MEMORIAL TRIBUTE

Remembering Steve Zucker

David A. Cox, Michael Harris, and Lizhen Ji

Figure 1.

Steven Zucker, "Steve" to everyone who knew him, died on September 13, 2019, the day after his 70th birthday. Steve was a versatile algebraic geometer who made important contributions to Hodge theory, L^2-cohomology, and compactifications of locally symmetric spaces.

Personal Life

Steve was born on September 12, 1949, in New York City and grew up in Queens. After attending Brown University as an undergraduate, he did graduate work at Princeton University, getting his PhD in Mathematics in 1974. He hoped to work with Phillip Griffiths, but when Griffiths went to Harvard, Steve switched to Spencer Bloch. His thesis on normal functions appeared in *Inventiones Mathematicae* as his first publication [1].

Steve's first job was at Rutgers University, where he wrote some significant papers, including [3], published in the *Annals of Mathematics* in 1979. In spite of this, Steve was denied tenure at Rutgers, a decision that left him "angry and anguished," as he writes in his mathematical autobiography [22].

After leaving Rutgers, Steve spent a year at the Institute for Advanced Study, where he attended Armand Borel's seminar. During this time, he wrote the paper [6] that contained the *Zucker conjecture*, to be described in Leslie Saper's contribution. In 1982, Steve moved to Indiana University and soon thereafter moved to Johns Hopkins University, where he became a full professor in 1985. In 2012, Steve was part of the inaugural class of Fellows of the AMS. Two years later, Steve's 65th birthday was celebrated with the conference *Hodge Theory and L^2-cohomology*, held at Johns Hopkins:

Figure 2. Conference on Hodge Theory and L^2-cohomology, Johns Hopkins University, November 21–23, 2014.

David A. Cox is a professor of mathematics emeritus at Amherst College. His email address is dacox@amherst.edu.

Michael Harris is a professor of mathematics at Columbia University. His email address is harris@math.columbia.edu.

Lizhen Ji is a professor of mathematics at the University of Michigan. His email address is lji@umich.edu.

Communicated by Notices *Associate Editor Steven Sam.*

For permission to reprint this article, please contact: reprint-permission@ams.org.

DOI: https://doi.org/10.1090/noti2310

Steve's official retirement was in 2019, but owing to ill health, he took a medical leave in 2017.

Steve had an active life outside of mathematical research. He loved to travel and was a gifted photographer and pianist (he had a baby grand piano in his apartment). While at Rutgers, he helped organize the Rutgers-Columbia relay race that was run along the tow path of the Raritan canal that connects New Brunswick to Princeton.

Steve was a thoughtful and demanding teacher. He was especially concerned with the transition from high school to the university. In 1996, he published "Teaching at the University Level" in the *Notices of the AMS*. The key point of the article, and something he practiced when teaching calculus, is that *students need to learn how to learn*. An expanded version of the article appeared in the second edition of Steve Krantz's book *How to Teach Mathematics* in 1999. Students found Steve's approach to be challenging but many came to appreciate what he was doing. He brought the same spirit of "learning how to learn" to his graduate seminars. Patrick Brosnan, now at the University of Maryland, College Park, notes that "His way of running the seminar was unique. He was learning the material with us, and you could see him really thinking about it in a way that a lot of people hide, but it was out in the open and inspiring." Richard Brown of Johns Hopkins University comments that "I have had many long discussions with Steven over the 14 years I have been the Director of Undergraduate Studies in the department here, on issues involving the education of undergraduates, how students learn, how to be effective in the classroom, and the differences between being a teacher and being an educator. We disagreed on many things, sometimes passionately. But we learned a great deal from each other. I will miss those interactions much. He thought deeply and understood much about how to effectively communicate the art of mathematics. Rest in peace, Steven."

In 1988, Johns Hopkins established JAMI, the *Japan-U.S. Mathematics Institute*, to further cooperation in mathematical research between Japan and the U.S. Steve played an active role in JAMI: he was a local organizer for the 2000–01 and 2004–05 programs and Director of JAMI for 2003–06. JAMI was awarded the Seki Takakazu Prize by the Mathematical Society of Japan in 2006. At the award ceremony, Steve commented that "We are very proud of the interactions with the Japanese that have developed and the bonds that have been strengthened through JAMI." The contribution by Masa-Hiko Saito will say more about Steve's links to Japan.

Mathematical Research

Steve Zucker was a highly original person both in mathematics and outside mathematics. He has made major contributions to the following three areas of mathematics and the interactions among them:

1. Hodge theory in algebraic geometry (e.g., normal functions, variation of mixed Hodge structure, degeneration of Hodge structure);
2. L^2-cohomology and also L^p-cohomology, $p \neq 2$;
3. compactification of locally symmetric spaces (Satake, reductive Borel-Serre, toroidal, etc.).

Though he made many contributions, there is no question that Steve was most famous for and will be remembered for the Zucker conjecture from 1982 which relates the second and third items above. It was resolved independently in 1987 by Saper-Stern and by Looijenga, using very different methods.

In this brief overview of Steve's major contributions to mathematics, we will try to explain the evolution of his work and the underlying unity through papers closely related to the above three topics. For a more comprehensive, detailed, and personal description of Steve's work, see his own narrative in [22] and other contributions to this memorial tribute as indicated below. Steve wrote joint papers with many people, and his autobiographical account [22] also contains vivid descriptions of these collaborations together with recollections on his career and related matters.

Steve was a complex algebraic geometer in the school of Griffiths, whose methods, influenced by ideas of Riemann, Poincaré, and Lefschetz, combine topology, geometry, and analysis in the study of algebraic varieties.

In his thesis in 1851, Riemann tried to use L^2 methods, i.e., the Dirichlet form and the Dirichlet principle, to prove the Riemann mapping theorem for proper and simply connected domains in $\mathbb{C}$. Later, in his very influential paper on abelian functions in 1857, Riemann also used harmonic functions and the Dirichlet principle to prove the Riemann inequality in the Riemann-Roch theorem for Riemann surfaces. In this paper, Riemann initiated the study of topology of Riemann surfaces and showed the power of topology in understanding the geometry and analysis of algebraic curves. An example is his complete solution to the Jacobi inversion problem, the dream problem of Weierstrass whose solution for hyperelliptic curves made him world famous. These works of Riemann showed the intimate connection between the analysis, geometry, and topology of Riemann surfaces. These results were made rigorous and systematically presented in Weyl's classical book *The Concept of a Riemann Surface* which was published in 1913.

Riemann's work was followed and generalized by different people in different directions. For example, Hodge theory was a direct generalization to higher-dimensional smooth projective varieties and more general compact

Kähler manifolds by Hodge, with improvements and refinements by Weyl and Kodaira. Picard generalized results on the geometry of algebraic curves to algebraic surfaces, which motivated later works of Poincaré and Lefschetz on the topology of algebraic surfaces, in particular the Hodge conjecture through the use of Lefschetz pencils.

Lefschetz used normal functions to prove a theorem about algebraic surfaces that represents the first instance of what is now known as the Hodge conjecture. Given a pencil of curves $\{C_t\}$ on S, Poincaré introduced the notion of a normal function associated with the pencil which is a function of t with values in the Jacobians of the curves C_t. He defined a normal function for every algebraic curve on S, and proved that every normal function arises in this way from an algebraic curve. Lefschetz proved that every normal function determines a class in $H^2(S,\mathbb{Z})$, which agrees with the fundamental class of the algebraic curve when the normal function arises from the algebraic curve as above. Furthermore, he proved that a class in $H^2(S,\mathbb{Z})$ arises from a normal function if and only if it lies in $H^{1,1}(S,\mathbb{Z})$.

Griffiths initiated a program to generalize Lefschetz's methods to higher-dimensional algebraic varieties through intermediate Jacobian varieties. Steve's thesis proved a generalization of Lefschetz's result: given a smooth projective variety Y of dimension $2n$ and a Lefschetz pencil of hyperplanes, every primitive integral Hodge class in $H^{2n}(Y)$ comes from a normal function. This was published in [1]. Poincaré's result above that normal functions come from algebraic curves depends on the positive solution of the Jacobi inversion problem. In the case of cubic fourfolds, Steve solved such an inversion problem and hence proved the Hodge conjecture for such fourfolds in [2].

Steve's work on normal functions led to his important paper on the variation of Hodge structure [3]. For his later work, one key aspect is that he used L^2-cohomology with respect to the Poincaré metric, which led to the Zucker conjecture mentioned earlier. Besides the application of Lefschetz pencils, understanding families of algebraic varieties is important to the problem of moduli, and one probably fruitful approach is to use the theory of variation of Hodge structures. Let $\bar{S}$ be a nonsingular projective variety, and X_s a family of projective algebraic varieties over $\bar{S}$, i.e., X_s are fibers of a morphism $\bar{f} : \bar{X} \to \bar{S}$. In general, not all X_s are smooth. Let $S \subset \bar{S}$ be the Zariski open subset consisting of points s for which the elements of X_s are smooth. For $f = \bar{f}_{|S}$, $R^m f_*\mathbb{C}$ defines a variation of Hodge structure of weight m. The main result of [3] states: *Assume that $\bar{S}$ is an algebraic curve. Let $\mathcal{V}$ be the locally constant sheaf over S that appears in the definition of a variation of Hodge structure over S, for example, $\mathcal{V} = R^m f_*(\mathbb{C})$ and $j : S \to \bar{S}$ the inclusion. Then $H^i(\bar{S}, j_*\mathcal{V})$ has a canonical Hodge structure of weight $m+i$.*

When $S = \bar{S}$, this result was proved earlier by Deligne, and Deligne's proof was the starting point of Steve's proof of the more general result. But as Griffiths pointed out in the review of this paper, "this extension is by no means routine, as what is required is a careful and precise analysis of what happens to the Hodge structure as s tends to a point of $\bar{S}-S\ldots$ the new analysis required to complete the proof of the above theorem represents a significant technical step in the theory." Basically, the idea is to use differential forms on S that are square-integrable with respect to the Poincaré hyperbolic metric on the punctured Riemann surface S to compute cohomology groups on $\bar{S}$.

Steve's interest in cohomology of locally symmetric spaces probably started with the article [5] where he applied the theory of polarized variations of Hodge structure to study certain cohomology groups attached to locally symmetric Hermitian spaces in a paper of Matsushima and Murakami (*Ann. of Math.*, 1963).

Though this paper [5] mainly studied compact locally symmetric spaces, his interest in L^2-cohomology of noncompact locally symmetric spaces of finite volume was indicated already. It is helpful to note that his interest in L^2-cohomology started earlier in the paper [3, §6], where he studied L^2-cohomology groups of the punctured disc with respect to the complete Poincaré metric. He also emphasized its connection with compactification [3, p. 417] (note that compactification and its connection with L^2-cohomology are crucial in this work):

> The Hodge theorem for this setting, then, really concerns L^2-cohomology on the non-compact manifold S, where standard theory does not apply. We are forced to work on S, since we wish to use Deligne's Kähler identities, and the "Hodge metric" on V (from a polarization of the variation of Hodge structure) has singularities where V degenerates. ... However, we use the presence of the compactification S to circumvent this difficulty.

Steve's work on L^2-cohomology of locally symmetric spaces of finite volume started with the article [6], which is also the first place where he stated the famous Zucker conjecture which identifies the L^2-cohomology groups with the intersection cohomology groups of the Baily-Borel compactification of the locally Hermitian symmetric space.

There are several reasons for the importance of this conjecture. The usual one, for example, as explained in later expository papers of Steve, is that the L^2-cohomology has a canonical Hodge structure, and this isomorphism should also give a "canonical" or "natural" Hodge structure to the intersection cohomology of the Baily-Borel compactification. Furthermore, due to the close connection between the L^2-cohomology and automorphic forms or representations, this isomorphism can be used to study the Hasse-Weil zeta function of the Shimura variety arising from the locally symmetric space. These expectations seem still to be open in general. We will concentrate on the motivations for this conjecture and its impact on other works of Steve. See the contribution of Leslie Saper in this memorial tribute for more discussion and detail on the Zucker conjecture.

Given the above assertions, it will be interesting to see how Steve described his original motivations for studying the L^2-cohomology of locally symmetric spaces and his conjecture. At the beginning of his paper [6], he wrote:

> Let G be a semi-simple algebraic group over $\mathbb{Q}$, Γ an arithmetic subgroup of G, and E a finite-dimensional (complex) representation space for G (hence for Γ). It is of interest to determine the cohomology groups $H^*(\Gamma, E)$. This seems to be a difficult problem in general, even for constant coefficients ($E = \mathbb{C}$, trivial representation).

He did not explain why the cohomology groups $H^*(\Gamma, E)$ are "of interest." Besides his work in the paper [5], attending Borel's seminar at IAS on topics related to the 1980 book by Borel and Wallach, *Continuous Cohomology, Discrete Subgroups, and Representations of Reductive Groups*, must be another interesting reason for him.

Furthermore, from the perspective of geometric group theory, arithmetic subgroups Γ of Lie groups provide the most natural and interesting class of finitely generated discrete groups, and their cohomology groups $H^*(\Gamma)$ are important invariants of these groups. According to the general theory of algebraic topology, one effective tool to compute these cohomology groups is to find explicit and low-dimensional models of the classifying space $B\Gamma$, or the first Eilenberg-MacLane space, of the group Γ, since $H^*(\Gamma) = H^*(B\Gamma)$. In the above setting, when Γ is a torsion-free arithmetic subgroup, the associated locally symmetric space $\Gamma\backslash X$ is a finite-dimensional model of $B\Gamma$.

Steve continued in his article [6, p. 169]:

> The cohomology groups above may be computed from an appropriate deRham complex with twisted coefficients $\mathbb{E}$ on the locally symmetric space $\Gamma\backslash X$ associated to G and Γ. A good place then to begin searching for cohomology class representatives is among the L^2-harmonic forms relative to natural metrics. However, these forms need represent non-zero classes only in the (intrinsic) L^2-cohomology. Thus, it becomes worthwhile to examine the mapping from L^2-cohomology to ordinary cohomology.

From this, it is clear that Steve's interest in Hodge theory was a major motivation. Some particular applications he had in mind are [6, p. 170]:

> i) It permits the transfer of vanishing theorems from the spaces of L^2 harmonic forms to the cohomology $H^*(\Gamma, E)$.
> ii) It allows one to conclude that known nonzero spaces of harmonic forms inject into $H^*(\Gamma, E)$.
> iii) It implies, by duality, assertions about the cohomology with compact supports $H_c(\Gamma\backslash X, \mathbb{E})$.

The main theorem of this paper is an isomorphism of L^2-cohomology and the usual cohomology in low degree [6, Theorem 3.20]. In the proof of this theorem, Steve introduced the reductive Borel-Serre compactification of locally symmetric spaces in [6, Proposition 4.2]. Its importance is that the sheaf of L^2-differentials defines a fine sheaf on the reductive Borel-Serre compactification. As we shall see later, this compactification was also used crucially in [18].

Though the paper [6] contains multiple interesting results, it will probably be remembered much more for the Zucker conjecture. The motivations for the Zucker conjecture were described in [6, p. 171]:

> It has been hoped that the L^2-cohomology would have a purely topological interpretation. We have been led to conjecture (6.20) an isomorphism between the L^2-cohomology, when X is Hermitian, and the Goresky-MacPherson intersection homology of the Baily-Borel-Satake compactification of $\Gamma\backslash X$. This is verified in §6 for some groups of $\mathbb{Q}$-rank one. The corresponding assertion is known to hold for spaces with conical singularities ($\mathbb{C}$ coefficients) [Cheeger's work], and also for degenerating local systems underlying variations of Hodge structure on curves [3]. The present work, and also [5], forms a part of the author's program to generalize [3, §12], to arbitrary Hermitian symmetric spaces.

After computation with some rank 1 locally symmetric spaces, Steve formulated the Zucker conjecture in [6, Conjecture 6.20] for locally symmetric spaces of general rank.

One approach to prove this conjecture was explained by Steve in [7, p. 313]:

> The most natural place on which to study the L^2-cohomology is the manifold with corners $\Gamma\backslash\overline{X}$ defined by Borel and Serre (*Comment. Math. Helv.*, 1973), or what comes to almost the same thing, on the maximal Satake compactification. ... The reason for this is that there are distinguished neighborhoods of compact subsets of the faces of the boundary of $\Gamma\backslash\overline{X}$; with respect to associated

> coordinates, the metric has explicit asymptotic formulas. Now, the verification of our conjecture is equivalent to certain vanishing assertions for the L^2-cohomology of neighborhoods of points on the Baily-Borel compactification. It seems to be a good idea, then, to express these neighborhoods in terms of the distinguished neighborhoods on $\Gamma\backslash\overline{X}$, for then we could try to patch together the local L^2 cohomology.

This and other reasons motivated Steve to "the idea of realizing the Baily-Borel compactification as the natural quotient of $\Gamma\backslash\overline{X}$" in [7, p. 313]. More generally, in the article [7], he realized "all generalized Satake compactifications as quotients of $\Gamma\backslash\overline{X}$."

Maybe another motivation to understand better Satake compactifications was related to a remark in [6, Remark 4.3], where Steve claimed that the reductive Borel-Serre compactification is the maximal Satake compactification, and this claim seems to be repeated in the above quote. Though it is true when the $\mathbb{Q}$- and $\mathbb{R}$-ranks of the semisimple linear algebraic group agree, it is not true in general. Maybe this is one reason for his writing the paper [7]. We note that if the reductive Borel-Serre compactification is indeed always and *obviously* isomorphic to the maximal Satake compactification, then the conclusion of this paper [7] seems to be obvious since the collection of all Satake compactifications is partially ordered with a unique maximal Satake compactification which dominates, i.e., is mapped surjectively onto, all other Satake compactifications.

The novelty of the construction of Satake compactifications [7, p. 313] is that: "We will reconstruct the Satake compactifications in such a way that they really do look like quotients of the manifold with corners." This construction is more adapted to the local geometric analysis near the infinity of locally symmetric spaces than the original constructions of compactifications by Satake, and by Baily-Borel. This paper [7] was mentioned proudly by Steve in his summary [22].

The Zucker conjecture inspired Steve's work in several directions. The paper Z10 in [22] proves the conjecture for several locally symmetric spaces of rank 2 and higher by making use of detailed analysis of the geometry of neighborhoods of infinity.

It is well known that it is often fruitful to study the special Hilbert space $L^2(M)$ associated to a Riemannian manifold M together with the natural family of Banach spaces $L^p(M)$, $p \in (1, +\infty]$. In this sense, after the L^2-cohomology groups, it is natural to study L^p-cohomology groups. The analogue of the Zucker conjecture for L^p-cohomology was carried out in [18] with some concrete applications in Steve's mind. Its result and applications (or rather the motivations) are well explained by the abstract of [18]:

> The L^2-cohomology of an arithmetic quotient of an Hermitian symmetric space (i.e., of a locally symmetric variety) is known to have the topological interpretation as the intersection homology of its Baily-Borel-Satake compactification. In this article, we observe that even without the Hermitian hypothesis, the L^p-cohomology of an arithmetic quotient, for p finite and sufficiently large, is isomorphic to the ordinary cohomology of its reductive Borel-Serre compactification. We use this to generalize a theorem of Mumford concerning homogeneous vector bundles, their invariant Chern forms and the canonical extensions of the bundles; here, though, we are referring to canonical extensions to the reductive Borel-Serre compactification of any arithmetic quotient. To achieve that, we give a systematic discussion of vector bundles and Chern classes on stratified spaces.

The article [18] established that the reductive Borel-Serre compactification is a natural compactification in view of the L^p-cohomology. But one remaining problem is to understand the relation between extensions of homogeneous bundles on locally symmetric spaces to the reductive Borel-Serre compactification and their extensions to the toroidal compactifications, a conjecture of Goresky and Tai (*Amer. J. Math.*, 1999).

It was conjectured by Harris and Zucker in [16] and proved by Ji (*Geom. Funct. Anal.*, 1998) that the greatest common quotient of toroidal and reductive Borel-Serre compactifications is the Baily-Borel compactification. On the other hand, Goresky and Tai proved in the paper cited above that "there is a mapping from the toroidal compactification to the reductive Borel-Serre compactification, whose composition with the projection to the Baily-Borel compactification agrees with the canonical projection up to an arbitrarily small homotopy." (Note that it is not a map of compactifications, i.e., restricting to the identity map on the interior.) Then they conjecture that under this map, extensions of homogeneous vector bundles to the toroidal compactification and the reductive Borel-Serre compactification are compatible under this map. The article [19] proves this conjecture by constructing two more new compactifications: the excentric Borel-Serre and the excentric toroidal compactifications.

The papers [18, 19] show that the reductive Borel-Serre compactification is a very natural space enjoying many good properties. For example, though it is only a topological space, it behaves like an algebraic variety. This is especially supported by the sequel [20], which constructs a mixed Hodge structure on the cohomology of the reductive Borel-Serre compactification as an application of "a procedure for constructing compatible mixed Hodge structures for the cohomology of various topological compactifications of locally symmetric varieties."

A close follow-up of [20] is the joint paper with Ayoub, Z27 in [22], which tries to reveal the algebro-geometric meaning of the reductive Borel-Serre compactification, i.e., to understand an underlying motive of the mixed Hodge structure of this compactification. One of the main results of this paper is a construction of a motive on the reductive Borel-Serre compactification whose Betti realization is the cohomology of the reductive Borel-Serre compactification. It was expected in the paper that the mixed Hodge structure of the compactification in [20] coincides with that coming from this motive.

Partially motivated by Steve's work on the Hodge structure of cohomology groups of locally symmetric spaces and detailed study of neighborhoods of compactifications of the locally symmetric spaces, Harris and Zucker extensively studied the boundary cohomology of Shimura varieties, or rather, locally Hermitian locally symmetric spaces, in [15–17].

Briefly, cohomology groups with coefficients in variation of Hodge structure have two structures: (1) mixed Hodge structures, which are more adapted to the toroidal compactifications (algebraic varieties), and (2) relations to automorphic forms, which are more adapted to Borel-Serre compactifications (topological spaces), for example, in order to understand behaviors of automorphic forms near infinity such as the constant terms. It is interesting and valuable to read and compare descriptions of these joint works by the two authors: Steve's summary in [22] and Harris' contribution in this memorial tribute.

One of the goals of the project of Steve and Harris was to prove the nonexistence of ghost classes in the cohomology of locally symmetric spaces, i.e., those nonzero classes which restrict to zero classes on boundary components of the Borel-Serre compactification. To do this, they need to understand cohomology groups of neighborhoods of the infinity of Shimura varieties in terms of two compactifications: the toroidal and the Borel-Serre compactifications. As pointed out above, these two compactifications are not compatible except that both dominate the Baily-Borel compactification. This difference makes both the results and proofs quite technical.

The above discussion and summary emphasize works of Steve which are related to the Zucker conjecture and compactifications of locally symmetric spaces. Many of his other papers deal with the above three topics. For example, [8–10] investigate basic issues in the theory of variation of Hodge structures such as characterizations of a good class of variation of mixed Hodge structure and conditions for extending normal functions of families of Kähler manifolds over a punctured disc across the puncture. See Hélène Esnault's contribution to this memorial tribute for more details about Steve's contribution to Hodge theory. Furthermore, the article [13] studies the Torelli problem, a basic problem in the application of variation of Hodge structure to moduli. See Masa-Hiko Saito's contribution to this memorial tribute for more details. The paper [11] is concerned with unipotent variations of mixed Hodge structure, and the paper Z23 in [22] is probably the furthest away from the three topics and deals with the comparison between the Cheeger-Chern-Simons invariants of a vector bundle provided with a connection and the corresponding characteristic classes in Deligne-Beilinson cohomology. See Richard Hain's contribution to this memorial tribute for more details about these papers.

Steve also wrote other articles not directly related to the above three topics. His joint paper with Cox on elliptic surfaces [4], which deals with generators mod torsion of the group of sections of elliptic surfaces, is one such example, but relations with automorphic forms and cohomology play an essential role in their work. See David Cox's contribution to this memorial tribute for more details about both the work and their unique collaboration process. Another example is Z05 in [22] which applies techniques in Hodge theory to reprove a basic result in algebraic geometry: the numerical positivity of the relative dualizing sheaf of a family of Kähler manifolds over a compact Riemann surface.

There are still other papers such as [12] and [21] which prove analogues of the Zucker conjecture for singular varieties with isolated singularities and the moduli spaces of Riemann surfaces.

Steve was a careful writer and explained ideas well. Because of this, he also wrote many expository articles. The most substantial one is the joint survey with Brylinski, Z17 in [22], which gives a comprehensive overview of the development in Hodge theory from 1975 to 1990. It is over 100 pages, and Steve's own contributions to Hodge theory and L^2-cohomology are essential parts of the new development.

Cohomology is a unifying concept of almost all of Steve's work. Lizhen Ji is reminded of a conversation with J. P. Serre, where he asked Serre how he could work on so many different subjects. Serre replied, "What do you mean? They are all cohomology!" Yes, indeed!

We conclude this tribute with remarks in roughly chronological order by David Cox, Hélène Esnault, Leslie Saper, Richard Hain, Masa-Hiko Saito, Michael Harris, and Lizhen Ji.

David Cox

I met Steve in the fall of 1970 when we were entering graduate students at Princeton. We both studied algebraic geometry, though I was more algebraic (à la Grothendieck) while Steve was more transcendental (à la Griffiths). This made for some lively conversations. A few weeks after we met, we realized that we had to write a joint paper because the combination of our last names, in the usual alphabetical order, is remarkably obscene.

In 1975, Steve and I were reunited at Rutgers, and our joint paper arrived three years later when we wrote "Intersection numbers of sections of elliptic surfaces" [4], published in *Inventiones Mathematicae* in 1979. The paper was inspired by the work of William Hoyt and his student Charles Schwartz, who wanted to show that certain sections of an elliptic fibration $\bar{f} : \bar{X} \to \bar{S}$ generated the group of all sections modulo torsion. Their methods required difficult calculations using the Eichler pairing of the automorphic forms associated to the sections.

Steve and I realized that this could be recast using an isomorphism of parabolic cohomology and the quotient L^1/L^2 coming from the Leray spectral sequence for $\bar{f}$. Steve's *Annals* paper [3] played a key role in the proof. Under this isomorphism, we showed that the Eichler pairing of sections coincides with the intersection pairing of the cohomology classes of the sections, suitably modified to lie in L^1. We also gave a simple algorithm to compute the pairing that only required knowing where the sections meet the singular fibers of $\bar{f}$. This was the first joint paper either of us had written, though Steve had a much better sense of what "joint work" means. I learned a lot from him.

When I arrived at Rutgers, Steve and I decided to run a seminar on the classification of algebraic surfaces, a subject I knew very little about at the time. We followed Shafarevich's 1967 Steklov monograph *Algebraic Surfaces*. I fell in love with elliptic surfaces and wrote papers about them for over ten years, beginning with [4]. I credit Steve with influencing an important part of my research career. Another example of his influence is the seminar he ran on toric varieties and toroidal compactifications. This provided valuable background for Steve's later work on compactifications of locally symmetric spaces. It was helpful for me as well, when about 15 years later I began doing research on toric varieties. As I said above, I learned a lot from Steve.

After I left Rutgers in 1979, Steve and I saw each other infrequently. I have fond memories of the postcards he would send me from various places around the world. We also spent time together in Hanoi in the winter of 2006 when he introduced me to Vietnamese coffee. Steve last visited me a couple of years ago and was pleased to see that his photo of Mount Rainier was still displayed in my living room. I will miss Steve and his unique sense of humor.

Hélène Esnault

At a time when traveling was less easy than what we experienced later (until the time of the pandemic), and email was in its infancy, I basically never had a serious direct face-to-face mathematical contact with Steve Zucker, even though he came to Bonn for a term on Hodge theory of which I was one of the organizers. I studied his 1979 paper [3] a long time after it was published. For me personally, it was a step before the theory of perverse sheaves. Steve's article remains the first foundational example, which in addition is nicely written.

I now say a few words about four of Steve's important papers on Hodge theory. I thank Pierre Deligne for kindly reading this report.

"The Hodge conjecture for cubic fourfolds," *Compositio Math.*, 1977 [2]. Steve proves the Hodge conjecture in codimension 2 for complex cubic fourfolds: rational classes in Betti cohomology H^4 of type $(2,2)$, which are called codimension 2 Hodge classes, are the cohomology classes of algebraic cycles of codimension 2. At the time, it was the first example which was not derived from the well-understood codimension 1 case. The proof uses "the method of normal functions, based on an outline presented by Phillip Griffiths" as Steve says in the introduction to [2]: a Hodge cycle produces a normal function with values in the intermediate Jacobian of the cubic threefolds which are the fibers of a Lefschetz pencil. Algebraicity in the intermediate Jacobian of cubic threefolds had been studied by Clemens-Griffiths (*Ann. of Math.*, 1972). At the same time Clemens gave a purely algebraic proof of the theorem based on the observation that there is a correspondence between the cubic fourfold and the family of Fano varieties of lines in the fibers of a Lefschetz pencil, which consists of the tautological $\mathbb{P}^1$-bundle above the latter, to which the problem is then reduced. This is the content of Appendix A of [2], written by Steve. Finally he writes in Appendix B an example of Mumford to the effect that the Hodge conjecture is not true in Kähler geometry. In addition, Murre (*Nederl. Akad. Wetensch. Proc. Ser. A*, 1977) observes that a cubic fourfold is unirational. This reduces the problem to the compatibility of the Hodge conjecture with blow-ups in dimension 4. Bloch observed that Murre's proof applies to quartic fourfolds as well.

Hélène Esnault is Einstein Professor of Mathematics at Freie Universität Berlin. Her email address is `esnault@math.fu-berlin.de`.

"Hodge theory with degenerating coefficients: L^2-cohomology in the Poincaré metric," *Ann. of Math.*, 1979 [3]. In the 70s, Deligne endowed the cohomology $H^i(S, \mathcal{V})$ of a polarized variation $\mathcal{V}$ of real Hodge structure of pure weight w on a smooth complex variety S, a notion due to Griffiths (*Amer. J. Math.*, 1968), with a mixed real Hodge structure of weight $\geq i + w$, which is pure of weight $i + w$ if S is projective. It is functorial in $\mathcal{V}$ and S. If $\mathcal{V}$ is a subquotient (which by Deligne's semisimplicity theorem is necessarily a summand) of a local system $R^j f_* \mathbb{C}$, where $f : X \to S$ is a smooth projective morphism between smooth varieties (in which case we say that $\mathcal{V}$ is geometric), then $H^i(S, \mathcal{V})$ is a subquotient of $H^{i+j}(X, \mathbb{C})$ as a mixed Hodge structure, or pure Hodge structure if X and S are projective. It is natural to ask how to geometrically recognize the pure weight $i + w$ subspace $W_{i+w}H^i(S, \mathcal{V}) \subset H^i(S, \mathcal{V})$. The main theorem of the article is that if $j : S \hookrightarrow \bar{S}$ is a good compactification, and S has dimension 1, then $W_{i+j}H^i(S, \mathcal{V}) = H^i(\bar{S}, j_*\mathcal{V})$. In addition, if $\mathcal{V}$ is geometric, and $\bar{f} : \bar{X} \to \bar{S}$ is a good compactification of f, then $W_{i+j}H^i(S, \mathcal{V})$ is a subquotient of $H^{i+1}(\bar{X}, \mathbb{C})$ as a Hodge structure via the map $R^j \bar{f}_* \mathbb{C} \to j_* R^j f_* \mathbb{C}$. The proof is purely analytic, using only L^2 and harmonic methods, and thus applies in the Kähler case as well. In his introduction, the author thanks Deligne both for the formulation of the theorem and for the methods developed and used. This article has been of great importance. In modern terminology one could summarize the purity result by saying that if $\mathcal{V}$ is any local system on S, then $j_*\mathcal{V}$ is the intermediate extension. If in addition $\mathcal{V}$ is a polarizable variation of pure real Hodge structure, then its intermediate extension is a pure sheaf, and cohomology of a pure sheaf on a smooth projective variety is pure. The existence of intermediate extensions and their purity in any dimension has been at the core of the theory of perverse sheaves initiated by Goresky-MacPherson, generalized by Deligne (*Invent. Math.*, 1983), then further developed by Beilinson-Bernstein-Deligne-Gabber (*Astérisque*, 1982).

"Variation of mixed Hodge structure, I & II," *Invent. Math.*, 1985 [9, 10]. In view of [3], El Zein's cohomological mixed Hodge complexes (*C. R. Acad. Sci.*, 1983), Schmid's SL_2-orbit theorem (*Invent. Math.*, 1973), and Steenbrink's definition of a limiting mixed Hodge structure (*Invent. Math.*, 1976), it is natural to ask whether there is a notion of a polarized variation of mixed Hodge structure $\mathcal{V}$ on a smooth curve S, such that $H^i(S, \mathcal{V})$ carries a mixed Hodge structure with the property that if $\mathcal{V} = R^j f_* \mathbb{C}$ comes from geometry via a not necessarily smooth or proper morphism $f : X \to S$, the mixed Hodge structure is compatible with the one on $H^\bullet(X, \mathbb{C})$. The problem is solved in [9] (written with Steenbrink) by the introduction of the relative weight filtration, a notion, as the authors explain in the acknowledgement, due to Deligne. It is characterised by a simple compatibility property. The rational local system $\mathcal{V}$ may be assumed to be locally unipotent, with associated nilpotent operator N; it is endowed with a Hodge filtration satisfying Griffiths' transversality, and a flat weight filtration W. The graded pieces Gr_k^W are polarizable and pure. The condition is then that there is a new filtration M on the fibers at the punctures of Deligne's extension of $\mathcal{V}$, such that N^i equates $Gr^M_{k+i}Gr^W_k$ with $Gr^M_{k-i}Gr^W_k$. If the filtration W is trivial, this is Deligne's nilpotent operator. The authors show using the methods of their previous articles, without using the new harmonic method, that this condition is the right one to yield a mixed Hodge structure on $H^i(S, \mathcal{V})$. The compatibility with the mixed Hodge structure of $H^i(X, \mathbb{C})$ in the geometric case is handled in [10]. The definition has had an important impact on Morihiko Saito's definition of the theory of mixed Hodge modules (*Publ. Res. Inst. Math. Sci.*, 1990), and beyond on the motivic enhancements of those. If S is higher dimensional, Cattani-Kaplan (*Invent. Math.*, 1979) show the uniqueness of the right monodromy filtration, called monodromy-weight filtration, an important step to show the algebraicity of the Hodge locus in a higher-dimensional variation (Cattani-Deligne-Kaplan, *J. Amer. Math. Soc.*, 1995), which very recently has been recovered and vastly generalized using o-minimality techniques.

Leslie Saper

I first met Steve when he was visiting the Institute in 1980 or 1981. I was a young graduate student at Princeton but I hung around the Institute a lot since my advisor Yau was there. I was studying L^2-cohomology and Steve kindly responded to my hesitant questions by sitting me down in the Common Room of Fuld Hall and patiently explaining to me the Poincaré metric on the punctured disk. Steve was a kooky guy but so is everyone in one way or another; I greatly valued our friendship. We met many times over the years, sharing our love of food (Zabaglione, crabs) and music (Najima Plays Ravel) as well as mathematics. He took a great interest in me, often pushing me to clarify what I was aiming at in my preprints and never letting me get away with a vague statement. He taught me to do calculations and to listen to what they were telling me. Sadly we never collaborated aside from a small expository paper.

The first player in the Zucker conjecture is the L^2-*cohomology* $H^*_{(2)}(\Gamma\backslash X; \mathbb{E})$. Let $\Gamma\backslash X = \Gamma\backslash G/K$ be a locally

Leslie Saper is a professor of mathematics at Duke University. His email address is saper@math.duke.edu.

Hermitian symmetric space with G defined over $\mathbb{Q}$ and $\Gamma \subset G$ arithmetic, and let $\mathbb{E}$ be a local coefficient system on $\Gamma\backslash X$ induced from a finite-dimensional complex representation $\rho: G \to \mathrm{GL}(E)$. The L^2-cohomology is defined analogously to de Rham cohomology except that the $\mathbb{E}$-valued differential forms are required to satisfy an L^2-growth condition with respect to the locally symmetric metric and a natural metric on $\mathbb{E}$; for $\Gamma\backslash X$ compact the L^2-condition is vacuous and one recovers de Rham cohomology. L^2-cohomology has a Hecke action and Borel (*Duke Math. J.*, 1983) showed $H^*_{(2)}(\Gamma\backslash X;\mathbb{E})$ is isomorphic to the $(\mathfrak{g},K)$-cohomology of $L^2(\Gamma\backslash G)^\infty \otimes E$; consequently L^2-cohomology is closely related to automorphic forms. Basic functional analysis and elliptic regularity show that Hodge theory holds for $H^*_{(2)}(\Gamma\backslash X;\mathbb{E})$ *provided, as Borel and Casselman* (*Duke Math. J.*, 1983) *proved in this case, it is finite dimensional*. So the L^2-cohomology is equal to the space of L^2-harmonic forms and has a (real) Hodge structure.

The next player is Goresky and MacPherson's (middle-perversity) *intersection cohomology* $IH^*(\Gamma\backslash X^*;\mathbb{E})$. Here $\Gamma\backslash X^*$ is the Baily-Borel-Satake compactification, a (usually) singular algebraic variety defined over a number field with a stratification indexed by Γ-conjugacy classes of rational maximal parabolic subgroups of G. Intersection cohomology is characterized by local vanishing in degrees $\geq c$ at points on the complex codimension c stratum when $c > 0$ (and a dual condition); for $\Gamma\backslash X^*$ smooth one recovers usual singular cohomology. The intersection cohomology of $\Gamma\backslash X^*$ is finite dimensional, has a Hecke action, can be defined algebraically, and (via ℓ-adic intersection cohomology if (ρ, E) is defined over $\mathbb{Q}$) has a Galois action.

When $\Gamma\backslash X$ is compact, the de Rham theorem shows that the two cohomologies above are isomorphic. The *Zucker conjecture* [6, (6.20)] states that $H^*_{(2)}(\Gamma\backslash X;\mathbb{E}) \cong IH^*(\Gamma\backslash X^*;\mathbb{E})$ even if $\Gamma\backslash X$ is noncompact, thus yielding a topological interpretation of L^2-cohomology. The isomorphism respects the Hecke action and endows $IH^*(\Gamma\backslash X^*;\mathbb{E})$ with an *L^2-Hodge structure*. An important potential application of the conjecture, which Steve mentions in the paper Z10 in [22] together with his thanks to Langlands, is to define a variant of the Hasse-Weil zeta function for $\Gamma\backslash X^*$ using intersection cohomology and relate it to the L-functions of automorphic representations of G which could be studied using L^2-cohomology.

The example of the upper-half plane $\mathfrak{h}$ (discussed by Steve in [3, §12]) illustrates the significance of the Zucker conjecture to the theory of automorphic forms. Consider $H^*_{(2)}(\Gamma\backslash\mathfrak{h};\mathbb{E}_k)$ where the representation on $E_k = (\mathbb{C}^2)^{\otimes k}$ is induced from the standard representation of SL(2). (Note that the metric in a neighborhood of a cusp is the Poincaré punctured disk metric.) For $k \geq 0$, the map $f \mapsto \phi_f = f(\tau)\binom{\tau}{1}^{\otimes k} d\tau$ defines an isomorphism from the space $S_{k+2}(\Gamma)$ of cusp forms of weight $k+2$ with respect to Γ to the space of L^2-holomorphic (and hence harmonic) $\mathbb{E}_k$-valued 1-forms on $\Gamma\backslash\mathfrak{h}$; the cuspidal condition corresponds to the L^2-condition. Thus we have a Hodge decomposition $H^1_{(2)}(\Gamma\backslash\mathfrak{h};\mathbb{E}_k) \cong S_{k+2}(\Gamma) \oplus \overline{S_{k+2}(\Gamma)}$. On the other hand, the vanishing condition on intersection cohomology shows that $IH^1(\Gamma\backslash\mathfrak{h}^*;\mathbb{E}_k)$ is the parabolic cohomology. Thus the Zucker conjecture in this case is the Eichler-Shimura isomorphism.

Special examples of the conjecture with $\mathbb{Q}$-rank up to 3 were proven by Zucker in [6] and the paper Z10 in [22]. The general $\mathbb{Q}$-rank 1 case was proven by Borel (*Emmy Noether in Bryn Mawr*, 1983) and the $\mathbb{Q}$-rank 2 case was announced by Borel and Casselman (*C. R. Acad. Sci.*, 1985). The Zucker conjecture in general was proven in 1987, independently by Looijenga (*Compositio Math.*, 1988) and by Stern and myself (*Ann. of Math.*, 1990).

The goal in both proofs is to establish the local vanishing of L^2-cohomology near a singular point as required by the characterization of intersection cohomology. The key problem is that the L^2-condition disallows local cohomology with high weight, however, intersection cohomology disallows local cohomology with high *degree*. (By "weight" we roughly mean the rate of growth of the norm of a form as one approaches a singular point; since the metric on X is G-invariant these weights correspond to Lie-theoretic weights.) A proof must show these two truncations are the same. Looijenga's proof uses algebraic geometry—the weights are related to those of mixed Hodge structures via a local Hecke operator and the equivalence of truncations is deduced from Morihiko Saito's version of the decomposition theorem (*Publ. RIMS*, 1990). (Later Looijenga and Rapoport (*Proc. Sympos. Pure Math. 53*, 1991) gave another version of this proof and highlighted the equivalence of the several types of "weight" that appear.) The proof with Stern is more analytic—we prove an estimate by methods based on Matsushima and Murakami (*Ann. of Math.*, 1963) and Raghunathan (*Osaka J. Math.*, 1966, 1979) and then show the estimate yields the desired vanishing by a careful consideration of roots and weights. For a more detailed description of the two proofs one can read Zucker's own comparision in the paper Z11 in [22].

The two proofs each have advantages. Looijenga's proof fits within important current ideas in algebraic geometry and the ideas could possibly be applied to other varieties. The methods in my proof with Stern can be applied to non-algebraic locally symmetric spaces as I now indicate.

In the papers Z10 and Z11 in [22], Steve discussed an extended conjecture made by Borel in 1983 and expressed the wish that there be a unified proof of both conjectures. Instead of requiring $X = G/K$ to be Hermitian symmetric, Borel just assumes it is *equal rank*, $\mathrm{rk}_{\mathbb{C}} G = \mathrm{rk}_{\mathbb{C}} K$; this is natural from the viewpoint of representation theory though X may no longer have a complex structure. Instead of $\Gamma\backslash X^*$ being the Baily-Borel-Satake compactification, Borel considers any Satake compactification in which all real boundary components are also equal rank. Stern and I verified case-by-case that our methods applied here. Later (in *Pure Appl. Math. Q.*, 2005) I gave a unified proof of both conjectures by using the theory of $\mathcal{L}$-modules. When restricted to the Hermitian case, the underlying idea is essentially the same as in my proof with Stern.

Another wish of Steve's was to identify the L^2-Hodge structure on intersection cohomology with the canonical Hodge structure obtained by Morihiko Saito. Steve proved this in [12] for the special case of a variety with isolated singularities endowed with a Kähler metric I created (blended from locally symmetric models) and for which I had proved the analogue of the Zucker conjecture (*J. Differential Geom.*, 1992); the proof also applied to several rank 1 locally symmetric spaces. At the banquet for his 65th birthday conference Steve proposed we solve this problem in general and we spent the last hours of my visit eating sandwiches at a deli and working on that. Sadly that was the last time I saw him.

Richard Hain

I first met Steve in the summer of 1984 at a conference at the University of British Columbia. Deligne had posed a conjecture to each of us about variations of mixed Hodge structure with unipotent monodromy. The conjecture had two parts. As Steve tells it in [22], he told me that one part was reasonable, the other not, and that I responded with the opposite assessment. Thus began our first collaboration. It resulted in the paper [11] in which the conjecture is proved.

Steve was a singular character. I do not think I am unique in making this assessment. He loved music, especially piano music, and strong coffee. His opinions were generally like his coffee, strong. These included his opinions on undergraduate education, which you can read about in his opinion piece "Evaluation of our courses" in the August 2010 issue of these *Notices*. He was certainly "old school." I don't know what his students thought of him, but from where I sat, he seemed to really care about his students and their learning.

At one point, I do not remember when, Steve told me that he was unhappy with his salary at Hopkins and that he had gone to see the Dean. He told the Dean that, if his salary was not increased, he would charge his calculus students a supplement for attending his lectures. This, he said, was perfectly reasonable as, when J. Sylvester taught at Hopkins in the 19th century, he did the same. I don't know how much of this story is true, but it is quintessential Steve.

One of the main mathematical themes in his work was the study and applications of variations of *mixed* Hodge structure. Variations of pure Hodge structure are local systems over a smooth complex algebraic variety that are abstractions of the local systems of cohomology groups associated to a family of smooth projective varieties. Wilfried Schmid (*Invent. Math.*, 1973) had established remarkable theorems about these in the 1970s. Also in the 1970s, Deligne showed that the cohomology groups of all complex algebraic varieties carry a canonical *mixed* Hodge structure that generalizes the familiar bigrading of the complex cohomology groups of a smooth projective variety. So it was natural to consider the mixed analogue of Schmid's work. This arises, for example, when considering locally topologically trivial families of algebraic varieties. These days, one might say that they are the Hodge analogue of "motivic local systems." His paper [9] with Steenbrink is the first paper on the subject and is fundamental. This was followed by its sequel [10] and then our paper [11] on unipotent variations of mixed Hodge structure.

By his own admission, his paper [3] on the L^2-cohomology of complex curves in the Poincaré metric was his favorite and his best. It led him to formulate the "Zucker conjecture," which I will not discuss other than to say that it stimulated a lot of important work and was eventually proved independently by Saper-Stern and Looijenga. I was not a participant in that story. However, I am a great admirer of this beautifully written paper, which I have used to generalize our work [11] from unipotent variations of mixed Hodge structure (VMHS) to all VMHS over a smooth base. This I needed to apply to problems related to moduli spaces of curves, especially modular curves. The point is that, by a Lefschetz argument, it suffices to prove the result for variations over curves. Without getting into technicalities, everything I needed but did not already know is in [3].

My second main collaboration with Steve, and also Johan Dupont, was an (unsuccessful) attempt in the paper Z23 in [22] to work out the precise relationship between the Borel and Beilinson regulators and Chern-Simons classes of flat vector bundles. While we did not completely resolve the problem, we did succeed in isolating the remaining difficulties and clarifying what needed to be done to resolve them.

The second to last time I saw Steve was at the Hodge theory conference in Vancouver in the summer of 2013. I was shocked to see how much his health had deteriorated.

Richard Hain is a professor of mathematics at Duke University. His email address is hain@math.duke.edu.

It is enough to say that after falling one night, he ended up in hospital in Vancouver, where he remained for about a week. The last time I saw him was at his 65th birthday conference at Hopkins in 2015. His health had further deteriorated.

Steve was a lot like an olive: an acquired taste. His passing was premature. I will miss him.

Masa-Hiko Saito

Two years after my PhD, in October 1987, I had the opportunity to stay at the Max Planck Institute in Bonn for a year and participate in a project to celebrate the 60th birthday of the director, Friedrich Hirzeburch. From autumn to winter, Esnault and Viehweg organized activities on Hodge theory, and Steve was invited as a big name in this field. Although I had been reading his papers [3], [4], [5], [9], [10], I was reluctant to talk with such a big name by myself. So, I was surprised when Steve suddenly came to my office and said "Konnichiwa" (Hello) in Japanese. To my further surprise, he knew my work, including the infinitesimal Torelli problem for elliptic surfaces and he proposed collaborations on the Torelli problem for fiber spaces. In this way, our collaboration began.

Figure 3. Zucker and Saito in Bonn, 1987.

Torelli problems are concerned with the relation between deformation theory (moduli) and period mappings (Hodge structures). Let $f : X \to S$ be a projective morphism from a smooth projective variety X to a smooth projective curve with connected fibers, and let $\Sigma \subset S$ be the set of all critical values of f and $Y = f^{-1}(\Sigma)$ such that $g \coloneqq f_{|X-Y} : X - Y \to S - \Sigma$ is smooth. We assume that Y is reduced and normal crossing for simplicity. We note that we can formulate the Torelli problem for these pure or mixed Hodge structures for X and $X - Y$. In [3], Steve showed that the natural Hodge structure on $H^{k+1}(X,\mathbb{Z})$ induces a Hodge structure on its Leray quotient

$$(L^1/L^2)H^{k+1}(X,\mathbb{Z}) \simeq H^1(S, R^k f_*\mathbb{Z}),$$

where the last cohomology group has an internal Hodge structure of pure weight $k+1$. Steve's main motivation was to formulate the Torelli problem with pure or mixed Hodge structures on

$$H^1(S, R^k f_*\mathbb{Z}), \quad H^1(S-\Sigma, R^k g_*\mathbb{Z}).$$

For the infinitesimal Torelli problem, one should consider the deformation theory of X or pair (X,Y), of the morphisms f, g. The infinitesimal deformations of (X,Y) were governed by the logarithmic tangent sheaf $\Theta_X(-\log Y)$. The set of all infinitesimal deformations of (X,Y) is given by $H^1(X, \Theta_X(-\log Y))$ and the differential of the period mapping from the local Kuranishi moduli space of (X,Y) to the period domain is given by the cup product with contraction

$$H^1(\Theta_X(-\log Y)) \otimes H^q(\Omega_X^p(\log Y)) \to H^{q+1}(\Omega_X^{p-1}(\log Y)).$$

In our main result, we defined a natural three-step filtration on $H^1(X, \Theta_X(-\log Y))$ which fits into the differential of the period mapping for fiber spaces and by using this we obtained the infinitesimal mixed Torelli theorem for elliptic surfaces with singular fibers, all of type I_1. After four years, with the help of an email connection, we had the joint paper [13].

After Steve left Bonn, I was interested in a special type of deformation of fiber spaces, namely deformations of X fixing the base data (S,Σ). Arakelov theory shows that the nonisotrivial fiber space of curves of genus $g \geq 2$ has no such deformation, that is, they are rigid. Chris Peters came to MPI and gave a seminar talk about an extension of Faltings' result of abelian varieties (VHS of weight 1) to general polarized variations of Hodge structures (VHS). Then my head was filled with finding an example of a nonrigid family of K3 surfaces. After a while, I realized that the Kuga-Satake construction of a family of abelian varieties from special quaternion algebras over number fields can be modified to obtain a nonrigid family of K3 surfaces, or equivalently, a nonrigid VHS of weight 2, and nonrigid families of K3 surfaces may be classified. I sent an air mail letter to Steve from Bonn to Baltimore about this construction and asked him to work on the classification problem for nonrigid families of K3 surfaces together with me. I was delighted when he agreed.

After I returned to Japan from Bonn, I moved to Hokkaido University in Sapporo in 1989. I invited Steve to that university in the summer, for I knew that Steve

Masa-Hiko Saito is a professor of mathematics at Kobe University and Director of the Center for Mathematical and Data Sciences. His email address is `mhsaito@math.kobe-u.ac.jp`.

loved cool and fresh weather like in the summer in Sapporo, and also there was good food. We were working on both projects in an office, and when I explained to him about the Kuga-Satake construction of a nonrigid family of K3 surfaces using a quaternion algebra, he made an impressive remark to me: "This is a quaternionic nightmare." Even so, we were happy to write our second joint paper [14].

In 1990/1991, I was invited to the third JAMI program and had a chance to visit Johns Hopkins University. After taking some time to get used to American life, our family found it enjoyable. I interacted with Steve on a daily basis using the email system that was just becoming popular at the time. Without doubt, Steve improved my English vocabulary in various fields at that time.

In the middle of the JAMI program, in 1991, my position changed to associate professor at Kyoto University and then in 1996, I became a full professor at Kobe University. After I hosted his visit to Kyoto University in 1993, Steve then visited Kyoto many times, and he could visit many Japanese friends in various places in Japan. One day, we invited him to a dinner in the small garden at our home in Hieidaira. Our two children thought Steve was a big uncle. I believe he felt at home when he stayed in Japan.

In 2006, we received very good news. The 3rd Seki Takakazu Award of the Mathematical Society of Japan was given to JAMI. Steve was the director of JAMI at that time, and he gave a very impressive speech at the big conference site of the biannual meeting of the Mathematical Society of Japan. The celebration party at the University of Tokyo was very nostalgic and fun, with the participation of JHU members and many Japanese mathematicians who participated in the JAMI program. One photo can be seen below.

After I invited him to a colloquium talk in Kobe in 2009, we did not have a chance to have further contact with each other. In 2015, I had a sudden phone call from Steve while he was visiting Kyoto, and we invited him to dinner at our home. Very sadly, that was the last time we met him.

Figure 4. Zucker with Saito's children in Hieidaira, 1993.

Figure 5. Mathematical Society of Japan celebration in Tokyo, 2006.

Michael Harris

To start the story of my friendship with Steve in the middle—as most of our conversations started—our collaboration began in 1988, at the two-week-long conference on automorphic forms in Ann Arbor. The Zucker conjecture had inspired a lot of work among specialists, and since it had just been proved—in two different ways—Steve was in Ann Arbor as a kind of visiting dignitary. My chief memory of the conference was that it took place during an excruciating heat wave, so our collaboration inevitably started over beer in an air-conditioned bar.

The Shimura variety $S(G,X)$ attached to a reductive group G over $\mathbb{Q}$ with hermitian symmetric space X is an algebraic variety, over a canonical number field, whose complex points are given by a union of quotients $\Gamma_i\backslash X$, where $\{\Gamma_i\}$ is a collection of arithmetic subgroups of $G(\mathbb{Q})$ indexed by a subset of the adeles of G. It follows from the theory of Griffiths that any irreducible algebraic representation $\rho : G \to GL(V)$ gives rise canonically to a variation of Hodge structure (VHS) $\mathcal{V}$ over $S(G,X)$ with general fiber isomorphic to V. Steve's paper [5] had studied the

Hodge structure of the cohomology $H^*(S(G,X),\mathcal{V})$ with coefficients in such a VHS when $S(G,X)$ is compact; he identified the graded pieces of the cohomology with coherent cohomology with coefficients in sheaves (now called automorphic vector bundles) attached to representations of a maximal compact subgroup of $G(\mathbb{R})$, and showed that the Hodge decomposition was given by harmonic forms for the invariant metric, which are automatically square integrable. This decomposition had the appealing property that the automorphic vector bundles that arose were matched naturally with the members of Harish-Chandra's discrete series L-packet attached to the original V; Faltings later gave a more conceptual explanation of this relation in terms of the Bernstein-Gelfand-Gelfand resolution of V. The Zucker conjecture generalized this to noncompact Shimura varieties: H^* was replaced by middle intersection cohomology and $S(G,X)$ was replaced by its minimal (Baily-Borel-Satake) compactification $S(G,X)^*$. This made it possible to apply the methods of perverse sheaves to cohomological automorphic forms.

The boundary strata of $S(G,X)^*$ are indexed by rational maximal parabolic subgroups of G, whereas the boundary strata of the Borel-Serre compactification $S(G,X)^{\mathrm{BS}}$, which (as Steve showed) is the maximal compactification of $S(G,X)$, are in bijection with **all** rational parabolic subgroups P. Borel and Harder had shown that the cohomology of the open $S(G,X)$, which is the same as that of $S(G,X)^{\mathrm{BS}}$, has a boundary part that is computed by an E_1 spectral sequence starting with the cohomology of the locally symmetric spaces attached to all P. The first vague question I asked Steve in that bar was whether the terms of this spectral sequence had a natural mixed Hodge structure (MHS)—although some of the spaces involved have no complex structure—that explained the MHS on the cohomology of the open variety. In Ann Arbor, I had explained how to relate automorphic forms to the coherent cohomology of toroidal compactifications $S(G,X)_\Sigma$, attached to combinatorial data, with coefficients in Mumford's canonical extensions of automorphic vector bundles. The space $S(G,X)_\Sigma$, unlike $S(G,X)^{\mathrm{BS}}$, is an algebraic variety, whose boundary strata are also indexed by all rational parabolics, and my second vague question was whether the restriction of a cohomology class to the stratum attached to P could be computed by taking the constant term of the corresponding automorphic form, in spite of the bad singularities the automorphic form acquires along the boundary.

We found the correct formulation of the first question and solved it in our second paper [16], using Morihiko Saito's brand new theory of mixed Hodge modules. The second question was answered in our first and third papers [15] and [17], mainly by finding a way to interpret the simplicial structure of the toroidal boundary in terms of differential forms on the deleted neighborhood. I was glad to see that the intuitions we developed were of use in the initial stages of my paper with Lan, Taylor, and Thorne on the apparently unrelated problem of the construction of automorphic Galois representations. At about the same time, Richard Pink was developing an analogous theory for ℓ-adic cohomology of the $S(G,X)$ in terms of boundary strata of the minimal compactification; and some years later Burgos and Wildeshaus found a Hodge module analogue of Pink's construction. Wildeshaus has since identified the requirements for a motivic structure that would give rise to all these results by taking the corresponding realizations; he has proved the existence of such a structure in some cases, but a general motivic theory of intersection cohomology is not yet available.

We worked together for more than 10 years. Steve visited me at Brandeis and later in Paris, and I made several trips to Baltimore, where Steve put me up at his place and where our meals were invariably Asian. During these visits Steve corrected my many misunderstandings about mixed Hodge theory, explained some useful subtleties about homological algebra, and improvised virtuosically when the geometry didn't work according to plan. For example, the plan to interpret the Borel-Harder spectral sequence in terms of the toroidal compactification assumed that the neighborhoods of the P-strata in the two compactifications could be nudged continuously onto one another. The problem was that they could not: Steve was the expert on the topology of these compactifications, and he knew the two kinds of compactifications were deeply incompatible. This was a source of frustration for several months, until Steve found a way to ignore the incompatibility, using Whitehead's theorem on homotopy equivalence, which I think has not appeared before or since in a paper on automorphic forms.

When we were too tired to think about mathematics, Steve resumed the role of musical educator that he had occupied when we first met in Princeton, when I was an undergraduate and he was a graduate student. Mathematics graduate students were heavily involved in organizing Vietnam war protests; both of us followed the leaders, and that's probably where Steve and I began talking. Most of our conversations in those days were about classical music, though. The glass doors on the first floor of Fine Hall, past the elevator, now lead to an ordinary office space, which in our time was an unbelievably dusty room with an old upright piano that smelled of cigarette smoke and had reached a state where the distinction between tuned and untuned no longer applied; the piano's action was progressively breaking down and keys were acquiring a purely decorative function. In spite of that, Steve managed credible renditions of many classics of the piano repertoire on that piano, and convinced me that I could learn to play Bach's Italian Concerto. In Baltimore, nearly 20 years later, his passion for music was unabated, and I still treasure the CDs he insisted on giving me as a way of sharing his enthusiasm.

Although we collaborated fully on most of the details of our papers, it was understood that I would write the sections about automorphic forms, group theory, and everything having to do with canonical models of Shimura varieties, while Steve would write the parts about MHS and most of the geometry. Inevitably we each used our own familiar notation, so that it changed not only from one paper to the next but from one page to the next. Our second paper had a submission deadline, and the notational mischief that we did not have time to correct made some of the statements literally wrong as written, so we published an erratum. Otherwise the capricious notation merely posed a challenge to the unwary reader. This is why Google Scholar can still find 10 versions of our third paper with the unofficial subtitle "The nightmare continues."

When we were not trying to reconcile notation the collaboration was a joy, not a nightmare. From time to time, Steve and I thought there might be a Part IV. Our three papers had left several loose ends. In the first place, the Hodge part of the Zucker conjecture asks whether the natural L^2 Hodge decomposition on L^2 harmonic forms coincides with the canonical Hodge decomposition on intersection cohomology defined by Morihiko Saito. Our methods settled the problem in the affirmative for the most general coefficients, but the hard cases remain unsolved. We had also hoped that we could use MHS to establish restrictions on the existence of ghost classes in the cohomology of Shimura varieties, i.e., those nonzero classes which restrict to zero classes on boundary components of the Borel-Serre compactification. Steve worked this out in a few cases, and the project has been revived recently by my student Matias Moya Giusti and his collaborators, but in general the problem is completely open.

Steve and I never did get around to picking up where we had left off. But I still apply the lessons he taught me almost daily, and I still have so many questions for him!

Lizhen Ji

Steve Zucker was a highly original mathematician. But he was also a kind and highly original person. It makes me both happy and sad to contribute to this memorial tribute for Steve. To explain this, perhaps I should start with explaining how I met him and some later interaction I had with him.

I met Steve for the first time at his office at Johns Hopkins University around 1993 when I visited Steve Zelditch who was at Johns Hopkins at that time. How it happened is a bit of a long story.

I had two advisors: Shing-Tung Yau and Mark Goresky. Under Yau, my thesis was concerned with the spectral degeneration of Riemann surfaces with respect to the hyperbolic metric. Compact surfaces have only discrete spectra, while noncompact finite area hyperbolic surfaces have the continuous spectrum $[\frac{1}{4}, +\infty)$, which is described by Eisenstein series. The problem was to understand how the spectrum and related spectral data change when the Riemann surfaces become noncompact, or go to the boundary of the moduli space of Riemann surfaces.

Due to the nature of my thesis and related work, I met Steve Zelditch at a conference in 1993. Later we had a joint project on the spectral theory of Riemann surfaces, and he invited me to JHU to give a talk. At that time, I was starting to become interested in compactifications of symmetric spaces.

Indeed, after my thesis, I wanted to learn and do something broader. By accident, I read a new list of open problems by Yau and found a problem on identifying the Martin compactification of simply connected nonpositively curved Riemannian manifolds, in particular about the structure of positive harmonic functions on them. Since symmetric spaces of noncompact type are probably the most important class of such manifolds, I started to learn something about symmetric spaces and their compactifications, and determined the Martin compactification in terms of the maximal Satake compactification and the geodesic compactification. This resulted in my first book on compactifications of symmetric spaces (joint with J. C. Taylor and Y. Guivarch).

When I visited Zelditch at JHU for the first time, he told me that I would be definitely interested in meeting his distinguished colleague Steve Zucker, and introduced me to Steve at his office. I do not remember exactly what we talked about, but our conversations must have gone well, since we continued to meet and talk on later visits. At the first meeting, I probably explained my interest in compactifications of symmetric spaces, and he explained his interest in locally symmetric spaces and the L^2-cohomology.

A few years later, I went back to JHU, probably around 1996. In fact, I have been to JHU many times. At one point, I had three coauthors, besides Steve, at JHU. In addition to giving seminar talks, I also attended JAMI workshops at JHU three times.

At that time, I became more interested in locally symmetric spaces for several reasons. One reason was the first meeting with Steve. Another reason is that Mark Goresky was my second advisor. He spoke to me many times about locally symmetric spaces in connection with intersection cohomology, the Zucker conjecture, and the

Langlands program. They sounded like interesting topics to me. Since it seemed natural to extend the study of symmetric spaces to locally symmetric spaces, I decided to learn something about locally symmetric spaces.

At this second meeting with Steve, he gave me some reprints of his and mentioned a conjecture in his joint paper with Harris [Z21, Conjecture 1.5.8] about the incompatibility between the Borel-Serre compactification and the toroidal compactification, which states that their greatest common quotient is isomorphic to the Baily-Borel compactification. As mentioned earlier in the overview of Steve's work, I settled this conjecture a few years later (*Geom. Funct. Anal.*, 1998).

The situation was a bit more complicated than had been conjectured or described earlier. Let $\mathbf{G}$ be a semisimple linear algebraic group, and $G = \mathbf{G}(\mathbb{R})$. Let $K \subset G$ be a maximal compact subgroup, and $X = G/K$ the associated symmetric space of noncompact type. Let Γ be an arithmetic subgroup of $\mathbf{G}(\mathbb{Q})$. Assume that the $\mathbb{Q}$-rank of $\mathbf{G}$ is positive. Then the locally symmetric space $\Gamma\backslash X$ is noncompact. The results on this conjecture can be stated as follows:

1. *For every toroidal compactification* $\overline{\Gamma\backslash X}_{\Sigma}^{tor}$, *where* Σ *is a* Γ-*admissible polyhedral cone decomposition, its greatest common quotient with the Borel-Serre compactification* $\overline{\Gamma\backslash X}^{BS}$ *is a new compactification which is independent of the choice of* Σ, *dominates the Baily-Borel compactification* $\overline{\Gamma\backslash X}^{BB}$, *and can be determined explicitly.*
2. *In particular, if* $\mathbf{G}$ *is* $\mathbb{Q}$-*simple but not absolutely simple, then the greatest common quotient of toroidal compactifications and the Borel-Serre compactification is isomorphic to the Baily-Borel compactification of* $\Gamma\backslash X$ *and hence the conjecture is true.*
3. *If* $\mathbf{G}$ *is* $\mathbb{Q}$-*simple and* $\mathbb{Q}$-*split but not equal to* $\mathrm{SL}(2)$, *then the greatest common quotient of toroidal compactifications and the Borel-Serre compactification strictly dominates the Baily-Borel compactification, and hence the conjecture is not true.*

For example, the conjecture is true for Hilbert modular varieties, but fails for Siegel modular varieties and Picard modular varieties.

A related result was also proved in the paper: *In all cases, the greatest common quotient of toroidal compactifications and the reductive Borel-Serre compactification is isomorphic to the Baily-Borel compactification of* $\Gamma\backslash X$. As noted above, the reductive Borel-Serre compactification was introduced by Steve in [6].

Attending JAMI conferences at JHU was enjoyable and fruitful. I attended one organized by Steve and spent several weeks there. In that period, he would always lead a large group of participants to go to lunch and dinner. There were many young Japanese mathematicians. It was very lively, and he apparently enjoyed them. One weekend after lunch, we went to his house and he showed us his collection of books which contained some exotic Japanese books.

Whenever I went to JHU, I always saw Steve and we often went out to Chinese restaurants. Later I attended another JAMI workshop on Teichmüller theory, and Steve suggested we drive to a good Chinese restaurant. We had an unusual conversation starting on a freeway. I said that we have known each other for a long time but have never written a paper together, so maybe we should write one. At that time, I was very interested in the analogy between moduli spaces of Riemann surfaces and locally symmetric spaces. But the analogue of the Zucker conjecture for moduli spaces of Riemann surfaces had not been studied before. So we talked about this problem both in the car and at the restaurant. It seemed reasonable. After the dinner, we went to the main library of JHU and checked a few references. It indeed seemed OK, and the paper was basically finished (of course, the actual writing took a bit longer).

His conjecture in [16, Conjecture 1.5.8], his work, and our many conversations had a definite impact on me. This is the reason why I could be counted as a student of his, as he mentioned at the end of his autobiographical summary [22].

On one trip to JHU probably around 2011, I chatted with Steve. He seemed a bit sad. He told me that he went to Japan on a trip fully arranged by his Japanese friends as a birthday gift. He thought that there would be a birthday conference, but it did not happen. I responded right away without too much thinking, "No problem, we can arrange one for you." I was thinking of organizing such a conference at the Tsinghua Sanya conference center. After I returned from the trip, I inquired a bit and it did not seem easy to do so. But I did not mention this to Steve. Probably two years later, Steve called my office and said "You said that you were going to organize a birthday conference for me." I apologized and said, "Sure, we will do one for your 65th birthday." Then I told him that we could do one at JHU and needed a local organizer. He said that he could be the local organizer. I was hesitating but he insisted on it. Then we wrote an NSF conference proposal. It was a long wait till the middle of September 2014. Once we had the funding, we needed to invite people right away since the conference was to be held in November 2014. Zucker was very careful about the list of invitees. It turned out that most people quickly and happily accepted the invitation and we had a long and impressive list of speakers for a three-day conference (Steve even arranged a back-up speaker if some speaker did not show up): Pierre Albin, Edward Bierstone, Nero Budur, Alexandru Dimca, Osamu Fujino, Phillip Griffiths, Richard Hain, James Lewis, Takeo Ohsawa, Gregory Pearlstein, Colleen Robles, Leslie Saper, Mark Stern, Wilfried Schmid, and Tomohide Terasoma.

All things went well and the conference happened on November 21–23, 2014. But when I arrived at JHU one day before the conference, Steve told me that he was sick and fell on the ground and broke a spot on his nose. Speakers prepared their talks carefully and the conference went well mathematically except for the fact that Steve could not come to some of the talks. It was sad for me and others. At that time, one idea occurred to me. I proposed to the speakers to publish a proceedings in honor of Steve. Since I gave them the most convincing reason which people could see clearly, people agreed immediately.

The conference was held on Friday, Saturday, and Sunday, and there was a conference banquet on Saturday. Steve put a lot of effort into arranging it well since he was the local organizer of the conference. I had some conversations which still make me feel sad now while I am writing this sentence. I told Steve that this banquet was an important formal social event and asked whether some member of his family could come and join it. He said that he had a nephew but he probably could not come.

After the conference, the task of editing the proceedings started. Among all of the books I have edited, this was the easiest since it was relatively easy to collect all of the contributions. Besides the speakers, several other people were invited.

In the fall of 2016, he visited me at the University of Michigan. By that time, the book was basically done in the sense that most papers were collected and refereed. Right from the beginning, I suggested that we include the group photo from the conference in the book. Originally Steve liked the idea and agreed. But when he came to Ann Arbor, he changed his mind. I asked him for the reason. He said that he hurt his nose before the conference and he did not look too good in the picture. It was very difficult for me to see the problem he pointed out. It took some arguing to convince him to keep the picture. I also asked him to write a summary of his own work and he happily agreed and wrote the summary [22].

Some months after he went back, he wrote me an email saying that he just moved out of the hospital since he was sick. I was aware that he was not too healthy but did not realize that it was getting worse quickly. By that time, all the material for the book was collected and sent to the publisher, the Higher Education Press in Beijing. It was sitting in a busy queue of the publisher. I explained the situation to the senior editors of the Higher Education Press in China, Liping Wang and Peng Li, and asked them to publish it as soon as possible. They agreed and put in extra effort to speed it up. At the beginning of August 2017, the book *Hodge Theory and L^2-analysis* was published, and the editors sent a copy to Steve by express mail from Beijing.

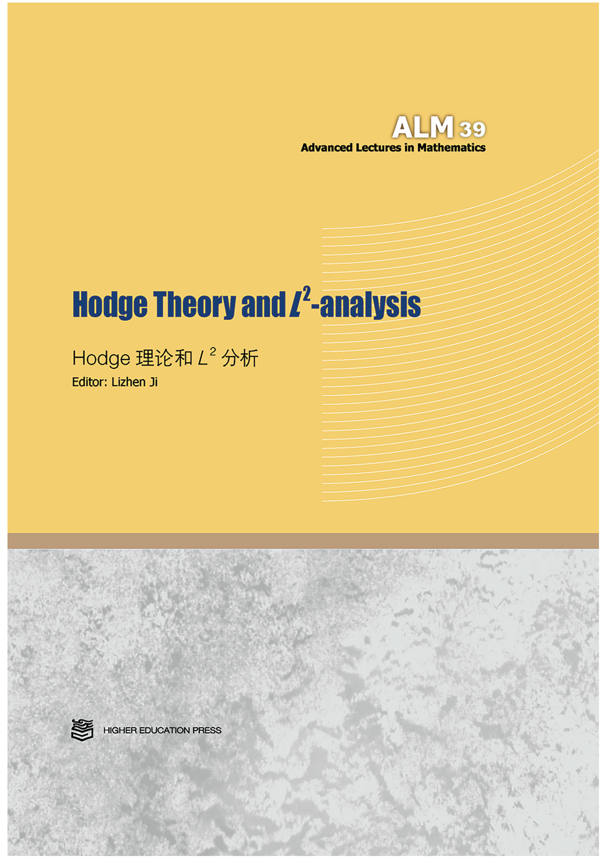

Figure 6. Cover of *Hodge Theory and L^2-analysis*.

On August 5, 2017, I received an email from Steve:

> Dear friends and family,
>
> I am now in rehabilitation for what I hope is the last time for the current reason.
>
> Issues of pain in walking were largely assuaged by physical therapy. I took to walking with a cane.
>
> More significant were issues of balance and falling; I have fallen (landed on the ground) numerous times in the past year. These have been diagnosed as Parkinson's disease, or something related. I have been put on medication, and that seems to be working.
>
> Best wishes, - Steve.
>
> PS - This message is being sent to a dozen recipients. Feel free to inform others who might care.

On September 8, 2017, I received a call from Steve that he went to the mathematics department and got the book and he was very happy with it. He sounded happy and healthy too.

Sadly, this email was the last email I got from Steve, and the phone call was also the last one.

Writing this article made me sad when I went over some of the conversations and experiences with Steve. On the other hand, the happy thing is that it is the mathematics which matters. People will always remember Steve's work, especially the Zucker conjecture. The publishing of the conference proceedings is concrete evidence.

This memorial tribute is even a better example. Besides mathematics, people will remember Steve's kindness and generosity.

ACKNOWLEDGMENTS. The authors would like to thank the referees for their helpful suggestions.

References

[1] Steven Zucker, *Generalized intermediate Jacobians and the theorem on normal functions*, Invent. Math. **33** (1976), no. 3, 185–222, DOI 10.1007/BF01404203. MR412186

[2] Steven Zucker, *The Hodge conjecture for cubic fourfolds*, Compositio Math. **34** (1977), no. 2, 199–209. MR453741

[3] Steven Zucker, *Hodge theory with degenerating coefficients. L_2 cohomology in the Poincaré metric*, Ann. of Math. (2) **109** (1979), no. 3, 415–476, DOI 10.2307/1971221. MR534758

[4] David A. Cox and Steven Zucker, *Intersection numbers of sections of elliptic surfaces*, Invent. Math. **53** (1979), no. 1, 1–44, DOI 10.1007/BF01403189. MR538682

[5] Steven Zucker, *Locally homogeneous variations of Hodge structure*, Enseign. Math. (2) **27** (1981), no. 3-4, 243–276 (1982). MR659151

[6] Steven Zucker, *L_2 cohomology of warped products and arithmetic groups*, Invent. Math. **70** (1982/83), no. 2, 169–218, DOI 10.1007/BF01390727. MR684171

[7] Steven Zucker, *Satake compactifications*, Comment. Math. Helv. **58** (1983), no. 2, 312–343, DOI 10.1007/BF02564638. MR705539

[8] Fouad El Zein and Steven Zucker, *Extendability of normal functions associated to algebraic cycles*, Topics in transcendental algebraic geometry (Princeton, N.J., 1981/1982), Ann. of Math. Stud., vol. 106, Princeton Univ. Press, Princeton, NJ, 1984, pp. 269–288. MR756857

[9] Joseph Steenbrink and Steven Zucker, *Variation of mixed Hodge structure. I*, Invent. Math. **80** (1985), no. 3, 489–542, DOI 10.1007/BF01388729. MR791673

[10] Steven Zucker, *Variation of mixed Hodge structure. II*, Invent. Math. **80** (1985), no. 3, 543–565, DOI 10.1007/BF01388730. MR791674

[11] Richard M. Hain and Steven Zucker, *Unipotent variations of mixed Hodge structure*, Invent. Math. **88** (1987), no. 1, 83–124, DOI 10.1007/BF01405093. MR877008

[12] Steven Zucker, *The Hodge structures on the intersection homology of varieties with isolated singularities*, Duke Math. J. **55** (1987), no. 3, 603–616, DOI 10.1215/S0012-7094-87-05530-X. MR904943

[13] Masa-Hiko Saito and Steven Zucker, *On the Torelli problem for fiber spaces*, Complex geometry and Lie theory (Sundance, UT, 1989), Proc. Sympos. Pure Math., vol. 53, Amer. Math. Soc., Providence, RI, 1991, pp. 269–282, DOI 10.1090/pspum/053/1141205. MR1141205

[14] Masa-Hiko Saito and Steven Zucker, *Classification of nonrigid families of $K3$ surfaces and a finiteness theorem of Arakelov type*, Math. Ann. **289** (1991), no. 1, 1–31, DOI 10.1007/BF01446555. MR1087233

[15] Michael Harris and Steven Zucker, *Boundary cohomology of Shimura varieties. I. Coherent cohomology on toroidal compactifications*, Ann. Sci. École Norm. Sup. (4) **27** (1994), no. 3, 249–344. MR1272293

[16] Michael Harris and Steven Zucker, *Boundary cohomology of Shimura varieties. II. Hodge theory at the boundary*, Invent. Math. **116** (1994), no. 1-3, 243–308, DOI 10.1007/BF01231562. Erratum, Invent. Math. **121** (1995), 437, MR1346216. MR1253194

[17] Michael Harris and Steven Zucker, *Boundary cohomology of Shimura varieties. III. Coherent cohomology on higher-rank boundary strata and applications to Hodge theory*, Mém. Soc. Math. Fr. (N.S.) **85** (2001), vi+116, DOI 10.24033/msmf.398. MR1850830

[18] Steven Zucker, *On the reductive Borel-Serre compactification: L^p-cohomology of arithmetic groups (for large p)*, Amer. J. Math. **123** (2001), no. 5, 951–984. MR1854116

[19] Steven Zucker, *On the reductive Borel-Serre compactification. II. Excentric quotients and least common modifications*, Amer. J. Math. **130** (2008), no. 4, 859–912, DOI 10.1353/ajm.0.0010. MR2427003

[20] Steven Zucker, *On the reductive Borel-Serre compactification. III. Mixed Hodge structures*, Asian J. Math. **8** (2004), no. 4, 881–911. MR2127953

[21] Lizhen Ji and Steven Zucker, *On the L^p-cohomology and the geometry of metrics on moduli spaces of curves*, Handbook of Teichmüller theory. Volume III, IRMA Lect. Math. Theor. Phys., vol. 17, Eur. Math. Soc., Zürich, 2012, pp. 747–775, DOI 10.4171/103-1/14. MR2952775

[22] Steven Zucker, *The research career of Steven Zucker: An autobiographical account*, Hodge theory and L^2-analysis, Adv. Lect. Math. (ALM), vol. 39, Int. Press, Somerville, MA, 2017, pp. i–xii. MR3751286

Credits

Figure 1 is courtesy of Steve Zucker.
Figure 2 is courtesy of International Press of Boston.
Figure 3 is courtesy of Yukihiko Namikawa.
Figure 4 is courtesy of Masa-Hiko Saito.
Figure 5 is courtesy of Yukiyoshi Nakajima.
Figure 6 is courtesy of Higher Education Press.