Emma Previato

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Figure 1. Emma in 2005 (Tokyo).

1. Preface

Ron Donagi

Emma Previato (11/29/1952 Padova, Italy—6/29/2022, Boston, Massachussetts) was a mathematical pioneer, who made major contributions to numerous mathematical subjects, including her two main areas, algebraic geometry and integrable systems. She was among the first women to do research in both areas. And her work in both has been deep and influential.

Emma received a Bachelor's degree from the University of Padova in Italy, and a PhD from Harvard University under the direction of David Mumford in 1983. Her thesis was on hyperelliptic curves and solitons. The work on hyperelliptic curves has evolved and expanded into Emma's lifelong interest in algebraic geometry. The work on solitons has led to her extensive research on integrable systems. She was a faculty member in the Department of Mathematics at Boston University for her entire post-PhD career, since 1983. She has published nearly a hundred research articles, edited six books (including [EP11, Pre01, DDFP96]), and directed seven PhD dissertations. Her broader impact extends through her renowned teaching and her extensive mentoring activities. She ran AFRAMATH, an annual outreach symposium, and worked tirelessly on several on- and off-campus mentoring programs. She also founded and led the activities of the Boston University chapters of MAA and AWM. She served on numerous advisory boards. She traveled widely, and visited the Institute for Advanced Study in Princeton, the Mittag Leffler Institute in Sweden, the Institut des Hautes Etudes Scientifiques outside of Paris, the Max Planck Institute in Bonn, and the Mathematical Sciences Research Institute in Berkeley, among others. Some of Emma's pioneering accomplishments have been celebrated recently in the two-volume set [DS20] *Integrable Systems and Algebraic Geometry: A Celebration of Emma Previato's 65th Birthday*, published by Cambridge University Press.[1]

Emma Previato worked in quite a range of different areas, using methods from algebra, algebraic geometry, mechanics, differential geometry, analysis, and differential equations. The bulk of her research belongs to "integrable equations." She is noted for often finding unexpected

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connections between integrability and many other areas, often including various branches of algebraic geometry.

1.1. Early activity. As an undergraduate at the University of Padova, Emma wrote a thesis on group lattices, followed by six journal publications, e.g., [Pre77]. With methods from algebra, initiated by Dedekind in the 19th century, this area's goal is to relate the group structure to the lattice of subgroups, and provide classifications for certain properties. An excellent overview is the article by Freese [Fre94], a review of the definitive treatise by R. Schmidt, where results from all of Emma's papers are used to give one example, a lattice criterion for a finitely generated group to be solvable.

1.2. PhD thesis and main area. Emma's thesis [Pre85], submitted at Harvard in 1983 under the supervision of D. B. Mumford, is still her most cited paper. Her thesis advisor was among the pioneers of this beautiful area, "integrable equations," which grew and unified disparate parts of mathematics over the next twenty years, and is still very active. Emma's original tool for producing exact solutions to large classes of nonlinear PDEs, the Riemann theta function, remained one of her main interests.

1.3. Theta functions. She later pursued more theoretical aspects of "special functions," such as Prym theta functions [PV93, vGP92], and also some surprising connections to numerical results in conformal field theory, the Schottky problem [PP01], and Thetanulls.

1.4. Algebraically completely integrable systems. The area of integrable PDEs is surprisingly related to "algebraically completely integrable Hamiltonian systems," or ACIS, in the sense that "algebro-geometric" (aka "finite-gap") solutions of integrable hierarchies linearize on Abelian varieties, which can be organized into angle variables for an ACIS over a suitable base, typically a subset of the moduli space of curves whose Jacobian is the fiber. Thanks to this discovery, the area integrates with classical geometric invariant theory, surface theory, and other traditional studies of algebraic geometry. With the appearance of the moduli spaces of vector bundles and Higgs bundles over a curve, at the hands of N. Hitchin in the 1980s, large families of ACIS were added to the examples, as well as theoretical algebro-geometric techniques. In [AHP88], Emma took up the challenge of generalizing the connection between ACIS and integrable hierarchies to curves beyond hyperelliptic.

1.5. Higher rank and higher-dimensional spectra. On the PDE side, the challenges were of two types. When the ring of functions on the (affine) spectral curve can be interpreted as differential operators with a higher-dimensional space of common eigenfunctions, the fiber of the integrable system is no longer a Jacobian: it degenerates to a moduli space of higher-rank vector bundles, possibly with some auxiliary structures [PW89]. Neither the PDEs nor the integrable systems have been made explicit in higher rank in general. Some cases, however, are worked out in a series of papers with Latham and Wilson [PW89] and elsewhere. The other challenge is to increase the dimension of the spectral variety, for example from curve to surface. Despite much work, this problem too has arguably no explicit solution in general. An attempt to set up a general theory over a multi-dimensional version of the formal Universal Grassmann Manifold of Sato which hosts all linear flows of solutions of integrable hierarchies, is given in a paper with Min Ho Lee. More concrete special settings are mentioned below, under the heading of "Differential Algebra."

1.6. Special solutions: Coverings of curves. An important aspect of theta functions is their reducibility, a property whose investigation goes back to Weierstrass and his student S. Kowalevski. Given their special role in integrability, reducible theta functions are invaluable for applied mathematicians to approximate solutions, or even derive exact expressions and periods in terms of elliptic functions. To the algebro-geometric theory of elliptic solitons, initiated by I.M. Krichever and developed by A. Treibich and his thesis supervisor J.-L. Verdier, Emma contributed almost a dozen papers [EEP07, CPP94, PV93], while [DP01] generalizes the reduction to Abelian subvarieties.

Another type of special solution is the one obtained by "self-similarity" in a work with Benes; the challenge here is to find an explicit relationship between the PDE flows and the deformation in moduli that obeys Painleve-type equations: this is one reason why Emma's work has turned to a special function which is associated to Riemann's theta function but only exists on Jacobians: the "sigma function" (cf. the eponymous section below).

1.7. Generalizing ACIS: Poncelet and billiards. Classical theorems of projective geometry can be generalized to ACIS, (cf. Previato's works with Accola, Oxbury, and Pauly) while the challenge of matching them with integrable hierarchies is presented in a paper with van Geemen and is still ongoing.

1.8. Generalizing ACIS: Hitchin systems. In general, explicit Hamiltonians for the Hitchin system are only available in theory. In some cases they are given explicit algebraic expression in [vGP96] and several related works.

1.9. Differential algebra. Differential algebra is younger than algebraic geometry, but they have many features in common. Mumford gives credit to J.L. Burchnall and T.W. Chaundy for the first "spectral curve," the spectrum of a commutative ring of differential operators. This is arguably the reason behind algebro-geometric solutions to integrable hierarchies. On the differential-algebra setting, Emma published [BP00], connecting geometric properties of the curve with differential resultants, a major topic of elimination theory which is currently being worked out and naturally leads to the higher-rank solutions: the higher-dimensional spectral varieties are addressed in [Pre08] and their Grassmannian aspects are taken up in works with Kasman, Spera, Dupré, and Glazebrook. Other aspects of differential algebra are connected to integrability, the action of an Abelian vector field on the meromorphic functions on an Abelian variety, a p-adic analog, and the action of deformations on modular forms.

1.10. The sigma function. Klein extended the definition of the (genus-one) Weierstrass sigma function to hyperelliptic curves and curves of genus three. H.F. Baker developed an in-depth theory of PDEs satisfied by the hyperelliptic sigma function, which plays a key role in work on integrable hierarchies (e.g., KdV-type). Beginning in the 1990s, this theory of Kleinian sigma functions was revisited, originally by V.M. Buchstaber, V.Z. Enolski and D.V. Leykin, much extended in scope, eventually to be developed for "telescopic" curves (a condition on the Weierstrass semigroup at a point). Previato, with Matsutani and others, go beyond the telescopic case, while they investigate the higher-genus analog of classical theorems in [MP14, EEM$^+$08]. Other works by these authors give the first algebro-geometric solutions to a dispersionless integrable hierarchy. It is not a coincidence that its integrable flow on the universal Grassmann manifold "cuts across" the Jacobian flows of traditional hierarchies, and this is where the two variables of the sigma function (the Jacobian, and the modular ones) should unite to explain the mystery of Painlevé's equations.

1.11. Algebraic coding theory. Emma's primary contribution to this area is through mentoring undergraduate and graduate theses or funded-research projects. In fact, this research strand began at the prompting of students in computer science who asked her to give a course on curves over fields of prime characteristic, which she ran for years as a vertically integrated seminar. Together with her PhD student Drue Coles, she published research papers pursuing Trygve Johnsen's innovative idea of error-correction for Goppa codes implemented via vector bundles [CP10]. This was followed by several overviews and extensions of Goppa codes to surfaces.

1.12. Other. Emma edited or coedited a number of books [EP11, Pre01, DDFP96]. In addition to book and journal publication, Emma published reviews (BAMS, SIAM), entries in mathematical dictionaries or encyclopaedias, teaching manuals, and online research or teaching materials; she also published on the topic of mentoring in the STEAM disciplines.

2. Emma Previato: "Be Strong"

Shigeki Matsutani

I collaborated with Emma since first meeting her in Montreal in May 2004, publishing 16 papers (e.g., [MP14, EEM$^+$08]) with other coauthors. At the time of Emma's death, we were writing a book which was almost completed.

I would like to reminisce about my research activities with her.

In the middle of 2000, John McKay read one of my preprints on the arXiv and immediately sent me an email. The content of the preprint was that the partition function of statistical mechanics describing the supercoiled structure of DNA could be calculated by constructing algebraic curve solutions of the mKdV hierarchy in terms of the sigma function of higher genera, i.e., the generalization of the Weierstrass sigma function for elliptic curves (of genus one) to algebraic curves with higher genera, and showing their mathematical structure. It was an ambitious attempt to modernize the theory of elastic curves, which had been studied by Euler and Bernoulli with the discovery of the elliptic function, and to generalize it based on algebraic geometry. John, who saw the depth of the work, pointed out that it should be related to his research theme, the structure of replicable functions associated with the monster group.

We decided to hold an informal meeting in Montreal in May 2004. For the meeting, he asked me to name people I would like to invite, and I mentioned Emma Previato and Victor Enolski. Emma was an expert in algebraic solutions of integrable systems and had done pioneering work on the mKdV flow, so she seemed like a good fit for the meeting. Victor was an expert on sigma functions of higher genus, and although he could not make it, we corresponded by email. I had four days of fruitful discussions with Emma and John. Since Emma was already working on the sigma function with Victor and Chris Eilbeck and had published papers in this area, and I had published some papers on the sigma function and had already sent them to her, the discussion went smoothly. Emma liked my research plan right away.

I was also impressed by the way Emma politely soothed a young post-doctoral researcher in algebra who attended the meeting but was depressed because she did not understand the content of the discussion. Emma gently taught her how to digest the content.

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Emma and I talked a little in between meetings and decided to start a joint research project. The goals of our research were twofold.

1. To construct a theory of algebraic functions in terms of the sigma function along the lines of Weierstrass rather than Riemann, or a higher-genus version of the Weierstrass elliptic function theory (which was a refinement of her dissertation). Since the Weierstrass elliptic function theory with the elliptic sigma function has powerful rigorous applications to various fields in mathematics and physics, it is highly desirable to have such a theory for general curves.

2. To clarify the geometrical structure of supercoiled DNA and describe its shape in terms of algebraic geometry by generalizing Euler's elastic curve of genus one (which was an embodiment of her 1992 work on the mKdV flow).

Toward these goals, Emma and I started with an analysis of algebraic functions using classical methods. Hyperelliptic curves are straightforward, so it is crucial to analyze non-hyperelliptic curves for a full understanding. Aiming at these, curves of "trigonal" type became our main subject.

The following year, I met Victor and Chris at an international conference on the sigma function at Tokyo Metropolitan University, where I also met Emma again. Then our collaboration was in full swing.

She was a strong member of the research group on the sigma function because of her work with Victor and Chris. We met at similar occasions after that. Our further investigation of the sigma function also involved Victor, Chris, C. Athorne, T. Ayano, E. D. Belokolos, J. Bernatska, H.W. Braden, S. Baldwin, V.M. Buchstaber, E. Y. Bunkova, M. England, Y. N. Fedorov, J. Gibbons, D. V. Leykin, D. Korotkin, A. Nakayashiki, Y. Ônishi, V. Shramchenko, K. Yori, and others. Our work on the first goal with Jiryo Komeda and Yuji Kodama was part of our research on the sigma function. Taking advantage of the fact that this research group was not so large, we had friendly discussions with each other in conferences and by email. Emma played a leading role in the group and wrote several papers with various members. When Victor Enolski, who hugely contributed to the development of the sigma function theory, passed away in 2019, Emma organized his obituary in the *Notices*.

Since Emma had many other interests as well, I basically wrote the main part of each topic, and she and I refined the results after discussions. For a new topic, sometimes she would start with a suggested issue; sometimes, I would select one of her interests to pursue the above goal. With her extensive knowledge of algebraic geometry, she always gave appropriate pointers. As one might guess, she often shortened proofs, revised propositions, and rewrote our papers extensively. Although our work relied on email, with her significant advice and discussion, before long, the manuscripts were firmly revised in an acceptable style. It was a very pleasant time. Above all, Emma sometimes gave me breakthrough suggestions. It was an irreplaceable collaboration.

During this process, on several occasions we saw critical connections with McKay's profound insight concerning the replicable functions.

Emma had empathy and encouragement for others going through hard times. After I lost my father, when I was cleaning out the belongings from his house in my hometown, I wrote her an email about a paper, and I let it slip that I was depressed. She replied immediately. After writing about the paper, she said that she also had to consider something similar in her house in her home country, and ended her mail with "Be strong. He loved you and would want you to be strong and do what you love." I think these were words that she told herself on a regular basis. In particular, this was true after her mother's passing in early 2020. Her mother was the only family she had left, and Emma told me that spending time in her hometown was the only thing that kept her going outside of mathematics. She worked as if obsessed, but she was also sensitive and vulnerable. She was repeatedly in and out of the hospital in recent years. However, every time she left the hospital, she sent me a cheerful email, so when I heard that she was going to be hospitalized in mid-June, I believed that she would be discharged in a short while.

I later learned from Chris that on June 22, she had sent him an email saying, "I'm in a bit of a bad way, still hoping (but not very much)." She knew everything.

On one of our goals, although some problems remained, she and I drew a shape similar to the electron-micrograph image of DNA in 2021 with a hyperelliptic function of genus two; she liked it.

For the other goal, the two papers submitted in July were on a solution to the problem posed by Emma in 2018 to construct the sigma function algebraically for general compact Riemann surfaces based on Weierstrass's canonical form. The two that formed the pair constitute a critical signpost toward the goal, which was a part of the book we were writing. One of the two was requested by her as editor of a special issue of a journal, where she was active in mathematics until the end of May. She was delighted to see their almost completed version ready in May before the submission. Indeed, on April 28, she gave her final very short talk on the topics in a workshop at Harvard.

When I shared her obituary with some members of the research group on the sigma function by email, many of them wrote me about their sadness on her passing. Although I am devastated by her death, I want to keep the words "Be strong" in my heart.

3. Emma the Light…

Volodya Roubtsov

Very strangely and unexpectedly, Emma Previato, whom I saw and met only a few times in my life, became a close person to me, and her unexpected departure was a serious loss, the loss of one of my friends, whose opinion and help supported me both in my mathematical reasoning and in various everyday projects and situations.

If I were to characterize Emma Previato briefly, then I would choose her description as a "light person." And this not only completely reflects the image of Emma, but, it seems to me, would completely convey her attitude to science, her beloved work, and the people around her. I probably do not know a more friendly, even, I would say enthusiastically friendly, person both among colleagues and among other people I know more generally.

I don't remember the exact date and moment of our first meeting, but I remember very well that I first learned the name "Emma Previato" in 1994. We (some of my Russian colleagues and myself) tried to understand and to give a conceptual description of the "Ruijsenaars duality" in multiparticle integrable systems, its generalization to such systems on moduli spaces of bundles, and its connection with quantum field-theoretic models. The project had many coauthors, all of us traveled at that moment to different Universities and Institutes, so the writing process progressed very slowly, although the results were reported periodically in various meetings and seminars both in the USA and in Europe.

Once I discovered (and in those years it was possible to surf through the entire daily new articles on the arXiv in a matter of minutes), Emma's article (together with B. van Geemen)—"On the Hitchin systems," where a brilliant algebro-geometric analysis was carried out of an example of so-called "self-dual Hitchin system," related to the Narasimhan–Ramanan moduli space of semistable bundles with trivial determinant over curves of genus 2. A few years later we met at one of the seminars (or at a conference?) in Paris. Our long-writing duality article was finally published and Emma amazed me by identifying me as one of many authors. She had started instantly to ask very valid and interesting questions. She very quickly accepted the main ideas, which she later formed as the basis of her conceptual article (with S. Abenda).

Our acquaintance and, I would say, friendship began from this meeting, despite the fact that we rarely met, but we exchanged letters regularly. Emma was unusually touching and charming in her correspondence. She knew how to make even the most casual business exchanges of peer review warm and friendly.

By the way, here I should note her amazing commitment and remarkable scientific "honesty" and decency. Once I had to search (long and unsuccessfully) for a reviewer for an article in a journal where I was one of the editors. The article was extremely technically overloaded, but it seemed to me that it would not be very appropriate to reject the paper only based on these grounds. Emma was certainly ideally suited scientifically for the reviewer role. However, it turned out that she and the author had a joint article many years ago. So first she tried to evade the peer review invitation. But, having learned from me that I had been unsuccessfully trying to find a suitable referee candidate for a very long time, she decided to help me. She did this hard work with exceptional care, showing the author how to make the article really attractive, which required a serious revision of the text. The author made a second attempt, but Emma rejected this version as well. When the author sent a third, still equally ponderous essay, she strongly and adamantly recommended that the article should be rejected. That was done with a light heart and deep gratitude to Emma. We have never worked together, but in almost thirty years of acquaintance, I have often come across scientific "greetings" from Emma—her results and works, which I have repeatedly used in my research. I should also mention Emma's fantastic *Mathematical Review's* reports which were in fact small surveys of a subject. Between 1985 and 2014 she wrote 186 reviews, which were consistently of high quality, very carefully composed, instructive and a pleasure to read, critical when necessary, laudatory in the opposite case, and always extremely reliable. In addition, they often contained real insight and ideas to study and develop.

Our last contacts were overshadowed by problems related to the Covid pandemic. I was very sad to learn that Emma was working in her empty department practically alone, and that her solitary trips to the university had a psychologically heavy effect on her mood.

Nevertheless, she reacted very warmly and positively to my invitation to join the Scientific Program Committee of the Satellite Conference "Mathematical Physics, Geometry and Integrable Systems" at the Mathematical Congress in St. Petersburg in 2022, which was to be held in Moscow. Emma, with full seriousness and responsibility, accepted this invitation. She very fruitfully joined in all the discussions and invitations of future speakers. Our application with her active participation was supported by the organizing committee of the Congress…But the terrible events that happened in Russia (the invasion of Ukraine in

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February 2022) made it impossible both to hold the Congress itself in Russia and for Emma's and my participation in the organization of the Conference. We exchanged sad and painful letters. Even more painful and sorrowful for me was to know that they were the last...

4. Memories of Emma

Malcolm R. Adams

The academic year of 1984–1985 was certainly a magical time for me. I was only a few years out of graduate school and I was lucky enough to land a one-year membership at the Institute for Advanced Study. The delicious lunches in the cafeteria and the fresh-made cookies at afternoon tea provided perfect opportunities for the members to discuss their mathematical projects, to share ideas, and to form collaborations and lasting friendships. Here I was able to join forces with Emma Previato and John Harnad to write the first of a series of papers on integrable Hamiltonian systems [AHP88].

Emma was the algebraic geometry expert on the team. Our work involved spectral curves and linear flows on Jacobi varieties and so her expertise was invaluable to the project. My own background in algebraic geometry was sketchy at best, and so I was confronted with a steep learning curve in that area. Emma graciously provided me with many hours of tutoring on the subject of curves, their Jacobi varieties, and their connection to soliton-type equations. Over the years since then, I have often found myself grateful for her patient explanations that served as a foundation for much of my work in that area. Thinking back on those days, it does not surprise me that in later years, Emma was recognized as an outstanding mentor.

Besides the hours discussing mathematics, our small group of early career mathematicians at the Institute enjoyed the rich intellectual opportunities in the Princeton area. Occasional evening concerts at the Institute, walks in the Institute woods, and traveling to conferences in my 1975 Dodge. At the end of the year, we all went off to various other positions, but our paper was barely yet in preprint stage. Those were the days before email, etc., so our work continued through "snailmail" correspondence and lengthy long-distance phone calls.

After that first paper, Emma found herself wrapped up in a number of other projects and collaborations, and so decided not to continue with our project, but this was certainly not the end of our friendship. Over the years we would occasionally find ourselves attending the same conferences. As always with good friends, the conversations would pick up immediately as if there had not been a gap of a few years since the last time we talked. Emma was always excited about her own recent mathematical projects as well as those of her students. Equally, she was eager to hear about my projects.

Only a year ago, I heard from Emma when she decided to teach from the linear algebra text I coauthored with Ted Shifrin. As she taught the course, she read through the text and solution manual in detail, sending us comments and corrections as she came upon them. In her usual fashion, as she pointed out the errors (some of which might be considered egregious) she would be apologetic and even self-deprecating. Of course, none of us suspected that we would lose her this past summer, but now I am so grateful for that last opportunity to exchange a few emails with a dear friend and colleague.

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5. Emma Previato

David Mumford

Emma was one of my last graduate students in pure math. When she arrived, I had begun to be fascinated by applied math, especially the connection of the Korteweg–de Vries equation and its solitons to hyperelliptic curves that I learned from the papers of Henry McKean. It seemed clear that the theta functions on particular abelian varieties, especially Jacobians of curves, could be used to solve various well-known partial differential equations arising in applied math. Moreover, the curve could be singular, leading to solutions using the generalized Jacobian. So when Emma knocked on my door, I thought I'd lay out this area and see if she was game to work on this. How amazingly she has fulfilled this dream, with an extraordinary lifetime of work on PDEs of many types. She wrote in the beginning of her thesis "My research (is) concerned with the applications of algebraic geometry in the theory of nonlinear wave equations, a subject which I now see to be extremely rich and beautiful." Indeed, she made it richer and more beautiful. She took the nonlinear Schrödinger equation:

$$u_{xx} + iu_t = \pm|u|^2 u$$

and found explicit formulas for solutions using the theta function associated to the hyperelliptic curve where two

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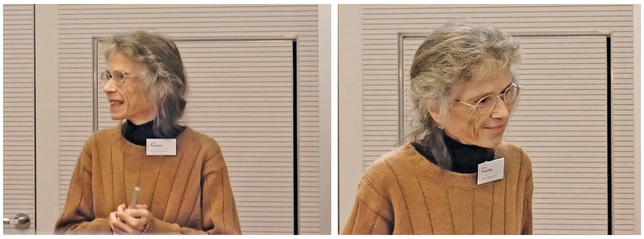

Figure 2. Emma in 2012 (Bad Honnef, near Bonn in Germany).

distinct points at infinity are identified. This was her beginning, and she went far beyond that specific area.

I enjoyed advising her but I often worried about her too. She describes herself in the acknowledgements to her thesis as being "tormented with sluggishness!" On the contrary, she was a workaholic and almost never left the department. I felt she needed to get outside in the sun. At one time, I convinced her to come with me to have an ice cream cone and, when we showed up at a fancy new Italian ice cream place in Harvard square, she tricked me by asking only for sample of gelato in a small paper cup while I went whole hog. But when we corresponded in 2007, she was in Sweden and was clearly enjoying the Scandinavian fresh air.

I recently had a correspondence with Al Osborne, an expert in water waves, especially rogue waves, that he believes are solitons. He has developed the use of theta functions to find such solutions and specifically used the same approach that Emma pioneered in her thesis, a wonderful confirmation of the applications of her work. She has left a big mark on the field of nonlinear PDEs.

6. Remembering Emma

James Glazebrook

I was deeply shocked and saddened when learning of Emma's passing away. A wonderful and kind soul has now departed from our community. My first meeting with Emma was in Washington, D.C. in 1995. At that time Emma's work (mainly on integrable systems) was only partially known to me, and vice-versa. Anyway, I tried to explain the sort of research I was then pursuing, while Emma was enthusiastically jotting down notes like a zealous newspaper reporter. It was sometime after, however, and via email correspondence, that we realized my ongoing work (with Maurice Dupré) could connect fruitfully with her study of differential algebras, when adopting a $C*$-algebra context. Further results in this direction followed up from the latter context in applications to integrable PDEs (KP hierarchy) that included methods of K-homology theory applied to algebraic (spectral) curves and their Jacobians [DGP13a].

These were challenging projects for Emma, but she suggested some splendid ideas such as those culminating in [DGP13b]. Although leaving the door open for future developments and collaboration, we drifted away to pursue different topics as is often the case with curious minds. For Emma these included coding and number theory to which she contributed substantially with other coauthors. How she could juggle so many projects at one time, and in such a masterly fashion, was indeed quite something — a sheer token of her brilliance. As an accomplished mathematician, and as a lady of profound dignity and grace, she will sadly be missed by us all.

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7. Emma

Ben Thompson

With the exception of her mother, whom she adored, math was Emma's everything. This was embodied not only by her devotion to research, but also by her devotion to her students.

Though I was one of her graduate students, I know she was a tremendous undergraduate advisor as well. It was clear to them how deeply she cared. The joy she found in math was positively infectious. She was at once both clearly formidable and entirely approachable—one of the most endearing people you could meet. Friends of mine that met her briefly would continue to ask after her years later. She could build a connection that quickly.

Her days were spent teaching, but her evenings too were devoted to supporting the many undergraduates she mentored and student clubs she advised. Which meant that just about every Thursday for seven years, it wasn't until 9p.m. that Emma could meet with me. It was a ritual I at first slightly resented, but soon came to enjoy, and which would eventually ensure that I would inherit her nocturnal ways (her late nights were when I presume she finally was able to do research).

Personally, I have Emma to thank for having graduated at all. She stopped me from walking away from my PhD on more than one occasion. So I hope I have also been able to inherit a fraction of her determination.

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I want to share some funny stories that capture her spirit, her stubbornness, her sweetness—and there are a thousand Emma stories to be shared—but the memories I'll cherish most are the simple ones. Emma hefting a tome half her size onto the table to recall some gem she wanted to share with me. A twinkle in her eye as she describes her favorite passages from Tolkien or C. S. Lewis. An email forcefully insisting that I mention the codeword, Painlevé , when we next meet, so she can share some important thought with me. "Ben, don't forget the codeword: PAINLEVÉ!"

Emma was one of a kind and I feel grateful and honored to have had her unfailing personal and professional support throughout my graduate career, and her dear friendship afterward.

Well, Emma, as it turns out I have forgotten the significance of Painlevé. But I know I won't soon forget what you did for your students, and the masterclass you taught us all in ingenuity, perseverance, and service.

8. Friend, Colleague, and Mentor

Lisa Jeffrey

Emma Previato was an inspiration to me. Her work on integrable systems (notably her joint work with Malcolm Adams and John Harnad [AHP88]) had a particular impact on my own work.

I interacted with Emma Previato because she was the editor responsible for two volumes where I published articles. One was the proceedings workshop "Geometric Methods in Physics: XXXV Workshop 2016 Trends in Mathematics" (Bialoweža, 2016). The second was "Grassmannians, Moduli Spaces and Vector Bundles" (Clay Mathematics Institute, 2006, [EP11]), which followed a CMI workshop in October 2006. Emma Previato was an editor of each of these volumes. Her meticulous attention to detail made my contributions better.

Emma Previato was a role model to me and her work has been a guiding influence on some of my own work. Her death is a great loss for the mathematical community.

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Credits

Figure 1 and Figure 2 are courtesy of J. Chris Eilbeck.