The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day

Reviewed by Andrew I. Dale

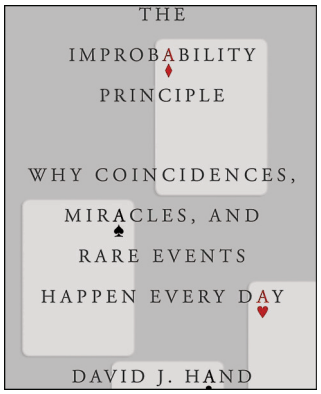
The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day
David J. Hand
Scientific American/Farrar, Straus and Giroux
288 pages, US$17.08
ISBN-13: 978-0374175344

The only law of chance is this:
extremely unlikely phenomena do not occur.

Thus writes Émile Borel in his *Les Probabilités et la Vie* [3, p. 5]. But, as we all know, such events *do* occur; we have all had the experience of bumping into an old friend, long unseen, soon after we had thought of him or of reading that the whereabouts of the body of a missing person had been correctly located by a "psychic." And even a politician can be trusted at times—though Sir Thomas Browne, writing in 1646, spoke for many when he said that statists (i.e. those skilled and expert in state affairs) and politicians "do hold, that truth is to be concealed from them; unto whom although they reveal the visible design, yet do they commonly conceal the capital intention" [5, p. 139].

In his book *Éléments de la théorie des probabilités* of 1950, translated by Freund in 1965, Borel went even further, examining those cases in which "the probability is so small that it is practically negligible" [p. 57], with a further subdivision into probabilities negligible from the human perspective (a value of 10^{-6}), the terrestrial perspective (10^{-15}), and the cosmic perspective (10^{-45}).

The drawing of attention to events of small probability was by no means original to Borel: Jacob Bernoulli [2], Richard Price (in his appendix to [1]), and Georges-Louis Leclerc, Comte de Buffon [6] all had something to say on the matter, and the idea of events being "morally certain," "morally impossible," or "physically certain" had been introduced.



Antoine Augustin Cournot wrote, "*The physically impossible event is thus one whose mathematical probability is infinitely small*" (italics in the original) [7, p. 78]. Shafer and Vovk [11], in fact, distinguish between two forms of Cournot's Principle: the *strong*, according to which an event distinguished in advance of a single trial and of very small or zero probability will not occur on that specific trial, and the *weak*, according to which an event of very small probability will happen very rarely in repeated trials.

"Science," wrote Robert Louis Stevenson in his *Pulvis et Umbra* of 1888, "carries us into zones of speculation, where there is no habitable city for the

Andrew I. Dale is professor emeritus at the University of Kwazulu-Natal in Durban, South Africa. His email address is dale@ukzn.ac.za.
DOI: http://dx.doi.org/10.1090/noti1191

mind of man." It seems particularly fitting then that I. J. Good should have chosen the title *The Scientist Speculates* for an edited book. All the more fitting to our present review is the fact that Christopher Scott, in questioning the normality of anomalies in his contribution to Good's anthology, should have remarked that "if the laws of nature are constantly being broken, we should *expect* each person to find the particular anomaly he is looking for. I suggest a new law of nature: natural anomalies of all kinds occur with perceptible frequency" [9, p. 149]. In *The Improbability Principle*, and *pace* Stevenson, Hand has found a habitable city for speculation to dwell.

Hand's thesis is in a sense contrary to Borel's and even to the weak form of Cournot's: it is that not only do extremely unlikely events occur but that they keep on occurring, and are, in fact, commonplace. Much of his book is taken up with the advancement of cogent reasons for the occurrence of such rarities, and once such reasons have been advanced and accepted, the next question is how to take advantage of such occurrences.

The Improbability Principle is a thread woven of a number of strands, the main ones being the Laws of Inevitability, of Truly Large Numbers, of Selection, of the Probability Lever, and of Near Enough. But there is also discussion of lesser matters; for instance, in his second chapter, "A Capricious Universe," Hand discusses such things as superstitions, prophecies, gods and miracles, parapsychology, and Jung's synchronicity.

The reasons for superstitions are at times difficult to fathom. If we look at Sir Thomas Browne's *Pseudodoxia Epidemica* we may find his mention of the reliance of thieves on the right eye of a hedgehog, boiled in oil and preserved in a brazen vessel, to allow them to see in the dark or that the left testicle of a weasel "wrapt in the skin of a she Mule, is able to secure incontinency from conception" [1904, p. 167] to be written off as expressions of ignorance. But are such views any more ridiculous than the firm belief of some even in the twenty-first century that powdered rhinoceros horn excites a man's "amorous propensities" (to borrow a delicate phrase from Dr. Samuel Johnson)—a belief that has had dire and perhaps irreversible consequences for the rhinoceros population in Africa—or the belief that an infusion or ointment made of certain herbs or animal parts has the ability to render one impervious to bullets?

Hand mentions the predictions of Nostradamus as an example "of the ambiguity in prophecies" [p. 22]. I would suggest that one might also see therein a touch of hindsight, as followers of the apothecary/occultist might well exclaim after a certain important event has occurred, "Ah, but this was certainly foretold in Quatrain N in the Master's writings." "Coming events cast their shadows before," wrote Thomas Campbell in his poem *Lochiel's Warning*, though it is perhaps the opacity of the shadow that makes correct and precise prophecy so difficult.

Defined by Hand as "an inexplicable event (normally a welcome one) attributed to a god: a supernatural event" [p. 27], a miracle turns out to be perhaps less of a surprise than something almost to be expected. Events that seem rare, or at any rate extraordinary, are, Hand convincingly argues, seen on careful thought to be expected. (Perhaps what makes it a miracle, or at any rate worthy of my particular attention, is that it occurs to *me*. A similar view was expressed by Fisher [8, §7] on the "one chance in a million" of an occurrence.) The reader wanting to learn more about the relationship between miracles and statistics can consult Kruskal [10].

Almost everyone will be aware of experiments to investigate things like telepathy, clairvoyance, and ESP (of which J. B. S. Haldane once said, "I daresay it does happen, but it's still a damned intrusion on one's privacy" [9, p. 164]). Hand reports the result of a metastudy of 380 studies designed to see whether subjects could psychically influence the results of coin tossing. Although the result was not altogether negative, one interesting explanation proposed for this fact is that of publication bias, that is, that it is much easier to get positive results published than negative ones.

When it comes to psychic phenomena, the investigator may well approach the matter in a spirit of criticism (i.e. with a measure of initial unbelief) or with a firm belief in the existence of such things. Hand, for instance, notes that some investigators have suggested "that psychic powers are influenced by the attitudes of those carrying out the experiments, and are thus less likely to manifest themselves if observed with a critical eye" [p. 35]. However, in *New Horizons*, Vol. 1, No. 3 (January 1974) a group of psychic investigators came down strongly in favor of the importance of initial belief, concluding that "one does have to believe implicitly that the phenomena can happen, and will happen, and not be surprised when something unusual does happen" [p. 12]. Can one investigate such phenomena without initial beliefs either strongly "for" or "against"? A. K. Talbot, in his "Partly-baked Idea" in Good [9], concluded that one cannot, writing: "After many years during which I have studied and observed many strange, apparently parascientific, phenomena, I have come to the shocking conclusion that the psychical researcher, *while carrying out his investigation, cannot afford the luxury of a neutral and unbiased attitude* if he

wishes to advance beyond the vicious circle of endless repetition of half-satisfactory experiments which are the usual reward of the half-convinced experimenter" (italics in the original) [pp. 175–176].

Chapter 3 is concerned with a general discussion of chance. Here, among other things, we find a discussion of the butterfly effect: the idea here is that a (very) slight change in the initial conditions of a physical system (say) may lead to our being completely unable to say in what state the system will be after some time. For example, this year sees the centenary of the start of World War I, an explosion often seen as being set off by the assassination of Archduke Franz-Ferdinand of Austria. But what would have happened if the driver of the car in which the archduke was travelling had not taken a wrong turn after lunch on that fateful day?

The first main strand in the Improbability Principle, the Law of Inevitability, is discussed in Chapter 4. This is succinctly given by Hand as follows: "It's the simple fact that something must happen" [p. 75]. That is, if one has correctly taken account of all possible outcomes, some one of these, no matter how unlikely, must occur, a statement well illustrated with a discussion of "amazing" (repeated) wins by buyers of lottery tickets.

Another feature of lotteries is the quantity of tickets sold, and this brings Hand to a chapter on the Law of Truly Large Numbers (LTLN). Here is an example from my own experience (Hand has a similar one): I read in the paper recently of a woman (call her X) whose daughter (Y) managed to find her a secondhand copy of a book about Australia that X had been given in 1944 by her father (Z) on his return from the Second World War. Nothing surprising about that, but X lived in Australia, and the book (with Z's inscription) had been bought by Y from a bookshop, picked almost at random, in New York. The LTLN states that "with a large enough number of opportunities, any outrageous thing is likely to happen" [p. 84]. As one of his examples here, Hand considers the case of a large number of people (say 100,000) each throwing a die and supposes that one person throws a string of six 1's. This being obtained simply by chance, one should not be surprised if in a further six throws Fortunatus does *not* produce a sequence of identical numbers—an example of regression to the mean.

As another interesting example Hand discusses crashes of F-14 fighter airplanes in the mid-1990s. Using scan statistics (with a month-long window over the period from 1970 to 2006 and an appropriate Poisson distribution) he shows that "the chance of having three crashes within some month over a five-year period is well over one-half" [p. 99].

The selection of data after the event of interest has occurred (e.g. the attention that is given to a psychic, one of whose predictions has materialized, while no attention is given to the many predictions she has made which were not fulfilled) comprises the Law of Selection. The importance of taking account not only of things that occur but also of things that have not occurred has long been realized. Pierre-Simon Laplace, in writing of astrologers, diviners, and augurs in the first edition of his *Essai philosophique sur les probabilités*, drew the attention of mathematicians to this when he wrote, "Nobody reflects on the large number of non-coincidences that have made no impression, or that are unknown. However, it is only the ratio of one to the other that can give the probability of the causes that one attributes to the coincidences" [1814, p. 77] (the wording changed slightly in subsequent editions).

Another important example concerns the role of apparent selection bias in science. Hand instances research of scientists from William Withering (who examined the use of foxgloves in the treatment of dropsy and who was careful to state that he "mentioned every case in which I have prescribed the Foxglove, proper or improper, successful or otherwise") to Louis Pasteur (infectious diseases and microorganisms) and Robert Millikan (examination of the charge on the electron). (Withering had learned of the use of foxgloves in the treatment of dropsy from an old woman in Shropshire, and he identified the active ingredient as digitalis. He later felt he had been short-changed by Erasmus Darwin [Charles Darwin's grandfather], whose tract on the subject was published in the same year [1785] as Withering's.) There is also a connection here with the publication bias mentioned before.

The Law of the Probability Lever essentially says "that a slight change in circumstances can have a huge impact on probabilities" [p. 142]. This has connections with chaos theory and the butterfly effect (again). Hand finds a further manifestation in the application of the probability associated with an average person to a specific one (e.g. the chance of the average man's being struck by lightning is very much smaller than that of a lumberjack's).

The final main strand in the Improbability Principle is the Law of Near Enough, which is concerned with regarding things that are sufficiently similar as identical. Here Hand considers S. G. Soal's attempted repetition of J. B. Rhine's telepathic experiments. Soal in fact found that one of his 160 subjects (with 128,350 observations in all) gave evidence of precognition, correctly predicting cards one ahead of that currently being shown. Soal's

experiments, however, seemed to be incapable of repetition.

In Chapter 9 Hand discusses some human aspects of the Improbability Principle, including the difficulty our intuition has in assessing probabilities, the prosecutor's fallacy (difficulties with conditional probabilities and Bayes's Theorem), the base rate fallacy (a failure to take account of background probabilities), an inclination to take account of evidence that supports one's beliefs (or pet hypotheses) and forget about contrary evidence, and hindsight bias.

Chapter 10 is entitled "Life, the Universe, and Everything." Here, among other things, Hand mentions the Copernican Principle and its extension to the principle of mediocrity, which says that "earth, and by implication humanity, is not located at a special position in the universe, *and moreover that there's nothing special about the condition of humanity in other ways*" (italics in the original) [p. 210]. There is also a discussion of fundamental constants in physics (these constants, it appears, have to be either exactly what they are or very similar for the existence of stars, planets, and humanity). And, finally, there's some discussion of the anthropic principle, which, in one sense, says that "the universe in which we live is one in which we *can* live" [p. 218].

The final chapter is devoted to ways in which we can, and should, use the Improbability Principle, illustrating these ways with examples from science, medicine, business, etc. Here we also find discussions of Bayesianism and statistical vs. practical significance.

Hand reveals several flashes of wit in *The Improbability Principle*. For example, in discussing the possibility of Shakespeare's deliberate use of alliteration in his plays, Hand writes, "Could the set of similar sounds in Shakespeare's sonnets simply surface serendipitously?" [p. 226]. The reader will find many other passages displaying a quick mind.

There is, I'm afraid, a difficulty about *The Improbability Principle*, and that is that almost every section prompts one to further and wider reading. Hand's excellent discussion of the various laws (in a style that makes them readily accessible to the general reader), his wealth of illustrative examples (from coins to fighter airplanes), the occurrence of situations ranging from physics to parasensory phenomena in which seemingly inexplicable results are accounted for make this a book that should be required reading for every serious student of probability and also for everyone who from time to time entertains "a suspicion that the universe is making fun of you" [p. 92]. I recommend it wholeheartedly.

References

[1] T. Bayes, An essay towards solving a problem in the doctrine of chances, *Philosophical Transactions* **53** (1764), 370–418.

[2] J. Bernoulli, *Ars Conjectandi*, Thurnisiorum, Basileæ.

[3] É. Borel, *Les Probabilités et la Vie*, Presses Universitaires de France, Paris, 1943.

[4] ———, *Elements of the Theory of Probability*, translated by J. E. Freund, Prentice-Hall, Englewood Cliffs, NJ, 1965.

[5] T. Browne (C. Sayle, ed.), *The Works of Sir Thomas Browne*, Vol. I. Grant Richards, London, 1646/1904.

[6] G.-L. L. de Buffon, Essai d'arithmétique morale, vol. 4 of *Supplément à l'Histoire naturelle*, Imprimerie Royale, Paris, 1777.

[7] A. A. Cournot, *Exposition de la theorie des chances et des probabilités*, L. Hachette, Paris, 1843.

[8] R. A. Fisher, *The Design of Experiments*, Oliver and Boyd, Edinburgh, 1935.

[9] I. J. Good (general editor), *The Scientist Speculates. An Anthology of Partly-baked Ideas*, Heinemann, London, 1962.

[10] W. Kruskal, Miracles and statistics: the casual assumption of independence, *Journal of the American Statistical Association* **83**, no. 404 (1988), 929–940.

[11] G. Shafer and V. Vovk, The sources of Kolmogorov's *Grundbegriffe*, *Statistical Science* **21**, no. 1 (2006), 70–98.