Robert Strichartz

Kasso Okoudjou, Luke Rogers, and Alexander Teplyaev

Robert Strichartz was an influential analyst known for his profoundly original results in harmonic and functional analysis and his dedication to growing the mathematical community, especially by encouraging school-aged childrens' interest in mathematics and involving undergraduate students in research. One word that comes to mind when thinking about Bob's work is that he served as a guide for many people who followed their own path in analysis. Indeed, even the titles of Bob's most well-known books are telling: "The Way of Analysis" [Str00b] and "A Guide to Distribution Theory and Fourier Transforms" [Str03b].

In his most famous paper [Str77], Bob introduced a tool now widely known as a Strichartz estimate. Estimates of this type govern the regularity of wave-type equations in terms of mixed Sobolev norms and are an essential technique in the modern analysis of PDE. According to Bob, he obtained these remarkable results in a single weekend, after reading a paper of Segal. With characteristic modesty, he wrote in [Str77] that "none of the methods used in this paper are new" and attributed the main ideas to Segal, Stein, and Tomas. Moreover, throughout his career, he generously ascribed the significance of the method to the contributions made by others, and said that he had just "dealt with an elementary case." A close examination of his work reveals otherwise: the major insight in [Str77] established a significant and fruitful connection between the decay of wave equations and a well-studied class of problems in harmonic analysis regarding the restriction of the Fourier transform to a surface. Specifically, Bob recognized that Segal's decay estimates for the Klein-Gordon equation were equivalent to asking for which $p \in [1,2)$ and $f \in L^p$ the Fourier transform $\hat{f}$ admits a well-defined restriction to $L^2(d\mu)$ where μ is the natural measure on a hyperbola. At that time, major new results on Fourier restriction estimates were reshaping harmonic analysis, and Bob had made contributions to related problems both in his Princeton PhD thesis [Str67] (written under the legendary Eli Stein) and a decade of subsequent papers. Generalizing Segal's idea to consider restrictions of the Fourier transform to quadratic surfaces, and analyzing the resulting problem using the approach of Stein and Tomas led Bob to his famous estimates.

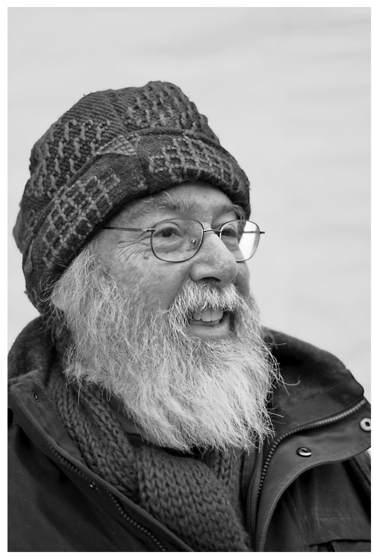

Figure 1. Robert Strichartz.

The connection Bob made between decay estimates for PDE and restriction estimates in harmonic analysis is just one example of the way in which his broad interests allowed him to bring ideas from different areas into insights that reshaped the fields in which he worked. He was particularly interested in the centrality of the Laplacian operator in analysis and geometry, once teaching a graduate course where he referred to it as "The triangle at the center of mathematics." For this reason, he sought to

Kasso Okoudjou is a professor of mathematics at Tufts University. His email address is `kasso.okoudjou@tufts.edu`.

Luke Rogers is a professor of mathematics at the University of Connecticut. His email address is `luke.rogers@uconn.edu`.

Alexander Teplyaev is a professor of mathematics at the University of Connecticut. His email address is `alexander.teplyaev@uconn.edu`.

Communicated by Notices *Associate Editor Reza Malek-Madani.*

For permission to reprint this article, please contact: `reprint-permission@ams.org`.

DOI: https://doi.org/10.1090/noti2496

generalize the use of the Laplacian to analyze smoothness and the behavior of PDE via spectral theory (Fourier analysis) as in [Str89], and to understanding curvature and the geometry of manifolds via the heat semigroup (e.g., in the celebrated Li-Yau estimates). Among his most influential works in this direction are his extension [Str83] of classical analytic results regarding the Laplacian to corresponding results for the Laplace-Beltrami operator on a complete Riemannian manifold, his contributions to the early development of sub-Riemannian geometry [Str86], and his role in the development of analysis on fractals [Str99, Str03a].

Along with the Laplacian, self-similarity was an abiding interest for Bob. He came to this from his study of the asymptotics of averages of Fourier transforms of measures [Str90], where he discovered a rich interplay between properties of self-similar measures and their Fourier transforms [Str94], including a fractal Plancherel theorem. This interest then echoed throughout his career, from early work [Str00a] regarding measures μ that admit an orthogonal basis of exponentials for $L^2(\mu)$, to work on wavelets and their associated tilings, and finally to foundational contributions to Kigami's theory of analysis on fractals during the last three decades of his career. The latter combined two of his main mathematical loves, the Laplacian and self-similarity, in developing an analytic structure on self-similar sets in which the fundamental differential operator is a Laplacian. His dream, as he expressed it, was that we should eventually be able to study physical phenomena on fractal sets via PDEs with a set of tools as rich as those used in Euclidean analysis. He did not expect this dream to be fully realized in his lifetime, but more than 80 of his papers developed large parts of such a theory. Bob's monograph "Differential equations on fractals" [Str06] epitomizes his unique ability to write and communicate deep mathematics to a diverse audience ranging from undergraduate students to researchers who want to learn about this area that Bob help popularize.

Another area in which Bob's influence on mathematics was profound and well known to the people involved, was his mentoring. Bob had a great passion for mathematics and enjoyed talking about it with anyone, from children to experts. He was an enthusiastic volunteer with mathematics programs for elementary and middle school students and for about thirty years he devoted his summers to his Research Experience for Undergraduates program. The latter involved an unusual symbiosis: Bob provided mathematical knowledge and insight, proposing problems and guiding their investigation, while the students, in addition to their pure mathematics research, contributed computer code, made numerical investigations, and created pictures of previously unexplored fractal phenomena. Bob considered his undergraduates to be essential partners in research; he was fond of saying that other mathematicians would tell him they did not know how he did research with undergraduates and he would respond that he did not know how they did research *without* undergraduates. Bob's mentorship, introducing undergraduates to research and offering a belief in the value of their ideas and capacity to contribute to mathematics and other disciplines, had a very broad impact. He published papers with more than one hundred undergraduate students, and many of his mentees went on to earn PhD degrees and win fellowships, prizes, and accolades for their work. This is a part of Bob's wider impact, which includes three textbooks [Str03b, Str06, Str00b], several expository papers, of which [Str82] won a Mathematical Association of America's Lester R. Ford Award, and his involvement in the creation of the Journal of Fourier Analysis and Applications.

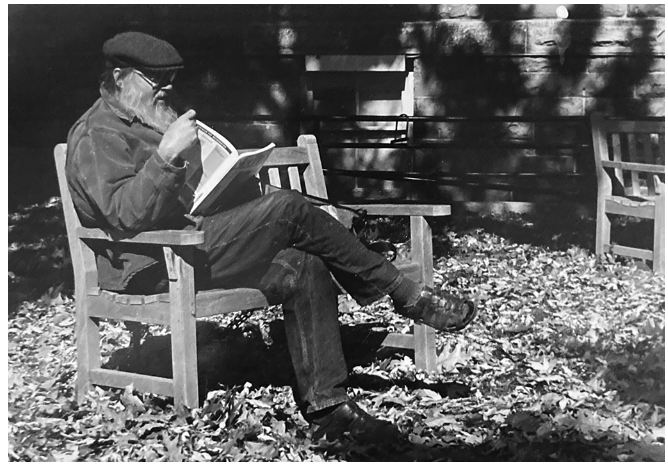

Figure 2. Robert Strichartz reading the *Notices* outside of the Cornell Mathematics Department, in the mid to late 1990s.

Bob's greatest insights were achieved by collecting several ideas together and presenting them in a way that was most useful for many applications. Since his first paper in 1965, Bob provided the analysis community with a great number of important insights, connecting classical and new ideas in harmonic analysis, multi-resolution analysis, and wavelets, geometric and functional analysis on Riemannian and sub-Riemannian manifolds, self-similarity, and fractals. He supervised a very large number of undergraduate students through the Cornell REU and later the Cornell SPUR program, many of whom later became successful mathematicians.

Bob was an original thinker, a great colleague, a wonderful mentor and coauthor, and a true and close friend for many. He will be irreplaceable in mathematics because of his research and work translating profound classical ideas in a way that influenced thinking about many areas of analysis.

References

[Str67] Robert S. Strichartz, *Multipliers on fractional Sobolev spaces*, J. Math. Mech. **16** (1967), 1031–1060. MR0215084

[Str77] Robert S. Strichartz, *Restrictions of Fourier transforms to quadratic surfaces and decay of solutions of wave equations*, Duke Math. J. **44** (1977), no. 3, 705–714. MR512086

[Str82] Robert S. Strichartz, *Radon inversion—variations on a theme*, Amer. Math. Monthly **89** (1982), no. 6, 377–384, 420–423. MR660917

[Str83] Robert S. Strichartz, *Analysis of the Laplacian on the complete Riemannian manifold*, J. Functional Analysis **52** (1983), no. 1, 48–79, DOI 10.1016/0022-1236(83)90090-3. MR705991

[Str86] Robert S. Strichartz, *Sub-Riemannian geometry*, J. Differential Geom. **24** (1986), no. 2, 221–263. MR862049

[Str89] Robert S. Strichartz, *Harmonic analysis as spectral theory of Laplacians*, J. Funct. Anal. **87** (1989), no. 1, 51–148, DOI 10.1016/0022-1236(89)90004-9. MR1025883

[Str90] Robert S. Strichartz, *Fourier asymptotics of fractal measures*, J. Funct. Anal. **89** (1990), no. 1, 154–187. MR1040961

[Str94] Robert S. Strichartz, *Self-similarity in harmonic analysis*, J. Fourier Anal. Appl. **1** (1994), no. 1, 1–37. MR1307067

[Str99] Robert S. Strichartz, *Analysis on fractals*, Notices Amer. Math. Soc. **46** (1999), no. 10, 1199–1208. MR1715511

[Str00a] Robert S. Strichartz, *Mock Fourier series and transforms associated with certain Cantor measures*, J. Anal. Math. **81** (2000), 209–238. MR1785282

[Str00b] Robert S Strichartz, *The way of analysis*, Jones & Bartlett Learning, 2000.

[Str03a] Robert S. Strichartz, *Function spaces on fractals*, J. Funct. Anal. **198** (2003), no. 1, 43–83, DOI 10.1016/S0022-1236(02)00035-6. MR1962353

[Str03b] Robert S. Strichartz, *A guide to distribution theory and Fourier transforms*, Reprint of the 1994 original. World Scientific Publishing Co., 2003. MR2000535

[Str06] Robert S. Strichartz, *Differential equations on fractals: A tutorial*, Princeton University Press, Princeton, NJ, 2006. MR2246975

Kasso Okoudjou

Luke Rogers

Alexander Teplyaev

Credits

Figure 1 is courtesy of the Cornell Mathematics Department.

Figure 2 is courtesy of Miranda Strichartz.

Photo of Kasso Okoudjou is courtesy of Alonso Nichols/Tufts University.

Photo of Luke Rogers is courtesy of Luke Rogers.

Photo of Alexander Teplyaev is courtesy of Alexander Teplyaev.