ADDENDUM: DERIVATIVES OF INFINITE ORDER

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We gave an incomplete proof¹ that, if $f(x)$ belongs to a quasi-analytic class $C\{M_n\}$ in $a < x < b$ and if $f^{(n)}(x_0) \rightarrow L$ for one x_0 in (a, b) , then $f(x)$ is analytic in (a, b) (and consequently $f^{(n)}(x) \rightarrow L e^{x-x_0}$ in $a < x < b$); the proof was completed by S. Mandelbrojt.² We now wish to point out that in fact T. Bang³ had already shown that if $f(x)$ belongs to a quasi-analytic class on $a < x < b$ and $g(x)$ is analytic, then $f^{(n)}(x_0) = g^{(n)}(x_0)$ for all n and $a < x_0 < b$ implies $f(x) \equiv g(x)$, which is precisely the result which we needed for our proof.

Bang has also pointed out to us that a function constructed in his thesis⁴ answers a question raised by us.⁵ We asked whether it is possible to have $\lim_{n \rightarrow \infty} f^{(n)}(x)/\lambda_n = g(x)$, $a \leq x \leq b$, with $\liminf |\lambda_{n-1}/\lambda_n| = 0$, and $g(x) \not\equiv 0$ in $a < x < b$. Bang constructed a function $f(x)$ analytic except for $x=0$, with $f^{(n)}(0)$ tending to ∞ arbitrarily rapidly; if we take $\lambda_n = |f^{(n)}(0)|$, we have $g(x) = 0$ for $x \neq 0$, $g(0) = 1$.

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¹ *Derivatives of infinite order*, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 523-526.

² R. P. Boas and K. Chandrasekharan, *Correction: Derivatives of infinite order*, Bull. Amer. Math. Soc. vol. 54 (1948) p. 1191.

³ T. Bang, *Om quasi-analytiske Funktioner*, Copenhagen thesis, 1946, p. 84.

⁴ Bang, loc. cit.

⁵ Boas and Chandrasekharan, op. cit., p. 525.