BOOK REVIEW

Or/And: A Review of *Figuring Fibers*

Reviewed by Elizabeth Wilmer

AMS, 2018.

Figuring Fibers
ed. Carolyn Yackel and sarah-marie belcastro

1. Mathematics and Fiber Arts? YES.

When I entered the Westinghouse Science Talent Search in 1987, the registration form asked how my interest in science had been sparked. I wrote about learning to knit and crochet as a child: I loved how beautiful patterns and complex structures accreted as I worked, and I read my local library's pattern books over and over again. Teachers at my technical high school, which had first admitted girls in 1969, told me to change that answer: fiber work was too domestic, too silly—and, I inferred, too feminine—to discuss.

Since then I've met many mathematicians and scientists in related areas whose technical interests were sparked by early experiences with fiber arts. Over the last twenty years there has been a broad explosion of interest in crafting, fostered by the Internet: Ravelry, Instagram, Etsy, Facebook, blogs, and even Twitter allow us to find inspiration, buy specialized tools and supplies, learn new skills from experts, discuss technical challenges, meet friends with shared interests, show off our finished objects, and admire others' masterpieces.

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Fortunately for mathematicians, there has been a simultaneous expansion of mathematically inspired fiber arts and fiber arts-inspired mathematics. Carolyn Yackel and sarah-marie belcastro have been at the center of efforts to bring mathematical fiber arts into mainstream mathematics institutions. *Figuring Fibers*, published by the AMS, is the third collection of papers they have edited; the two earlier volumes, *Making Mathematics with Needlework* (2007) [MM] and *Crafting by Concepts* (2011) [CC] were put out by A K Peters. All three are primarily based on work presented at the Special Sessions on Mathematics and Mathematics Education in the Fiber Arts that belcastro and Yackel organized at the 2005, 2009, and 2014 Joint Meetings; at the 2014 Joint Meetings they also ran a juried fiber arts exhibition.

Fiber arts are also gaining prominence in the broader mathematical art and recreational mathematics worlds, with fiber media work regularly featured in the Annual Mathematical Art Exhibitions at the Joint Meetings, the Bridges conferences celebrating mathematical art, and the Gatherings 4 Gardner. Physicist Elisabetta Matsumoto's NSF-funded work on the mechanics of knitted fabric has been featured in the *New York Times*; she has also organized a session of talks at an American Physical Society annual meeting.

It may not be obvious to those unfamiliar with fiber arts how they can support such a rich relationship with mathematics. (References below are to paper numbers in the three belcastro-Yackel volumes; every paper is listed at least once.)

Knitting and crochet both produce two-dimensional fabric consisting of arrays of slipknots. One linear (or circular) row of stitches is worked at a time. Varying the specific stitches and the colors of yarn can produce a range of textures and patterns, which can be studied in their own right [MM 3, 4; CC 2; FF 3], or used to illustrate a variety of mathematical phenomena [MM 4, 6, 8; FF 2]. Number theoretic issues also arise when fitting together discrete stitches in different patterns or orientations [MM 2; FF 5]. Traditional techniques for changing the number of stitches alter the shape of flat patches of fabric [CC 1]; precisely shaped and sized pieces can be assembled in mathematically interesting ways [CC 7; FF 1]. Changing stitch counts can also be used to approximate intrinsically curved or knotted fabric [MM 10; CC 3; FF 6] (Taimina [Tai] gives a comprehensive discussion of negatively curved crochet).

Weaving constructs regular two-dimensional lattices, while cross-stitch and needlepoint take place on the grids formed by woven substrate fabric [MM 5; CC 4, 6]. The physical process of stitching can impose additional constraints, leading to questions in graph theory and discrete combinatorial geometry [MM 9; CC 5, 8]. The piecework of quilting can be viewed as geometric decomposition [CC 9; FF 4]; of course the existence of a covering by flat coordinate patches does not imply the global topology is trivial [MM 1, 7; FF 8]. Fabric is flexible and durable, so quilted models can demonstrate geometric transformations [CC 7, FF 7].

Beyond the explicit mathematical connections based in the underlying techniques, I suspect that many mathematically inclined fiber artists enjoy crafting for some of the same reasons they enjoy mathematics: although the work is largely rule-based and detail-oriented, it can generate startling beauty. To succeed, innovations must satisfy a wide range of physical and logical constraints, and patience can be required to see how a chain of steps plays out.

2. *Figuring Fibers*

belcastro and Yackel's volumes have all been designed to invite in broad and varied audiences. Each paper in *Making Mathematics with Needlework* included not only mathematical exposition and project instructions, but also guidance on pedagogical uses of the material. This structure was relaxed somewhat in *Crafting by Concepts*, which included some "mini" writeups of projects. *Figuring Fibers*'s back cover promises:

> Amazing exposition transports any interested person on a mathematical exploration that is rigorous enough to capture the hearts of mathematicians… If you love mathematics or fiber arts, this book is for you!

Each chapter includes both mathematical exposition and detailed crafting instructions; separate abstracts for mathematicians and crafters appear at the front of the book. The current paper structure is more readable than that of the earlier volumes; each article simply has fewer moving pieces.

The first five chapters of *Figuring Fibers* are delightful. Each presents a sophisticated mathematical treatment of a problem and includes detailed and accessible instructions for a related project. Wildstrom [FF 1] adapts traditional crochet "granny squares" into polyominoes; Calderhead [FF 2] develops a space-filling curve that can be crocheted by carefully interlacing dual triangular and hexagonal grids in contrasting colors. Yackel [FF 3] adapts a traditional knitting pattern, Templeton squares, to allow for a range of Truchet tiles in the style of Bosch and Colley [BC]. Givens [FF 5] considers closely the mechanics of combining knitting stitch patterns with different periods, while Shepherd [FF 4] gives algorithms for building a "perfect" version of a traditional snake trail quilt. Of these, all but Calderhead provide general guides for future designers to adapt their techniques, and Calderhead includes charming mini-projects as stepping stones to the large and complex throw at the center of his work.

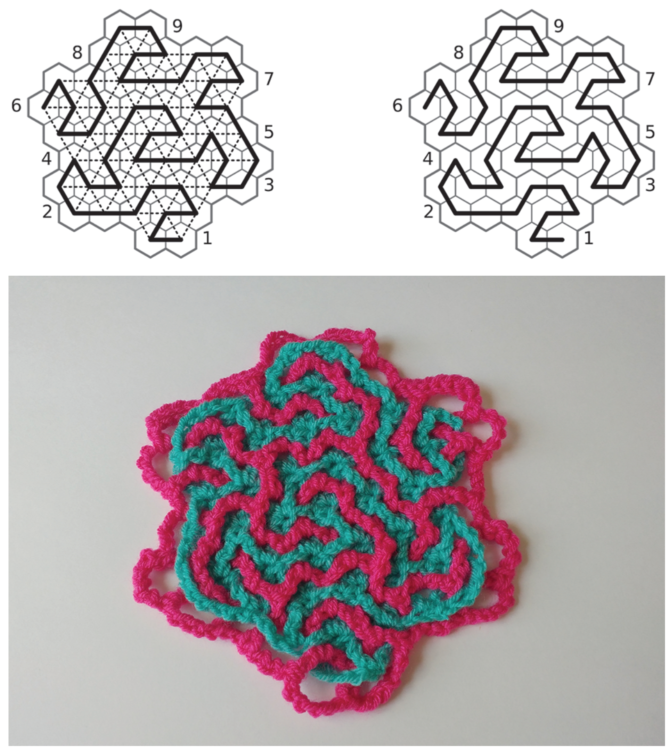

Figure 1. [FF 2, Figure 36 (left) and Figure 37] Kyle Calderhead's warm-up project illustrating a novel space-filling curve construction in intermeshed crochet.

As a knitter, I tried out Givens's cowl pattern. The instructions were clear and correct, and it was fun to work up; there are some surprises in how the colorwork is handled, and the textural contrast between the stitch patterns is very satisfying.

The most exciting papers in *Figuring Fibers* are, however, the final three, each of which develops novel crafting techniques to explore sophisticated mathematics. The resulting objects are at once gorgeous expressions of the authors' mathematical imagination and technical masterpieces.

The cover of *Figuring Fibers* shows an (8,6) torus link. Using long, flexible knitting needles, which are themselves tied in knots, belcastro [FF 6] produces knotted fabric. This is the knitting equivalent of gymnastics! Fortunately there are detailed instructions (in terms of braid presentations) for setting up projects.

Gould [FF 7] has chosen the medium of layered fabric with embroidery for its physical advantages as she models infinite semiregular polyhedral surfaces. Bonding two layers of fabric in different colors distinguishes the two sides of a surface; embroidery highlights symmetries. Her work extends the long tradition of building models of polyhedra (see, e.g., Wenniger [Wen]) to infinite surfaces. Even small sections of these highly connected surfaces require many pieces, and Gould works at the edge of what can be considered handcrafting. She draws templates in Geometer's Sketchpad, then follows computer-controlled cutting and embroidery with hand assembly of the models. The narrative of her process is enthralling.

Figure 2. [FF 7, Figure 31 (left), p. 161] A model of a portion of the murhombicuboctahedron (4,4,4,6) by S. Louise Gould. Each face consists of two bonded layers of fabric, one machine embroidered; the bound edges are stiffened with boning.

Finally, Nimershiem's [FF 8] pair of quilts modeling boundaries in Thurston's construction of a hyperbolic structure on the complement of the Borromean rings is virtuosic crafting. She combines traditional quilting with an array of idiosyncratic techniques: channels made of commercial eyelet ribbon hold both drawstrings and rigid sticks, while bits of flexible tubing become joints. The resulting quilts can be reconfigured from a flat half-plane configuration to symmetric octahedra.

3. Onwards!

As mathematical fiber arts gradually matures into a professional pursuit, we should ask: what are the central questions? What are the goals?

Figure 3. [FF 8, Figure 2, both photos, p. 176] One of a pair of quilts designed by Barbara E. Nimershiem to model Thurston's decomposition of the complement of the Borromean rings can be reconfigured from a boundary between half spaces to the boundary of an octahedron.

Yackel and belcastro lay out in the introduction to *Figuring Fibers* a taxonomy of question types:

1. using mathematics to solve fiber arts problems,
2. designing and fabricating a fiber arts piece to display a mathematical concept,
3. proving that a specific fiber art can be used to exemplify a mathematical concept,
4. mathematically analyzing an aspect of a specific fiber art

and explain how work set in each of these paradigms can be found in *Figuring Fibers*.

I would add two additional categories. First, on the border with history: what mathematics do crafters already know, or must they necessarily know, to carry out their work? In *Figuring Fibers* Shepherd and Givens touch on these issues for quilt design and stitch pattern selection, respectively. This question can also be asked about particular experts: for instance, the great twentieth-century knitting authors Elizabeth Zimmerman and Barbara Walker, who broadly systematized techniques, but also current designers, such as Cat Bordhi or Norah Gaughan, who explicitly use mathematical themes. ([CC 1] riffs beautifully on Zimmerman's classic Pi Shawl pattern [Zim].)

Second, on the border with psychology and complexity: what structures are possible for human beings to construct, using a given craft? What makes patterns satisfyingly intricate, but not overwhelmingly difficult? How does automation fit in? Why is it so difficult to follow randomized patterns? How do crafters modify instructions as they work, and, in the spirit of Gowers's question on when two proofs are essentially the same [Gow], how far can the changes go before it becomes a different pattern?

Yackel and belcastro have maintained an admirable commitment to broad accessibility throughout their series of volumes. I wonder, though, whether this area is ready for somewhat more specialized channels: more targeted at fiber arts-experienced readers and mathematically mature readers, respectively. In some sense the split in mood between the first five and last three papers in *FF* already corresponds to this division.

What would it take to position a volume on mathematical fiber arts for a broad crafting audience? More photographs, shot more professionally and printed better, to start. The lack of color saturation in the hard-copy *Figuring Fibers* is heartbreaking; the warmer tones and brighter reds in the electronic version (why is it not available on Amazon?) show that some fault lies in the printing. But more work-in-progress photographs, more careful arrangement of displayed work, non-distracting background settings, and better lighting would all be appreciated. Consistently including accessible mini-projects (as Calderhead and Gould do) and guidance on locating unusual tools or materials would be appropriate, as would be marketing through crafting channels: Ravelry groups? Knit (or…)-a-alongs? Kits?

For a technical audience: are we ready for a journal of mathematical crafting? Or some regular series of proceedings, expecting authors to display novelty in at least one of: the underlying mathematics, the connection between the mathematics and the craft, and the craft techniques used? I leave it to any future editors to decide what media to include (fiber? origami? 3D-printing? any physical realization?), and whether work on theoretical crafts should be allowed—that is, discussions of objects that *could* be produced, but haven't actually been.

Back in 1987, more out of honesty than any ideological commitment, I ignored my teachers' advice and left the discussion of knitting and crochet on my Westinghouse entry forms. In 2019, I was truly delighted that the spectacular Gould and Nimershiem chapters of *Figuring Fibers* draw mathematical inspiration from, respectively, the work of J. Richard Gott [Got] and Bill Thurston [Thu]—both of whom were judges for the 1987 Westinghouse competition. The appearance of these specific mathematical topics is, of course, accidental—low-dimensional geometry is what's susceptible to physical realization!—except that Gott and Thurston shared both vivid visual imaginations and strong commitments to outreach. The editors and authors of *Figuring Fibers* do too, as they braid together fiber arts and serious mathematics.

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Credits

Figure 1 graphic is courtesy of Kyle Calderhead. Erika Ward crocheted and photographed the piece in Figure 1.

Figure 2 is courtesy of S. Louise Gould.

Catalina Betancourt made the quilts photographed by Paul Samuel Ignacio in Figure 3.

Author photo is courtesy of Jennifer Manna.