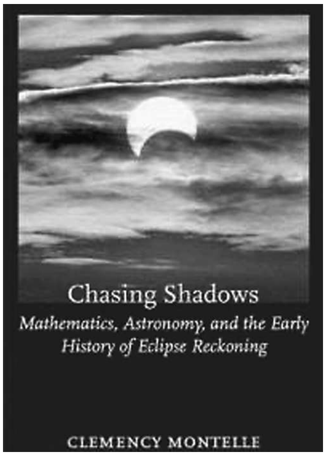

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning

Reviewed by Christopher Linton

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning
Clemency Montelle
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It is not hard to appreciate the enormous impact that a solar eclipse must have had on those witnessing such an event in times when the cause of eclipses was not understood. Lunar eclipses are much less dramatic but much more common (as far as an individual observer is concerned), and even here it is easy to imagine the sense of fear and wonder that they would have generated. The facts that a great deal of mythology grew up around eclipses, that people sought to be able to predict when they would occur, and that there was a desire to understand the causes behind the phenomena thus come as no surprise. But the details of how mathematics displaced superstition and how our physical understanding of the cosmos supplanted tradition and folklore are less well known.

There is an extensive literature on the history of ancient mathematical astronomy, much of it rather technical, and the contribution and influence of Otto Neugebauer (1899–1990) cannot be overstated. (Incidentally, Neugebauer founded *Mathematical Reviews* in 1940.) Neugebauer's monumental *History of Ancient Mathematical Astronomy*, published in three large volumes in 1975, set the standard for future scholarship and research. Neugebauer founded the History of Mathematics department at Brown University in 1947, and his and future generations of students have continued the program that he began. The author, Clemency Montelle, is a direct descendant in this line, having been supervised in her doctoral studies by Neugebauer's student David Pingree.

It was the Greeks who first understood eclipses as being caused by the spatial alignment of three bodies, and it is worth describing this geometry so as to introduce some terminology. The Moon's form changes over a period of about $29\frac{1}{2}$ days from thin crescent to full moon back to thin crescent again and then disappears for two or three nights. This time period is known as a lunation or synodic month, and it formed the basis of many ancient calendars. When the Moon is full, it is diametrically opposed to the Sun and we say that the Sun and the Moon are in opposition; whereas when the Sun and the Moon are in the same direction, at new moon, they are said to be in conjunction. The Moon's orbit is inclined to the plane of the Earth's orbit around the Sun (the ecliptic plane), and it crosses this plane at two points called the nodes, labelled as ascending or descending depending on whether the Moon is moving from south to north or vice versa. A lunar eclipse occurs when the Moon is in opposition to the Sun and simultaneously at (or near) one of its nodes. Similarly, a solar eclipse occurs when the Moon's passage through one of its nodes corresponds to a conjunction with the Sun.

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The period between successive ascending (or descending) nodes is known as the draconitic month, the word deriving from the Greek for dragon. In medieval times that part of the Moon's orbit south of the ecliptic was known as the dragon (which devoured the Moon during eclipses), and from this we get the terminology "dragon's head" for the ascending node and "dragon's tail" for the descending node. The speed with which the Moon moves across the sky relative to the stars varies between about 12° and 15° per day, and the period of time it takes for the Moon to return to the same speed is called the anomalistic month. This period is related to the elliptical nature of the lunar orbit and hence to variations in the apparent diameter of the Moon (which has a significant effect on eclipse phenomena).

Montelle's book focuses on the contributions of four different cultures to the understanding of eclipse phenomena: those of the ancient Near East, ancient Greece, India, and the Islamic Near East, all of which played a key role in shaping the broader tradition of "Western" astronomy. Attention is also given to the transmission of ideas and practice between the distinct scientific environments that existed. It is significant that the space devoted to Indian eclipse reckoning is more than double that afforded to any of the other cultures, which suggests that readers will find much more here than simply a rehash of material that is readily available elsewhere, and this is indeed the case. Most of the research which underpins the book was done during the past fifty years, and it is no exaggeration to say that our understanding of the achievements of Mesopotamian, Indian, and Islamic scholars has been transformed during that period. For example, Anton Pannekoek's classic *History of Astronomy*, originally published in 1951, brushes over Indian contributions in a couple of paragraphs within a short chapter covering the whole of Arabic astronomy.

Beginning in the ancient Near East, the omen compendium *Enūma Anu Enlil*, compiled roughly 4,000 years ago, contains about 7,000 omens, both auspicious and inauspicious, in the form *if* [something is observed], *then* [something will happen], which cover both astronomical and meteorological phenomena. Eclipses are a prominent theme, and many aspects of these phenomena are considered: time of impact, duration, even color. What is of particular interest though, from the perspective of the development of understanding, is the recognition that some aspects of eclipses are periodic (or at least roughly so) and therefore that they may plausibly be predicted. By the seventh century BCE, the facts that lunar eclipses can occur only on the fourteenth or fifteenth days of each lunation (counting from the new moon) while solar eclipses occur on the twenty-ninth or thirtieth day appear to have been established. Similarly, it was observed that lunar eclipses are spaced at six-monthly intervals, with occasional deviations from this rule.

The real flowering of the mathematical approach to heavenly phenomena in the ancient Near East came in the Seleucid era (311–125 BCE). The astronomers of that period had access to records of eclipses dating back over many centuries, and they appreciated that this was a rich source of data from which to extract patterns that could then be manipulated into predictive tools. The nature of the deviations from the six-month separation between lunar eclipses suggested that a more sophisticated cycle could be constructed for times at which eclipses might occur; in this cycle, a succession of six-month intervals was interspersed with an occasional five-month gap. This is particularly impressive, as it is now known that at no time during the centuries that these observations were made was there a pair of lunar eclipses separated by five months where both eclipses were potentially visible to the observers. The most notable such pattern is the so-called Saros cycle, containing 223 months and 38 eclipse possibilities, the accuracy of which is related to the rough equivalence between 223 synodic months, 242 draconitic months, and 239 anomalistic months. With modern average values, we have, in days,

$$\begin{aligned}223 \times 29.531 &\approx 242 \times 27.212\\ &\approx 239 \times 27.555 \approx 6585\tfrac{1}{3}.\end{aligned}$$

Montelle provides a detailed description of typical astronomical texts that were produced by Babylonian astronomers during the last three centuries BCE, showing how combinations of simple periodic schemes were used to model many recurring astronomical phenomena, including eclipses. Two distinct approaches were used: so-called system A, in which the Sun's nonuniform motion was modelled as a periodic step function; and the more sophisticated system B, in which alternate increasing and decreasing arithmetic progressions (often illustrated as a zig-zag function) were employed. The level of sophistication and predictive power of these techniques is hugely impressive, especially when one considers that there appears to have been no attempt to understand the causes of eclipses or to use a geometrical picture of the heavens to guide enquiry. It was ancient Greek thinkers who took on this challenge, but the legacy of observational records, astronomical conventions, and arithmetical techniques that the Greeks inherited from Mesopotamia were invaluable and instrumental to their success.

From a mathematical perspective, one of the most significant developments that arose from attempts to create a geometrical picture of the heavens was the emergence of what we now call trigonometry, beginning with the work of Hipparchus in the second century BCE. By the time Ptolemy wrote the *Almagest* three hundred years or so later, techniques of plane and spherical trigonometry were well understood. The *Almagest* includes complete geometrical models for the motion of the Sun, the Moon, and the planets, including tables with instructions for their use as well as details of their construction, and it dominated astronomical thought for more than a millennium. In terms of eclipses one of the key barriers to accurate predictions is the concept of parallax (the effect of the fact that the observer is displaced from the centre of the Earth), and the first detailed and complete account of parallax computations that we have is Ptolemy's.

The *Almagest* is an awe-inspiring work, but it is hardly user friendly, and Ptolemy appears to have appreciated this, since he later produced a much more practical reference for astronomers in the form of the *Handy Tables*. It appears that this handbook was constructed so as to be particularly useful in computing eclipses. The *Handy Tables* was commented on, reproduced, and modified many times over the centuries so as to adapt the contents to users' needs, but challenges to the fundamental theoretical basis on which it was built took a long time to materialize.

Outside of scholarly journals, relatively little has been written about Indian astronomy and virtually nothing which addresses in any depth the technical details contained within the considerable corpus of Sanskrit writings on the subject. *Chasing Shadows* is thus a significant contribution. Indian astronomy was built on a wealth of sources from other civilizations, initially Mesopotamian and Greek, but this material was expanded, modified and molded into a distinctly different cultural tradition. The format in which Indian astronomical works were preserved is certainly noteworthy, as they were written in verse. This would appear to have been to make them easier to memorize, as they were to be used in a largely oral environment, but as Montelle points out, there was a price to pay for this "scientific poetry". The author was forced to exclude key material or to use rather obscure expressions so as to conform to the strict metrical rules of the verse. For example, two systems were devised to represent numbers: the *bhūtasaṅkhyā* system, in which common objects were associated with numbers (eye = 2, gods = 33, for example), and the *kaṭapayādi* system, in which each number was assigned a set of letters so that meaningful words could be created to represent strings of numbers.

As well as a general discussion covering the various different Indian schools of astronomical thought that existed during the millennium from about 400 CE, the significance and impact of their work, and the general features of eclipse reckoning in India, Montelle gives a detailed description of a number of texts. For example, the *Pañcasiddhāntikā* of Varāhamihira (sixth century), a compilation of five earlier works, shows clear evidence of Greek and Babylonian sources, but the level of sophistication of the rules for accounting for parallax (which do not use the spherical trigonometry found in Ptolemy) compared with that in later Indian works shows that the transmission of ideas from the Near East to India was not limited to a single period in history. Other texts include Āryabhaṭa's *Āryabhaṭīya* (ca. 500 CE), the *Brāhmasphuṭasiddhānta* and the *Khaṇḍakhādyaka*, written about forty years apart during the seventh century by Brahmagupta, and Vaṭeśvara's *Vaṭeśvarasiddhānta* (ca. 900 CE). Montelle's elucidation of these texts is accompanied by numerous transliterations and translations of the original Sanskrit, many by the author herself.

Early trigonometry was built around the Greek chord function. Given a circle of radius R, the chord is related to the modern sine function via the relation $\operatorname{crd} 2\alpha = 2R \sin\alpha$. In particular, it is a length rather than a ratio. It was Indian astronomers who recognized that a lot of time could be saved in calculations by tabulating the half-chords of double angles ($\frac{1}{2}\operatorname{crd} 2\alpha$), a recognition that ultimately led to the modern sine function. They also introduced functions related to the modern cosine function and the now largely defunct versine ($\operatorname{versin}\alpha = 1 - \cos\alpha$). However, while these and a number of other technical improvements were implemented, there was little attempt to understand and improve the underlying models and parameters until perhaps the work of Parameśvara and the astronomical school that grew up in the southwestern region of Kerala in the fourteenth and fifteenth centuries.

Early Indian mathematical astronomy did exert a significant influence on the Islamic empire that grew up around the Mediterranean and flourished between the eighth and fourteenth centuries. The unifying feature of the scholarly tradition that emerged was the Arabic language, and a massive translation program was initiated very soon after the establishment of a new capital at Baghdad. Over a period of more than two hundred years, Indian, Persian, and, most significantly, Greek works were translated and assimilated. The *Almagest* was translated several times—the definitive version for the time being that of Isḥāq ibn Ḥunayn, as revised by Thābit ibn Qurra in the late ninth century—and served as the bedrock of Islamic astronomy.

Many astronomers produced works, each known as a *zīj*, a word of Persian origin which was used originally to mean a set of astronomical tables (of which the *Handy Tables* was the prototypical example) but which was later used to refer to any astronomical treatise. Thousands of these works survive to this day, most unedited and/or untranslated. Indian mathematical advances, such as their trigonometric functions, the decimal place value system, and the concept of zero, were quickly adopted, as were some other aspects of Indian astronomy, notably techniques for the calculation of the effects of parallax. Taken alongside Ptolemy's work, Islamic scholars were then equipped with the necessary tools to develop a sophisticated and accurate theory of eclipses. Equally significantly, they appreciated that judicious simplifications to the complicated machinery that had been developed could reduce computational effort (and hence increase utility) with minimal loss of accuracy.

Montelle discusses the contributions to Islamic eclipse reckoning of figures such as al-Khwārizmī (ninth century), from whose name we get the word algorithm and whose work was very much influenced by Brahmagupta and Vaṭeśvara, and Naṣīr al-Dīn al-Ṭūsī (thirteenth century), who was one of the most prominent figures in Islamic intellectual history and whose Ṭūsī couple (a geometrical device which produced motion in a straight line from a combination of circular motions) undermined the Aristotelian distinction between circular heavenly motion and rectilinear terrestrial motion.

The stated primary purpose of this book is to shed light on the ways in which knowledge about eclipses was originated, developed, preserved, and transmitted. Measured against this criterion, it is certainly a success. Parts of the work are heavy going—there is a limit to how many techniques for calculating the effect of parallax one can absorb at a single sitting—but one cannot fail to be impressed by Montelle's mastery of the subject matter and her commitment to increasing our understanding of such a rich history.