Report on a US-Canadian Faculty Survey on Undergraduate Linear Algebra

Could Linear Algebra Be an Alternate First Collegiate Math Course?

Christine Andrews-Larson, Jason Siefken, and Rahul Simha

1.0. Introduction

Linear algebra is a stalwart of the undergraduate mathematics experience, one of the core areas of mathematics whose "unreasonable effectiveness" finds critical application in disciplines ranging from business analytics to quantum computing. Most universities offer a first course in linear algebra, although what such a course entails varies from one institution to another. Our goal in this paper is to review the status of a first course in linear algebra in the US and Canada through an instructor survey and then discuss ways in which an updated linear algebra curriculum might better serve its constituents.

In our survey, 70% of instructors report their course is a service course for computer science, data science, economics, or engineering majors (only about 25% of the institutions use linear algebra as an introduction-to-proof course). At the same time, universities have seen a recent dramatic increase in computing majors (a 200% increase between 2006 and 2015 [5]) and a rapid rise of interdisciplinary data science, with accompanying linear algebra curricular recommendations [1, 6, 10, 15] that include, for example, exposure to the singular-value decomposition (SVD) and its popular cousin, principal components analysis (PCA).

In 1993, the Linear Algebra Curriculum Study Group (LACSG [4]) published recommendations for a first course in linear algebra – emphasizing the importance of serving client disciplines and decreasing emphasis on abstraction by focusing on matrices, with a recommended core set of topics consisting of: operations on matrices, linear systems, determinants, properties of R^n, eigenvectors and eigenvalues, and orthogonality. Recently, the National Science Foundation provided financial support to convene LACSG 2.0, bringing together experts on the teaching and learning of linear algebra along with representatives from client disciplines. Their forthcoming recommendations additionally emphasize the importance of linear maps, their compositions and inverses, as well as diagonalization and the singular value decomposition [12]. Topics recommended for a second course by this group include an abstract treatment of vector spaces, matrix factorizations, inner product spaces, finite-dimensional spectral theorem, pseudo-inverses, operators, and Jordan form. The teaching of linear algebra, we note, has received considerable attention as a topic of research [7, 14].

We complement the above body of work with two useful perspectives. First, our survey, which gathers data on topic coverage and client populations, situates the above recommendations (and ours) in the context of what is

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actually being taught today. Second, the growth of new client populations together with the recently documented failure rates in calculus [2] bring fresh focus to the idea of having linear algebra provide an alternative to calculus as the first encounter with college math.

2.0. The Survey

We surveyed linear algebra instructors at 129 of the top-ranked universities (including doctoral research universities and 4-year liberal-arts institutions that do not offer doctorates in mathematics) across the US and Canada. Our survey included 25 questions aimed at understanding the prerequisites, content, points of emphasis, and disciplines served for introductory linear algebra courses (see online supplemental materials here: `https://www2.seas.gwu.edu/~simha/research/AMS-LinAlg-Supplemental.pdf`). We solicited responses from linear algebra instructors teaching at all US universities ranked to be in the top 100 of *US News*'s university rankings in 2020 (100 total, with 80% from the overall rankings and 20% from top liberal arts colleges) and all Canadian universities in the top 1000 of the Times Higher Education World University Rankings (29 total).

After soliciting via email one to two faculty members from each target institution, we received 70 responses, of which 64 related to a first course in linear algebra. Responses from research-intensive, doctoral-granting institutions comprised 69% of overall responses. About half of respondents were from US institutions, about a quarter were from Canadian institutions, and the rest did not identify a national affiliation. Most respondents reported significant linear algebra teaching experience, with 88% having taught linear algebra more than 5 times and 44% having taught linear algebra more than 10 times.

3.0. What Goes on in a First Linear Algebra Course?

Our survey data indicates: (a) there are eight "universally covered topics" in a first linear algebra course, (b) computer science or data science students make up a large portion of linear algebra students, (c) prerequisites vary but success in linear algebra courses is high, and (d) while the experience of teaching linear algebra comes with challenges, for most instructors who responded, it's among their favorite courses to teach. We examine each of these findings in more detail below.

3.1. Core topics

Our survey revealed that a set of eight core topics is included in at least 90% of all surveyed courses. We call these *universally covered topics*. In contrast, there are seven *often covered topics* (included in 55–90% of courses) and five *sometimes covered topics* (included in 35–54% of courses). The remaining 12 topics in the survey were included in fewer than 35% of courses (LU factorization, cross products, Markov chains, QR decomposition, spectral theorem, SVD/PCA, quadratic forms, differential equations, Fourier series, scalar fields other than R and C, numerical linear algebra, Jordan forms). Note only 8% of respondents report covering SVD/PCA in a first course. Additional details are provided in Figure 1.

Universally covered topics	1. Solving systems using row reduction (97%) 2. Eigenvectors/values (97%) 3. Determining linear independence/dependence of a set (94%) 4. Dot products (92%) 5. Characteristic polynomials (92%) 6. Diagonalization (92%) 7. Determinant formulas (91%) 8. Producing bases for subspaces (91%)
Often covered topics	1. Fundamental subspaces of a matrix (83%) 2. Similar matrices (77%) 3. Geometric / algebraic multiplicity (72%) 4. Non-diagonalizable matrices (69%) 5. Gram-Schmidt orthogonalization (64%) 6. Function spaces / polynomial spaces (63%) 7. Change of basis (58%)
Sometimes covered topics	1. Determinants as volumes (52%) 2. Least squares (50%) 3. Complex numbers (48%) 4. Abstract vector spaces (42%) 5. Inner product spaces (36%)

Figure 1. Percent of respondents who report covering each topic.

Breaking down the data by institution and class type, topic coverage appears similar across doctoral granting and liberal arts universities and across courses serving math majors and general science, technology, engineering, and mathematics (STEM) courses. The only notable differences appear in the often/sometimes covered topics of Canadian vs US schools. For example, 82% of Canadian institutions include cross products as a topic, compared to 10% of US institutions. In contrast, fewer than 55% of Canadian institutions include the topics of similar matrices, non-diagonalizable matrices, and Gram-Schmidt orthogonalization, while more than 75% of US universities include those topics.

We also asked instructors to what extent they emphasized (i) applications, (ii) computer programming, (iii) rote computation, (iv) geometry, and (v) introduction to proofs. The vast majority (72–82%) of respondents report a medium or high level of emphasis on introduction to proofs, rote computation, geometry, and applications. In contrast, 83% of instructors report no or low emphasis on computer programming—providing evidence that potential points of synergy between linear algebra and computer programming are not widely leveraged in the teaching of a first course in linear algebra.

3.2. Who takes linear algebra and what texts are used?

Current implementations of linear algebra play the role of a service course for engineering and computer science

(72% of respondents), science and economics majors (63% of respondents), and as a core course for mathematics and statistics majors (80% of respondents). About one fourth of respondents indicated that linear algebra functions as an introduction-to-proofs course for their students.

The most common textbook used is a version of Lay's text, used by about $^1/_3$ of respondents. About 8% of respondents reported using each of the following texts: Nicholson, Poole, and Norman & Wolczuk. All other texts were reported as being used by only 2–3 respondents. References to the named textbooks above are provided in the online supplemental material.

3.3. Widely ranging prerequisites; largely positive outcomes

The most common highest-level prerequisite for linear algebra was either single-variable calculus (~45% total with ~15% requiring Calc I and ~30% requiring Calc II) or high-school mathematics (~30%). About 10% of the time multivariable calculus was a prerequisite, and only 3% required a different advanced math course as a prerequisite.

What is striking is that, among our respondents, failure rates for linear algebra courses were lower than failure rates for calculus courses regardless of prerequisites to the linear algebra courses. Namely, of the instructors surveyed, no one reported a failure rate of higher than 30%, with three quarters of all respondents reporting a failure rate of 10% or lower, and about 98% reporting a failure rate below 20% (see Figure 2). Further, over half (12 of 22) of the instructors who taught a linear algebra class that did not require calculus as a prerequisite reported a failure rate below 10% and 91% reported a failure rate below 20%. A large national calculus study found that failure rates in calculus range between 22–37% depending on institution type [2]. Taken together, this provides strong evidence that linear algebra courses have lower failure rates than calculus courses, regardless of prerequisites.

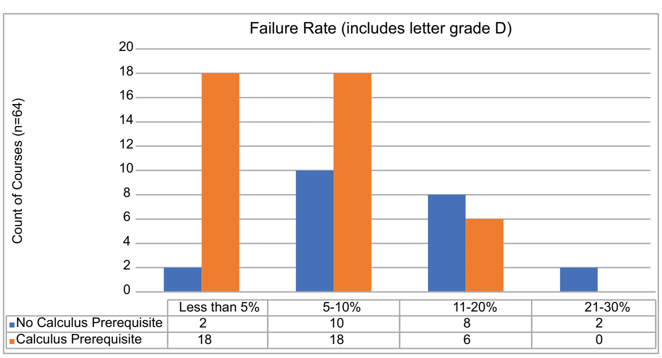

Figure 2. Failure rates for first linear algebra course disaggregated by prerequisite.

3.4. Challenges and instructor opinions

As part of our survey, we asked participants to characterize the biggest challenge in teaching linear algebra. One respondent made a particularly strong argument for moving linear algebra earlier in the curriculum:

> Perhaps it's that pre-calculus and calculus dominate the last several years of most students' math training. I wonder sometimes if I'd have better luck teaching high school sophomores, who haven't (yet) had their geometric intuition beaten out of them to make room for mechanical calculus procedures!

While this response was not a dominant theme, it is related to the broader argument we make in this paper—namely that linear algebra should appear earlier in the college curriculum in a way that could function as an alternative path to STEM. Other participants' responses offer valuable insights into the challenges inherent in the teaching of linear algebra, which are critical to consider along with significant shifts in the location and role of linear algebra in the undergraduate curriculum.

About half of respondents described the biggest challenge of teaching linear algebra in relation to the nature of the content itself. These responses highlighted how ideas fit together, the need to bridge and balance computation and theory, and challenges related to abstraction and proof. One respondent explained this challenge in the following way:

> It is like a puzzle. Every piece (that is, every topic in the course) is important, and if any are missing you won't be able to finish the puzzle. But you also can't see what you are trying to accomplish until you have assembled enough of the pieces, making it difficult to motivate students to keep working on every piece.

Another explained:

> So many of the concepts depend on each other to really make sense; it takes a good ¾ of the term before I feel like I can really communicate the big picture of what's going on to my students, and until then I'm leaving a lot of hanging ideas floating around.

Yet another linked this to the way in which students experience the flow of content:

> Our course starts with a lot of really basic row reduction/matrix multiplication, inverses etc. that students tend to find easy. Then it quickly moves to Linear Transformation and abstract stuff. Students tend to think they are awesome

> and doing great and stop working as hard by the time they really need to.

And others situated challenges in relation to students' prior coursework:

> Linear algebra is often one of the first classes students see that is not procedural. It is full of new mathematical language, abstract and spatial thinking, algebraic arguments, visual arguments, real-life numerical applications, and other analyses that students must internalize in order to succeed. It is also a class where a computer algebra software system is required. Students who did well in earlier classes through short term memorization often struggle in linear algebra.

Other common themes included challenges related to variations in students' needs and backgrounds (reported by 20% of respondents) and limited resources (e.g., time, large class size, and the need for more high-quality problems and exercises; reported by about 20% of respondents). *Only about 12% of respondents reported that the biggest challenge in teaching linear algebra was related to student interest, effort, or capability*. We interpret this to mean that the instructors surveyed tended to have productive beliefs about their students' capabilities in linear algebra, and there is ample evidence to suggest that this has important implications for more positive and equitable outcomes [3]. Indeed, about four times as many respondents pointed to the nature of the material as the biggest source of challenge, rather than any inherent issue with students and their preparation.

4.0. Could Linear Algebra Be an Alternate First Collegiate Math Course?

Calculus is currently the gateway to many STEM majors, commonly required in students' first-year coursework for engineering, physics, mathematics, and statistics programs. Unfortunately, more than half of students entering college with the intent to major in STEM fail to complete a STEM major, according to a report by the President's Council of Advisors on Science & Technology [11]. The same report squarely identifies the "math barrier" to STEM fields, and hints at the need for alternatives to the status quo, explaining that "introductory mathematics courses often leave students with the impression that all STEM fields are dull and unimaginative, which has particularly harmful effects for students who later become K–12 teachers" (p. vi).

As mentioned earlier, a recent survey paper [2] on the state of collegiate calculus calls for rethinking introductory calculus' harmful role on student motivation, given its high 22–37% DFW rate (with a remarkable 18% DFW rate among students who previously took calculus in high school) for a group of students who come in with high confidence in mathematics (90%): "one of the clearest conclusions…[is] how effective this course is in destroying that confidence" (p. 182). Notably, women are 1.5 times more likely than men to leave STEM after taking a first college calculus course, a decision consistent with gendered differences in confidence but not performance [8].

Taken together, the considerations of a growing need for linear algebra knowledge in content disciplines and relatively high pass rates with limited prerequisite knowledge required lead us to the question: *Could linear algebra, remodeled as a service course and updated for its changing population, offer a parallel pathway to STEM?* To stimulate discussion of these issues in the undergraduate mathematics community, we take up this possibility in the remainder of the paper, and include discussion of what a first course might imply for a more advanced second course.

4.1. The case for linear algebra as a first collegiate encounter with mathematics

Note that the suggestion of broadly adopting a first-year linear algebra class isn't new. Tucker, in 1993, called for a "'redistricting' of the lower-division mathematics sequences" to include linear algebra [13 p.8]. Our survey indicates that this is being done with remarkable success in some places, and that linear algebra features higher pass rates than calculus regardless of whether calculus is required as a prerequisite or not. Furthermore, the high level of agreement on a relatively small set of core topics for a linear algebra class lends itself nicely to the proposal to adapt linear algebra courses for a lower division audience.

There are two important arguments to be made in support of first-year linear algebra courses: (a) such courses would better serve the rapidly increasing population of computer and data science majors, and (b) first-year linear algebra courses have the potential to increase STEM opportunity for underrepresented groups.

Point (a) is supported directly by enrollment data but point (b) needs more explanation. We argue that a first-year course in linear algebra has the potential to level the playing field for incoming students, regardless of whether their high school is well-resourced in mathematics, and thus open up additional pathways to STEM. Because most students do not take linear algebra in high school, by making linear algebra an entry-level course, students would experience a more even playing field with regard to their high school coursework. In contrast, many students take calculus in high school, then take it again in college… sometimes without success [2]. This can foster an uneven playing field depending on the resources available to students at the high school level—as well as leaving students feeling unmotivated and underprepared. Linear algebra, on the other hand, offers a chance to start afresh, as well as to usefully review more elementary topics in coordinate geometry and equation solving, together with an application-driven focus to motivate students.

4.2. Issues in the design of first-year linear algebra

Any consideration of moving linear algebra into the freshman year, without a calculus prerequisite (and, as we propose, also with computer and data science applications), is naturally enmeshed with broader issues surrounding the mathematics curriculum. Questions arise related to sequencing across courses such as: Which later courses require calculus for specific calculus content as opposed to merely seeking mathematical maturity? Could linear algebra be a suitable alternative? If a second, more abstract course in linear algebra is to be offered, what flexibility does this afford the first course, and what should go into the second? Questions related to what happens within a first, freshman-level linear algebra course arise as well: What are the roles of computer programming and applications in a first-year linear algebra course? What might be the role of proof in a first-year linear algebra course? And how can client departments adjust their curricula to make best use of a freshman course in linear algebra?

In the following sections, we outline ideas for two courses, an introductory freshman-level course open to all, driven by computing and data science applications, and a later second course for majors, ensuring that topics omitted from the first can be addressed in the second and that proofs take their central place in a math-major course. What makes this possible, we believe, is that linear algebra as a content area is *adaptable* to both audiences. Our suggestions are also informed by recommendations from Linear Algebra Curriculum Study Group's 1993 report (and the forthcoming update) [4, 12], the various data science curricular reports cited, as well as the authors' own experience with linear algebra. *We emphasize that these ideas are intended to begin discussion and are not intended as a prescription.*

4.3. What might a first-year linear algebra course look like?

In this section, we explore possibilities for a first-year linear algebra course. We take it as a given that a first-year linear algebra course is designed with the idea that math majors will also take a second linear algebra course (aimed at second or third-year students), and therefore provides room to deviate from "traditional" approaches. We aim to spark conversation by showing some ways that linear algebra can be adapted for a first-year audience. This section is not meant to describe any specific course (for an example outline of a specific course, see Appendix 3 of the online supplemental materials). Instead, we highlight the versatility of the subject through a few vignettes.

Linear algebra for computer science: computer graphics. Applications of linear algebra to computer graphics are both visually compelling and directly useful to the growing population of computer science students in linear algebra courses. For example, as the outline in Appendix 3 of the online supplemental materials shows, many ideas in linear algebra can be motivated with standard transformations (rotation, reflection, affine-extended translation) in 2D and 3D computer graphics. Thus, a reflection followed by a rotation can be both seen visually and easily computed through successive matrix multiplication; likewise "undoing" a transformation can serve to motivate and set up matrix inversion. However, including these applications raises questions about whether students and instructors should be expected to write code, whether this should occur in a separate lab, and whether computation can be supported at scale. We argue that it is relatively easy for instructors to demonstrate with software (without showing code) in class, and with a little help, students can run software to compute results that can then be viewed to confirm hand-calculated matrix-vector manipulations.

Making demonstrations of applications to computer graphics a key goal allows one to introduce Bezier curves and surfaces, a foundational element of computer animation. Because Bezier curves are constructed using linear combinations of polynomials, students get to see linear algebra at work with a different type of mathematical object and connect linear algebra with how animated movies are developed.

Linear algebra for data science: getting to PCA in a first-year course. At first glance, it might seem that the road to Principal Component Analysis (PCA) is a long one that winds through linear transformations, then to eigenvalues and characteristic equations, and from there to diagonalization. However, once the course is past the core of linear algebra (span, basis, rank, dot products, orthogonality), there is a way to demonstrate and explain PCA through a shorter, less intensive path. With the computer graphics applications having shown how a matrix transforms one vector into another, one can then demonstrate that some resulting vectors have the same direction, and that even when the result is a different direction, repeated application of the same matrix converges to a vector with this unusual property (the power method). By using software to compute eigenvalues and eigenvectors, one can avoid characteristic polynomials and go straight to explaining a change of basis using the eigenvectors as the new basis. With this background, students can take data matrices, perform mean-centering, calculate correlation and then apply the change of basis, after which they can plot the first two coordinates, a process sufficient for many datasets. In this sense, the focus is on change-of-basis rather than eigenvalue calculation.

Linear algebra that integrates review of high-school math. A first-year linear algebra instructor should generally anticipate a wide range of mathematical exposure amongst students in the course. Fortunately, linear algebra offers the opportunity to review many topics from high school, including some for which students' exposure may vary. These include coordinates, equations of lines and planes, polynomials, mean, variance, correlation, and sigma notation for summation.

It has been documented that high-achieving mathematics high school students are largely successful solving linear equations with a unique solution [9], but identifying when those equations are never or always true is more challenging. Thus, it is reasonable that linear algebra could provide a rich opportunity for such students to extend and build on their knowledge of linear equations and systems of linear equations and their solution sets - while also broadening students' conceptions of how lines and planes can be represented in 2 and 3 dimensions (e.g., with parametric vector equations). Additionally, opportunities to explore applications of polynomials in a realistic, meaningful context such as computer graphics could importantly build students' fluency with these mathematical objects and extend students' appreciation of their usefulness within and beyond mathematics.

A point we want to emphasize is that we don't advocate for including "more" in a first-year linear algebra course. Instead, we argue that specific tradeoffs can and should be made to *reimagine* linear algebra as a first-year course. For example, the course outlined in the online supplemental materials does not introduce characteristic polynomials or determinants (two topics often covered in a linear algebra course). This frees up time to explore PCA, which would be a justified tradeoff for a course aimed at a broader population. By breaking from the "traditional," there is an opportunity to create a new and useful experience for first-year students.

4.4. What might a second course in linear algebra look like?

Coupled with a first-year introduction to the subject, a second linear algebra course, suitable both for math majors as well as physics and theoretically inclined computer science students, provides an opportunity to dive more deeply into linear algebra theory. What topics might such a course cover if students have had the first course? We offer a potential organizing theme for commonly covered topics that is useful and compelling for students who enjoy both theory and seeing its application. This theme, we believe, also fills an inexplicable oversight in the current landscape of linear algebra teaching, the fact that nearly all linear algebra textbooks fail to cover its spectacularly successful application: *quantum mechanics*.

The core ideas of quantum mechanics and quantum computing are based on the linear algebra of complex vectors, with Hermitian and unitary matrices playing key roles. One can begin more approachably with qubits and finite-dimensional vectors before transitioning to Hilbert spaces. Several introductory books in quantum computing are now available that do not require any physics; the linear algebra topics from these books could serve as the starting point for a second course. Thus, students would get to see many parallels to help them grapple with abstraction: the parallels between reals and complex numbers, dot versus inner products, linear versus bilinear transformations, tensor products, finite versus infinite-dimensional, and an abstract treatment of operators. Mathematicians might benefit from the adoption of conventions from physics in the second course such as left-side conjugation for inner products, and even the dreaded Dirac notation. Once examples that demonstrate its power are presented, students will eventually get comfortable with the notation, and it will be extremely valuable for physics and computer science students who go further with quantum topics.

4.5. Further mathematics education research

One problem with papers like this is that much of the above advocacy is based on argument and author experience, a problem that also afflicts any kind of stance rooted more in theoretical speculation than in empirical evidence. While there has been a considerable amount of research published on the teaching and learning of linear algebra over the last 15 years, much of it has focused on how students reason about particular topics in the context of linear algebra courses (e.g., span, linear independence), and instructional innovations organized around use of technology or inquiry-oriented instruction [15]. The mathematics community would benefit from a careful and detailed study of how exactly various micro-topics from mathematics are used by its students in the application disciplines: For any subtopic X: who really uses X and why? How often do engineers use determinants or the concept of a null space, and for what purpose? How exactly do economists use eigenvalues and eigenvectors? Knowing precisely both the usage of a topic and the proficiency needed (is the proof needed, just the idea, or the calculation experience?) will greatly help the design of the standard math courses with the largest enrollments and whose impact is critical.

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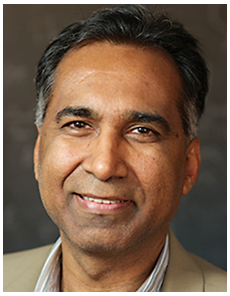

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