Teaching University Mathematics: One Mathematician's Contribution

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What makes a "Great Teacher"? Do we learn to be one, or is it intrinsic? In December 2013, Professor Jim McClure was inducted into Purdue University's Book of Great Teachers, providing us with a case study. In this article we present background information about McClure and introduce the domains of mathematical knowledge for teaching presented by Ball, Thames, and Phelps [1]. We then synthesize comments from his colleagues and students to uncover factors of McClure's success in teaching, factors that position teaching as a skill akin to mathematics that can be refined.

In thirty-eight years working as a topologist, McClure has produced more than thirty publications, including two papers in the *Annals of Mathematics* and one in the *Journal of the American Mathematical Society*. He earned his PhD from the University of Chicago in 1978 under Professor J. Peter May. After working at Johns Hopkins University and the University of Kentucky, he settled at Purdue in 1992, where he has been named to the student-determined College of Science Top Ten Teachers list three times and was selected as the Best Teacher in the College of Science in 1999. In 2014 McClure added associate head of the Department of Mathematics to his roles and is directing the academic functions of the department while continuing topology research and mathematics teaching.

Mathematical Knowledge for Teaching

Building on the work of Shulman [4], Ball, Thames, and Phelps [1] developed a framework for mathematical knowledge for teaching (see Figure 1), detailing the types of knowledge necessary for teaching mathematics. The framework consists of subject matter knowledge and pedagogical content knowledge, recognizing that both aspects are crucial for teaching mathematics. Subject matter knowledge includes three domains: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK). CCK refers to "the mathematical knowledge and skill used in settings other than teaching" ([1], p. 399); SCK refers to the mathematics that only teachers need to know; HCK includes knowledge about how mathematics is connected across the curriculum. Pedagogical content knowledge also contains three domains: Knowledge of Content

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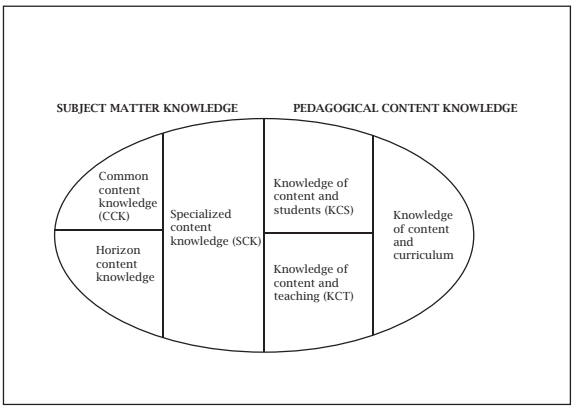

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball, Thames, and Phelps, Journal of Teacher Education, November/December 2008, volume 59, p. 403, Figure 5; ©2008. Reprinted by Permission of SAGE Publications.)

and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). KCS is knowledge about the relationship between mathematics and students in such a way that one can predict obstacles that may affect student learning; KCT includes the structure of the logical order of teaching mathematics and is helpful for teachers when planning curricular sequences; KCC is the knowledge of a variety of instructional materials used in teaching mathematics.

While McClure became familiar with "pedagogical knowledge" through reading Liping Ma's 1999 book *Knowing and Teaching Elementary Mathematics: Teacher's Understanding of Fundamental Mathematics in China and the United States*, we argue that McClure's work embodies this more current framework of mathematical knowledge for teaching. Below, we take a closer look at how McClure provides students opportunities to develop all domains of this framework.

Mathematical Passion and Understanding: Subject Matter Knowledge

McClure displays his passion for mathematics and desire for students to be mathematically stimulated and competent. One student recalled, "His class was very engaging and challenging. You can tell when you sit in his classroom that he loves what he does and is very passionate about mathematics." Colleague Professor David Goldberg noted McClure's desire to engage the students when teaching them how to transition from the pragmatic to axiomatic approach while introducing *Euclid [Elements] Book* I, giving students an opportunity to understand and experience CCK, thereby deepening their knowledge of mathematics.

McClure assists in the development of SCK, an understanding of mathematics for teaching. This was noted by a former student as she recalled learning from McClure, "It isn't sufficient to just know how to do the [mathematics] that one teaches, but…to know…the "whys" of the concepts and theoretical dispositions of the levels of mathematics that one shares with novice students." McClure's sharing of his mathematical insights gives students an opportunity to see SCK in action. Another student highlighted this, saying, "He always has great insight [into] the problem at hand, whether that be providing background and history of a topic or theorem, or being able to…explain the intricacies in depth…[McClure] inspired me."

In "Start where they are: Geometry as an introduction to proof," McClure [3] argued that one of the most successful ways to teach students how to do proofs is by making connections with secondary geometry concepts. Using familiar knowledge allows students to deepen HCK, the awareness of concepts spanning across mathematics. Another manner in which he aids students in developing HCK was recalled by a former student, saying McClure "was very good at relating what we were learning to other math ideas that we had learned before…he would show us how [the] same mathematical ideas/ways of thinking related to the new material we were learning."

"Start Where They Are…": Pedagogical Content Knowledge

McClure concentrates on conceptual and procedural understanding, focusing on meeting students where they are in their mathematical knowledge. This allows for the development of KCS, which requires knowing about students and knowing about mathematics. A former student commented, "I appreciated McClure's approach to teaching: He relied less on giving example problems [and] rather on the conceptual knowledge…Many times, I felt I would teach the same way he taught us when I became a teacher." Another student echoed, "He taught in a way that he knew students best understood, instead of just how he best understood the material," again showing alignment with KCS in considering students' and mathematical perspectives. Professor Guershon Harel, a friend and former colleague, noted:

> [McClure] has continually…evaluated his ideas in various mathematics classes he taught…and thought deeply about the pedagogy of mathematics… As an example, I mention his excellent paper, "Start where they are: Geometry as an introduction to proof" (McClure [3]). This paper includes strikingly innovative ideas about the role of geometry in advancing students' conception of proof.

McClure's continual reflection on the pedagogical and logical orders of teaching and

learning mathematics is also notable. This practice is aligned with KCT in that it requires considerations of task selection and sequencing. A former student mentioned McClure's pedagogical strategy in which he starts with concrete examples and connects them to prior knowledge before abstracting, stating that it "helped both with remembering the new material and being comfortable with it because he showed us it wasn't completely new, just the same ideas/thoughts applied in a different situation," demonstrating knowing about teaching and knowing about mathematics, or KCT.

McClure created a geometry course at Kentucky and has continued refining it at Purdue, sharing notes with colleagues to use when teaching the course; it is this course that is most often cited by future mathematics teachers as McClure's contribution to their future teaching. He explicitly discusses his use of Euclid's *Elements* with students in order to connect historical and foundational geometric knowledge with secondary geometry curriculum. This transparent explication of his curricular decision-making provides opportunities for students to engage with KCC.

Summary

Professor Jim McClure's pedagogical perspective, while always keeping mathematics as the focus, is what has gained him recognition as a "Great Teacher" at Purdue University. McClure provides students with opportunities to develop the domains of Mathematical Knowledge for Teaching, deepening subject matter knowledge and pedagogical content knowledge. He implements multiple characteristics of effective teaching, such as relaying his deep understanding of mathematics through his teaching approaches while considering the pedagogical and logical orders of all levels of mathematics and considering the background knowledge of his students when planning his lessons, and attempts to meet them where they are. His impact will be felt for many years through the mathematicians and the mathematics educators he has inspired.

References

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