Some why's and how's of programming DNA molecules.*

Mark Ellingham, Vanderbilt University, *Beyond the Map Color Theorem.*

Mauro Maggioni, Duke University, *Digital data sets: Geometry, random walks, multiscale analysis, and applications.*

Weiqliang Wang, University of Virginia, *What is super in representation theory of Lie superalgebras?.*

Special Sessions

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 2A), **J. Scott Carter**, University of South Alabama, and **Mohamed Elhamdadi** and **Masahico Saito**, University of South Florida.

Analysis in Metric Spaces (Code: SS 3A), **Thomas Bieske**, University of South Florida, and **Jason Gong**, University of Pittsburgh.

Applications of Complex Analysis in Mathematical Physics (Code: SS 9A), **Razvan Teodorescu**, University of South Florida, **Mihai Putinar**, University of California, Santa Barbara, and **Pavel Bleher**, Indiana University-Purdue University Indianapolis.

Asymptotic Properties of Groups (Code: SS 20A), **Alexander Dranishnikov**, University of Florida, and **Mark Sapir**, Vanderbilt University.

Combinatorics: Algebraic and Geometric (Code: SS 23A), **Drew Armstrong**, University of Miami, and **Benjamin Braun**, University of Kentucky.

Complex Analysis and Operator Theory (Code: SS 8A), **Sherwin Kouckekian**, University of South Florida, and **William Ross**, University of Richmond.

Computational Algebraic Geometry and Applications (Code: SS 25A), **Tony Shaska**, Oakland University, and **Artur Elezi**, American University.

Dirac Analysis (Code: SS 18A), **Craig Nolder**, Florida State University, and **John Ryan**, University of Arkansas.

Discrete Mathematics and Geometry (Code: SS 17A), **Eunjeong Yi** and **Cong X. Kang**, Texas A&M University Galveston.

Discrete Models in Molecular Biology (Code: SS 1A), **Alessandra Carbone**, Université Pierre et Marie Curie and Laboratory of Microorganisms Genomics, **Natasha Jonoska**, University of South Florida, and **Reidun Twarock**, University of York.

Extremal Combinatorics (Code: SS 13A), **Linyuan Yu**, University of South Carolina, and **Yi Zhao**, Georgia State University.

Finite Fields and Their Applications (Code: SS 15A), **Xiang-dong Hou**, University of South Florida, and **Gary Mullen**, Pennsylvania State University.

Graph Theory (Code: SS 14A), **Mark Ellingham**, Vanderbilt University, and **Xiaoya Zha**, Middle Tennessee State University.

Hopf Algebras and Galois Module Theory (Code: SS 7A), **James Carter**, College of Charleston, and **Robert Underwood**, Auburn University Montgomery.

Interaction between Algebraic Combinatorics and Representation Theory (Code: SS 4A), **Mahir Can**, Tulane University, and **Weiqiang Wang**, University of Virginia.

Inverse Problems in Partial Differential Equations (Code: SS 24A), **Xiaosheng Li**, Florida International University, and **Alexandru Tamasan**, University of Central Florida.

Low-Dimensional Topology (Code: SS 22A), **Peter Horn**, Columbia University, and **Constance Leidy**, Wesleyan University.

Modeling Crystalline and Quasi-Crystalline Materials (Code: SS 5A), **Mile Krajcevski** and **Gregory McColm**, University of South Florida.

Nonlinear Partial Differential Equations and Applications (Code: SS 19A), **Netra Khanal**, University of Tampa.

Recent Developments of Finite Element Methods for Partial Differential Equations (Code: SS 21A), **Bo Dong**, Drexel University, and **Wei Wang**, Florida International University.

Representations of Algebraic Groups and Related Structures (Code: SS 12A), **Joerg Feldvoss** and **Cornelius Pillen**, University of South Alabama.

Solvability and Integrability of Nonlinear Evolution Equations (Code: SS 6A), **Wen-Xiu Ma**, University of South Florida, and **Ahmet Yildirim**, Ege University and University of South Florida.

Spectral Theory (Code: SS 11A), **Anna Skripka** and **Maxim Zinchenko**, University of Central Florida.

Stochastic Analysis and Applications (Code: SS 16A), **Sivapragasam Sathananthan**, Tennessee State University, and **Gangaram Ladde**, University of South Florida.

Stochastic Partial Differential Equations and Random Global Dynamics (Code: SS 10A), **Yuncheng You**, University of South Florida, and **Shanjian Tang**, Fudan University.

Washington, District of Columbia

George Washington University

March 17–18, 2012

Saturday – Sunday

Meeting #1080

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: January 2012

Program first available on AMS website: February 9, 2012

Program issue of electronic *Notices*: March 2012

Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: Expired

For abstracts: January 31, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Jim Geelen, University of Waterloo, *Matroid minors*.

Boris Solomyak, University of Washington, *Some recent advances in tiling dynamical systems*.

Gunther Uhlmann, University of California, Irvine and University of Washington, *Cloaking: Science meets science-fiction* (Einstein Public Lecture in Mathematics).

Anna Wienhard, Princeton University, *Deformation spaces of geometric structures*.

Special Sessions

Analysis of Wavelets, Frames, and Fractals (Code: SS 11A), **Keri Kornelson**, University of Oklahoma, and **Judy Packer**, University of Colorado Boulder.

Computable Mathematics (in honor of Alan Turing) (Code: SS 8A), **Douglas Cenzer**, University of Florida, **Valentina Harizanov**, George Washington University, and **Russell Miller**, Queens College and Graduate Center-CUNY.

Convex and Discrete Geometry (Code: SS 9A), **Jim Lawrence** and **Valeriu Soltan**, George Mason University.

Difference Equations and Applications (Code: SS 18A), **Michael Radin**, Rochester Institute of Technology.

Dynamics of Complex Networks (Code: SS 7A), **Yongwu Rong**, **Guanyu Wang**, and **Chen Zeng**, George Washington University.

Homology Theories Motivated by Knot Theory (Code: SS 3A), **Jozef H. Przytycki**, George Washington University, **Radmila Sazdanovic**, University of Pennsylvania, and **Alexander N. Shumakovitch** and **Hao Wu**, George Washington University.

Mathematical Methods in Disease Modeling (Code: SS 15A), **Shweta Bansal**, Georgetown University and National Institutes of Health, and **Sivan Leviyang**, Georgetown University.

Mathematics Applied in the Sciences: From Statistics to Topology (Code: SS 12A), **James Carroll** and **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, Enfitec, Inc. and University of New Mexico.

Matroid Theory (Code: SS 1A), **Joseph E. Bonin**, George Washington University, and **Sandra Kingan**, Brooklyn College.

Nonlinear Dispersive Equations (Code: SS 10A), **Manousos Grillakis**, University of Maryland, **Justin Holmer**, Brown University, and **Svetlana Roudenko**, George Washington University.

Optimization: Theory and Applications (Code: SS 2A), **Roman Sznajder**, Bowie State University.

Relations between the History and Pedagogy of Mathematics (Code: SS 14A), **David L. Roberts**, Prince George's Community College, and **Kathleen M. Clark**, Florida State University.

Self-organization Phenomena in Reaction Diffusion Equations (Code: SS 5A), **Xiaofeng Ren**, George Washington University, and **Junping Shi**, College of William and Mary.

Structural and Extremal Problems in Graph Theory (Code: SS 4A), **Daniel Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Symmetric Functions, Quasisymmetric Functions, and the Associated Combinatorics (Code: SS 16A), **Nicholas Loehr**, Virginia Tech, and **Elizabeth Niese**, Marshall University.

The Legacy of Goedel's Second Incompleteness Theorem for the Foundations of Mathematics (Code: SS 17A), **Karim J. Mourad**, Georgetown University.

Tilings, Substitutions, and Bratteli-Vershik Transformations (Code: SS 6A), **E. Arthur Robinson**, George Washington University, and **Boris Solomyak**, University of Washington.

Topics in Geometric Analysis and Complex Analysis (Code: SS 13A), **Zheng Huang** and **Marcello Lucia**, City University of New York, Staten Island.

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: February 2012

Program first available on AMS website: March 8, 2012

Program issue of electronic *Notices*: March 2012

Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For abstracts: February 14, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Frank Calegari, Northwestern University, *Studying algebraic varieties through their cohomology—Recent progress in the Langlands program.*

Christopher Leininger, University of Illinois at Urbana-Champaign, *On complexity of surface homeomorphisms.*

Alina Marian, Northeastern University, *Title to be announced.*

Catherine Yan, Texas A&M University, *Enumerative combinatorics with fillings of polyominoes.*

Poster Session

We encourage everyone (undergraduate, graduate students, post docs, and faculty) to participate in the Kansas Poster Session to be held Saturday, March 31, 2012, 5:45 p.m.–6:45 p.m., on campus in the Summerfield Room, KU Alumni Center. A reception for all conference participants will be held simultaneously at the alumni center. Deadline for abstract for Kansas Poster Session: February 28, 2012. Register on-line (<http://www.math.ku.edu/conferences/2012/AMS/poster-sessions.html>).

Accommodations

Participants should make their own arrangements directly with a hotel of their choice as early as possible. Special rates have been negotiated with the hotels listed below. Rates quoted do not include taxes (8.85%). The AMS is not responsible for rate changes or for the quality of the accommodations. When making a reservation with a conference hotel, participants should state that they are with the **KU Math Department**. Cancellation and early checkout policies vary; be sure to check when you make your reservation. When making reservations please call the hotel directly and ask for “in house” reservations.

Hotel Shuttle. A free shuttle service will be provided by the University of Kansas from the conference hotels to the university. Please visit the mathematics department website at www.math.ku.edu/conferences/2012/AMS/index.html to view the schedule.

Baymont Inn & Suites, 740 Iowa Street, Lawrence, KS; 1.5 miles from Snow Hall. Call 785-838-4242 or visit their website at www.baymontinnlawrence.com. Double Queen/King Room: US\$80 per night on Thursday/ US\$116 per night on Friday and Saturday. Rate includes free WiFi, complimentary breakfast and indoor pool. Close to restaurants. **The deadline for reservations is February 29, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 24 hours prior to arrival without penalty.

Best Western Lawrence, 2309 Iowa Street, Lawrence, KS; two miles from Snow Hall. Call 785-843-9100 or visit www.bestwesternkansas.com/hotels/best-western-lawrence/. Double Queen/Single Queen: US\$55 per night plus tax. Rate includes free WiFi and breakfast. **The deadline for reservations is February 28, 2012, or when**

sold out. Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 72 hours prior to arrival without penalty.

Eldridge Hotel, 701 Massachusetts, Lawrence, KS; located downtown. Call 785-49-5011 or 800-527-0909 or visit www.eldridgehotel.com. Deluxe King rooms are US\$125 per night on Thursday, US\$159 per night on Friday-Saturday. Valet: US\$10 per vehicle per night. Rate includes free WiFi. **The deadline for reservations is February 28, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department or booking code number 733451. Rooms may be cancelled 48 hours prior to arrival without penalty.

Hampton Inn, 2300 West 6th Street, Lawrence, KS; 1.5 miles from Snow Hall. Call 785-841-4994. King/Queen standard rooms are US\$110 per night. Rate includes complimentary breakfast, fitness room, and indoor pool. **The deadline for reservations is March 8, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 48 hours prior to arrival without penalty.

Lawrence Holiday Inn Holidome, 200 McDonald Drive, Lawrence, KS; two miles from Snow Hall. Call 785-841-7077 or visit www.holidayinn.com/hotels/us/en/lawrence/lwrks/hoteldetail. King, single/double rooms are US\$89 per night. Rate includes continental breakfast and free WiFi. Two restaurants located on the property, indoor pool, exercise facility, and miniature golf. **The deadline for reservations is February 28, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 48 hours prior to arrival without penalty.

Oread Hotel, 1200 Oread Avenue, Lawrence, KS. The Oread Hotel is about a 15-minute walk from campus. For reservations call 785-843-1200 or 877-263-6347 or visit www.theoread.com. A classic King room is US\$115 per night on Thursday, US\$149 per night on Friday-Saturday. Valet: US\$12 per vehicle per night. Rate includes free WiFi. **The deadline for reservations is February 28, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department or confirmation number 733451. Rooms may be cancelled 48 hours prior to arrival without penalty.

Quality Inn is located at 801 Iowa Street, Lawrence, KS; 1.5 miles from Snow Hall. For reservations call 785-842-5100 or visit <http://www.qualityinn.com/hotel-lawrence-kansas-KS082>. Queen rooms are US\$68 per night. Rate includes free WiFi, lobby computer workstation with printer, and breakfast bar. **The deadline for reservations is February 29, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 72 hours prior to arrival without penalty.

SpringHill Suites, One Riverfront Plaza, Lawrence, KS; located downtown. For reservations call: 785-841-2700 or visit <http://www.marriott.com/lwcks> (group code KUMKUMC for a Queen suite or KUMKUMD for a King suite). Double Queen/King Room: US\$92 per night plus tax. Rate includes a complimentary hot breakfast buffet and free WiFi. **The deadline for reservations is February 29, 2012, or when sold out.** Reservations made after this date will be on a space-available and rate-available basis. Please state you are with the KU Math Department. Rooms may be cancelled 24 hours prior to arrival without penalty.

Food Service

There are a number of restaurants adjacent to the campus. Downtown Lawrence has a large number of restaurants, with many price ranges and a wide variety of cuisines. Located in the lower level of Wescoe, The Underground Food Court and the Pulse Coffee Bar operated by KU Dining Services, will be open Friday (until 5:00 p.m.) and Saturday (11:00 a.m. – 2:00 p.m.). A list of restaurants will be available at the registration desk.

Local Information

Please visit the website maintained by the Department of Mathematics at www.math.ku.edu/conferences/2012/AMS/index.html, the University of Kansas website www.ku.edu, or Lawrence Chamber of Commerce site at www.lawrencechamber.com.

Other Activities

Book Sales: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to 25 percent off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Parking is open and free on weekends (except in the obvious do not park zones and special reserved spots). Participants can park in any of the posted lots free of charge. (See map at <http://www.http://parking.ku.edu/visit.shtml/>).

Registration and Meeting Information

Registration and AMS Book Exhibit will be held in the Wescoe Hall Building on the fourth floor. Invited Addresses, the book sale, and all other sessions will be held in this building. Please refer to the campus map at www.maps.ku.edu for specific locations. The registration desk will be open on Friday, March 30, 2012 from 2:00 p.m.–6:00 p.m. and Saturday, March 31, from 7:30 a.m.–4:00 p.m. Fees are US\$52 for AMS members, US\$72 for nonmembers; and US\$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check or credit card.

Travel

By Air: The Kansas City International Airport (MCI) is served by most major airlines and is located approximately one hour from the University of Kansas. Please see the following options for airport transfers.

Ground Transportation Inc. has some scheduled arrival and departure times but they may arrange trips for non-scheduled times. It takes approximately one hour from MCI to Lawrence. The cost is US\$74 round trip (subject to change). Call for reservations and more information, 785-842-TAXI (842-8294) or 888-467-3721.

The Kansas City Roadrunner also has some scheduled arrival and departure times. This shuttle serves other cities and it may take longer to reach the airport. Call for reservations and more information, 800-747-2524.

Super Shuttle, shared-ride door-to-door shuttle service for up to ten passengers. For additional information visit their webpage or call 800-BLUEVAN or 800-258-3826.

Better Alternative Transportation Services (BATS) offers personalized service for up to five people with door-to-door delivery to any location in Lawrence. Reservations must be made at least two days in advance by email (mariokcishuttle@hotmail.com) or at least 24 hours in advance by telephone 913-634-5484.

KCI Express Shuttle provides door-to-door, non-stop shuttle services for groups and individuals. Make your reservations online or call 816-645-1815. After 8:00 p.m. CST call 816-372-1556.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount", and type in our convention number (CV): 04N30002. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Driving: If you are driving, Lawrence, Kansas is just off I-70, about 30 miles east of Topeka and about 50 miles west of Kansas City, MO.

Weather

Temperatures typically vary from 40° F to 55° F in late March. For up-to-date forecasts visit www.weather.com using 66045 as the reference zip code.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at <http://sites.nationalacademies.org/pga/biso/visas/> and http://travel.state.gov/visa/visa_1750.html. If you need a preliminary conference invitation in order to secure a visa, please send your request to pfs@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Rochester, New York

Rochester Institute of Technology

September 22-23, 2012

Saturday - Sunday

Meeting #1082

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: May 2012

Program first available on AMS website: July 19, 2012

Program issue of electronic *Notices*: September 2012

Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: February 22, 2012

For consideration of contributed papers in Special Sessions: May 15, 2012

For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgsectional.html.

Invited Addresses

Steve Gonek, University of Rochester, *Title to be announced.*

James Keener, University of Utah, *Title to be announced.*

Dusa McDuff, Barnard College, *Title to be announced.*

Peter Winkler, Dartmouth College, *Title to be announced.*

Special Sessions

Analytic Number Theory (Code: SS 5A), **Steve Gonek**, University of Rochester, and **Angel Kumchev**, Towson University.

Continuum Theory (Code: SS 3A), **Likin C. Simon Romero**, Rochester Institute of Technology.

Financial Mathematics (Code: SS 1A), **Tim Siu-Tang Leung**, Columbia University.

Microlocal Analysis and Nonlinear Evolution Equations (Code: SS 2A), **Raluca Felea**, Rochester Institute of Technology, and **Dan-Andrei Geba**, University of Rochester.

Special Session on Operator Theory and Function Spaces (Code: SS 4A), **Gabriel T. Prajitura** and **Ruhan Zhao**, State University of New York at Brockport.

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section

Associate secretary: Matthew Miller

Announcement issue of *Notices*: June 2012

Program first available on AMS website: September 6, 2012

Program issue of electronic *Notices*: October 2012

Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: March 13, 2012

For consideration of contributed papers in Special Sessions: July 3, 2012

For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgsectional.html.

Invited Addresses

Anita Layton, Duke University, *Title to be announced.*

Lenhard Ng, Duke University, *Title to be announced.*

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines.*

Milen Yakimov, Louisiana State University, *Title to be announced.*

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: August 2012

Program first available on AMS website: September 27, 2012

Program issue of electronic *Notices*: October 2012

Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 22, 2012

For consideration of contributed papers in Special Sessions: July 10, 2012

For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgsectional.html.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced.*

Tim Cochran, Rice University, *Title to be announced.*

Ronald Solomon, Ohio State University, *Title to be announced.*

Ben Weinkove, University of California San Diego, *Title to be announced.*

Special Sessions

Groups, Representations, and Characters (Code: SS 1A), **Mark Lewis**, Kent State University, **Adriana Nenciu**, Otterbein University, and **Ronald Solomon**, Ohio State University.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: August 2012

Program first available on AMS website: October 4, 2012

Program issue of electronic *Notices*: October 2012

Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 27, 2012

For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgsectional.html.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced.*

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced.*

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Title to be announced.*

Goufang Wei, University of California, Santa Barbara, *Title to be announced.*

Special Sessions

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona, Tucson.

Geometric Analysis and Riemannian Geometry (Code: SS 4A), **David Glickenstein**, University of Arizona, **Guofang Wei**, University of California Santa Barbara, and **Andrea Young**, Ripon College.

Geometrical Methods in Mechanical and Dynamical Systems (Code: SS 3A), **Akif Ibragimov**, Texas Tech University, **Vakhtang Putkaradze**, Colorado State University, and **Magdalena Toda**, Texas Tech University.

Harmonic Maass Forms and q -Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: October 2012

Program first available on AMS website: November 1, 2012

Program issue of electronic *Notices*: January 2012

Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Oxford, Mississippi

University of Mississippi

March 1–3, 2013

Friday – Sunday

Southeastern Section

Associate secretary: Matthew Miller

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 1, 2012

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: April 2013
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
 For consideration of contributed papers in Special Sessions: To be announced
 For abstracts: To be announced

*The scientific information listed below may be dated.
 For the latest information, see www.ams.org/amsmtgs/sectional.html.*

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday
First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the “Simion Stoilow” Institute of Mathematics of the Romanian Academy.

Associate secretary: Steven H. Weintraub
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: Not applicable
 Program issue of electronic *Notices*: Not applicable
 Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
 For consideration of contributed papers in Special Sessions: To be announced
 For abstracts: To be announced

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday
 Southeastern Section
 Associate secretary: Matthew Miller
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: To be announced
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
 For consideration of contributed papers in Special Sessions: To be announced
 For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday
 Central Section
 Associate secretary: Georgia Benkart
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: To be announced
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
 For consideration of contributed papers in Special Sessions: To be announced
 For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday
 Western Section
 Associate secretary: Michel L. Lapidus
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: To be announced
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
 For consideration of contributed papers in Special Sessions: To be announced
 For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
 Associate secretary: Matthew Miller
 Announcement issue of *Notices*: October 2013
 Program first available on AMS website: November 1, 2013
 Program issue of electronic *Notices*: January 2013
 Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday

The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2014

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: January 2015

Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2015

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: January 2016

Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at www.ams.org/meetings/.**

Meetings:

2012

March 3-4	Honolulu, Hawaii	p. 349
March 10-11	Tampa, Florida	p. 350
March 17-18	Washington, DC	p. 351
March 30-April 1	Lawrence, Kansas	p. 352
September 22-23	Rochester, New York	p. 354
October 13-14	New Orleans, Louisiana	p. 355
October 20-21	Akron, Ohio	p. 355
October 27-28	Tucson, Arizona	p. 355

2013

January 9-12	San Diego, California	p. 356
	Annual Meeting	
March 1-3	Oxford, Mississippi	p. 356
April 6-7	Chestnut Hill, Massachusetts	p. 356
April 27-28	Ames, Iowa	p. 356
June 27-30	Alba Iulia, Romania	p. 357
October 5-6	Louisville, Kentucky	p. 357
October 18-20	St. Louis, Missouri	p. 357
November 2-3	Riverside, California	p. 357

2014

January 15-18	Baltimore, Maryland	p. 357
	Annual Meeting	
June 16-19	Tel Aviv, Israel	p. 358

2015

January 10-13	San Antonio, Texas	p. 358
	Annual Meeting	
June 11-14	Porto, Portugal	p. 358

2016

January 6-9	Seattle, Washington	p. 358
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Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 111 in this issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of L^AT_EX is necessary to submit an electronic form, although those who use L^AT_EX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in L^AT_EX. Visit <http://www.ams.org/cgi-bin/abstracts/abstract.pl>. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see <http://www.ams.org/meetings/> for the most up-to-date information on these conferences.)

February 16-20, 2012: AAAS Meeting in Vancouver, British Columbia, Canada (please see www.aaas.org/meetings for more information.)

March 11-14, 2012: Fourth International Conference on Mathematical Sciences, United Arab Emirates (held in cooperation with the AMS). Please see <http://icm.uaeu.ac.ae/> for more information.

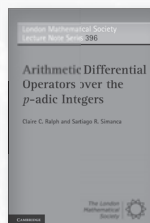
June 10-June 30, 2012: MRC Research Communities, Snowbird, Utah. (Please see <http://www.ams.org/amsmtgs/mrc.html> for more information.)

MATHEMATICS TITLES *from* CAMBRIDGE!

Algebraic Shift Register Sequences

Mark Goresky *and* Andrew Klapper

\$85.00: Hb: 978-1-107-01499-2: 520 pp.

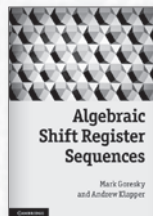


Arithmetic Differential Operators over the p -adic Integers

Claire C. Ralph *and* Santiago R. Simanca

London Mathematical Society Lecture Note Series

\$65.00: Pb: 978-1-107-67414-1: 220 pp.



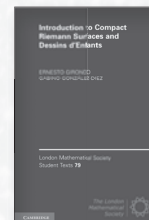
Introduction to Compact Riemann Surfaces and Dessins d'Enfants

Ernesto Gironde *and* Gabino González-Diez

London Mathematical Society Student Texts

\$115.00: Hb: 978-0-521-51963-2: 312 pp.

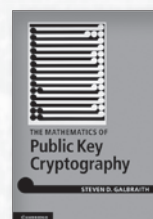
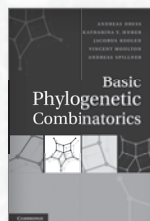
\$39.99: Pb: 978-0-521-74022-7



Basic Phylogenetic Combinatorics

Andreas Dress, Katharina T. Huber, Jacobus Koolen, Vincent Moulton, *and* Andreas Spillner

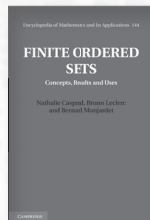
\$65.00: Hb: 978-0-521-76832-0: 276 pp.



The Mathematics of Public Key Cryptography

Steven D. Galbraith

\$70.00: Hb: 978-1-107-01392-6: 600 pp.



Finite Ordered Sets Concepts, Results and Uses

Nathalie Caspard, Bruno Leclerc, *and* Bernard Monjardet

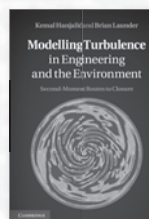
Encyclopedia of Mathematics and its Applications

\$90.00: Hb: 978-1-107-01369-8: 360 pp.

Modelling Turbulence in Engineering and the Environment Second-Moment Routes to Closure

Kemal Hanjalic *and* Brian Launder

\$130.00: Hb: 978-0-521-84575-5: 402 pp.



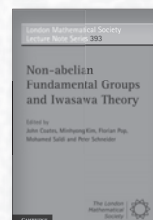
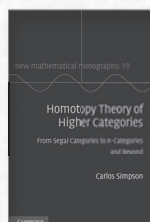
Homotopy Theory of Higher Categories

From Segal Categories to n -Categories and Beyond

Carlos Simpson

New Mathematical Monographs

\$105.00: Hb: 978-0-521-51695-2: 652 pp.

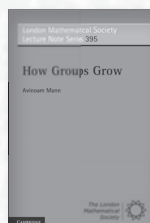


Non-abelian Fundamental Groups and Iwasawa Theory

Edited by John Coates, Minhyong Kim, Florian Pop, Mohamed Saïdi, *and* Peter Schneider

London Mathematical Society Lecture Note Series

\$90.00: Pb: 978-1-107-64885-2: 320 pp.



How Groups Grow

Avinoam Mann

London Mathematical Society Lecture Note Series

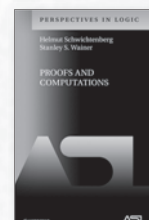
\$60.00: Pb: 978-1-107-65750-2: 210 pp.

Proofs and Computations

Helmut Schwichtenberg *and* Stanley S. Wainer

Perspectives in Logic

\$90.00: Hb: 978-0-521-51769-0: 480 pp.

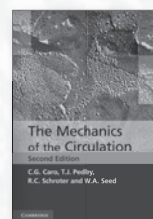
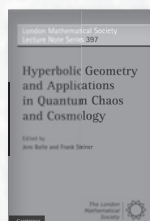


Hyperbolic Geometry and Applications in Quantum Chaos and Cosmology

Edited by Jens Bolte *and* Frank Steiner

London Mathematical Society Lecture Note Series

\$70.00: Pb: 978-1-107-61049-1: 284 pp.



Second Edition

The Mechanics of the Circulation

C. G. Caro, T. J. Pedley, R. C. Schroter, *and* W. A. Seed

Prepared for publication by K. H. Parker

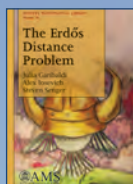
\$49.00: Pb: 978-0-521-15177-1: 552 pp.

Prices subject to change.



Recent Titles from

THE STUDENT MATHEMATICAL LIBRARY SERIES

**The Erdős Distance Problem**

Julia Garibaldi, Alex Iosevich, *University of Rochester, NY*, and Steven Senger, *University of Missouri-Columbia, MO*

This book...achieves the remarkable feat of providing an extremely accessible treatment of a classic family of research problems. ...The book can be used for a reading course taken by an undergraduate student (parts of the book are accessible for talented high school students as well), or as introductory material for a graduate student who plans to investigate this area further...Highly recommended.

—M. Bona, *Choice*

A problem-oriented analysis of results related to the classical Erdős problem, illustrating the relationships among several areas of mathematics

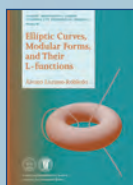
Student Mathematical Library, Volume 56; 2011; 150 pages; Softcover; ISBN: 978-0-8218-5281-1; List US\$29; AMS members US\$23.20; Order code STML/56

**Probability Tales**

Charles M. Grinstead, *Swarthmore College, PA*, William P. Peterson, *Middlebury College, VT*, and J. Laurie Snell, *Dartmouth College, Hanover, NH*

An in-depth examination of four popular real-world topics that illustrate the elements of probability theory

Student Mathematical Library, Volume 57; 2011; 237 pages; Softcover; ISBN: 978-0-8218-5261-3; List US\$42; AMS members US\$33.60; Order code STML/57

**Elliptic Curves, Modular Forms, and Their L-functions**

Álvaro Lozano-Robledo, *University of Connecticut, Storrs, CT*

A view of the surprising connections among three families of mathematical objects and their meaning for number theory

Student Mathematical Library, Volume 58; 2011; 195 pages; Softcover; ISBN: 978-0-8218-5242-2; List US\$37; AMS members US\$29.60; Order code STML/58

**Introduction to Representation Theory**

TEXTBOOK

Pavel Etingof, *Massachusetts Institute of Technology, Cambridge, MA*, Oleg Golberg, *Sebastian Hensel, Universität Bonn, Germany*, Tiankai Liu, *Massachusetts Institute of Technology, Cambridge, MA*, Alex Schwendner, *Two Sigma Investments, New York, NY*, Dmitry Vaintrob, *Harvard University, Cambridge, MA*, and Elena Yudovina, *University of Cambridge, United Kingdom*

with historical interludes by Slava Gerovitch, *Massachusetts Institute of Technology, Cambridge, MA*

The goal of this book is to give a “holistic” introduction to representation theory

The goal of this book is to give a “holistic” introduction to representation theory

Student Mathematical Library, Volume 59; 2011; 228 pages; Softcover; ISBN: 978-0-8218-5351-1; List US\$42; AMS members US\$33.60; Order code STML/59

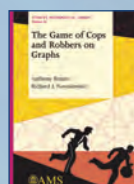
**Mostly Surfaces**

TEXTBOOK

Richard Evan Schwartz, *Brown University, Providence, RI*

Presents a number of interesting topics related to surfaces in an informal and friendly way

Student Mathematical Library, Volume 60; 2011; 314 pages; Softcover; ISBN: 978-0-8218-5368-9; List US\$47; AMS members US\$37.60; Order code STML/60

**The Game of Cops and Robbers on Graphs**

Applied Mathematics

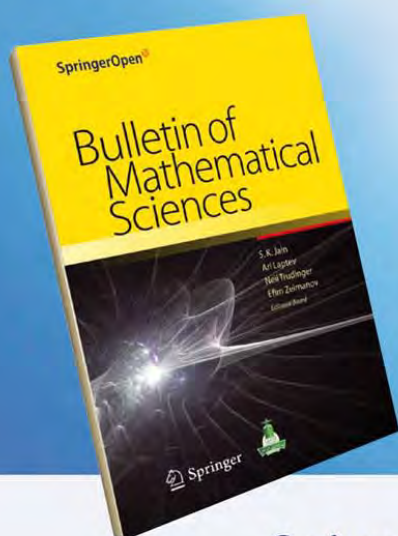
Anthony Bonato, *Ryerson University, Toronto, ON, Canada*, and Richard J. Nowakowski, *Dalhousie University, Halifax, NS, Canada*

The first and only book on Cops and Robbers games and on vertex pursuit games on graphs

Student Mathematical Library, Volume 61; 2011; 276 pages; Softcover; ISBN: 978-0-8218-5347-4; List US\$45; AMS members US\$36; Order code STML/61



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Aims and Scope

The Bulletin of Mathematical Sciences, a peer-reviewed open access journal, will publish original research work of highest quality and of broad interest in all branches of mathematical sciences. The Bulletin will publish well-written expository articles (40–50 pages) of exceptional value giving the latest state of the art on a specific topic, and short articles (about 10 pages) containing significant results of wider interest. Most of the expository articles will be invited.

Editorial Board

S. K. Jain (Algebra, Pure Mathematics), Ari Laptev (Analysis, Applied Mathematics), Neil Trudinger (Differential Equations, Applied Mathematics), Efim Zelmanov (Algebra, Pure Mathematics)

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Associate Editors

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Forthcoming articles include:

- Splines and index theorem, by C. Procesi
- The Möbius function and statistical mechanics, by F. Cellarosi and Ya. G. Sinai
- Majorana representation of A_6 involving 3C-algebras, by A. A. Ivanov
- On braided zeta functions, by S. Majid and I. Tomašić

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