CHOICE FUNCTIONS AND TYCHONOFF'S THEOREM

W. H. GOTTSCHALK

The purpose of this note is to point out that the following set-theoretic theorem [R. Rado, *Axiomatic treatment of rank in infinite sets*, Canadian Journal of Mathematics vol. 1 (1949) pp. 337–343] is an easy consequence of Tychonoff's theorem that the cartesian product of a family of compact spaces is compact.

THEOREM. *Let $(X_\alpha | \alpha \in I)$ be a family of finite sets, let $\mathcal{A}$ be the class of all finite subsets of I , and for each $A \in \mathcal{A}$ let ϕ_A be a choice function of $(X_\alpha | \alpha \in A)$. Then there exists a choice function ϕ of $(X_\alpha | \alpha \in I)$ such that $A \in \mathcal{A}$ implies the existence of $B \in \mathcal{A}$ such that $B \supset A$ and $\alpha\phi = \alpha\phi_B$ ($\alpha \in A$).*

PROOF. For $A \in \mathcal{A}$ let E_A be the set of all $\phi \in X = \prod_{\alpha \in I} X_\alpha$ such that $\alpha\phi = \alpha\phi_B$ ($\alpha \in A$) for some $B \in \mathcal{A}$ with $B \supset A$. Provide each X_α ($\alpha \in I$) with its discrete topology. Since X is compact and $\{E_A | A \in \mathcal{A}\}$ is a class of nonvacuous closed subsets of X with the finite intersection property, there exists $\phi \in \bigcap_{A \in \mathcal{A}} E_A$. The proof is completed.

COROLLARY. *A family of finite sets has a one-to-one choice function if and only if each of its finite subfamilies has a one-to-one choice function.* [Cf. C. J. Everett and G. Whaples, *Representations of sequences of sets*, Amer. J. Math. vol. 71 (1949) pp. 287–293.]

UNIVERSITY OF PENNSYLVANIA

Presented to the Society, April 22, 1950; received by the editors January 16, 1950.