

be equal by the Cauchy integral theorem (by [1] and a result in the author's dissertation not yet published).

Theorem 1 follows directly from Theorem 3.

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REMARK ON A FORMULA FOR THE BERNOULLI NUMBERS

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Some years ago Garabedian [1] proved the following formula:

$$(1) \quad B_{k+1} = \frac{(-1)^{k+1}(k+1)}{2^{k+1} - 1} \sum_{r=0}^k (-1)^r \frac{\Delta^r 1^k}{2^{r+1}} \quad (k \geq 0),$$

where the even suffix notation is employed for the Bernoulli numbers. The proof of (1) made use of the sum of a certain divergent series.

We wish to point out that (1) is not new. It can be found (in somewhat different notation) in [3, p. 224, formula (68)].

It may be of interest to give a short proof of (1). We use the formula [2, p. 28]

$$(2) \quad C_k = 2^{k+1}(1 - 2^{k+1}) \frac{B_{k+1}}{k+1},$$

where the C_k are the coefficients in the Euler polynomial:

$$(3) \quad E_k(x) = \left(x + \frac{C}{2}\right)^k = \sum_{s=0}^k \binom{k}{s} 2^{-s} C_s x^{k-s}.$$

Then in view of

$$(4) \quad E_k(x+1) + E_k(x) = 2x^k,$$

we have

$$(5) \quad E_k(x) = \left(1 + \frac{1}{2} \Delta\right)^{-1} x^k = \sum_{s=0}^k (-1)^s 2^{-s} \Delta^s x^k.$$

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If we take $x=1$ in (5) and use (3) and (4), we get

$$C_k = -2^k E_k(1) = - \sum_{s=0}^k (-1)^s 2^{k-s} \Delta^s 1^k.$$

Substitution in (2) leads at once to (1).

In a similar way we can prove

$$(6) \quad B_{k+1} = \frac{(-1)^{k+1}(k+1)}{2^{k+1}-1} \sum_{r=0}^k (-1)^r 2^{-r-1} \Delta^r 0^k.$$

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