JUMP Math: Multiplying Potential

John Mighton

When people complain about problems in American education, they often speak as if those problems would be solved if students in the U.S. were able to perform as well on international tests of reading and mathematics as students from countries that achieve the highest scores. Nations like Finland and Singapore are singled out in the media as having superior educational systems because their students do better on tests like PISA and TIMMS.

It's worth looking at the results of these tests closely, but more for what they reveal about our beliefs about children and their potential than for what the tests prove about education. From the way people talk about the tests, you can see clearly what they expect the average child to achieve at school.

In 2006 only 10 percent of American students scored above level 5 in mathematics on the PISA tests (this is the level of proficiency required to take courses involving math at university), compared to 30 percent in top-performing countries such as Taiwan, Hong Kong, South Korea, and Finland. However, in each of the top performing countries, roughly 40 percent of students scored at level 3 or below. Students at level 3 would have trouble holding a job that required fairly basic mathematics.

Many people have suggested that American educators should find out how math is taught in the top-performing countries so it can be taught in the same way in the U.S. I expect this is a good idea, but we might also want to find out how countries that produce such strong students still manage to teach so little to almost half their populations. Answering this question might do as much to help the U.S. improve the teaching of mathematics as any efforts to emulate the educational practices of other countries.

Wide differences in mathematical achievement among students appear to be natural: in every school in every country only a minority of students are ever expected to excel at or love learning mathematics. In the many schools I have visited on several continents, I've always seen a significant number of students who are two or three grade levels behind by grade five. In my home province of Ontario, where children do rather well on international tests, only 58 percent of grade-six students met grade-level standards on the provincial exams last year.

Fourteen years ago I started a charity called JUMP Math in my apartment because I wanted to help students who struggle in math. The first JUMP students were referred by local schools and were matched with volunteer tutors. Most of these students had serious learning disabilities and were years behind in math, so I believed that the best way to help them was to provide them with one-on-one instruction. But JUMP soon outgrew my apartment, and teachers in schools where it was offered began to ask me to teach some lessons in their classrooms. In my first lessons I was surprised to see that the weakest students often became more engaged in the classroom than they did in tutorials—they loved putting up their hands and coming up to the board when the lesson was taught in a way that they could understand.

In designing lessons that would work for the whole class, I had to learn to break explanations and challenges into small steps so students who were initially weaker could experience success, to provide adequate review and practice for those who needed it, and to raise the level of difficulty incrementally so children would get more excited and their brains would work efficiently. I soon began to design special "bonus" questions that didn't introduce any new skills or vocabulary so faster students could independently explore small variations on the concepts they had learned while I spent time with students who needed extra help. As weaker students became more confident and attentive, they began to work much more quickly so they could get their bonus questions too. Their excitement at succeeding in front of their peers seemed to greatly increase their rate of learning.

It was clear that teachers didn't have time to develop lessons of this type, so JUMP hired a team of mathematicians and educators to help me write online teachers' guides that cover the full curriculum from grades one to eight in great detail.

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JUMP is now used by about 100,000 students in Canada and the U.S. as their main resource for mathematics for grades one to eight. In the U.S. many school boards are piloting versions of our materials that are aligned to the Common Core State Standards.

In a randomized controlled study presented at the Society for Research in Child Development in 2011, cognitive scientists Tracy Solomon and Rosemary Tannock from the Hospital for Sick Children and the University of Toronto found that students from eighteen regular classrooms using JUMP showed twice the rate of progress on a number of standardized tests of math ability as students receiving standard instruction in eleven other classrooms. As randomized controlled studies rarely show such striking differences between students in different math programs, the U.S. Department of Education has funded a much larger multiyear study by the same team.

Based on my observations of thousands of students and on data gathered in studies of JUMP (see `jumpmath.org` for a summary of these studies), I am convinced that the vast majority of students have far more potential to learn and enjoy learning math than they exhibit at school. To fully appreciate the extent of this hidden potential and of the losses that we incur as a society when we fail to nurture this potential, it helps to consider a case study.

In the fall of 2007 fifth-grade teacher Mary Jane Moreau of Mabin School in Toronto gave her students a standardized math assessment called the Test of Mathematical Abilities (TOMA). The class average was in the 54th percentile, with a wide range of scores, including one student who ranked as high as the 75th percentile and another at just the 9th percentile. A fifth of the pupils in the class were identified as learning disabled. After testing her students, Mary Jane abandoned her usual teaching approach (which meant pulling together lessons with the best materials she could find) and followed the JUMP lesson plans with fidelity. After a year of JUMP, the average score of her students on the grade-six TOMA rose to the 98th percentile, with the lowest mark in the 95th percentile. At the end of grade six, Mary Jane's entire class signed up for the Pythagoras Math competition, a prestigious contest for sixth-graders. One of the most able students was absent on the day of the exam, but of the seventeen who participated, fourteen received awards of distinction (with the other three close behind). Students who write the Pythagoras competition are almost all in the top five percentile in achievement, but the average score for students in this (initially unremarkable) class was higher than the average for students writing the Pythagoras.

The most challenged ten-year-old student in Mary Jane's class improved her score on the TOMA from the 9th percentile to the 95th percentile after only one year of JUMP. But ten-year-old brains are more developed and less plastic than four-year-old brains, so grade five is not the ideal grade for an intervention. It seems reasonable to assume that Mary Jane's student could have achieved much more in grade five if she had been enrolled in a math program as good as or better than JUMP from an early age. Indeed, if every child were taught according to their true potential from the first day of school, then I would predict that by grade five the vast majority of students (over 95 percent) could learn and love learning as much as the top one or two percent do now.

I should point out that this is not a prediction about JUMP, as it requires that children be taught "according to their true potential." JUMP has produced some extremely strong results in pilots and studies, but the program may not, in its present form, produce the results I think are possible. JUMP has partnered with many distinguished cognitive scientists and educational researchers to try to determine what works in our approach and what needs to be improved. Better programs than JUMP will certainly be developed, and JUMP itself will continue to evolve. I hope that readers will not allow any doubts they have about JUMP in its present form to distract them from considering what may be possible for children in the future.

In the randomized controlled study, teachers used JUMP with varying degrees of fidelity but still managed to double the average rate of progress of their students. I expect the results of the study would have been stronger if every teacher had followed the program with fidelity. But even if I am wrong about how effective JUMP can be when it is implemented properly, my beliefs about what children can achieve are likely to be true, as they are well supported by independent evidence from cognitive science. One day this evidence will be more widely known, and educators will be inspired to set higher expectations for students and schools, whether or not they use particular programs such as JUMP.

The methods on which JUMP is based are ones that cognitive scientists are now promoting for the development of expertise in general. In "The expert mind", an article that appeared in *Scientific American* in July 2006, Philip Ross examines the implications of a century of research on how experts develop abilities in chess and other fields and how the expert mind processes and receives information. His conclusions lend strong support to the notion that abilities can be nurtured in students through rigorous instruction and practice:

> The preponderance of psychological evidence indicates that experts are made, not

> born. What is more, the demonstrated ability to turn a child quickly into an expert—in chess, music and a host of other subjects—sets a clear challenge before the schools. Can educators find ways to encourage students to engage in the effortful study that will improve their reading and math skills? Instead of perpetually pondering the question "Why can't Johnny read?" perhaps educators should ask: "Is there anything in the world he can't learn to do?"

H. Wu has warned against drawing false dichotomies in math education (for instance, between concepts and deep understanding versus procedures and algorithms). One dichotomy is particularly damaging to students: the false opposition between "explicit" or "direct instruction" versus "discovery" or "student-centered" instruction. Current research in cognitive science suggests that effective lessons should combine elements of both approaches. In 2011 A. Alfieri et al. conducted a meta-analysis of 164 studies of discovery-based learning and concluded that "unassisted discovery does not benefit learners," whereas discovery combined with "feedback, worked examples, scaffolding and elicited explanations do[es]." An effective lesson can be student-centered but still led by the teacher.

Research in cognitive science suggests that, while it is important to teach to the strengths of the brain (by allowing students to explore and discover concepts on their own), it is also important to take account of the weaknesses of the brain. Our brains are easily overwhelmed by too much new information, we have limited working memories, we need practice to consolidate skills and concepts, and we learn bigger concepts by first mastering smaller component concepts and skills.

Teachers are often criticized for low test scores and failing schools, but I believe that they are not primarily to blame for these problems. For decades teachers have been required to use textbooks and teaching materials that have not been evaluated in rigorous studies. As well, they have been encouraged to follow many practices that cognitive scientists have now shown are counterproductive. For example, teachers will often select textbooks that are dense with illustrations or concrete materials that have appealing features because they think these materials will make math more relevant or interesting to students. But psychologists such as Jennifer Kaminski have shown that the extraneous information and details in these teaching tools can actually impede learning.

To improve their practice, teachers must be made aware of the growing body of research in cognitive science that shows that higher-level abilities are grounded in practice and the acquisition of basic skills and knowledge and that overly complex lessons can overwhelm the brain. They must be allowed to innovate and test methods that are supported by solid research, and they must never be compelled to adopt programs that have not been rigorously evaluated.

The JUMP method is called "guided discovery". In a JUMP lesson students develop and explore ideas on their own, but the lesson is a carefully scaffolded series of questions and challenges in which one idea naturally leads to the next. Students are provided with many supports of the kind that research has identified as effective, such as immediate feedback and worked examples. They are also given many opportunities to practice and consolidate concepts and are assessed frequently so they can get excited about their success and so the teacher can be sure no one is falling behind.

Some lessons in JUMP allow for more open-ended exploration, but here is an example of a structured lesson on long division. I have found that this approach enables kids to both discover the steps of the algorithm and understand the underlying concepts while learning to perform the algorithm proficiently.

I tell students that the notation $3\overline{)72}$ can be interpreted to mean: 3 friends wish to share 7 dimes and 2 pennies (72 cents) as equally as possible. I then ask students to draw a picture to show how they would divide the dimes among the friends. If students use a circle for each friend and an X for each dime, the diagram would look like this:

XX XX XX X

I ask students to tell me the meaning of their diagram: Each friend gets two dimes and there is one dime left over. I then tell students that if they happened to see someone carrying out the first few steps of the long division algorithm, this is what they would see:

$$\begin{array}{r} 2 \\ 3\overline{)72} \\ \underline{-6} \\ 1 \end{array}$$

I challenge students to figure out what the steps in the algorithm mean by identifying where they see each number in their diagram. Students readily make the following connections between their diagram and the algorithm:

$$\begin{array}{rl} 2 & \Leftarrow \text{ each friend got two dimes} \\ 3\overline{)72} & \\ \underline{-6} & \Leftarrow \text{ 6 dimes were given away altogether} \\ 1 & \Leftarrow \text{ there was 1 dime left over} \end{array}$$

I ask students to complete their diagram to show me how much money still has to be divided among the friends. If students use a circle to represent a penny, their diagram looks like this:

XX XX XX XOO $\Leftarrow$ 1 dime and 2 pennies haven't been given out yet

I invite three students to come to the front of the class so I can demonstrate how I would divide the remaining coins among the three friends. I give two students a penny each and one student a dime. The students always protest that my way of dividing up the coins isn't fair: they tell me they would exchange the dime for ten pennies and divide the twelve pennies among the friends. I inform students that this process of "regrouping" the tens (dimes) as ones (pennies) is actually a step in the long division algorithm. Most adults call this the "bring down" step, but very few understand it:

$$\begin{array}{r l} 2 & \\ 3\overline{)72} & \\ -6 & \\ \hline 12 & \Leftarrow \text{ when you "bring down"} \end{array}$$

…the number in the ones (pennies) column, you implicitly change the number in the tens (dimes) column into the smaller unit (pennies). Then you combine all of your smaller units (to give twelve pennies altogether).

I then ask students to show me in their diagrams how they would divide the (twelve) remaining pennies among the friends. I also ask them to connect the numbers in their diagram with the remaining steps of the algorithm:

XXOOOO XXOOOO XXOOOO

$$\begin{array}{r l} 24 & \Leftarrow \text{ each friend received four pennies (24 cents altogether)} \\ 3\overline{)72} & \\ -6 & \\ \hline 12 & \\ -12 & \Leftarrow \text{ twelve pennies were given out altogether} \\ \hline 0 & \Leftarrow \text{ no pennies were left over} \end{array}$$

At each step in this process I give students several practice questions so I can verify that they understood the step.

Mary Jane loved teaching math and was recognized as an excellent teacher before she started using JUMP. But after reading the JUMP Teachers' Guides, she said she realized that many of the concepts she had previously taught in one step actually involved two or three steps or required skills or knowledge that she didn't normally assess or teach. She found that the more closely she followed the guides the better her students did.

Research has shown that many elementary teachers (unlike Mary Jane) are mathphobic or have very rudimentary knowledge of math. The JUMP writers and I wrote the guides, in part, because we saw that schools could not afford to provide enough professional development for teachers to make up for these deficits. In following the online lesson plans, teachers learn the math as they teach. Many have become excited about their new understanding of the subject and have formed volunteer networks to support and mentor other teachers. Two mathphobic teachers in a Vancouver network recently completed master's degrees in math education after they were inspired by their success with the program.

The principles on which JUMP lessons are built (adequate review and practice, rigorous scaffolding, continuous assessment, incrementally harder challenges, and differentiated instruction) are not new or even controversial in education, although we have tried to apply these principles with a great deal of rigor. If there is anything different about JUMP, it may lie in the belief that extreme hierarchies of ability are caused, at least in part, by the presumption that these hierarchies are natural.

Children are unlikely to fulfill their potential in math until math programs are designed to take into account the way academic hierarchies can inhibit learning. As early as grade one, children begin to compare themselves to their peers and identify themselves as "smart" or "dumb" in subjects such as math. When children decide they aren't talented in math, their brains work less efficiently: they stop paying attention, taking risks, and persevering in the face of difficulty, and they often develop anxieties or behavioral problems. By making all of her students feel capable from the first day of school, Mary Jane was able to produce a class of students who were, to a surprising degree, equally capable.

No method of teaching is likely to produce a school full of students who all have exactly the same capacity for success, but the results of teachers like Mary Jane suggest that students have far more potential in math than they exhibit at school. To bring about significant change in education, we must insist that every child has a right to fulfill their intellectual potential, just as they have a right to develop healthy bodies. We don't have to wait until we have recruited an army of superhuman teachers or invented some miraculous new technology to guarantee this right. We already have the teachers we need to transform our schools. We simply need to give them the means to teach children using effective methods that are backed by rigorous evidence.