Jacques Tits (1930–2021)

Richard M. Weiss

Introduction

Figure 1. Jacques Tits at age 19.

Jacques Tits was born in Uccle, a municipality of Brussels, on August 12, 1930, and died on December 5, 2021. The son of a mathematician, Tits displayed extraordinary mathematical ability at an early age. He received his doctorate at the University of Brussels in 1950 and spent the following year at the Institute for Advanced Study. In 1964, he moved from the University of Brussels to a professorship in Bonn, and then in 1973 to the Collège de France, where he remained for the rest of his career. For almost thirty years he held courses and seminars at the Collège de France and for nineteen years, Tits was editor-in-chief of the *Publications Mathématiques de l'IHES*.

Tits made many fundamental contributions to our understanding of the structure of semisimple algebraic groups and finite simple groups and did more than anyone to explore and reveal the geometric nature of these subjects.

When Tits was young, Chevalley had shown that semisimple algebraic groups over an algebraically closed field are classified up to isogeny by Dynkin diagrams. Tits made a careful study of various structural features of groups defined over an arbitrary field, much of which appeared in a famous collaboration with Borel. Tits then used this work to show that semisimple algebraic groups over an arbitrary field are uniquely determined by combinatorial data in the form of a *Tits index* (a Dynkin diagram endowed with certain decorations) and an *anisotropic kernel*, the two things knitted together by Galois descent as described in his lecture notes from a meeting that took place in Boulder in 1966. This deep result is in the spirit of the theorem of Wedderburn that says that a simple associative ring which is finite-dimensional over its center is isomorphic to $\text{Mat}_n(D)$ for some division ring D, n and D being analogs of the Tits index and the anisotropic kernel. Another important analog is the theorem of Witt that says that a finite-dimensional quadratic form is uniquely determined by its anisotropic part and the dimension of its hyperbolic part.

Tits is best known for the theory of buildings. A building is a geometric structure defined by a few simple axioms involving a notion of dimension called the *rank*. The simplest example of a building of rank n is the projective space associated with a vector space of dimension $n + 1$. Further examples arise when the vector space carries a quadratic or Hermitian form. Together these are the buildings associated to the classical groups.

Buildings have distinguished substructures called *apartments*. A building is *spherical* if its apartments are finite. A building is *irreducible* if it is not a direct product. The classical buildings are all spherical and irreducible.

Tits introduced the notion of a BN-pair (also known as a Tits system) and used it together with the structural features revealed in his work with Borel to show that to every absolutely simple algebraic group G of positive k-rank n for a given field k, there is an irreducible spherical building of rank n on which the group $G(k)$ acts. When G is classical, then so is the building. These buildings are particularly fascinating objects in the case when G is exceptional.

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Figure 2. At the induction of Jacques Tits into the Order Pour le Mérite, Bonn, 1996.

In a celebrated volume of Springer Lecture Notes published in 1974, Tits gave the classification of irreducible spherical buildings of rank at least 3. His classification shows that every such building is either classical or exceptional or belongs to a unique family defined over an imperfect field in characteristic 2.

Inspired by the work of Iwahori and Matsumoto, Tits went on to investigate *affine* buildings. Affine buildings are those in which the apartments have a natural representation as an affine space. Every affine building X of rank $n+1$ has a boundary which carries the structure of a spherical building of rank n (and is called the *building at infinity* of X). Tits observed that for every absolutely simple algebraic group of positive k-rank for a field k that is complete with respect to a discrete valuation, there is an affine building on which the group $G(k)$ acts and that its boundary is precisely the spherical building associated with $G(k)$. His famous lectures on the structure of absolutely simple groups isotropic over a local field and their affine buildings were delivered at a meeting in Corvallis, Oregon in 1977. In two monumental volumes of the *Publications Mathématiques de l'IHES* (and in lecture notes from a conference on Lake Como in 1984), Tits and Bruhat completed the classification of irreducible affine buildings of rank $n + 1$ for $n \geq 3$. Central to this classification is the notion of a *valuation of a root datum* of the building at infinity.

Tits and Borel had shown that a semisimple algebraic group of positive k-rank has a configuration of subgroups, which Tits called a *root datum*. Tits identified a corresponding property for spherical buildings he called the *Moufang condition* in honor of Ruth Moufang, a pioneer in the study of projective planes. Every irreducible spherical building of rank at least 3 satisfies this condition, and every spherical building satisfying this condition possesses, in a suitable sense, a root datum.

An irreducible spherical building of rank 2 is simply a connected bipartite graph in which every vertex has at least three neighbors and $g = 2m$, where g is the girth, m is the diameter of the graph, and the minimal circuits are the apartments. Tits called such graphs *generalized m-gons*.

The *residues* of a building are certain distinguished subbuildings. Every building is, in a suitable sense, an amalgam of its irreducible rank 2 residues; and if the building is spherical, then it is, in fact, uniquely determined by these subbuildings. The proof of this was a crucial step in Tits' classification result for spherical buildings.

There is now an enormous literature on the subject of generalized polygons, especially finite generalized polygons. Generalized polygons are, however, too numerous to classify (every projective plane can be viewed as a generalized triangle, for example), but Tits observed that the irreducible rank 2 residues of an irreducible spherical building of rank $n \geq 3$ all satisfy the Moufang property as do all the spherical buildings associated to an absolutely simple algebraic group of k-rank 2. In 2001, Tits and Weiss classified generalized polygons that satisfy the Moufang property. They are almost all the spherical buildings associated with an exceptional or classical group, but this time the list of exceptions is longer.

Affine buildings are CAT(0)-spaces uniquely determined by their boundary whenever the boundary satisfies the Moufang condition and the field is complete. It is this property that makes affine buildings a subject of great interest in geometric group theory. It also points toward a possible connection to physics through the holographic principle.

Jacques Tits and Mark Ronan introduced and developed the notion of a twin building. This notion was inspired by Tits' work on Kac–Moody groups which points to another possible connection with physics. Tits also extended the Moufang condition to buildings of rank 1 with the notion of a *Moufang set*. Moufang sets have proved to be an essential tool in the study of absolutely simple algebraic groups of k-rank 1.

Tits maintained a keen interest in the classification of finite simple groups as it unfolded. As a tool for identifying the finite groups of Lie type, spherical buildings played an essential role in the classification. Later a theory of "diagram geometries" based on older ideas of Tits' was introduced by Francis Buekenhout and others with the goal of including the sporadic groups in this geometric picture. This led, in turn, to Tits' "local approach" to buildings. Tits wrote papers on Griess's construction of the monster and moonshine and on several other sporadic groups as well and he proved the simplicity of ${}^2F_4(2)'$, now called the Tits group.

Figure 3. Jacques Tits wearing the medal of the French National Order of the Legion of Honor, Paris, 1995.

Tits introduced the notion of the Coxeter complex associated to a Coxeter group (and the term *Coxeter group* itself) including roots, projection maps, and other essential features of these complexes. He proved fundamental results about the structure of the automorphism group of a tree, the simplest of all affine buildings. Tits analyzed geometric structures associated with the Suzuki and Ree groups, showed that these groups are classified by "Tits endomorphisms" of the corresponding field, and proved their simplicity even when the field is imperfect.

In 1970, Tits proved that in characteristic 0, every finitely generated linear group contains either a solvable subgroup of finite index or a non-abelian free group. Now known as the *Tits alternative*, this result has inspired a host of generalizations.

In 1964, Tits proved a remarkable result about the simplicity of the subgroup of the group of rational points of a k-simple algebraic group generated by certain unipotent elements. Some remarks in this paper gave rise to the Kneser–Tits conjecture. His 1968 paper on quadratic forms became the starting point of the *Book of Involutions*. In 1971, Tits determined all the k-irreducible linear representations of a reductive group over an arbitrary field. He also took the first steps in the theory of pseudo-reductive groups.

In what is now known as the Tits-Kantor-Koecher construction, Tits obtained Lie algebras from arbitrary Jordan algebras. He devised the Freudenthal-Tits magic square which forges a Lie algebra out of a composition algebra and a degree 3 Jordan algebra, yielding all exceptional Lie algebras if the field is algebraically closed. He also produced the first and second "Tits constructions" which play a central role in the structure theory of Jordan algebras.

This brings to a close our attempt to name the highlights of Tits' mathematical career, but no brief summary can encompass them all, nor can we sufficiently describe the influence Tits' mathematics has had on group theory and all its many neighboring disciplines.

Figure 4. Jacques and Marie-Jeanne in Oslo, 2008.

Tributes

Jean-Pierre Bourguignon

Jacques Tits and I met for the first time in the early 1970s in Bonn. Friedrich Hirzebruch, who had convinced him to take a position there, introduced me to him in the tea room on the ground floor of Beringstrasse 1. This remains a special memory for me because of the many precious opportunities I had to meet him later in my life.

Indeed, after I became director of the IHÉS in 1994, we developed a close and trusting relationship, something I am highly grateful for because of the thoroughness with which he approached questions I posed to him. His sense of humour and his gentle way of talking to people were legendary.

Jacques Tits was one of the very first visitors to IHÉS, shortly after its creation in 1958. He lectured several times at the Institute's first location Rond-Point Bugeaud, near Place de l'Étoile in the heart of Paris's XVIth arrondissement. Here is what he wrote in a letter to Léon Motchane, the founder of IHÉS and its first director, dated 15 July 1961: *"Pour autant que l'on puisse juger de son propre travail, je compte les deux séjours que j'ai faits à l'Institut des Hautes Études Scientifiques parmi les périodes les plus productives de ma carrière scientifique."*

Later, when the Institute had moved to Bois-Marie in Bures-sur-Yvette, where it still is, Jacques came for several long visits, staying with his wife Marie-Jeanne at the Ormaille Résidence.

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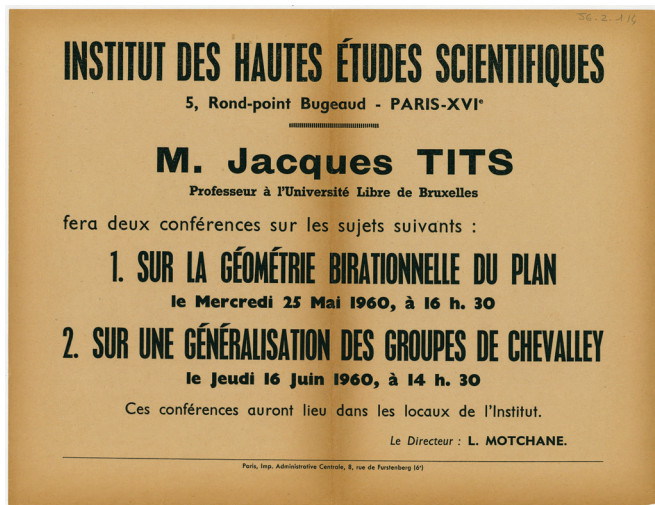
Figure 5. Announcement of Tits' first lecture at IHES.

Jacques Tits was asked to become editor-in-chief of the *Publications Mathématiques de l'IHÉS* in 1980, succeeding Jean Dieudonné who had held the position for 20 years. He was an extraordinarily dedicated editor-in-chief. In the interview Pierre Deligne gave on the occasion of the conference held in 2000 to honour Jacques[1] when he retired from the Collège de France and shortly after having left the helm of the *Publications Mathématiques*, Pierre Deligne says: "*C'était une situation idéale. Tits faisait énormément de travail. . . Il était un despote éclairé. Il jouait son rôle parfaitement et savait prendre des décisions quand il fallait mais il consultait d'abord.*" The advice Jacques Tits gave to Étienne Ghys when he took over the editorship from him is revealing: "*Vous savez, c'est très facile, il suffit d'aimer la revue.*"

His extremely careful checking of all articles to be published there contributed certainly to the high recognition the journal enjoys in the mathematical community. During his editorship, the journal attracted a number of landmark articles, some of them quite long.

On a number of occasions during my time as Director of IHÉS, I reached out to him for advice on issues related to scientific initiatives IHÉS should take concerning mathematicians to invite or events to organize. His in-depth knowledge of the mathematical community at a high level was very valuable. Earlier, he had been helpful in securing some financial support for the IHÉS from the Belgian government.

His later years were not easy due to a difficult health condition which confined him to his apartment. At the end of his life, Tits needed care the around the clock, but

his sense of humor persisted in spite of all the hardships and his mind remained clear and agile.

Several mathematicians paid him regular visits. Jean-Pierre Serre would come to see him every three weeks more or less. I accompanied Misha Gromov on some visits, as Jacques appreciated my providing some "translation" of what Misha said.

His funeral was very simple. Besides the testimony of a family representative, Étienne Ghys and Michel Broué presented accounts of their admiration for him. I had the privilege of reading short testimonies received from five Fields medallists. Jacques Tits made the Foundation Hugot of the Collège de France his sole legatee.

Pierre Deligne

It has been my good fortune that Tits was a professor at the ULB (Université Libre de Bruxelles) in the early sixties. He gave me two crucial pieces of advice: "Do what you want" and "Go to Paris." The latter was easier said than done. Tits made it possible by introducing me to Grothendieck (at the Fall 1964 Bourbaki seminar), who, together with his colleagues, enabled me to become "pensionnaire étranger" at the ENS (École Normale Supérieure in Paris).

Every Thursday afternoon during one of my last years of high school, I would bicycle to the ULB to attend Tits' course on Lie groups. I vividly remember the day he wanted to define the adjoint group. He began a pedestrian proof that the center is an invariant subgroup, then stopped to say (rough translation): "In fact, this is obvious. As I can define the center, it is stable by any automorphism, hence by inner automorphisms." For me, this interrupted proof was a revelation of the power of "transport of structures." It also shows how symmetry was never far from his mind.

I also fondly remember the day when I bicycled to his house in the pouring rain to show him some mathematics, and how, arriving unannounced, I was warmly received–and dried–by him and Marie-Jeanne.

In 1964, Tits left Brussels for Bonn. It was only in 1973 that I again saw him regularly. Attending his course at the Collège de France was one of the highlights of my week.

Tits was a perfectionist. When he succeeded Dieudonné as editor of the *Publications Mathématiques de l'IHÉS*, he devoted a lot of energy to it, but he enjoyed the result, and the beautiful typography. Tits resigned when the composition was computerized.

He had a great interest in languages. He learned Japanese to better enjoy his visits and Chinese to read classical

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Figure 6. The Tits family: Léon (nicknamed Pap), Jacques (Yaak), Jean (Coc), Yvonne (Vonne), Ghislaine (Ghaine), and Louisa (Mam).

poetry. In French, he suffered when I failed to use a needed imperfect subjunctive, and regularly chided me for my Belgicisms, correcting my "rouler en vélo" to "rouler à vélo."

He and Marie-Jeanne were inseparable. When walking became difficult, he leaned on her. Her death was a shock from which he did not recover.

His death, during the covid epidemic, came unexpectedly. I could only find some solace by reading from his *Collected Works*, where his spirit remains.

Jean-Pierre Tignol

As a thesis advisor, Jacques Tits was always supportive and benevolent to me, and I benefited immensely from his approachable demeanor and generous personality. In our infrequent work sessions, I had the privilege to witness the workings of his mind and to appreciate his unfailing, often self-deprecating, sense of humor. Even though the problem he suggested to me was purely algebraic, his line of thought was infused with geometric insights.

This unique opportunity bestowed on a student in Belgium by an illustrious mathematician from Bonn University who was about to move to the Collège de France was a result of Tits' attachment to his country of birth. While he lived abroad, he regularly returned to visit not only his family, but also his colleagues in the mathematics department of the Université Libre de Bruxelles, from which he had graduated and which had offered him his first position. Tits had to become a French citizen in order to take his chair at the Collège de France, but he kept an enduring connection with Belgium. He once recounted that on an official visit at the Collège de France the French president

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Valéry Giscard d'Estaing asked him where he came from (*"Et vous, d'où sortez-vous ?,"*) expecting as a reply the name of any of the prestigious French *grandes écoles*. Tits replied: "Er... from Belgium."

Pierre-Antoine Absil

Jacques Tits was the brother of my maternal grandfather. I met him on rare occasions, but my mother Janine Tits was his closest relative during the latter period of his life. Together with her brother André Tits, she was of great help in gathering the family memories that are shared in this contribution.

Born on August 12, 1930 Jacques was a lively, joyful child, curious about everything. He lived in the family home at 21 Avenue Victor-Emmanuel III in Uccle, Belgium, with his parents, his older brother Jean, and his older sisters Ghislaine and Yvonne. As a child, he dreamt of becoming a tramway driver: he loved watching the driver doing his thing. Jacques' father, Léon Tits (born in February 1880), was employed as an assistant in the mathematics department at Université Catholique de Louvain (UCL). At the time, he was a Catholic priest, like most professors and many assistants back then at UCL. By 1914 though, he was in disagreement with the clergy. He left the priesthood and was forced to resign from the university. The Catholic Church made it difficult for him to find employment elsewhere. His parents and many of his relatives rejected him as well. In 1917, he married Louisa André, a remote cousin, a warm, honest person, who worked as a piano teacher. The family lived happily, though with limited means. Léon died of Parkinson's disease in 1943, in the midst of World War II. After Léon's passing, Jacques' older brother Jean became the family's breadwinner and took over his father's private tutoring.

In 1941, Jean was starting as an engineering student at the Université Libre de Bruxelles (ULB). As Jacques would hear his dad and brother discuss integration, he wanted to understand. It was shortly before Léon's death, when Jean told his bed-ridden father "now I know" that Jacques is truly exceptional. Jacques started to teach university-level mathematics to his brother's classmates who were encountering difficulties. He soon decided, on the encouragement of his mathematics teacher, Charles Nootens, to attempt the entrance examination to the ULB's engineering school.

In preparation for the entrance examination, Jacques had to learn trigonometry, so Jean lent him his 60-page textbook. The next day, Jacques *knew* it all. His secret: start

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Figure 7. Marie-Jeanne and Jacques, Rome, 1953.

from the end; if you understand the end, you can reconstruct the rest. He passed the entrance examination with flying colors. At age 14, he entered the ULB as a student in mathematics.

In his third year at the ULB, under the direction of Paul Libois, he obtained startling results in algebra, in particular on 3-transitive groups. He obtained his bachelor's degree at age 18, then his PhD before age 20. I recall seeing a framed newspaper article at my grandparents place praising his accomplishment as the youngest Belgian Doctor of Science.

In 1949, Jacques met then-Princeton mathematician Emil Artin at the *Colloque d'algèbre et théorie des nombres* in Paris. Artin invited him to visit Princeton. This would be Jacques' first trip outside of Europe.

Also in 1949, Jacques' first niece, my mother Janine, was born. "Yaak" became "Oncle Yaak." He loved taking care of Janine. In his late years, he confided to Janine that he would have loved to have children of his own and that he considered her as his daughter.

In 1953, in Rome, Jacques met Marie-Jeanne Dieuaide, herself an FNRS Fellow from Belgium. Her field was history. Jacques and Marie-Jeanne were housed in the same dormitory building. Marie-Jeanne later confided to us that, before Jacques' arrival, she had joked with other FNRS fellows: "A mathematician is joining us? I hate math! Too serious and boring for me!" She soon changed her mind and proceeded to make him see other horizons. Marie-Jeanne and Jacques got married in Brussels on September 8, 1956.

From 1956 to 1962, Jacques taught extensively at the ULB. In 1964, Jacques and Marie-Jeanne left for Bonn, where they would remain for ten years. Jacques kept close contacts there for the remainder of his life.

A close friendship had been formed between Jacques and Jean-Pierre Serre. Serre wished to have Jacques with him at the Collège de France, and succeeded after Jacques changed his citizenship to French, at that time a requirement for obtaining a professorial position at the Collège de France. In 1975, Jacques gave his inaugural lecture at the Collège de France. This lecture was addressed to a general audience, and Jacques succeeded in making it seemingly understandable, even exciting, to the "person in the street," bringing to life the central role played by symmetry in mathematics.

In 2008, several family members had the privilege of attending the Abel Prize award ceremony in Oslo. Jacques, in a wheelchair, peppered his speech with the humorous touch that characterized him.

In the latter portion of his life, Jacques had health problems. Multiple times, Marie-Jeanne contacted Janine, head-pharmacist at Verviers Hospital, asking her to consult with Jacques' doctors concerning his ailments. Several times Janine, in close consultation with her cousin Claude, had both Jacques and Marie-Jeanne urgently hospitalized in Paris. In spite of all these travails, Jacques never complained. Always accepting his fate, smiling, full of great charm and humor, he had an amusing anecdote for everyone.

My wife Tatiana Sirbu recently accompanied my mother to Paris. Originally from Moldova, she speaks fluent Russian. Jacques wanted to hear about her home country, her youth in the USSR, her current research work on deportations and transfers of populations during the Soviet era; they even had long conversations in Russian together. That day Jacques was especially witty. Beside being fluent in English and German, he could converse in Russian and Italian, and was in the process of learning Spanish (he wanted to read *Don Quixote* in the original) from his then chief homecare person, Madame Rodriguez, plus an Assimil book. He also studied Chinese and Japanese. He once confided to my mother and André that he still had some to-be-written mathematics papers in his head.

Jacques and Marie-Jeanne never had children. Their child was their research, their life was the Collège de France. An idea emerged: Would Jacques bequeath his entire estate to the Collège de France, specifically to its Fondation Hugot? Jacques was delighted at such a thought. Jean-Pierre Serre contacted the Fondation Hugot and soon Jacques wrote a will, before two witnesses: his dear friends Jean-Pierre Serre and Jean-Pierre Bourguignon (then President of the European Research Council). This being settled, Jacques was serene. He received the promise that he would never have to leave his apartment, and Florence Terrasse-Riou, director of Fondation Hugot, told him that, down the road, his apartment would remain the "Apartment Jacques Tits" and would be made available as housing for visitors.

Jacques never showed interest in using new technologies. He lived without a TV or even a radio; newspapers,

Figure 8. Jean-Pierre Bourguignon, Jean-Pierre Serre, Jacques Tits, Florence Terrasse-Riou, Claude Piret, and André Tits in the Tits' apartment in Paris, 2017.

Figure 9. Franz Bingen and Jacques Tits at the wedding of Tits' sister Yvonne and Bingen's brother Roald in Brussels, June 19, 1954.

magazines, and books were sufficient for him. For his 91st birthday (on August 12, 2021), André offered him a laptop so that, with the help of Stéphanie, his chief homecare person at the time, he could read emails we sent him and interact with us on Skype.

On December 1, 2021, Janine had a pleasant Skype conversation with Jacques and Stéphanie. Jacques sent her a virtual kiss. He would leave us four days later at dawn.

Always generous, charming, smiling, and joking, Jacques expressed interest for all things. We keep from him enchanted memories, a life model for future generations.

Franz Bingen

Jacques Tits was born in 1930 as the youngest in a family of four surviving children. As a child, he played a lot with his sister Yvonne, who preceded him by eighteen months. The two felt like twins. They kept this special complicity throughout life. There was a mathematics gene in the family. Jacques' father was a high school mathematics teacher. He taught Jacques how to calculate at the age of four. Jacques made rapid progress and skipped grades in elementary school. His father quickly realized his uncommon mathematical gift and did his best to develop it. Unfortunately, he died as Jacques was approaching thirteen. Jacques found his own way to help his mother to make ends meet. He gave lessons in mathematics to students preparing for the entrance exam to the Faculty of Applied Sciences at the Université Libre de Bruxelles (or ULB). He took that opportunity to take the exam himself. Jacques

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came out first in the exam, and this allowed him to start early working on a bachelor's degree in mathematics at the Faculty of Sciences of the ULB. He got his BA at eighteen. Two years later, he defended a PhD thesis prepared under the direction of Paul Libois, who had been his geometry professor throughout his studies. After that, he obtained postdoctoral support from the science foundation in Belgium (FNRS). This gave him the opportunity to present his habilitation and to start an academic career at the ULB. In particular, he assisted Paul Libois by contributing to the teaching of the projective geometry course for second-year students in mathematics.

I became a student at the ULB in 1950. Projective geometry was one of the courses I had to take and Jacques Tits was the professor. I enjoyed his very personal style of teaching. In the tradition of Enriques as later developed in a two-volume book by Veblen and Young, one usually started from the axiomatics of the projective plane and deduced its main properties. After this one climbed a dimension higher and if time allowed one reached general projective space. Jacques Tits began with the projective line and the characterization of the group of projectivities on the line among triply transitive groups. In dimension 2, he introduced the nearly fourfold transitive groups and again established the relationship with the group of projectivities of the projective plane. Then he could deduce the traditional properties more easily. In spite of the difficulty of the material, he succeeded in keeping his lectures understandable to the students. We owed this to his very communicative style of teaching, which was very different from the distant manner adopted by most professors at that time in universities. His eyes sparkled with intelligence and above all he radiated kindness.

Three years later, I met Jacques again in very different circumstances. My brother Roald went to a summer camp in Austria. Holidays did not go according to plan for him, as an angina forced him to take to his bed. One young lady paid special attention to him during his convalescence. She was, by a nearly miraculous coincidence, Jacques' sister Yvonne. She and Roald soon became engaged and they married on June 19, 1954. At the wedding, Jacques (24 years old) and myself (two years younger) were of course present. We raised our glass to the health of the newlyweds and also, to a certain extent, to a new friendship that would last for the rest of our lives. Obviously, Jacques felt very comfortable in our family. We were soon separated, however, by our various scientific stays abroad.

The early sixties was an exciting time at the ULB. Jacques became a full professor, while I was appointed lecturer in the Flemish section of the university. We found it important, at a time when the number of students, and therefore of professors, researchers, and assistants, was growing significantly, to activate research in mathematics at the university. Our contribution took the form of a seminar devoted to a current scientific subject. The first year was devoted to Banach algebras. Jacques wanted to better understand the link between a commutative Banach algebra with unit and its compact spectrum. Lucien Waelbrouck, who had studied continuous inverse algebras, was playing the third wheel at the seminar, which was rapidly named the BTW seminar (BTW is the acronym for value added tax in Dutch). Through Georges Papy, we got to know another young mathematical prodigy, Pierre Deligne, still on the benches of secondary school. To interest him, we oriented the second and third seminars towards algebraic geometry and Lie algebras, this time with the help of Guy Valette and Firmin Bratslavski, two geometers.

In 1964, Jacques Tits obtained a chair of mathematics at the University of Bonn, better tailored to his mathematical interests than his assignment in Brussels. Jacques came to Belgium regularly to visit family, in particular his dear sister Yvonne and her children. Here is how Christine, a daughter of Yvonne, describes her relationship with her uncle: "…for us, Uncle Jacques was above all this super-funny uncle, extremely simple and kind, who came to visit us once a year according to the availability of his conference life, staying with us for the weekend. He told us extraordinary stories, experienced during his travels around the world. Magical moments for the children that we were, where he had this mischievous side, disarming with candor alongside an immense sweetness. This is the image that, I am sure, my sister and brother will keep with me of this uncle we loved a lot and who made us laugh and dream so much."

Figure 10. Hendrik Van Maldeghem, Gopal Prasad, Pierre-Emmanuel Caprace, Jef Thas, Bertrand Rémy, Jean-Pierre Serre, Ernie Shult, Bernhard Mühlherr, Jacques Tits, Francis Buekenhout, Marie-Jeanne Tits, Richard Weiss, and Mark Ronan at a colloquium in honor of Jacques Tits' 75th birthday at Ghent University, October, 2005.

Much, much later, around 2008, I started meeting him again, this time in his apartment in Paris. My wife and myself went several times a year to the ballet at the Paris Opera and always took the opportunity to visit Jacques and his wife Marie-Jeanne Dieuaide. Jacques had developed Parkinson's disease. He had his *Complete Works* on his bedside table. He leafed them through with us and asked for the latest news in his family. Jacques passed away peacefully in December 2021. His friends retain the image of a brilliant mathematician with a charming personality.

Michel Broué

I would just like to tell here how sad many of us are, who have known Jacques Tits professionally and personally. A peculiar intuition, a source of exceptional ideas, an original and quite productive point of view, and even a kind of library, have disappeared. This is quite a loss.

But the first feeling which comes to my heart when I think of him is: kindness. To chat with him was always pleasant, reassuring, quiet. He also expressed — I am not sure I find the right word — a kind of unusual modesty. A kind of modesty always spiced up with a soft and constant sense of humor. Once he was giving a lecture at the Bourbaki Seminar, and at one point he had to mention a theorem known all around the world as "Tits' Theorem;" he talked about *"le théorème de moi."*

He had been very precocious, defending the equivalent of a Habilitation at the age of 20. Years later, he explained to me that the main hardship for a mathematician is

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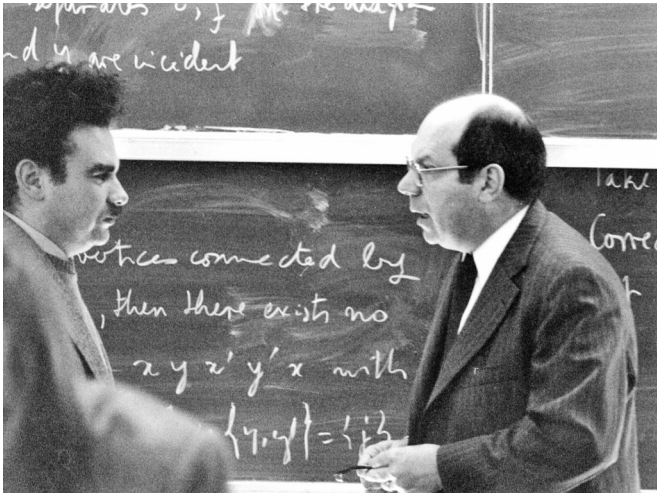
Figure 11. Michel Dehon and Jacques Tits at the conclusion of a talk by Tits in Ghent, October 25, 1979.

always to understand (and to smoothly accept) that one day you will find someone quicker, brighter, "better." Once this is accepted, a mathematician's life is marvellous, he added.

He was both attached to traditions and opened to rational reasons (he was a mathematician…). A student of mine submitted a "Note aux Comptes-Rendus de l'Académie des Sciences" which was good. But the author insisted on writing *"je définis," "je démontre,"* etc. and at first Tits would not accept this. He wanted the usual *"nous définissons," "nous démontrons,"* etc. The student insisted that no one else but he had defined and proved, and he added that only the late Kings of France would speak of themselves with "nous." Tits accepted "je."

The Collège de France was profoundly renovated at the end of the last century. One day the room where he was supposed to deliver the first lecture of his annual course was unavailable, and there were signs on the main door which directed the audience to another room. Tits arrived from the rear and did not see the signs. *"Voilà, personne pour mon cours, je savais que cela arriverait un jour"* was what he immediately thought, and when he eventually found the right room he was still pale. Needless to say, though, that Jean–Pierre Serre, among others, never missed one of his lectures.

Alain Valette

The late seventies were an exciting time to study mathematics in Brussels. The two universities (the French-speaking one, ULB, and the Dutch-speaking one, VUB) were

sharing the same campus, and there was intensive collaboration between the two mathematics departments, with a number of professors enjoying dual affiliations. People like Jean Bourgain and Ingrid Daubechies were active at VUB, and at ULB we enjoyed regular visits of extraordinary alumni like Pierre Deligne and most frequently Jacques Tits. We were lucky, as undergrads, to have two young geometry teachers, Francis Buekenhout and Jean Doyen, who strongly encouraged us to attend research seminars. So, from my 4th and final undergraduate year (1979–1980), I enjoyed following Tits' seminar talks, in Brussels and in Ghent. Even if I did not always have the prerequisites, I was always impressed by his clarity, and there was always something deep to extract from his beautiful lectures. That same year, in spring 1980, my mathematical inclinations were leading me towards operator algebras; I applied for a PhD thesis scholarship from the Belgian Fund for Scientific Research (FNRS), and I was lucky to get it. Simultaneously, my official thesis supervisor Lucien Waelbroeck had a severe accident that kept him away from academia for a full year. So I found myself in the embarrassing situation of having a scholarship but no supervisor. To help me out of this unpleasant situation, Buekenhout arranged an appointment for me with Tits. I was extremely intimidated, and trying to make me more comfortable Tits said, waving his hands about 60 cm from each other: "Oh but I know you, you were like that first time I saw you." He was alluding to the fact that he met me as a baby boy, back in 1959, when my father Guy Valette was doing his PhD thesis with him. (My father, born in 1934, was Tits' first PhD student.) Even more intimidated, I nevertheless succeeded in explaining my thesis project. Tits exclaimed: "Young man, if you want to do operator algebras today, there is one saving grace: go to Paris and work with Alain Connes!" With the recklessness of youth, I went to find Connes in Paris and indeed became his unofficial PhD student. Two years later Connes got the Fields medal. In retrospect, Tits gave me the best advice in my career.

Since my thesis was on C^*-algebras associated with real or p-adic simple Lie groups, I frequented group theory conferences where I would occasionally meet Tits. Sometimes I had the good fortune to be invited to his table for lunch or dinner and got to experience how sweet and gentle he was, but also how funny and witty he could be, with a typically Belgian sense of self-mockery. Tits' style of writing was akin to his style of lecturing: a model of clarity and exposition. Un grand monsieur.

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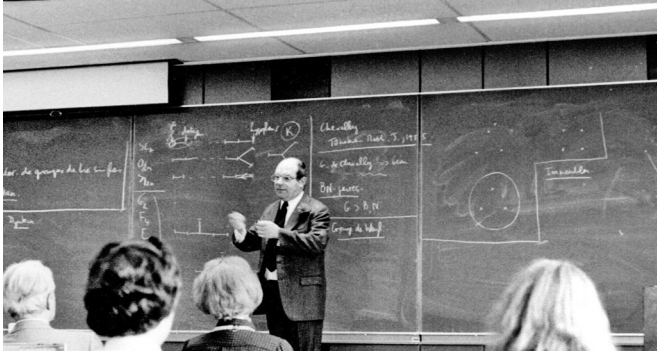
Figure 12. Tits lecturing at the 80th birthday conference for Tits' advisor Paul Libois at the Université Libre de Bruxelles, April 1, 1981.

Michel Racine

Jacques Tits was a kind and witty friend. I met Jacques and Marie-Jeanne in the fall of 1966. He was a visiting professor at Yale and I, a beginning graduate student. I did not have the prerequisites to attend his course on algebraic groups but was looking for the opportunity to speak French. They were both extremely kind with a good understanding of what belonging to a French-speaking minority meant. This developed into a life-long friendship. A few years later while they were visiting Ottawa, I introduced my future wife to them. I had tried to explain to her Jacques' mathematical importance without much success until I told her he received his PhD at 19. She asked Jacques if this was true. "Yes." "But you are a genius!" His answer was yes but with a connotation of there are things in life that can't be helped. Marie-Jeanne looked aghast and said "Jacques!" All three of us broke into laughter and Lise looked nonplussed. What made it so funny was that it was really out of character.

At the 1974 ICM in Vancouver, he began his talk with: Pick a group. Any group. Your favorite group. Let's say E_8. Looking around at those who were laughing or smiling, you could tell who would enjoy the talk. In 1988–89, we spent a sabbatical in Paris. Marie-Jeanne was helpful in finding us a place to stay. Early on, Jacques made the rounds of the mathematical libraries to introduce me to the librarians. I thought, what a waste of time, we could have discussed math instead. But, of course, he knew what he was doing. Without his personal intervention, I would not have been allowed to use these institutions.

Tits was proud to be a foreign member of the German order Pour le Mérite founded by Frederick the Great. There are no more than 40 German members and 40 foreign ones.

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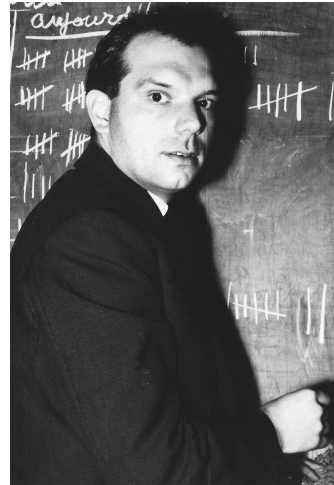
Figure 13. Tits teaching at the Université Libre de Bruxelles his first year as an Assistant Professor, 1957.

In the preface of his book [2], Nathan Jacobson wrote: "I am greatly indebted to Jacques Tits who took time off from his own important researches on algebraic groups to derive, via the theory of algebraic groups, the elegant constructions of exceptional Jordan algebras which we have given in Chapter IX."

Jacques and Marie-Jeanne were inseparable. Her illness and death were the great tragedy of the end of his life. When they spent a few weeks in Ottawa, I rented a two-bedroom suite in a nearby hotel. The staff cleared one of the bedrooms and installed two banquet tables side by side so they could work together. In our conversations, "What are you reading?" was a frequent question. Once Jacques answered "We are rereading Proust." "Are you reading the same thing?" "Of course. We read in bed; one of us reads aloud until we feel sleepy."

Roger Howe

When I think of Jacques Tits, I think of a kind and generous person. Among the leaders of French mathematics of his generation, he stands out as the one who saw value in what I was doing, and took steps to further my career.

I met Jacques somewhat by accident, for me a very happy accident. We both spent 1971–72 at IAS and we both had apartments in the IAS visitors apartment cluster. My walk to Fuld Hall took me past the Tits' apartment. Apparently, my whistling while walking by attracted the attention of Jacques and his wife Marie-Jeanne. (Perhaps because it was off key. I am not at all musical, but they were too polite ever to say that.)

Jacques arranged to have me invited to visit the Sonderforschungsbereich run by Friedrich Hirzebruch at the University of Bonn for 1973–74. In Bonn, I had two significant mathematical interactions with Jacques. Günter Harder was also party to these. The first concerned the orbit structure of pairs of classical groups acting on the tensor product of their standard modules. This led me to the idea of dual pairs in the symplectic group, which has been

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my main research focus in the half century since. I first reported on the basic ideas at the Arbeitstagung conference that marked the end of the academic year for the Sonderforschungsbereich.

The second was about a question of Harish-Chandra, who was trying to establish properties of character distributions, which had been so central to his work on real reductive groups, in the p-adic situation. Harish-Chandra wanted to know if certain constructions could be guaranteed to be well-behaved relative to nice maximal compact groups. I knew how to show this was so when the size of the residual field was large enough. Jacques, with his understanding of algebraic groups as group schemes, was able to show that this meant it could always be done. He communicated the results to Harish-Chandra, who incorporated them into his work on characters.

I saw Jacques next in the summer of 1977, at the AMS Symposium on "Automorphic Forms, Representations and L-Functions" in Corvallis, Oregon. He gave a set of plenary lectures on buildings and their implications for reductive algebraic groups over local fields. I also gave a talk in Corvallis, sketching how the ideas conceived in Bonn had developed since 1974 and some implications for p-adic representation theory. The main facts were mostly still quite conjectural, but Jacques again was supportive, and in the following year he invited me to give a talk at the Collège de France.

Over the following decade plus, I had the pleasure of seeing Jacques and Marie-Jeanne in New Haven, when Jacques would visit Yale. He had substantial interests in common with Nathan Jacobson (Jordan algebras) and Walter Feit (finite groups). A most enjoyable feature of these visits was the farewell dinner at the Union League Cafe, generally considered the best restaurant in New Haven.

I regret that I hardly saw Jacques after 1990. What remains strong is gratitude for the substantial help and encouragement he gave, and appreciation for the person he was.

Jef Thas

In 1969, I followed a series of lectures on "Groupes de Chevalley" at the University of Brussels; the main organizers were Francis Buekenhout and Franz Bingen. There I learnt about the work of Jacques Tits on BN-pairs. A few years later, I read the book of Peter Dembowski on finite geometries, and learned about generalized polygons. These objects were defined by Tits in his famous 1959

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paper on triality. In Dembowski's book, I found the description of certain Tits' generalized quadrangles $T(O)$ arising from ovoids. I generalized the construction of these $T(O)$ and gave a lecture on it at a summer school in Italy in 1972. I sent the paper to Jacques Tits, and he answered me from Princeton that he would present my work to *Geometriae Dedicata*. So my first contacts with Jacques Tits started in the period 1972–1974. That was the beginning of the many handwritten letters I received from him. It was also the starting point of my research on generalized polygons, one of the topics on which I am still working.

In 1976, Jacques Tits sent a letter to Stanley Payne, Francis Buekenhout, and me. In his letter he included the preprint "Quadrangles de Moufang, I." He also mentioned that due to his moving from Bonn to the Collège de France his collection of reprints and preprints was "in a shamble" and so he was not sure that the results in his text were new. He asked us if the results were known up to the best of our knowledge. He also said that "the interest of the paper is certainly quite limited." In fact this paper is part of his huge achievement, the classification of all Moufang polygons. This letter also shows his kind way of dealing with much younger researchers. (I was 32 and was of course very honored.) In February 1979, Jacques Tits was awarded the title of Doctor Honoris Causa by Ghent University. It was a great honor for me to be his promoter and to introduce him during the ceremony. It was the first time that my wife and I met Jacques Tits and his wife Marie-Jeanne Dieuaide. We talked about a lot of things, not only mathematics. We told him that we liked Italy very much and that we stayed twice at the Academia Belgica in Rome. It appeared that in 1953 Jacques stayed at the Academia and that he met Marie-Jeanne there while she was doing research on Medieval History.

In the winter of 1979, Jacques Tits and his wife visited Ghent University again, and Jacques gave a talk on diagram geometries. Jacques Tits and his wife were very fond of Ghent. While being in Ghent, Marie-Jeanne consulted the archives of the city in the frame of her research about the Flemish cities in the Middle Ages. They always stayed in their favorite hotel, the "Cour St Georges" in an eighteenth-century building.

In 1981, they visited Ghent again, and now Tits talked about groups and Kac–Moody algebras. At the same time my colleague Stanley E. Payne, then professor at Miami University, Ohio, was visiting me. We were busy working on our book *Finite Generalized Quadrangles*. In 1977, Payne published two long papers proving the uniqueness of the generalized quadrangle of order 4. Before including it in the book, we discovered that the proof was incomplete. We mentioned this to Tits who immediately started to write on the blackboard in my office. The next day he

came up with a proof that certain configurations could not exist. This allowed us to complete the missing part in the proof. His reasoning was very original, and I am not sure that Payne and myself would have found it. Tits was just in time visiting me in Ghent to save us!

For Tits' 60th birthday, the Belgian Mathematical Society organized a conference in 1990 at the Palace of the Academies in Brussels. The rector of our university invited Jacques, Marie-Jeanne, my wife and myself for an informal lunch, just the five of us in a room adjacent to the meeting room of the University Board. The rector had to preside at a meeting of the Board, which was supposed to start right after the lunch, but the rector was so charmed by the Tits family that he let the Board know that they could start without him.

For my 50th birthday, some of my colleagues organized a two-day conference as a surprise. An even bigger surprise was that Jacques and Marie-Jeanne showed up. Jacques gave a beautiful talk on Moufang polygons.

In 1996, we organized a conference in honor of the 65th birthday of Jacques Tits. Then in 2003, I had the opportunity to see him again in Brussels at a conference in honor of my good colleague Francis Buekenhout. Some years later, my colleagues Van Maldeghem and Mühlherr organized a meeting for the 75th birthday of Jacques.

The last time I spoke briefly to Jacques was in 2008 at the Palace of the Academies in Brussels, during a ceremony in honor of Jacques Tits and Pierre Deligne being awarded the Abel prize respectively the Wolf Prize. His health was not good anymore and he needed a wheelchair.

Jacques Tits was a great man, not only as a mathematician but also as a human being. We all will miss him.

Hendrik Van Maldeghem

The first time I heard about Jacques Tits was in a lecture for second-year undergraduates at Ghent University. It was a course in projective geometry, and the professor (Julien Bilo), nearing his retirement, was more concerned about telling stories than presenting mathematical results. One story was about a little boy wearing short pants that he met at Brussels University, who amazed his professors with his knowledge and mathematical insight. Jacques was barely 14 when he entered university.

Some years later—I think I was still a student, or perhaps a first-year PhD—Jacques Tits visited Jef Thas and I saw Jacques in real life. Jacques was an honorary doctor at our university (on the initiative of Jef Thas) and paid regular visits. The first talk I heard him present was in Mons in

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the same year, I think it was 1983. I remember him writing down the correct order of the monster on the blackboard, excusing himself for knowing it by heart by pretending that this huge number consisted of his telephone number, then his bank account number, then his social security number, etc. The talk was in French, as far as I remember, but it did not matter. In fact, that is one of the many things I always liked about Jacques' talks: he spoke and pronounced very clearly, using simple words; it did not matter in which language he was speaking—and he spoke many languages! His explanations always made the audience feel that they understood everything; that was his special gift.

One especially charming feature about Jacques was that he always made people feel important; for him all mathematicians were equal, he never looked down on lesser gods. I experienced this myself several times. For example, after finishing my PhD, which was about the special class of affine buildings of type $\widetilde{A}_2$, I wrote a letter to Jacques explaining what I did (no email at that time; it was 1984). In the same year, Jacques classified all affine buildings of irreducible type and rank at least 4. His reply to my letter started with the sentence "It seems that we have been working along the same lines this year." As a second example, many years later, in 1994, I invited Jacques Tits to present a talk at the conference celebrating the 50th birthday of Jef Thas, my mentor at Ghent University. We were publishing the proceedings and I persuaded Jacques to submit a paper. He wrote one about the Moufang condition for generalized polygons and the relation with root systems. He wrote this by hand, and I committed myself to put the text into LᴬTEX. Doing this, I discovered a small oversight in one of the formulations. (He'd overlooked that the root groups of the smallest Suzuki group are abelian.) Jacques was very pleased, and at the conference he started his talk by thanking the organizers, as usual, but added, "if you ever write a paper full of mistakes, just send it to Van Maldeghem to type it out, and he will not only do this, but also correct all your errors."

I remember Jacques as someone who was very generous and thankful. In the 90s, I followed several courses of his at the Collège de France. Every Tuesday in winter, I drove 630 kilometers from Ghent to the center of Paris and back to follow his lectures. Sometimes PhD students joined me, either for the full course, or on a sporadic basis; one time my two sisters even joined me (they were math teachers). And at the end of every course, Jacques invited me and everyone else who joined me regularly, to an extended lunch in Paris (more like a dinner at noon). He was so thankful that we came from so far just for him—but of course the pleasure and the added value were entirely ours.

Jacques' lectures were very pleasant to follow.

Figure 14. Francis Buekenhout, Jean-Pierre Serre, Jacques Tits, and Hendrik Van Maldeghem in the first row, at the conference in honor of Jacques' 75th birthday in Ghent, 2005.

His style was informal but in a way very efficient. For instance he talked about "a path with hair on" to mean a path together with all neighbors of its inner vertices, which was immediately clear to everyone and fine for us. But Serre did not agree with his informality and often interrupted him asking for more precise and mathematically sound definitions and expressions. Jacques explained everything in a rather geometric way, which I liked. He only struggled when there was a choice between two alternatives, like plus or minus, inside or outside, left or right. I remember I corrected him once (no big deal in fact), and then for the rest of the lecture he called on my help to decide every dichotomy he encountered. . . .

There were three conferences in honor of Jacques held in Belgium, one in Brussels and two in Ghent. The first was on the occasion of his 60th birthday, the second for his 65th birthday, and the third for his 75th birthday. I was not involved in the organization of the first one (that was Jef Thas), but I was the main and local organizer of the other two. We celebrated his 65th birthday in fact one year late, in 1996. At that conference he gave a double talk, one on Friday and one on Saturday. In the first talk, my sister (a teacher in mathematics in high school) came into the room with two dozen school girls, on a school trip to see one of the greatest mathematical minds in action. Jacques was absolutely not disturbed by that and made his young audience feel welcome with a few jokes. The next day, he continued his lecture, but he was so into it, that he lost track of time. When he looked at his watch after one hour and a half (he was supposed to speak for 50 minutes, but nobody minded), he exclaimed "My God, look at the time, when did I start?" To which Francis Buekenhout dryly replied "Yesterday."

Soon after that conference, we celebrated Francis's 60th birthday with a special session in the one-week conference "Finite Geometry and Combinatorics." Jacques came over to give a talk on the new class of Moufang quadrangles that

Richard Weiss had just discovered. This triggered the following example of Jacques' humor (a humor that, in contrast with some professional jokers, complimented people instead of insulting them or making fun of them). With this new class, Jacques confessed with a little bit of drama "my friend Richard disproved my old conjecture, and so he proved me wrong," and then he continued along the line "but luckily I have two other friends, Bernhard and Hendrik, because they saved my conjecture by showing that the new quadrangles fit into the broad picture of generalized Galois descent, so all Moufang quadrangles are of algebraic origin after all."

At all conferences that I organized and invited Jacques, I had the pleasure of accompanying him to lunch and dinner, and even of inviting him to my home. These were always very joyful experiences for me.

The conference celebrating his 75th birthday was the last one in which I saw him participate. When I took him to the train station, he immediately asked for a wheelchair, and he apologized to me saying "It must be awful to see a friend be discharged in a wheelchair like that, but do not worry, I am getting used to it." His Parkinson's had become worse (during one of the conferences that he organized on Algebraic Groups in Oberwolfach, he confided in me that this illness prevented him from riding a bicycle, which he would have loved to do). It was also the last time I saw Jacques in Ghent.

One of the greatest honours in my scientific career was to be a co-editor of Jacques' *Collected Works*. One of the highlights for me was the day that the four editors spent in Paris together with Jacques asking him all sorts of questions. Jean-Pierre Tignol produced a transcript of these interviews. We didn't use it for the *Collected Works*, but it is now an invaluable treasure to me. Jacques talked nineteen to the dozen about all kinds of aspects of his life and career. Near the end of the production process of the *Collected Works*, I was the one making contact with Jacques through Jean-Pierre Serre. I delivered two copies of the four books of his *Collected Works* to his apartment in Paris, on Thursday January 30, 2014. That day I had lunch with Serre at 13:30 and coffee with Tits and his wife Marie-Jeanne at 16:00. Marie-Jeanne told an interesting story. She said that when historians meet (she was an historian) and discuss scientific matters, at the end of the day when they separate they each have their own original ideas and beliefs. When mathematicians meet and started discussing various matters, at the end of the day they all agree, no matter what their original belief was.

Marie-Jeanne and Jacques were together at many conferences. She accompanied Jacques as frequently as possible. When I had an appointment with Jacques after a lecture at the Collège, I noticed that he always first called

Figure 15. Jef Thas, Arjeh Cohen, Dan Hughes, Francis Buekenhout, Jacques Tits, Ernie Shult, and Antonio Pasini at a conference in honor of Francis Buekenhout in Brussels, November, 2003.

Figure 16. Jacques Tits, Bertrand Rémy, Gopal Prasad, Bernhard Mühlherr, and Jean-Pierre Serre at the meeting in honor of Jacques' 75th birthday in Ghent, 2005.

Marie-Jeanne just to say his lecture went well and ask how she was doing. She also took great care of Jacques when his illness became worse. Sadly, Jacques' life companion passed away too soon, on Tuesday February 2, 2016.

On Tuesday April 23, 2019, Bernhard Mühlherr and I presented Jacques with a hard copy of the *Complement to the Collected Works of Jacques Tits* [6]. This was the last time that I saw him.

A few months before Jacques died, there was an initiative among my department to compose a booklet containing trivia about the math professors. One of the items was what is considered their greatest scientific achievement. You could read great theorems there, proofs, prizes, and other concrete accomplishments in that rubric. On my page, it just mentioned my friendship with Jacques Tits.

Thank you, Jacques, for your beautiful mathematics, for your beautiful personality, and for your beautiful friendship. An architect died, but what he built will live on.

Bernhard Mühlherr

My first encounter with Jacques Tits was in January, 1989. Tits was giving a course on twin buildings at the Collège de France. I was in Brussels working on buildings for my Diplom thesis and my advisor, Francis Buekenhout, recommended that I attend Tits' course. I was surprised that I was able to understand so much of his lectures despite my rudimentary knowledge about buildings. Only much later, did it become clear to me that Tits possessed an extraordinary talent for describing the central ideas of his mathematics on a very concrete level. Since Buekenhout had let Tits know that I would be attending his course, Tits offered that we could meet for an hour after one of his lectures so

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that I could ask him questions about buildings. I looked forward to this hour, but I was anxious that I didn't understand enough and the meeting would just waste his time. It turned out that my fears were completely unfounded. Tits listened carefully to the ideas I was working on, made valuable suggestions and encouraged me to continue with my project.

Looking back, it is clear that those lectures influenced me more than anything else in my mathematical training. Around this time, Tits formulated several open questions about twin buildings and in the following years I made a number of contributions to their solution. Throughout this time, we stayed in regular contact. Whenever I had some progress to report, it sufficed for him that I would give the general idea; we never talked about the details. For me, these discussions were principally a kind of mathematical compass. At one point they were decisive in suggesting that I should pursue a vague idea I'd described. This idea brought a breakthrough in the classification of twin buildings. Tits invited me to give a lecture about my results at the Collège de France. This invitation and the fact that on another occasion Tits referred to me as his student, are among the greatest honors of my research career.

I remember well a number of meetings with Tits. Once, when I was in Paris with Hendrik Van Maldeghem for one of Tits' lectures, the three of us met in Tits' office, where there was a table piled high with manuscripts and preprints. He told us that these were all the things that he still needed to work his way through and we'd better not steal anything! This was just one of the many small moments when I got to enjoy Tits' very special sense of humor. Tits was always interested in languages. We generally spoke in German to each other and he would say something if I used a construction that wasn't familiar to him. Because of my South German origins, this occurred

fairly often and he liked to joke in these situations that my French was better than my German.

Jacques Tits' mathematics opened exciting perspectives to mathematicians of the next generation. He enjoyed seeing others work with his ideas and he was generous with his support and encouragement. I remember well a conversation he had with my first PhD student Pierre-Emmanuel Caprace at the conference in Ghent in honor of Tits' 75th birthday. Caprace had given a talk about his thesis in which he combined and modified many of Tits' ideas to solve the isomorphism problem for Kac–Moody groups. Tits asked questions about the details, but as usual only a few general remarks were all he needed to appreciate what Caprace had accomplished. Tits' pleasure at seeing his mathematical ideas woven together in new ways and bearing valuable fruit was particularly clear that day.

Figure 17. Arjeh Cohen, Marie-Jean Tits, and Jacques Tits on a boat during the conference on buildings and diagram geometries by Lake Como, 1984.

Richard Weiss

I first "met" Jacques Tits in the late 70s in the math library of the Free University of Berlin, where I was a post-doc. Killing time looking through recent journals on display, I came across a paper entitled "Non-existence de certains polygones généralisés, Part I" in the latest issue of *Inventiones*, where Tits was editor. In this paper, Tits began the proof that Moufang n-gons exist only for $n = 3$, 4, 6, and 8. I didn't know what a Moufang polygon was, but I had been working on generalizations of a theorem of William Tutte that says that finite trivalent graphs whose automorphism group acts transitively on paths of length s but not on paths of greater length exist only for $s = 4$, 5, and 7. I knew that special attention was needed to rule out the case $s = 9$. The coincidence in these numbers was striking and within hours I understood how to prove a more general version of Tits' result by combining a lemma in his paper with results that I had in my drawer. Tits reacted to news of my result with charm and generosity. Not to leave things hanging, he wrote a much shorter version of his Part II using ideas from my paper and the two papers appeared quickly back to back.

In 1992, I was spending a couple of months of a sabbatical visiting Hendrik Van Maldeghem in Ghent. One afternoon Hendrik stuck his head in my office and said he was driving to Paris the next morning to hear a lecture of Jacques Tits about Moufang polygons and would I like to come. In fact, the subject was the theorem that $n = 3$, 4, 6, or 8. This was before Thalys, and Paris was far away. We were on the road at 5:00 AM, merged into the daily traffic jam on the Périférique just as the sun was rising, and then drove through the city, arriving at the lecture room just on time for the 9:00 AM lecture. Tits came in and started to write on the board, but when he turned around and

noticed me, he made a startled expression and said "Oh, this is like lecturing on the Riemann hypothesis and discovering that Riemann is in the audience." Tits always knew how to be witty and generous at the same time. Hendrik and I went down every Tuesday for the remaining lectures and on the last day, Tits invited the two of us to a merry and lavish lunch in a nearby restaurant.

In 1993 Dina Ghinelli invited me to hold a series of lectures on Tits' work on Moufang polygons in Rome. For the last part of his course, I worked through his unpublished notes on the Moufang quadrangles that he called "indifferent." At that time, Tits had classified Moufang triangles and octagons and announced the classification of Moufang hexagons, but this unpublished manuscript was all that he'd done with Moufang quadrangles apart from describing examples coming from groups of type E_6, E_7, and E_8 in lectures at the Collège de France. Once I thought I'd really understood Tits' proof in the indifferent case, I grew ambitious and wanted to go farther. After much hesitation, I wrote a letter to Tits proposing that we collaborate to finish the classification and write the whole thing up as a book. I was proposing coming in on a project in which he'd invested years of effort and was quite certain that my offer would be rebuffed. In fact, months went by with no reply. Tits was at Yale for the semester visiting his old friend Nathan Jacobson. Still no reply. I'd mentioned my letter to Diego Benardete who was at Trinity College at that time. Later, I learned that at a tea after a colloquium talk at Yale, Diego marched up to Tits with the words "Professor Tits, you're keeping Weiss waiting!" This did the trick. Days later, shortly before his return to Paris in December, Tits called me at home in Boston to say he agreed to work together. "But I'm very busy with many other projects," he warned, "and it might take us five years!" He was wrong. In the end it took seven.

These were a thrilling seven years. Our collaboration consisted mostly of written exchanges. Laptops were not yet common and Tits never used anything but a fax machine for his communications. Our first goal was to complete the classification of Moufang quadrangles. Pushing the ideas in Tits' indifferent paper, we arrived at the situation where the exceptional Moufang quadrangles should turn up. In this case, we had to invent and classify structures that we later called "quadrangular algebras." When the classification was essentially complete, I noticed a mistake in a lemma asserting the existence of an element of order 4 in one of the root groups when the characteristic is 2. Each repair to the proof fell apart. It turned out that there was, in fact, a new family of Moufang quadrangles whose root groups were all abelian. Tits was thrilled. Within a week of hearing about them, Bernhard Mühlherr and Hendrik Van Maldeghem showed that these new quadrangles filled in a gap in Tits' picture. They arise by descent from a group of type F_4, but not one associated with an absolutely simple algebraic group, rather from a split pseudo-reductive group of type F_4 defined over a purely inseparable field extension.

I think that Tits was particularly pleased with these developments because they confirmed his well-known attitude about the importance of characteristic 2. Characteristic 2 was, as a rule, historically excluded in the study of quadratic forms, Jordan algebras, and composition algebras. This offended Tits' understanding of the geometric nature of these things. Here is what the authors of the *The Book of Involutions* wrote in their introduction: "Not only was Jacques Tits a constant source of inspiration through his work, but he also had a direct personal influence, notably through his threat—early in the inception of our project— to speak evil of our work if it did not include the characteristic 2 case."

Once or twice a year I was able to spend a month in Paris and often had the use of a small windowless storage room in the Collège de France Annexe as my office. Tits' office was down the hall, but as he'd warned me at the start, he was a very busy man and our meetings were always by appointment. Tits knew my limitations as a mathematician. I think, though, that he had respect for my persistence and trusted me to get around the technical problems that arose on my own. But his guidance about what *ought* to be true was the real driving force behind the project. Tits often joked about seeing things in his crystal ball, but what he was really referring to was his uncanny ability to see a whole world hidden in a Dynkin diagram.

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In the bibliography we have included all the works of Jacques Tits as well as a few other books alluded to in this article. Volume I of [8] also includes a Curriculum Vitae and surveys of Tits' work written during his lifetime, including one by Tits himself.

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[10] Hendrik Van Maldeghem, *Generalized polygons*, Modern Birkhäuser Classics, Birkhäuser/Springer Basel AG, Basel, 1998. [2011 reprint of the 1998 original] [MR1725957]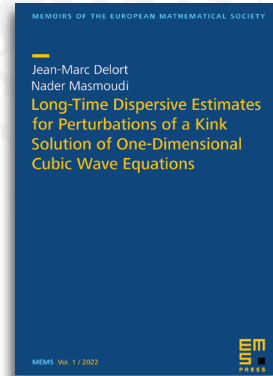, DOI 10.1007/978-3-0348-0271-0. MR3014920