MEMORIAL TRIBUTE

Jean-Pierre Demailly (1957–2022)

Junyan Cao, Ya Deng, and Jian Xiao

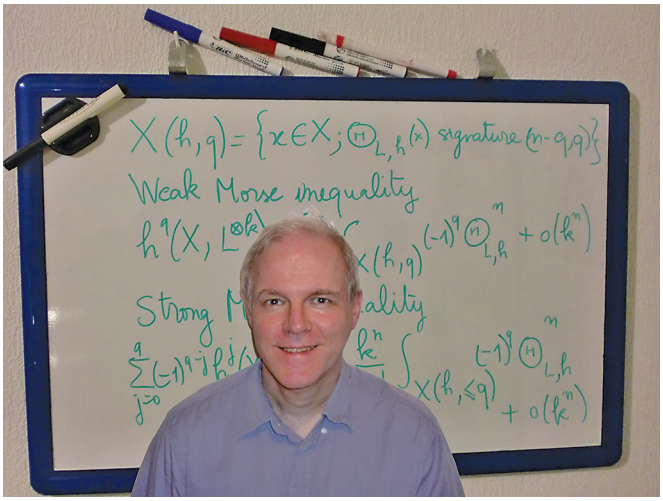

Figure 1. Jean-Pierre Demailly with his favorite work—holomorphic Morse inequalities in 2010.

Professor Jean-Pierre Demailly passed away on March 17, 2022. His most important mathematical contributions concern analysis and complex geometry. He was particularly innovative in his ability to develop powerful analytic tools and apply them to deep problems in algebraic geometry. This memorial article is a tribute to him, with contributions from many of his former students and colleagues.

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Demailly was born on September 25, 1957 in Péronne, France. He attended the Lycée de Péronne from 1966 to 1973 and the Lycée Faidherbe from 1973 to 1975. He entered the École Normale Supérieure in 1975, where he received his agrégation in 1977 and graduated in 1979. During this time, he received an undergraduate licence degree from Université de Paris VII in 1976 and Thèse de 3ème Cycle (within three months) under the supervision of Henri Skoda at the Pierre and Marie Curie University in 1979. He received his Doctorat d'État in 1982 under the direction of Skoda at the Université de Paris VI, with thesis "Sur différents aspects de la positivité en analyse complexe."

In 1979, Demailly was nominated to be a chargé de recherche of CNRS at the Université de Paris VI. Then in 1983, he became a professor at the Université Joseph Fourier (now called Université Grenoble Alpes) at the age of 26 (the youngest ever professor of mathematics in France) until his retirement in 2020.

He served as the editor-in-chief of the *Annales de l'Institut Fourier* from 1998 to 2006 and the editor-in-chief of *Comptes Rendus Mathématique* from 2010 to 2015. He was also an editor for *Inventiones Mathematicae, Crelle, Journal of Geometric Analysis*, and *Journal de Mathématiques Pures et Appliquées*.

As a junior and senior member of the Institut Universitaire de France (IUF) and as former Director of the Institut Fourier, Jean-Pierre Demailly received many distinctions and prizes during a particularly brilliant international career. Demailly received the CNRS Bronze Medal in 1981, the Prix Mergier-Bourdeix from the French Academy of Sciences in 1994, the Humboldt Prize in 1996, the Simion Stoilow Prize from the Romanian Academy of Sciences in 2006, the Stefan Bergman Prize from the American Mathematical Society in 2015, and the Heinz Hopf Prize from ETH in 2021. He was elected a correspondent of the French Academy of Sciences in 1994 and then became a

permanent member in 2007. He was an invited speaker at the International Congress of Mathematicians in 1994 and a plenary speaker in 2006.

Demailly's mathematical works concerned complex geometry in a very wide sense, using techniques from complex analysis with applications to algebraic geometry and number theory. There is probably a general consensus that they should include:

- His work on holomorphic Morse inequalities [Dem85] is a fundamental result in complex geometry. This is a refinement of the Riemann–Roch formula. Among its applications, it was applied to give an alternating proof of Siu's solution of Grauert–Riemenschneider conjecture, and played important roles in the study of hyperbolicity of algebraic varieties (Green–Griffiths–Lang and Kobayashi conjectures), and the characterization of uniruled varieties.
- His theorem about the regularization of closed positive currents [Dem92] is an important tool in complex analysis and complex geometry. For example, his regularization theorem allows the L^2-estimate to be used in a very general setting. It has tremendous applications in complex geometry, for example, his groundbreaking work on Fujita's very ampleness conjecture [Dem93].
- In his joint works with F. Campana, T. Peternell, and M. Schneider, they studied the structure of compact Kähler manifolds with semipositive curvatures by using the analytic methods systematically. It built a bridge between the algebraic geometry and analytic geometry.
- His paper on the hyperbolicity of algebraic varieties [Dem97] launched a vast program of study of the Kobayashi and Green–Griffiths–Lang conjectures (algebraic degeneracy of entire curves). He introduced the techniques of invariant jet differentials, which can be seen as algebraic differential equations on algebraic varieties such that entire curves are solution of these equations, refining previous work by Green and Griffiths. These techniques have a far-reaching impact in this subject, and are an essential building block for the recent proof of the Kobayashi conjecture.
- His joint work with M. Păun [DP04] on the numerical characterization of the Kähler cone of a compact Kähler manifold gave a beautiful resolution of a basic problem in Kähler geometry. This theorem generalizes the classical Nakai-Moishezon criterion of ampleness to a numerical characterization of Kählerness of a compact Kähler manifold. The ideas used by the authors had been further developed by others to solve conjectures on the relations between solvability of certain geometric PDEs (e.g., J-equations, deformed Hermitian-Yang-Mills equations...) and algebro-geometrical stabilities.

He was also among the pioneers of free circulation of knowledge. His online free book *Complex analytic and differential geometry*, a reference still today, has an important impact on complex geometry, and it is highly recommended for young researchers. He was one of the founders of Episciences, a platform providing the scientific guarantee of an editorial committee to articles published in open archives.

His personality and his passion for mathematics have left their mark on generations of students. He contributed to the development of mathematics in Tunisia, India, and China through regular visits. His articles, his lectures, and his public presentations were always distinguished by their clarity. He supervised about twenty doctoral students, most of whom are recognized mathematicians today.

Demailly was much appreciated by his colleagues, students, and friends. He was not only an excellent mathematician, but also a kind, generous and modest person. We all sorely miss him.

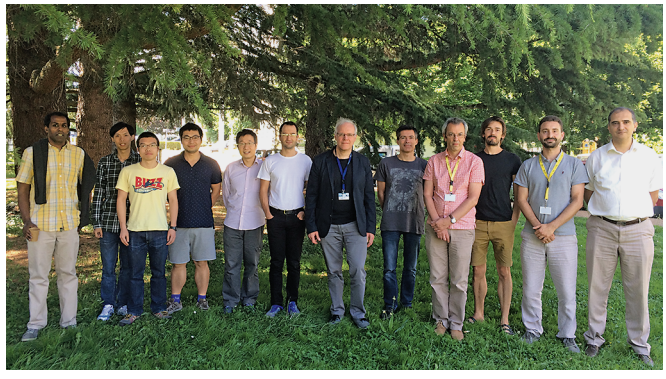

Figure 2. Demailly with many of his former students at his 60th birthday conference in Grenoble in 2017.

Sébastien Boucksom

My relationship with Jean-Pierre somehow began with a false start. In spring 1997, he was delivering a course on one-variable complex analysis to freshmen at the Ecole Normale Supérieure de Lyon which I was supposed to take. But a misplaced sense of self-confidence led me to skip classes, candidly thinking that I already knew enough of

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the subject, and it was only after hearing my better-advised friends' praises of his remarkable teaching—and after getting a less-than-great grade on the exam—that I realized what I had missed.

A year later, I was developing a keen interest in complex analytic and algebraic geometry, and I had the good fortune to cross paths again with Jean-Pierre through the advice of Etienne Ghys. I remember the first time I nervously climbed the stairs of the Institut Fourier in Grenoble to office 314 (what else?) which he then occupied, and being handed a copy of his beautiful "Panoramas et Synthèses" lecture notes on Hodge theory. I devotedly studied it and still enthusiastically recommend it to all of the master's students I advise. Another favorite of mine is of course his famous "open content" book *Complex Analytic and Differential Geometry*. Jean-Pierre's lectures and writings had a distinctive quality owing to a perfect balance between elegance and clarity. It is heartening to think that future generations of mathematicians will still be able to enjoy the treasure of lecture notes he has left us.

Jean-Pierre was also a fantastic advisor—patient, supportive, and always available despite the many parallel activities he was already engaged in as a Linux programmer and an activist reformer of elementary mathematics education. To add to the father figure effect, he had also grown up in the north of France, like me, and even bore the same first name as my dad. I would always leave Jean-Pierre's office with renewed enthusiasm and broadened perspectives on the problems I was working on. I particularly enjoyed watching him getting fully absorbed for sometimes worryingly long minutes by some technical question I might have, typically about currents, before he would come up with either the reassuring declaration that he did not see any obstruction or with a nasty counterexample!

During my PhD years with Jean-Pierre, I was fortunate enough to witness the birth of the celebrated Demailly–Paun theorem, providing a numerical characterization of nef classes on compact Kähler manifolds, whose technique of proof turned out to be just what I needed to study the volume of big classes. I would also regularly listen to Jean-Pierre's accounts of his joint progress with Peternell toward a similar characterization of pseudoeffective classes via movable curves. Then in the summer of 2002, to my great surprise and admiration, the problem was suddenly solved through a particularly tricky use of his beautiful holomorphic Morse inequalities.

After I left Grenoble, our interactions naturally became less frequent, but each meeting was a new occasion to be captivated by Jean-Pierre enthusiastically telling me about his latest bold idea to tackle a major math problem (the 'transcendental' Morse inequalities being one favorite theme), but also about his deepening interest in Thorium-based nuclear reactors or his long-time passion for table tennis, to name just a few. Jean-Pierre's capacity to maintain such an intensive commitment in so many different things while at the same time producing top-level mathematics has always been a source of amazement and inspiration for me, and for most people around him.

Jean-Pierre was an extraordinary mathematician who transformed the field of complex geometry, and whose deep insights and technical strength were only matched by his great kindness and accessibility. He left us much too early, and will be missed very much.

Figure 3. Claire Voisin, Sébastien Boucksom, Elisha Falbel, Xiaonan Ma, Jean-Pierre Demailly, Thomas Peternell, Andreas Höring, and Junyan Cao at the banquet of Cao's Habilitation à diriger des recherches.

Christine Laurent-Thiébaut

I first met Jean-Pierre during the academic year 1976–77. He was still a student at the Ecole Normale Supérieure in Paris and we both attended the graduate course on L^2-theory in complex analysis given by Henri Skoda, who had just started a professorship in Paris. Jean-Pierre appeared to be a very bright student. One year later, when he started his PhD under the supervision of Henri Skoda, we would meet every Tuesday at the Lelong seminar. Jean-Pierre was a very kind guy and we became friends. My mathematical interests were more related to integral formulas and their applications to the Cauchy–Riemann equation, while at that time Jean-Pierre's interests were in L^2-theory and vector bundles. When I had a problem which could be solved using vector bundles, I turned to him and I discovered his remarkable talent as a teacher. His explanations were incredibly clear and precise. Our conversations led us to write a joint paper. In 1983, he left Paris to take up a professorship in Grenoble, but we met regularly during conferences on Complex analysis. Some years later, I also moved to Grenoble where it was a great pleasure to join Jean-Pierre. We spent more than 30 years as colleagues at the Institut Fourier.

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Jean-Pierre was much appreciated both by the students as an exceptionally good teacher and by the members of the institute. He was always ready to answer any mathematical question. For him the transmission of knowledge was extremely important. So in 1992, he created the Summer School of Grenoble which welcomes between 50 and 100 students from all over the world. It is always organized by a member of the Institut Fourier and is on a different topic each year. Despite his intensive research activity, he also regularly took part in the administrative side of things. Jean-Pierre was director of the Institut Fourier for four years from the beginning of 2003.

Outside of our work relationship, let me recall two good memories. I remember a table tennis game in Oberwolfach where I lost miserably to Jean-Pierre, who was a very good player. During the conferences in Trento, he introduced table tennis as an alternative to the traditional soccer matches of Italy agains the rest of the world, with the hope that the rest of the world would win more frequently. In Spring 2015, we both spent some time at Beijing University. During the 1st of May holiday, we decided to go hiking and camping on the Great Wall. It was really a very nice moment. I will miss him for his kindness and his great humanity. He was a great researcher, a great teacher, and an outstanding colleague.

Robert Lazarsfeld

Jean-Pierre and I shared an interest in questions involving positivity, so it was natural that we were in semi-regular contact for much of our mathematical careers. But his influence on my own work was particularly decisive for about fifteen years starting from the early 1990s. In 1993, Demailly published a seminal paper in the *Journal of Differential Geometry* on Fujita's conjecture, in which he demonstrated the power of multiplier ideals for studying linear series in higher dimensions. This paper stunned the algebro-geometric community, and it inaugurated a period during which, largely under the leadership of Demailly and Siu, complex analytic methods made fundamental contributions to higher-dimensional algebraic geometry.

Together with Ein and others, I got interested in trying to understand and apply some of the techniques involving multiplier ideals to other questions in commutative algebra and algebraic geometry. It turned out that they had already been used in passing by Esnault and Viehweg, but still it was great fun to talk with Demailly in an attempt to translate aspects of his viewpoint into more traditional algebro-geometric language. He made a particularly fruitful visit to Ann Arbor in March 1999, which planted the seeds for the one joint paper we wrote (with Ein), on the subadditivity property of multiplier ideals.

In addition to his raw mathematical power and creativity, Jean-Pierre was uniquely skilled in explaining to algebraic geometers the intuition behind his analytic tools. In the JDG paper mentioned above, for instance, Demailly introduced a non-closed current that didn't have a simple cohomological interpretation. However he suggested that it should be understood as measuring asymptotically the excess self-intersection of a system of divisors. This hint eventually grew into an appealing story about asymptotic invariants of linear series.

I was always very impressed by the range of Demailly's interests and passions. He played table tennis at a very serious level, and he showed me with pride his membership card in the relevant professional society. Jean-Pierre was also fond of sharing horror stories about the state of mathematical education in French schools, a topic to which he devoted considerable thought and energy. Well before the advent of the Stacks Project, he was a fervent believer that the future of scientific communication lay in the direction of open source.

Demailly was an inspiring and polished lecturer, and I was fortunate on a few occasions to join him at schools aimed at graduate students. One of the last times I was with him for an extended period was in 2014, when Jean-Pierre, Steve Zelditch, and I gave courses in Milan on the topic of asymptotic methods in complex and algebraic geometry. The conference photo appears below; Demailly is at the far right.

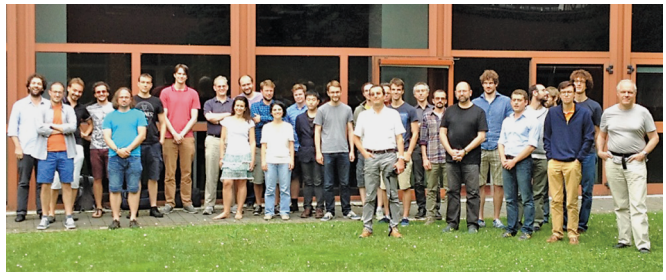

Figure 4. A conference on asymptotic methods in complex and algebraic geometry in Milan in 2014.

Jean-Pierre was an amazing mathematician and an inspiring colleague. I will miss him greatly.

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Takeo Ohsawa

As far as I remember, it was from Grauert that I first heard of Demailly. It was in September of 1980. In a train named Otto-Hahn which we took to go back to Göttingen from a conference at Oberwolfach, he made an impressive comment on a discovery of Demailly and Skoda which says that $E \otimes \det E$ is Nakano positive if E is Griffiths positive. Its immediate consequence is the vanishing of $H^q(X, K_X \otimes E \otimes \det E)$ $(q \geq 1)$ for any Griffiths positive bundle E over a compact complex manifold X. At that time, I understood this vanishing as a corollary of Kodaira's vanishing on the projectivization of the dual of E, as was stated in a paper of S.Kobayashi and T.Ochiai. So I made a remark that the result had already been known. Grauert told me that the vanishing itself is trivial, but the positivity is much deeper. Grauert was right, as one can see from the development of the studies on the positivity properties of various sheaves in recent years, for instance. Next year, when I was staying in Wuppertal, Diederich gave me a preprint of Demailly on the nonexistence of nonconstant bounded holomorphic functions on certain coverings of the complex plane with punctures. This paper gave me an impression that the author had a very strong sense on complex analytic objects. It reminded me of the feeling I got from Oka's first paper.

In 1981, I mainly worked on a generalization of Kodaira's vanishing theorem inspired by Fujita's semipositivity theorem for the direct image of the relative canonical bundle by a surjective morphism from a Kähler manifold to a curve. For that purpose, I formulated a vanishing theorem with an L^2 estimate. The paper was completed and submitted in 1981, but it was not published until 1984. Soon after its publication, I received a letter from Demailly saying that the main result of the paper was contained in his thesis, which had been published in 1982. So, an L^2 vanishing theorem on complete Kähler manifolds, which I had noticed in 1978 but only partially wrote down the proof of in a paper published in 1980, was completely hidden under the celebrated L^2 vanishing theorem of Demailly, which was perfectly formulated and proved in a much more comprehensive style than mine. Afterwards, the generalized Kodaira's vanishing for a proper morphism from Kähler manifolds in the same paper of 1984 became also obscure because of Kollár's torsion freeness theorem for the direct images of relative canonical bundles by projective morphisms, which was later generalized by Takegoshi to Kähler morhisms.

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In 1986, there was a big conference in Wuppertal organized by Diederich. Demailly was there. I was also there with Kensho Takegoshi. Demailly presented a characterization of affine manifolds and I talked about a joint paper with Takegoshi on the extension of L^2 holomorphic functions. Takegoshi looked very happy when he was handed a reprint from Demailly. In a bus to the banquet, Demailly told me some updates on hyperconvex domains, containing the hyperconvexity of bounded pseudoconvex domains in $\mathbb{C}^n$ with Lipschitz continuous boundary. Some of my later works on hyperconvex domains have been inspired by this conversation. For instance, I found a locally pseudoconvex domain in $\mathbb{CP}^n$ with Hölder continuous boundary which is locally hyperconvex but globally not, which I wrote in a joint paper with Diederich published in 1998. It was in 2017 that Harrington succeeded in showing the hyperconvexity of Lipschitz pseudoconvex domains in $\mathbb{CP}^n$.

In 1990, the ICM took place in Kyoto. I was supposed to help the local organizers of the meeting. For instance, I was informally asked to organize the music program for the reception, from which I could manage to escape. This gave me the chance to look at the provisional list of invited speakers. Demailly's name was there. However, shortly after that I was asked to give an invited talk instead of him because Demailly somehow declined the invitation. I gave a talk on the L^2 extension and an application of the L^2 estimate to the topology of isolated singularities. It might be worthwhile to note here that a trick in our proof of the L^2 extension theorem was inspired by Demailly's paper on the complex Morse ineuqalities, in the sense that it showed how to obtain complex analytic information from deformed families of Laplacians. In fact, the decisive step of our proof of the L^2 extension was to derive an L^2 estimate for the $\bar{\partial}$ operator from a "twisted" Nakano identity for a kind of deformed Laplacian.

In 1992 in Kyoto, Steven Lu told me that Demailly found a remarkable application of the L^2 extension theorem. It was on the approximation of plurisubharmonic functions by the averaged logarithms of the weighted Bergman kernels. The L^2 extension was with weight factors only because the proof went in the same way as in the non-weighted case. Significance of the independence on the weights of the constant appearing in the estimate for the extended function was first revealed by Demailly's approximation theorem. It was quite unexpected (to me at least) because the motivation for our paper was to obtain a very modest estimate for the Bergman kernel from below. For such an application, the extension without the weight factor was enough.

Anyway, Demailly's approximation theorem refreshed my interest in the L^2 extension. In 1993, while formulating

an extension theorem which extends certain non-L^2 functions to L^2 ones, I noticed that an extension theorem of such a type partially solves a question on the estimate for the Bergman kernel from below by the logarithmic capacity, which had been asked by N. Suita in 1972. Although it was far from the expected estimate, the result caught the attention of people in Krakow including W. Zwonek and Z. Błocki. The accurate estimate was first proved by Błocki's paper in 2013 and a related conjecture was completely solved by Q. Guan and X.-Y. Zhou in 2015.

Demailly was also interested in extending the L^2 extension theory. He gave three survey lectures on it in an Italian workshop "CIRM-ICTP Complex Analysis and Geometry - XXV" (June 7–11, 2021). While watching the conference in Nagoya on my PC, I had the great surprise of hearing him say that "the fundamental results are all due to Ohsawa." I almost felt that my life could end happily at that moment. When I saw the mail informing me of his death, I cried "NO!" from the bottom of my heart.

Mihai Păun

I feel honoured and at the same time deeply saddened to write this in commemoration of the death of Jean-Pierre Demailly in March 2022. For everyone who met him, his premature passing was a terrible shock. He was an extraordinarily gifted and generous mathematician and had such endearing qualities that he would have well-deserved the nickname "pater seraficus" of complex geometry.

Before I met Jean-Pierre for first time in September 1993, I was warned about his enormous reputation by Paltin Ionescu, with whom I took a course in algebraic geometry at the University of Bucharest. I was about to join ENS Lyon (as "pensionnaire roumain de l'ENS"), when Paltin suggested that I should go to Demailly for a PhD problem at the Institut Fourier in Grenoble instead of Lyon. I still remember him saying "the exact sequences you saw in my lectures will be replaced by metrics... but other than that, his recent contributions to algebraic geometry are fantastic, he is very active, at the height of his career." I followed his advice, and in the fall of 1993 I joined Jean-Pierre's impressive team of students.

Institut Fourier seemed like the place to be, primarily because of its intense mathematical activities. This was attributed to the presence of leading mathematicians in various fields (complex and algebraic geometry, Riemannian and spectral geometry...) and their bright students. I was instantly charmed by the advanced course in L^2 theory that Jean-Pierre gave that year: the results presented were beautiful and he explained them in a very elegant, convincing manner. The impression I had in assisting at his lectures was that not only did he completely master the subject, but he also enjoyed sharing his vast knowledge with the audience. In his expert hands, everything turned so crystal clear, and all the epsilons were put in the right place!

During the following years, I studied his monumental book "Complex analytic and differential geometry" and afterwards his work in collaboration with T. Peternell and M.Schneider (1993–1994), since part of the problems Jean-Pierre assigned me were discussed in these articles (about the properties of compact Kähler manifolds with nef tangent or anticanonical bundle). They are written in a very clear manner, but because of my limited mathematical understanding as student, it took Jean-Pierre an infinite amount of time and patience to help me navigate through the subject. He was an outstanding PhD advisor, very inspired and generous with the subjects he proposed to his students. Despite his numerous interests and involvements beyond complex-geometry activities, he was always available for his students, especially when they were at a mathematical dead end with their problems and in need of advice for new directions or approaches.

For the half decade that I was at the Institut Fourier, Jean-Pierre was mainly interested in problems surrounding Kobayashi hyperbolicity and the Fujita conjecture, and therefore, many of his seminars were centred on these themes. Seeing that I was doing my best to follow, he suggested reading his article *A numerical criteria for very ample line bundles* (1992) in which techniques from Monge–Ampère equations and closed positive currents were used to establish a statement in the framework of algebraic geometry. Even if the effective bounds obtained were far from Fujita's predictions, this paper is a pure masterpiece, and the techniques developed by Jean-Pierre turned out to be fundamental to the study of the positivity properties of real $(1,1)$-classes on Kähler manifolds.

After graduating, I was not able to enjoy the pleasure of meeting Jean-Pierre in person as often as before. Nevertheless, we exchanged emails regularly, and every now and then I was able to see him at conferences or on other occasions. Life went on and things changed, and still one thing remained unchanged: it was always very inspiring to discuss mathematics with him. As years passed, many of the problems that caught Jean-Pierre's attention were the same, and the ideas and enthusiasm with which he was trying to solve them stayed equally unbounded as well, which can be seen in his contributions to the field of Kobayashi hyperbolicity. In his article, *Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials* (1997), Jean-Pierre offered a vigorous survey of the field and he also

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introduced the so-called invariant jet differentials as main tools in the study of families of discs in varieties of general type. He mentioned on many occasions that he was exploring several possibilities of constructing these objects. Then, the right idea came to him later: in the article "Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture" (2011) he established the existence of jet differentials on manifolds of general type as consequence of another of his prominent achievements, namely the holomorphic Morse inequalities (in combination with spectacular curvature computations).

Figure 5. Nessim Sibony, Jean-Pierre Demailly, Kazuko Matsumoto, and Mihai Păun in Kyoto in 2011.

Jean-Pierre Demailly was an outstanding researcher, who was highly regarded and esteemed by his peers and students for his mathematical work. Despite his respected standing in the mathematical society, he always maintained a down-to-earth, unpretentious demeanour, which instilled a level of modesty and *bon enfant* atmosphere among mathematicians. His departure came way too early, and it is an irreplaceable loss for complex geometry and for all of us who remember him for who he was.

Thomas Peternell

My first contact with Jean-Pierre was on the occasion of a conference on complex analysis in Trento, held in the beautiful Villa Madruzzo in June 1983—Jean-Pierre gave a lecture on positive closed currents. Apart from mathematics, I still remember at least one joint soccer match during one of these meetings. We were on the same team, a hint for the future. At that time—1983—he already held a professor position in Grenoble. One of his first visits to Bayreuth was in 1990 to speak at a conference "Complex Algebraic Varieties" which I organized with Klaus Hulek, Michael Schneider, and Frank-Olaf Schreyer. This was—shortly after the dramatic political changes in 1989—one of the first international meetings with participants from both the west and the east. Jean-Pierre lectured on *"A numerical criterion for very ample line bundles;"* one of his numerous breakthroughs in Complex Analysis and Algebraic Geometry and the starting point for all developments around the Fujita Conjecture. His contribution to the proceedings of this conference, appearing as Springer Lecture Notes 1507, is the extremely influential article "Singular metrics on positive line bundles." Not only did he introduce the important notion of a singular hermitian metric on a holomorphic line bundle which allows us to characterize pseudoeffective line bundles by the positivity of the curvature current of a singular metric, but he also invented the notion of *Seshadri constants* for nef line bundles, a measure for positivity, which have been studied in numerous research articles since then.

Around 1990, we started joint research projects, together with Michael Schneider. Our first paper studied compact Kähler manifolds X whose tangent bundle are *nef*, a geometric version of having a metric with semipositive holomorphic bisectional curvature. A priori, the nefness assumption is weaker than the metric assumption. We proved structure theorems which gave a rather precise picture of these manifolds up to a central conjecture which states a *Fano* manifold with nef tangent bundle must be rational-homogeneous. Despite many efforts and results in special cases, this conjecture is still very much open. At that time, our collaboration was sponsored by a French-German program, called PROCOPE, which financed collaborative visits. In the following decades, we studied compact Kähler manifolds with nef anticanonical bundles, followed by pseudoeffective line bundles, in particular in connection with the nonvanishing and the abundance conjecture, in the minimal model program, and line bundles with some partial positivity, as well as the geometry of special varieties.

Actually, despite a unified Europe, travel from Grenoble to Bayreuth or the other way around was and is not that easy and takes some time. Usually Jean-Pierre took a night train from Paris in a couchette, a compartment with six people. In revenge, we enjoyed the hospitality of Hotel Gloria or Hotel des Alpes during visits in Grenoble. Of course, Grenoble has the advantage of being a real city,

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being located in a beautiful landscape, and last but not least, not being too far from Burgundy.

Of course, we met at many conferences. A very sad occasion was the Bayreuth memorial conference in 1998 for our friend Michael Schneider. Here Jean-Pierre lectured on "*Almost complex projective embeddings of compact real symplectic manifolds*" a very broad and general subject outside Kähler geometry.

Jean-Pierre had special connections to Göttingen (my Alma Mater), equivalently to Hans Grauert. Shortly after the solution by Yum-Tong Siu, he showed that the Grauert–Riemenschneider conjecture is a consequence of his famous holomorphic Morse inequalities which he developed in the late 80s. He further received the Dannie Heinemann Prize from the Academy of Science in Göttingen in 1991. Needless to say, he was one of the main speakers in the conference in honour of Grauert's 70th birthday, held in Göttingen in 2000, with a lecture on "*Subadditivity of multiplier ideal sheaves and asymptotic Zariski decomposition*." He also contributed an extremely elegant article "*On the Frobenius integrability of certain holomorphic p-forms*," which had its origin—and accomplishment—during the conference. I was studying projective contact manifolds X with second Betti number at least two jointly with Stefan Kebekus, Andrew Sommese, and Jaroslaw Wiśniewski. In order to prove our structure theorem for these manifolds we needed the fact that the canonical bundle of X is not nef. So, we asked Jean-Pierre and he solved the problem (in a much more general setting and establishing an integrability theorem) during the conference—without even missing talks, I guess.

The Complex Analysis meetings in Oberwolfach—the institute located more or less in the middle between Grenoble and Bayreuth—in late August every other year have a great tradition going back to the early 50s. For a long period, they were conducted by Grauert–Remmert–Stein and later Grauert–Remmert–Barth. It was absolutely natural that Jean-Pierre followed as an organizer, and I had the pleasure to work with him and Klaus Hulek in this function from 1996 to 2012.

The mathematical influence of Jean-Pierre cannot be overestimated. He introduced important techniques such as his regularization of positive closed current, his holomorphic Morse inequalities (as already mentioned), and enforced the study of multiplier ideal and the use of Monge–Ampère equations. His new methods in the study of hyperbolicity are indispensable.

The scientific and personal appreciation of Jean-Pierre was manifested in a conference in Grenoble in June 2017 in his honour, with a large number of outstanding mathematicians and an illustrious program.

Of course, these were just a few personal remarks and views. It would take many pages to give a detailed overview of the extraordinary mathematical work of Jean-Pierre, not to speak of his mathematical school with many extremely successful students and his numerous other activities: organizing science, publication of science (the Episcience project), school education in France, and nuclear energy problems.

The mathematical community will commemorate Jean-Pierre as a brilliant mathematician, but also as a person of great empathy and humanity. For me, he was a true friend.

Yum-Tong Siu

The passing of Jean-Pierre Demailly is a great loss to the field of several complex variables. The first time I heard about Jean-Pierre was from Henri Skoda who told me in the early 1980s that he had a wonderful student with great technical prowess who was full of innovative ideas.

Jean-Pierre made fundamental contributions in a very broad range of topics and injected new vitality into many areas. Personally, his work that impacted my own research most is his holomorphic Morse inequality in *Ann. Inst. Fourier* in 1985 and his numerical criterion for very ample line bundle published in *J. Diff. Geom.* in 1993, especially the latter.

His numerical criterion for very ample line bundles was the first step and opened up a new vista in the study of effective results for compact algebraic manifolds of general dimension. Thanks to his pioneering work, in the last couple of decades, effective results such as Fujita's conjecture have been a very fruitful active direction of research.

Kodaira's vanishing theorem for line bundles with positive curvature has been one of the most important tools in complex geometry. For a line bundle whose curvature is not positive, when there is still some information on the signs of its eigenvalues, the holomorphic Morse inequality can still perform the task of Kodaira's vanishing theorem to a certain extent. It produces asymptotic results concerning truncated alternate sums of cohomology groups of the power of the line bundle, as the power goes to infinity, in terms of the curvature integral on the set where the curvature has not more than a certain number of negative eigenvalues. It is a very powerful tool, applicable to a very general setting.

With his collaborators and students, Jean-Pierre had been implementing a very successful program of transporting techniques and methods from more restrictive

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known formulations to more general situations; for example, from compact algebraic manifolds to compact Kähler manifolds, from smooth forms to currents, from nonnegatively curved line bundles to nef line bundles, etc. Such transportation pushes known techniques and methods to their natural limits and provides a better understanding of why they work.

We will all miss sorely the pioneering ideas and inspiration of Jean-Pierre.

Figure 6. Jean-Pierre Demailly and Yum-Tong Siu in Hong Kong in 2003.

Henri Skoda

The first time I met Jean-Pierre Demailly was in October 1976, when I was first elected full professor at Paris VI University. He had been admitted to Ecole Normale Supérieure in 1975. By chance or by an act of fate, probably with the help of Michel Hervé, deputy director of Ecole Normale Supérieure and of Jean-Louis Verdier, director of mathematical studies, one of the first who attended my lessons for advanced studies was the young student J.-P. Demailly. I was teaching Pierre Lelong's and Lars Hörmander's methods. For, since 1970, spectacular results given by Enrico Bombieri on the algebraic values of a meromorphic map of several variables [Bom70], then by Yum Tong Siu on the analyticity of level sets associated with Lelong's numbers of a closed positive currents [Siu74] have highlighted the depth and the efficiency of these methods in the theory of holomorphic functions of several variables. At the beginning of 1977, J.-P. Demailly also attended Pierre Lelong's lessons about plurisubharmonic functions and closed positive currents. The benefit was great for him and had a deep influence on the focus of his research activities.

J.-P. Demailly immediately took up the torch. I had the great pleasure of supervising J.-P. Demailly's thesis. He was an exceptional, dedicated, efficient, deep-thinking doctoral student.

In 1953, Jean-Pierre Serre raised the question of whether a fiber bundle with Stein basis and Stein fiber is itself a Stein space one. That problem remained in the 70s, one of the important open questions in complex analytic geometry. In spite of many positive partial results, I built a counterexample in February 1977 [Sko72], with $\mathbb{C}^2$ fiber, the basis of which is a multiply connected, open subset of the plane. In his PhD on the growth of plurisubharmonic functions on a fiber bundle with Stein basis and $\mathbb{C}^n$ fiber and on a Riemann surface, which he defended in 1978, J.-P. Demailly [Dem78] considerably expanded the range of the counterexample by showing the same process also happened in the case of a fiber bundle, with fiber $\mathbb{C}^2$ and with a simply connected basis (disk, plane) or without a simply connected basis and with polynomial automorphisms acting on the fiber. These buildings used a plurisubharmonic functions convexity inequality due to P. Lelong in a decisive manner.

In his doctoral thesis (thèse d'Etat) on different aspects of positivity in complex analysis, which he defended in 1982, J.-P. Demailly extended and made more flexible Lelong's number concept. He studied the behavior of this number by the action of a proper analytic morphism and he drew new applications to the geometry of analytic sets and currents. Then and above all, he made the connection with other important problems of that time, as numerical vanishing theorems and especially Hodge conjecture which connects the cycles and the cohomology of a projective manifold.

He decisively observed the cone of closed positive currents on a projective manifold must have extremal elements different from analytic cycles for, if not, it would lead to a much too strong and clearly wrong version of Hodge conjecture. Then he explicitly built such positive closed extremal currents on $\mathbb{P}^n$ and $\mathbb{C}^n$ as limits of analytic cycles answering thus Pierre Lelong and Reese Harvey's important question [Dem82]. Then such currents naturally appeared in the iteration of polynomial endomorphisms of $\mathbb{C}^n$ or $\mathbb{P}^n$ and now play a very important role in this area of research. J.-P. Demailly has been a pioneer in this field. Furthermore, thanks to this building, we can more easily identify the specific difficulties connected to a potential solution of the Hodge conjecture by analytic methods.

J.-P. Demailly was immediately elected full professor at the University in Grenoble. He intensively developed his

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work through many articles which had also a great impact on other connected fields like algebraic geometry.

The proof of the Grauert–Riemenschneider conjecture, given by Y.T. Siu, in 1984, the new analytic proof of classical Morse inequalities given by Edward Witten, and the spectral theory techniques developed by Yves Colin de Verdière led J.-P. Demailly to obtain optimal holomorphic Morse inequalities for an arbitrary hermitian holomorphic line bundle on a compact manifold without any positivity assumption in 1985 [Dem85]. It is one of the most original and spectacular results of J.-P. Demailly.

At that time, two results deeply reinforced Hörmander's L^2 methods efficiency: in 1987, L^2 holomorphic extension Ohsawa-Takegoshi's Theorem [OT87], and in 1989, coherence Alan Michael Nadel's one [Nad89]. With numerical vanishing Kawamata–Vieweg's theorem, they had a strong impact on J.-P. Demailly's research in algebraic geometry. For instance his deep results on the Fujita conjecture about the ampleness of line bundles on a compact complex manifold [Dem96] are closely linked to the key problem of the classification of compact algebraic manifolds.

For more than thirty years, he supervised an impressive number of theses, not only due to the high level of his mathematics, but also because he was able to find so many new research topics accessible to students. Most of them have been elected associate professors, full professors, or researchers at our National Scientific Research Center.

The election of J.-P. Demailly in Grenoble leads to many memories about mathematicians deeply involved in Science. In 1945, Grenoble University was the first position of P. Lelong as a full professor. In 1968, B. Malgrange came to Paris and gave a talk on L. Hörmander's works, the interest of which immediately became decisive for me. In the same period, I met P. Lelong for the first time at the seminar he organized at the Henri Poincaré Institute. In October 1976, I shared the managing of the Complex Analysis Seminar jointly with P. Lelong. In 1977, J.-P. Demailly gave a talk in this seminar on his first results about J.-P. Serre's problem. In 1982, J.-P. Demailly was elected in Grenoble and could join B. Malgrange. There he founded the famous school we all well know and developed it for 35 years.

In 1992, Henri Cartan honored J.-P. Demailly by attending the lessons he gave at Ecole Normale Supérieure where he talked about Nadel's results on L^2 multiplicative ideal sheaves. He gave a clever synthesis of two important theories, Henri Cartan's coherent sheaves one and L. Hörmander's L^2 $\bar{\partial}$ operator one.

In December 2007, J.-P. Demailly was elected a Member of French Academy of Sciences.

J.-P. Demailly is for all of us a model of engagement in Science, University, and International Scientific Community. We have a great deal of admiration not only for his extensive and various mathematical work but also for his attachment to humanistic values, which is reflected through devotion to his teaching and all the vocations he has raised.

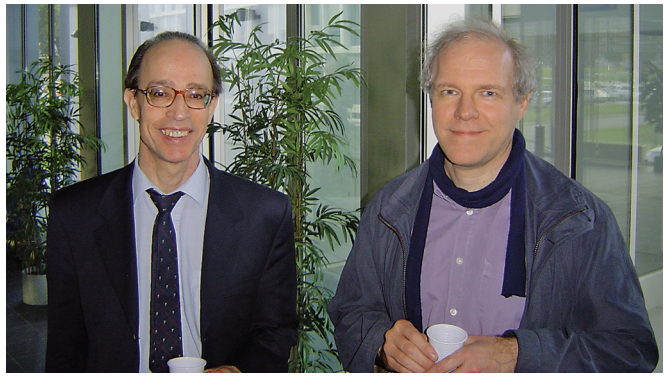

Figure 7. Henri Skoda and Jean-Pierre Demailly in Toulouse in 2006.

Valentino Tosatti

I first met Jean-Pierre Demailly at the workshop "Complex Monge–Ampère Equation" that took place at BIRS in Banff in October 2009:

Figure 8. Group photo of the workshop "Complex Monge–Ampère Equation" at BIRS in October 2009.

I had known Jean-Pierre's mathematics for some time, having spent considerable time during my PhD studying his monumental online book "Complex Analytic and Differential Geometry" and some of his papers. When I

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finally met him, I was very impressed by how kind, open, and friendly Jean-Pierre was in person. During that conference Jixiang Fu presented his recent work with Zhizhang Wang and Damin Wu [FWW10] where they introduced certain "form-type Calabi-Yau equations," reminiscent of the complex Monge–Ampère equation on Kähler manifolds, but where the unknown quantity is not a scalar function but an $(n-2,n-2)$-form (where $n \geq 3$ is the dimension of the manifold), and whose solvability in general remained open. In the break after the talk, Jean-Pierre was having animated discussions with Jixiang about possible applications of these equations to the problem of showing that deformation limits of compact Kähler manifolds are bimeromorphic to Kähler. I did not make much of this back then, but four years later after my friend Ben Weinkove discussed these topics again with Jean-Pierre at another meeting, we set out to work on this in earnest and eventually succeeded in solving the form-type Calabi–Yau equation [TW17]. Jean-Pierre's influence on our work was enormous.

I would like to mention another instance of this. One of my favorite results of Jean-Pierre's is his celebrated numerical characterization of the Kähler cone, proved together with Mihai Păun [DP04], which is a far-ranging generalization of the classical Nakai–Moishezon ampleness criterion. One way to state this is that if X is a compact Kähler manifold and $[\alpha] \in H^{1,1}(X,\mathbb{R})$ is a $(1,1)$-cohomology class which is a limit of cohomology classes of Kähler metrics (we say that $[\alpha]$ is *nef*) then $[\alpha]$ itself contains a Kähler metric if and only if the intersection numbers

$$\int_V [\alpha]^{\dim V}$$

are all strictly positive, as V ranges among all closed irreducible positive-dimensional analytic subvarieties $V \subset X$. Their proof marvelously combines Jean-Pierre's famous regularization procedure for closed positive $(1,1)$-currents [Dem92] and a new technique of "mass concentration" along subvarieties for currents that arise as weak limits of solutions of families of complex Monge–Ampère equations (which generalizes earlier pioneering work of Jean-Pierre's where the mass concentration happened at just one point [Dem93]), among other ideas. From their result we see that if $[\alpha]$ is nef but not Kähler, then there is at least one such subvariety V with vanishing intersection number, and the (nonempty!) union of all such subvarieties is then an interesting object associated to $[\alpha]$ (called its *null locus*). When the manifold is projective and $[\alpha]$ is the first Chern class of a line bundle, a famous result of Nakamaye [Nak00] in algebraic geometry shows that the null locus is equal to the augmented base locus (an asymptotic invariant of the line bundle). In 2007, I realized that the null locus of nef classes also arises naturally when studying the singularities of limits of solutions of geometric PDEs on Kähler manifolds (including Yau's Ricci-flat Kähler metrics and the Kähler–Ricci flow), and that it would be desirable to prove an extension of Nakamaye's theorem to this transcendental setting. With Tristan Collins, we finally succeeded in proving such a result in 2013 [CT15], and combining our results with Chiose's new proof of the Demailly–Păun mass concentration result [Chi16], we obtained in particular another proof of the Demailly–Păun's numerical characterization theorem. Again, the influence of Jean-Pierre's work cannot be overstated.

Jean-Pierre was not only an incredible mathematician, he was also one of the most kind and generous people that I had the fortune to meet. I still recall fondly the memory of Jean-Pierre playing table tennis (his favorite sport) at his club in St-Martin-d'Uriage with my three-year old daughter Chloe during a family visit in 2016. He is very sorely missed.

Claire Voisin

I first met Jean-Pierre Demailly in 1989, during the award ceremony of the IBM France Prize that was bestowed on him (this prize, financed by the IBM company, existed between 1984 and 1990 but seems to have disappeared since). I still have the leaflet where each recipient indicated the keywords and strong points of their research. Here is what Jean-Pierre had written (it is rather spartan, and in fact there was no photograph):

"Analysis of the structure of positive currents. Characterization of affine algebraic varieties by conditions on volume growth and curvature. Holomorphic Morse inequalities and solution to the Grauert–Riemenschneider conjecture characterizing Moïshezon spaces. Vanishing theorems for the cohomology of ample vector bundles."

Of course, he later received prizes that were more important, as well as great international recognition, but I am moved by the recollection of this bright young mathematician (who seemed old to me at the time) who already had a substantial oeuvre under his belt.

At that time, I had not realized that we were scientifically close. Algebraic geometry and complex analysis (or complex differential geometry) had developed separately since the days of Henri Cartan, although fundamental results of complex algebraic geometry, such as the embedding theorem of Kodaira or the theory of Hodge structures due to Griffiths, provided important bridges between the

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two fields. One of the major contributions of Jean-Pierre Demailly, that was critical for our understanding of the positivity of divisors in algebraic geometry, was his characterization of "large," or "pseudo-effective," divisors in terms of the positivity of the closed currents of type (1,1) that represent them. This contribution, published in the 1990s, can be seen as a version of Kodaira's theorem, but with weaker positivity hypotheses. It marked a decisive moment in the history of analytic methods applied to algebraic geometry, of which Jean-Pierre was a major figure. I remember him delivering a talk on this topic in the algebraic geometry seminar at Jussieu at the beginning of the 1990s. His talks were powerful and focused, without any showing off, and he delivered them with a well-balanced mix of enthusiasm and mastery.

I have a huge amount of esteem and admiration for Jean-Pierre, and I believe that he also appreciated me. He was a mathematician in the classic style, demanding, rigorous, and deep. His scrupulous intellectual honesty was combined with great kindness, which could be seen in his commitment to the University as well as to his students of all levels. He was devoid of any snobbism or feelings of superiority. More than his success, it was his joy and pride of doing good mathematics that gave him his quiet confidence. His real and important influence within the academic world never involved any power games. I believe that he was just uninterested in such things.

Xiangyu Zhou

Jean-Pierre Demailly was an outstanding mathematician who worked on complex analysis and complex geometry. When I was young, I benefited from his work on Serre's problem about Stein fiber bundles and holomorphic Morse inequalities. He was a member of the panel committee of Section Real and Complex Analysis ICM 2002 (this information was public after the ICM), at which I gave an invited talk. After several meetings at international conferences, Demailly and I began to establish a friendship.

At the turn of the new century, I gave a graduate course based on his ebook *Complex Analytic and Differential Geometry*. I also invited him several times to visit our institute and give colloquium talks and talks at my seminar on several complex variables. Each visit lasted about half a month. Participants of the seminar mainly consisted of graduate students. These lectures helped us understand his work on various topics including singularities of plurisubharmonic functions, singular hermitian metrics, multiplier ideal sheaves, L^2 methods including L^2 existence and L^2 extension, applications of analytic methods in algebraic geometry, holomorphic vector bundles and curvatures, complex Monge–Ampère, pluripotential theory, hyperbolicity problems, structure theorem of compact Kähler manifolds with nef tangent bundles or anticanonical bundles, and so on.

During the gaps between his lectures, I often drove him to do some sightseeing in Beijing. He enjoyed visiting many famous scenic spots. Of course, we pay for tickets and fuel ourselves when accompanying foreign guests to visit scenic spots. But he often preempted buying tickets. While sightseeing, we talked about math or other things. In this way, I understood parts of his work better and learned about what is known and unknown related to his work. I learned from him what an HK integral is. He also told me about the Pirate Party in some European countries and his opinions on free circulation of knowledge and on math education in France. He was a thoughtful man. He was a founder of Episciences, which is a platform providing the scientific guarantee of an editorial committee to articles published in open archives. As chair, he invited me to join the Episciences committee. A few days before he passed away, he was arranging the election of the new chair. I think he is a responsible man.

We benefited from him both by talking with him and by studying his papers and books. He usually posted his papers, books, and lecture slides on his webpage so that his work is easily accessible. He published his book *Analytic Methods in Algebraic Geometry* in China. His books are popular and influential in the community of several complex variables and complex geometry in China. Many young Chinese scholars and students have visited him during the last ten years.

Jean-Pierre was easy to communicate with. One day in the winter, when taking him to a suburban scenic spot located in the mountains, the road was frozen and my car skidded. It was a bit dangerous. He didn't show any nervousness or fear. He helped me push the car and we eliminated the risk. I regretted that I hadn't anticipated that this might occur beforehand. However, he didn't complain and on the contrary he felt the trip was interesting.

Jean-Pierre loved playing table tennis and was good at it. He told me that he regularly played at the local club. We sometimes played it at my institute or Oberwolfach. Every time he visited China, I found that his table tennis skill increased. Sometimes I arranged for some good players to play table tennis with him, which he enjoyed.

In 2011, he invited me to visit him in Grenoble, where I gave a seminar on my recently finished paper with my students Qi'an Guan and Langfeng Zhu. We had developed a method to approach the optimal L^2 extension

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problem, which is an optimal estimate version of the Ohsawa–Takegoshi L^2 extension theorem. We introduced an undetermined function method to try to find better estimates and reduced the problem to determining the undetermined function. Jean-Pierre was quite interested in our work. In talking with him after my lecture, we realized that the undetermined function satisfies an ODE and our main result may be reformulated by solving the ODE. Jean-Pierre told me how to solve the ODE numerically with the help of a computer, and wrote the computer program (see Remark 6.2 in [ZGZ12], which was handled by Jean-Pierre). This paved the way for further developments.

Jean-Pierre was pleased to accept our institute's invitation to give an invited online lecture in Nov. 2020, in memory of Hua Loo-keng's 110th birthday. Hua was a founder of modern mathematics in China. The title of his talk was "Hermitian–Yang–Mills approach to the conjecture of Griffiths on the positivity of ample vector bundles." I chaired the lecture and we met online. I last saw him online when I gave an online talk at an Oberwolfach workshop in 2021 and he gave helpful comments after my talk. He had promised to visit China after his retirement. This was postponed because of the pandemic. Unfortunately this will not be realized.

About his trips to China, he said, "Needless to say I have been extremely impressed by the current development of China!"

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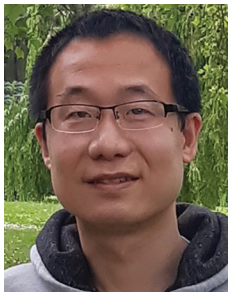
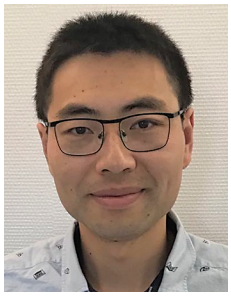


Junyan Cao



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